Development and Validation of an Algebraic Turbulence Model for Outdoor Airflow and Contaminant Simulations around a Building

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Abstract

The design of outdoor environment requires a tool that could provide fast and reliable simulations for outdoor airflow and contaminant dispersions. This investigation has developed and validated a new algebraic turbulence model for outdoor environment design. The new model was tested by a natural ventilation case and two contaminant dispersion cases. The computed air velocities and contaminant concentrations agree well with the wind tunnel measurements. The computation time for the new model is about two times faster than the time needed by the standard k-ε model with an even better accuracy for the three validation cases.

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Keywords: Algebraic model, Pollutant dispersion, Concentration

1. Introduction

Accurate predictions of the outdoor airflow are important for many engineering problems such as outdoor contaminant dispersions. Many different kinds of contaminants contribute to the pollution of the outdoor environment. For contaminant dispersion around buildings, the convection transport mechanism plays an important role because the Reynolds numbers of outdoor airflow are high. Therefore, to accurately model contaminant dispersions around buildings, the airflow field has to be correctly predicted. However, accurate simulations of outdoor airflow and contaminant dispersions have challenging problems. Outdoor airflow stochastically changes its velocity, direction, and turbulence characteristics with respect to time and space. Therefore, it is crucial to simulate transient airflow and contaminant dispersions. Furthermore, in urban areas, the wind usually impacts many obstacles such as buildings and other structures. These interactions lead to turbulence dissipation and create additional complexities for outdoor airflow numerical simulations.

The origin of turbulence modeling dates back to the year of 1895, when Reynolds decomposed the velocity field into a time averaged motion and a turbulence fluctuation. This further led to the development of Reynolds-Averaged Navier-Stokes (RANS) equations. Boussinesq eddy-viscosity approximation has historic importance in turbulence modeling. Most turbulence models are based on different simulations of eddy viscosity. The early progress in attempting to solve RANS equation was made by Prandtl, who discovered the boundary layer and introduced the mixing-length hypothesis. This important contribution formed the basis of turbulence modeling for the next several decades. A turbulence model based on the mixing-length hypothesis is referred to as an algebraic model or a zero-equation model.

To provide a more realistic model for turbulence description, Prandtl postulated a model with the kinetic energy of the turbulence fluctuation, k . An additional partial-differential equation was proposed to solve this parameter, and create a one-equation turbulence model. In addition, turbulence length scale is necessary for a complete modeling of turbulence. Kolmogorov introduced the first complete model of turbulence. This model introduced another new parameter – the rate of dissipation of energy in unit volume and time, ω . Two similar partial-differential equations were used to define these two parameters, k and ω. This type of model, therefore, is defined as a two-equation turbulence model.

The above three types of turbulence models are based on Boussinesq eddy-viscosity approximation. The stress-model, on the other hand, obviates this assumption, which modeled the Reynolds-stress with differential equations. Therefore, four main categories of turbulence models were formed, that is: Algebraic (Zero-Equation) Models, One-Equation Models, Two-Equation Models, and Stress-Transport Models. Since 1960's, hundreds of new models have been developed within these four classes.

Theoretically, the algebraic models are computationally most efficient turbulence models because only a simple algebraic equation instead of one or more differential equations needs to be solved. However, the computation time saved is usually paid by the decrease in accuracy. Therefore, model validations are necessary for the algebraic equation model to assess accuracy of simulating a certain type of flow. The algebraic turbulence models had successful applications in the simulations of free shear flow, pipe flow, and channel flow. However, with the inexpensive computations, the more complex turbulence models are more popular than the algebraic equations. Generally, the algebraic equation models are considered to be fast, but not accurate enough for engineering applications.

For the indoor airflow modeling, an algebraic model developed by Chen and Xu (1998) offered new life to algebraic equation models. This model calculates the eddy-viscosity empirically by an algebraic equation. It was validated with three different cases, including natural convection with infiltration, forced convection with a partition wall, and mixed convection with displacement ventilation (Srebric, Chen, & Glicksman 1999). The model was proved to be very successful for the prediction of complex indoor airflow. It can accurately predict the indoor airflow within minutes, which is about an order of magnitude faster than simulations with a two-equation model. However, this algebraic equation model does not work for outdoor airflow since the length-scale for outdoor airflow is extremely large. In addition, the flow characteristics between outdoor and indoor airflow are very different. The Reynolds numbers for outdoor airflows are about two orders of a magnitude higher than that of indoor airflows. Therefore, it is important to develop a new algebraic model with an appropriate model constant and length scale to simulate outdoor airflow and contaminant dispersions. As a result, the overall objective of this study was to develop and validate a new algebraic turbulence model for the simulations of outdoor airflow and contaminant dispersions.

2. New algebraic model

A new algebraic model for outdoor airflow simulations is first developed directly from wind tunnel data. Data regressions for turbulence intensity and length scale in different regimes around the building resulted in the preliminary version of the model. However, the original model was too complicated and not tested on other independent airflows and turbulence data. Therefore, two assumptions are proposed for the simplification of turbulence intensity and turbulence length scale. The final model is developed based on different sets of wind tunnel data from the existing literature, which is:

$$\mu_t = 0.2 \cdot \left(\frac{10^5}{Re_b}\right) \cdot \rho \cdot T_{i\_inflow} \cdot U \cdot l \qquad (1)$$

where

Re_b = inflow Reynolds number at the building height

T_{i_inflow} = inflow turbulence intensity at the building height

U = local mean velocity

l = length scale (the nearest distance to the building surface)

ρ = air density

This new algebraic turbulence model was incorporated into commercial software - (CHAM, 2005). This study created the following simulations of outdoor airflow and contaminant dispersions:

- Cross Ventilation (Jiang et al., 2003)
- Gaseous Diffusion around a Cubic Building (Tominage, Murakami, & Mochida 1997)
- Pollutants Dispersion around a Rectangular Building (Huber, Synder, & Lawson 1980)

3. Validation of the new algebraic model

Although the standard k-ε model is not accurate in simulating the outdoor airflow around buildings, it is still the most widely used turbulence model for design applications. Therefore, the simulation results by the standard k-ε model could be used to evaluate the new model. As long as the simulation results by the new algebraic model are as accurate as the standard k-ε model, but they involve less computational time, the new model would be a great improvement for outdoor airflow simulations.

3.1 Cross ventilation

A cross ventilation experiment was conducted at the wind tunnel in Cardiff University, U.K. (Jiang, 2002). The building model has two same size openings on the opposite walls, as shown in Figure 1. The model dimensions are 0.82ft x 0.82ft x 0.82ft (0.25m x 0.25m x 0.25m), and the size of the opening is 0.28ft x 0.41ft (0.084m x 0.125m). The inflow mean velocity along the stream-wise direction follows a logarithmic law equation, and the mean velocities in the other two directions are zero. The Reynolds number is 1.4 x 10^5 based on the inflow velocity at the building height. The inflow turbulence intensity is 10% by superposing random perturbation to the mean velocity. Mean velocities were measured along 10 vertical lines, and their locations are shown in Figure 2. The wind

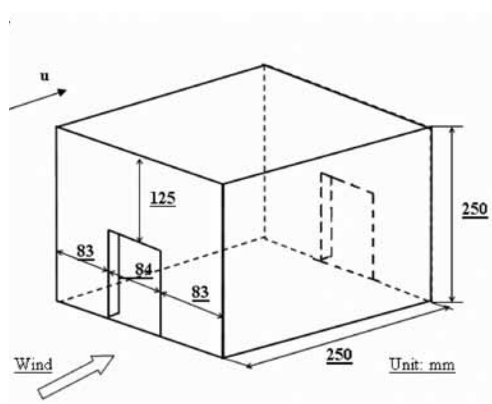
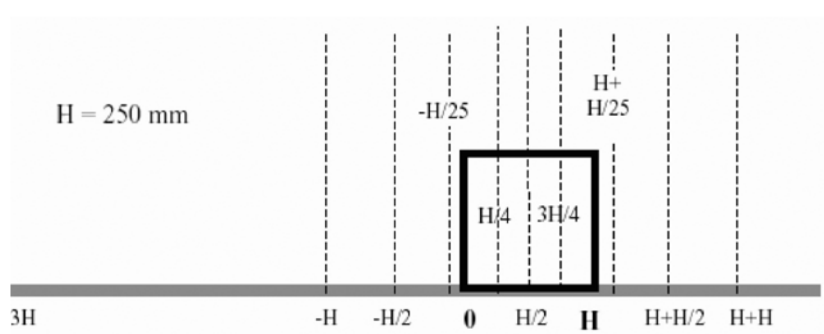


Figure 1. Schematic view of the building model for cross ventilation (Jiang et al., 2003).

Figure 2. The positions for the vertical measurement locations (Jiang et al., 2003).



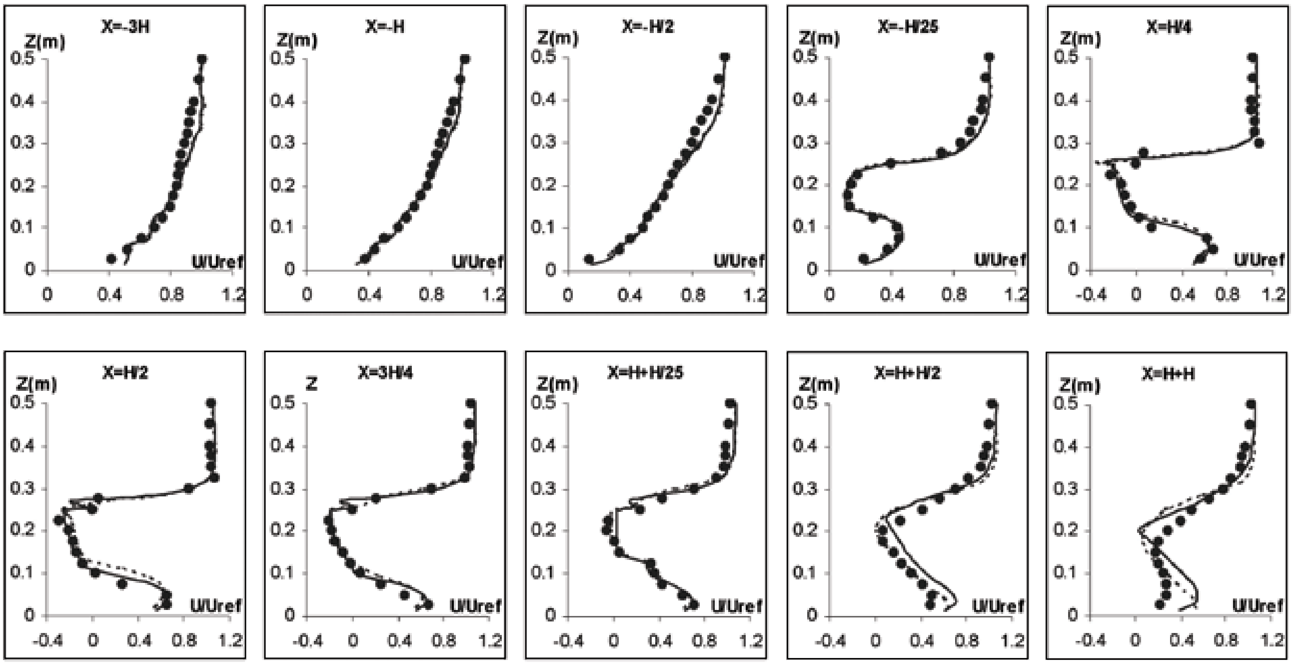


Figure 3. Stream-wise mean velocity distributions. Black Dots: experiments; dashed line: standard k-ε model; solid line: new algebraic model.

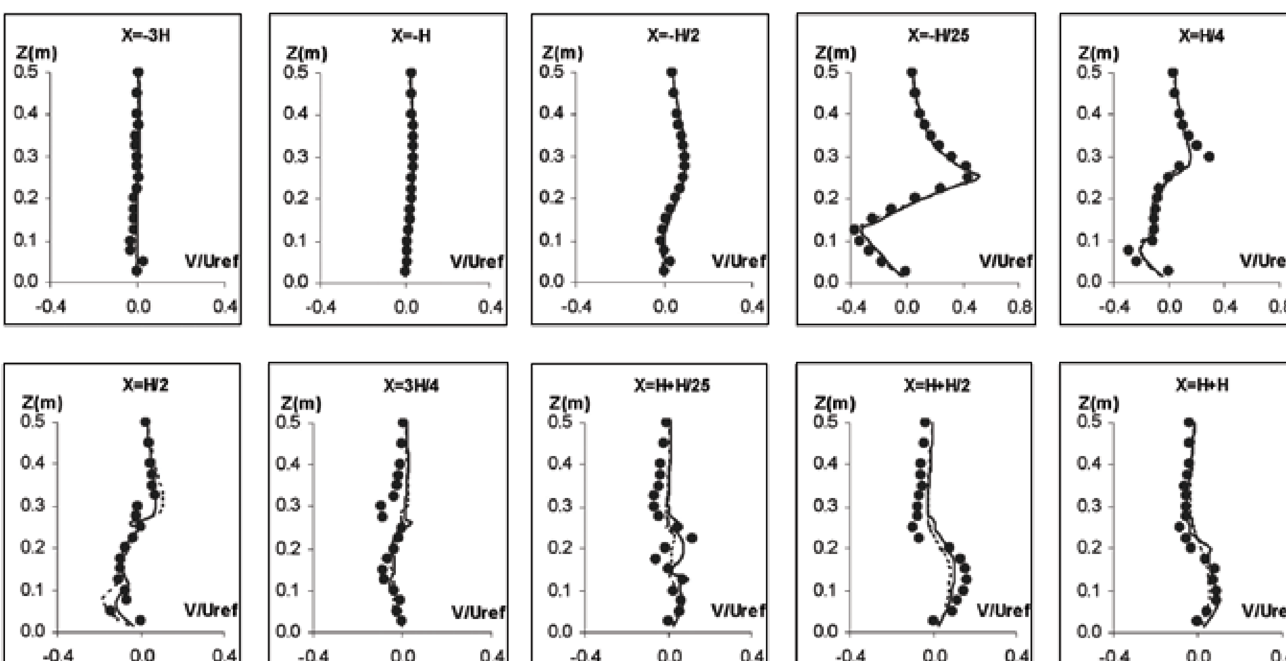


Figure 4. Vertical mean velocity distributions. Black Dots: experiments; dashed line: standard k-ε model; solid line: new algebraic model.

tunnel data are compared to the simulation results by the new algebraic model and the stand k-ε model. The horizontal mean velocities and vertical mean velocities along these ten lines are compared in Figure 3 and Figure 4, respectively. Both the new model and the stand k-ε model accurately predict the mean flow field around the building, compared to the wind tunnel results. In most regions, the discrepancies are less than 5% between the wind tunnel data and numerical simulations. Large Eddy Simulation (LES) provides similar accuracy as these two turbulence model and its computation time was about one week in a powerful workstation (Jiang, 2002). The simulations by the new model took about one hour in a Pentium® 4, CPU 2.26GHz personal computer, which is much fast than LES, and is about two times faster than the standard k-ε model. Hence, the new algebraic model could simulate the natural ventilation case with acceptable accuracy and computation time.

However, there are still differences between the numerical results and the experimental data, which appear mostly at the wake area behind the building. For example, as shown in Figure 3, at H/2 distance behind the leeward wall, the stream-wise velocities close to the ground are underestimated by both of the models. At H distance behind the leeward wall, the stream-wise velocities close to the ground are overestimated. Therefore, both numerical models are not very accurate in predicting complex phenomena such as recirculation at the wake area. Small discrepancies also exist for mean vertical velocities predicted by both models. As compared with wind tunnel data, both models have similar accuracy levels.

Therefore, the new algebraic model provides a similar accuracy as the standard k-ε model with respect to the mean velocities predictions.

3.2 Gaseous diffusion around a cubic Building

The cubic building model case with the gaseous contaminant source behind the building is simulated, and the schematic view of the building is shown in Figure 5 (Tominage et al., 1997). The building model height H_b is 0.66ft (0.2m), and the Reynolds number based on the inflow velocity at the building height $<u_b>$ is 5.7 x 10^3. The inflow stream-wise velocities follow the ¼ power law, and the turbulence intensity is assumed to be 10%. A gas contaminant source (C_2H_4) with a 0.25H_b x 0.25H_b square-shape is located at the ground level in the recirculation area behind the cube, and the distance to the leeward wall is 0.5H_b.The exit gas speed is 0.5$<u_b>$.

Gas concentration is normalized by the standard gas concentration as $<C_g>/<C_{g0}>$, where $<C_g>$ is the mean gas concentration, $<C_{g0}>$ is the standard gas concentration. The standard gas concentration is defined as:

$$<C_{g0}>=q / <u_b> / H_b^2 \quad (2)$$

where, q is the gas emission rate; $<u_b>$ is the inflow velocity at building height; H_b is the height of the building model.

The simulation results by the standard k-ε model and the new algebraic model are compared with the wind tunnel experiments, as shown in Figure 6. As revealed by the experimental results, the tracer gas is dispersed upstream from the source, which is caused by the reversed flow in the wake area behind the building. Therefore, the largest gas concentration appears between the leeward wall and the gas exit. Both the standard k-ε model and the new algebraic

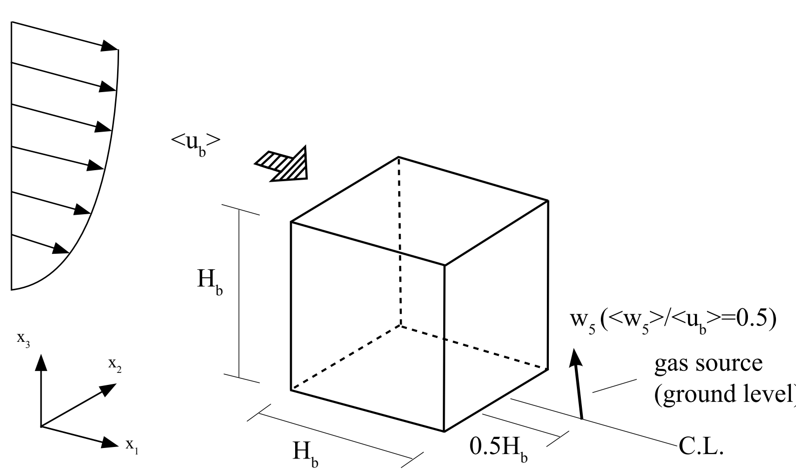


Figure 5. Schematic view of cubic building and gas source (Tominage, Murakami, & Mochida 1997).

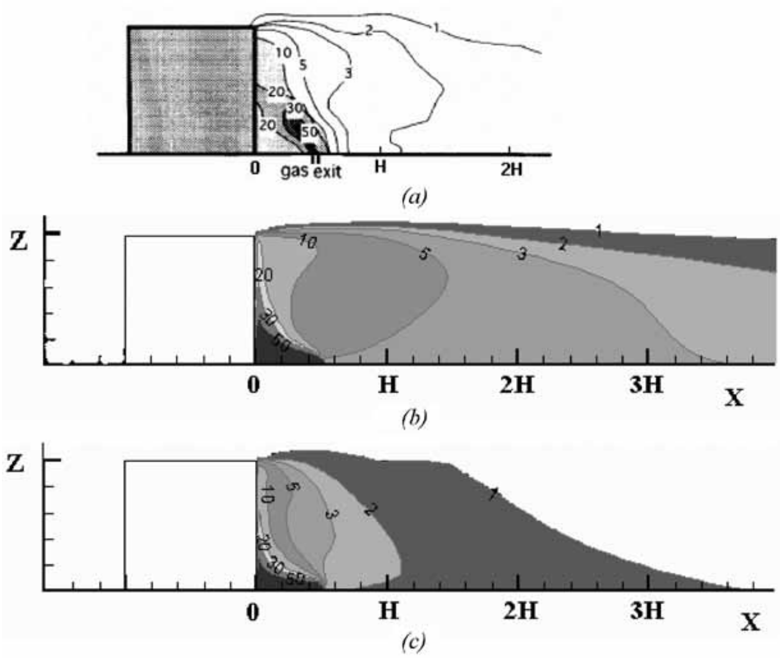


Figure 6. Non-dimensional Concentration (at the center section): (a) experiment; (b) standard k-ε model; (c) new algebraic model.

model correctly predicted this phenomenon. This means that both models are able to simulate the recirculation phenomenon behind the building.

However, both turbulence models over-estimated the concentrations in the area near the leeward wall face. The size and center of the vortex behind the building, therefore, are not accurately reproduced by these two turbulence models. LES with the standard Smagorinsky model (S model) has a similar problem, which could be slightly improved with composite grid settings. LES with Dynamic Mixed model (DM model), however, could provide more accurate results for concentrations near the leeward wall face (Tominaga, Murakami, & Mochida 1997). Therefore, the simulation results by the two turbulence model are acceptable, since their accuracy is comparable to LES results with the standard S model.

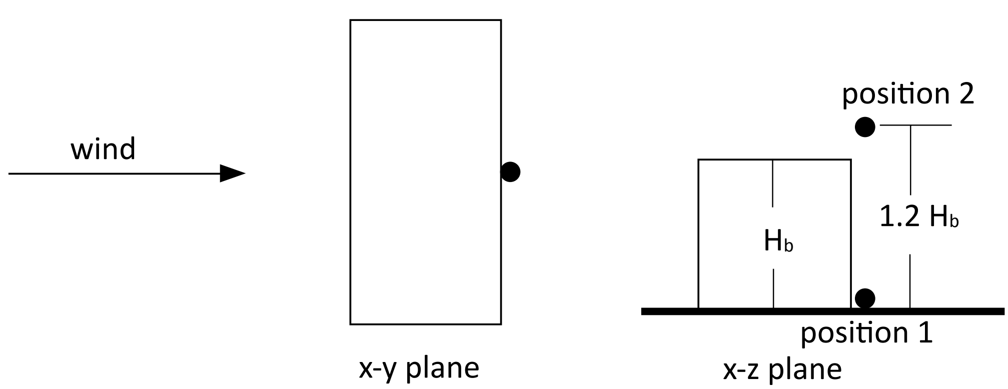
In summary, the standard k-ε overestimates the concentrations downstream the gas exit. The new algebraic model, on the other hand, underestimates the concentrations a little. Compared with the experimental results, the new algebraic model shows better agreement than the standard k-ε model for this area.

3.3 Pollutants dispersion around a rectangular building

A rectangular building case is simulated with pollutant sources at different positions, as shown in its schematic view in Figure 7. The building model height H_b is 0.82ft (0.25m), and the length is twice of its height and width. In one case, the point source locates at the ground (position 1) just behind the center of the leeward building wall; in another case, the point source is at a height of $1.2H_b$ above the ground (position 2). The performance of the simulation results are compared with measured concentrations from the wind tunnel data of Huber, Synder, and Lawson (1980).

The velocity profile U/U_r is set to fit a one-sixth power law. The reference velocity, $U_r = 2.3\text{m}/s$, is the velocity at $1.5H_b$ above

Figure 7. Source position (Li & Stathopoulos, 1997).



the ground. The non-dimensional concentration is defined as: $CU_r H_b^2 / Q$, where Q is the emission rate of the point source.

Figure 8 illustrates the longitudinal profiles of concentrations at the ground downstream from source 1. With the same numerical settings, numerical scheme, and convergence criteria, the new algebraic model shows that it is in a much better agreement with the wind tunnel than the standard k-ε model. The new algebraic equation model under-predicts the concentrations close to the leeward wall in the near wake zone. Beyond the distance $x/H_b = 1$, the new algebraic model fits well with the measurement data. However, the standard k-ε model overestimates the concentrations all the way along the ground.

Using the simulation flow-field by the k-ε model, Selvam (1997) computed the pollutant concentrations for the same case by upwind and Streamline Upwind Petro Galerkine (SUPG) finite element procedure (FEM). SUPG procedure overestimates the longitudinal concentrations resulting in approximately double the wind tunnel values. The upwind procedure underestimates the values close to the wall, but it is much closer to the wind tunnel data away from the wall. In these simulations, a hybrid scheme is used for both the standard k-ε model and the new algebraic models. Therefore, a different numerical scheme could be a solution to improve the accuracy of the standard k-ε model.

The vertical concentration profile at $3H_b$ downstream from the leeward building wall for source position 1 is shown in Figure 9. The results of the new algebraic model are much closer to the wind tunnel data than the standard k-ε model, although it under-predicted the concentrations for $z/H_b < 1$. The standard k-ε model overestimates the concentrations for $z/H_b < 1.4$ while under-estimates the values for the rest of area. This phenomenon is similar to the previous cubic building case that the standard k-ε model tends to over-predict while the new algebraic model may under-predict the concentrations downstream from the

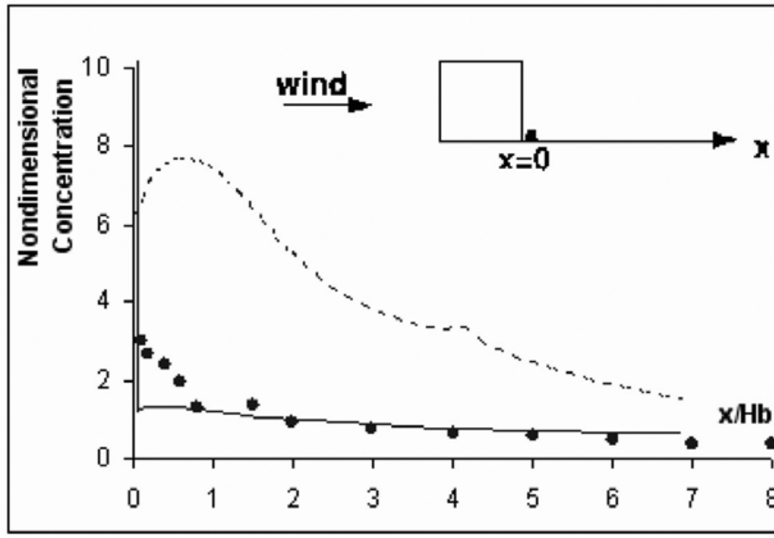


Figure 8. Longitudinal ground-level concentrations along the centerline for source position 1. Black Dots: wind tunnel experiments; dashed line: standard k-ε model; solid line: new algebraic model.

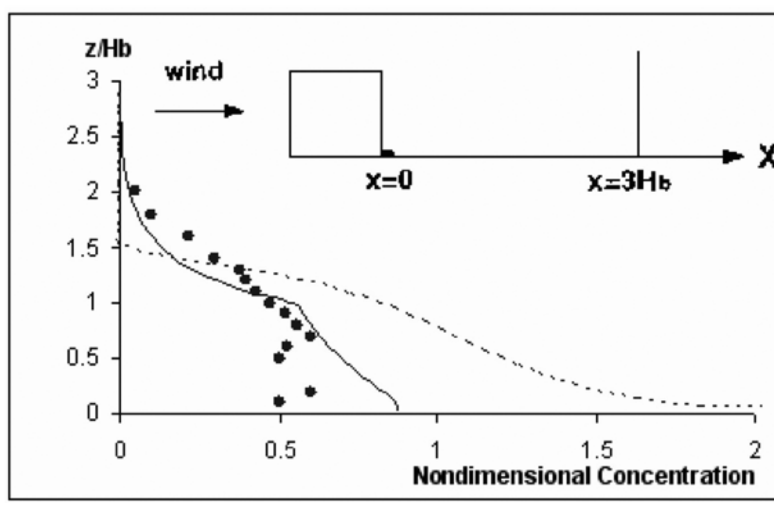


Figure 9. Vertical profiles of concentrations along the centerline at for source position 1. Black Dots: wind tunnel experiments; dashed line: standard k-ε model; solid line: new algebraic model.

source. The FEM results by Selvam (1997) show that the upwind procedure is more accurate than the SUPG procedure in this particular vertical concentration profile.

The longitudinal ground-level concentrations along the central line for source position 2 are illustrated in Figure 10. The results of the new algebraic model fit very well with the wind tunnel measurements. The concentration values by standard k-ε model are close to the wind tunnel data, but the trend of the curve does not fit well with the measurement results. Figure 11 shows the vertical profiles of concentrations along the centerline at $3H_b$ for source position 2. Both the standard k-ε model and the new algebraic model fail to predict the concentrations. The standard k-ε model over-predicts the concentrations, while the new algebraic model under-predicts the concentrations in most areas. Therefore, although the new algebraic model shows a better agreement than the standard k-ε model in the previous comparisons, it still has problems in predicting the contaminant dispersion around the building, especially at the wake area.

The flow field is complex in the region behind the building. Both the standard k-ε model and the new algebraic model could not accurate predict the flow field behind the building. Therefore, the results of the concentration field behind the building, which is solved based on the flow field simulation, is difficult to reach a satisfactory level. Further work is necessary to improve the new algebraic model in order to get better results for the wake area.

4. Discussion

The three cases used to validate the zero-equation model include (1) Cross Ventilation, (2) Gaseous Diffusion around a Cubic Building, and (3) Pollutants Dispersion around a Rectangular Building. Each simulation case had from 200,000 to 300,000 simulation grids, and used both the standard k-ε model and the zero-equation model. All simulations used the hybrid numerical scheme and

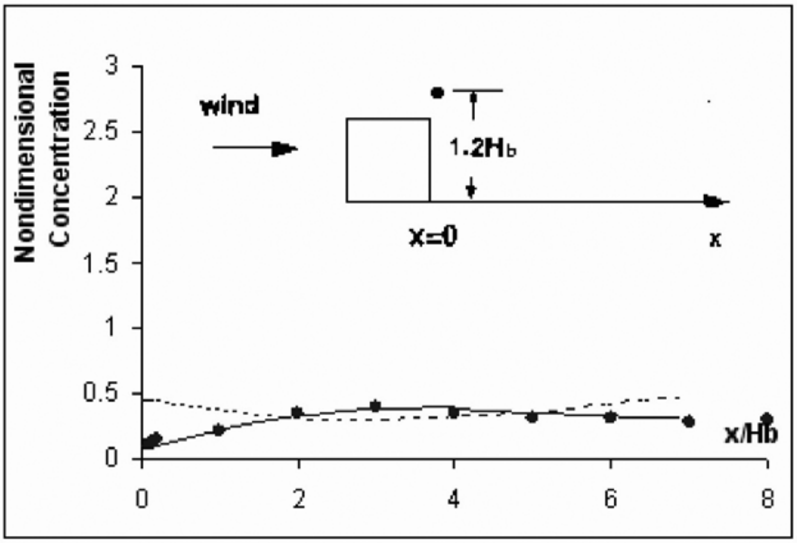


Figure 10. Longitudinal ground-level concentrations along the centerline for source position 2. Black Dots: wind tunnel experiments; dashed line: standard k-ε model; solid line: new algebraic model.

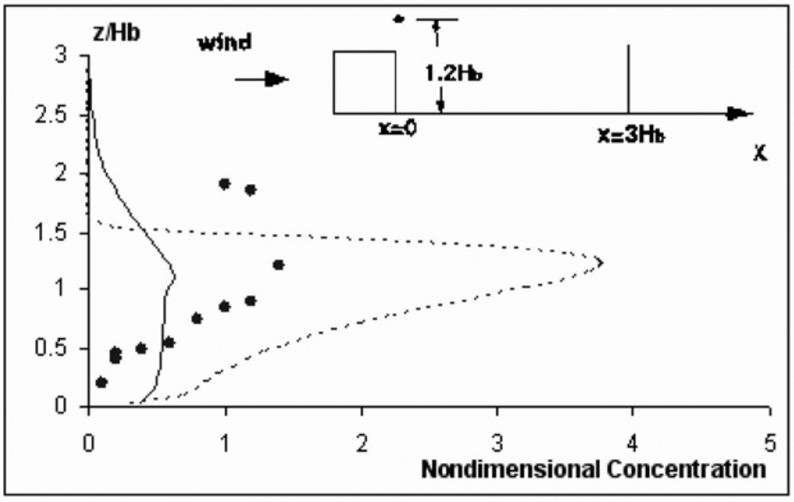


Figure 11. Vertical profiles of concentrations along the centerline at for source position 2. Black Dots: wind tunnel experiments; dashed line: standard k-ε model; solid line: new algebraic model.

the convergence criterion requiring that the total mass residual is lower than 0.1% of the incoming mass flow rate. The inflow stream-wise velocities followed the ¼ power law, and the turbulence intensity was assumed to be 10%. In all of the validation case, performance of the zero-equation model was similar or slightly better than the standard k-ε model when compared to the measured wind tunnel data.

Based on the validation case studies, the zero-equation model can be used for predictions of velocity fields as well as concentration fields around a building. In practical applications, this model can be used for design of natural ventilation, including outdoor and indoor airflows as demonstrated in the cross ventilation case. The other two case studies indicate that the model can be used to address design issues for known pollutant sources such as pollution from the nearby traffic or industrial complexes. The model is currently being tested for much larger simulation domains to assess its full benefits in reducing required computational time for outdoor airflow simulations.

5. Conclusions

Outdoor airflow was simulated for a rectangular building case with the new algebraic model and the standard k-ε model. Using the results of the standard k-ε model as the benchmark, the new algebraic model accurately predicted the flow patterns and pressure field. With a quantitative comparison of the mean velocity distributions to the wind tunnel data, both the standard k-ε model and the new algebraic model show discrepancy in the wake area behind the building. The agreement for numerical results and the wind tunnel data is good for the rest of the simulation field. Therefore, the new algebraic model has a similar accuracy in predicting the airflow around a building as the standard k-ε model. This level of accuracy is acceptable for design applications.

Two more cases are further simulated to validate the new model in predicting the concentration field around a building. Although the simulation results do not fit very well with the wind tunnel data, the concentration field predicted by the new algebraic model is in a better agreement than the field predicted by the standard k-ε model for these two cases. Additional cases need to be simulated in order to evaluate the current new algebraic model. Since discrepancies still exist for both the new algebraic and the standard k-ε models, further improvements are necessary for this new model.

With similar accuracy, the new algebraic model only requires as a half of the computation time necessary for the standard k-ε model, which can be significant for large simulation domains usually required for outdoor simulations. Therefore, the new algebraic model provides a promising way of outdoor environment simulations in a fast, reliable, and cost-effective manner.

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