Review Article:
Structural Morphology: An Effective Solution for Generating Efficient Structural Forms

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Abstract

Many natural forms have been imitated by architects and designers and applied to the design process in order to achieve the most efficient solution for structural arrangement and architectural form. The integration of knowledge in biology and structure in nature is an approach to structural morphology that offers the possibility of developing innovative forms and efficient structural prototypes. This review introduces the fundamental concept of structural morphology based on form-finding and structural optimization through the works of the biologist D'Arcy Thompson, the theory of Michell and the experimental studies of pioneers like Antoni Gaudi, Frei Otto and Heinz Isler. Some examples of buildings using structural morphology as effective design solutions are illustrated to substantiate the potential applications in architecture.

Keywords: structural morphology, form-finding, structural optimization, topology optimization, structural form

1. Introduction

In the early twentieth century, many architects and builders used nature as a source of inspiration to establish new creative processes in architectural design (Theissen, 2011). One of the most intriguing was the concept of organic architecture, which refers to the use of organic or live materials to create architectural forms. The concept of organic architectural form emerged through the works of several renowned architects, including Antoni Gaudi, Frank Lloyd Wright and Rudolf Steiner (Figure 1), who are considered to be the pioneers of organic morphology in contemporary architecture.

(a) Casa Milà designed by Antoni Gaudi (Berger, 2021)

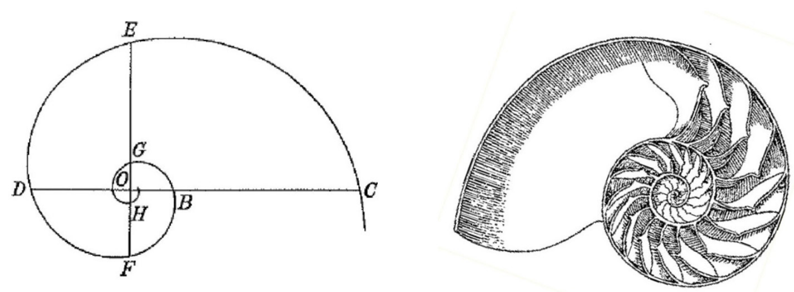
(b) Johnson Wax Headquarters designed by Frank Lloyd Wright (Yu, 2016)

(c) Goetheanum designed by Rudolf Steiner (Gray, 2010).

Figure 1. Examples of organic architecture

The word "morphology", introduced by Johann Wolfgang von Goethe[1] , is derived from the Greek 'morphé', meaning 'form', and 'lógos', meaning 'study'. Hence, the meaning of morphology is the science of form (Eekhout, 2001). There are different interpretations in different fields of morphology. In the morphology of language, it refers to the composition and systems of words within language, whereas in biology, it is the study of the forms of organisms.

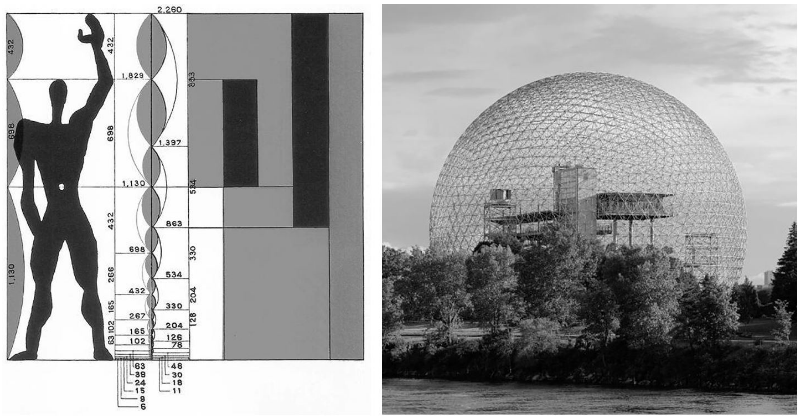
One of the most famous works in the field of morphology was carried out by D'Arcy Wentworth Thompson[2] . Thompson developed his ideology based on a combination of the structuralism of Goethe and the functionalism of Bernard Russell. In his famous book entitled On Growth and Form, Thompson attempted to explain the biological form in terms of mathematics and physics via numerous examples, for instance, using the idea of logarithmic spiral (known as the equiangular spiral) to explain the nautilus shell geometry (Figure 2).



(a) (b)

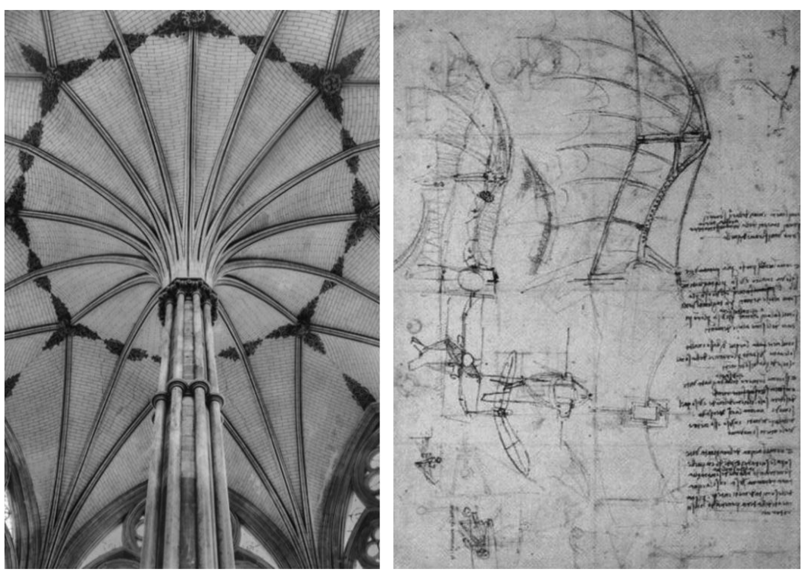
Figure 2. (a) A logarithmic spiral. (b) Half shell of a Nautilus (Thompson, 1917).

The spiral of a Nautilus shell is an example of a simple geometric algorithm in nature, where its growth increases but it still maintains the same proportions. The self-similarity of the Nautilus shell spiral is associated with the architectural property of the golden section. Furthermore, Thompson also proposed that the form of any biological object is the consequence of the 'diagram of force' that acted upon it (Thompson, 1917). For example, the shapes of individual cells are the result of the surface tension of their cell membranes, like soap bubbles. The many analogies from structural to organic forms explained via Thompson's theory reveal the affinities between organic and structural morphogenesis.



(a) (b)

Figure 3. (a) The Modulor by Le Corbusier (Corbusier, 1998).
(b) Geodesic dome by Buckminster Fuller (Bau, 2016).



(a) (b)

Figure 4. (a) Tree-like column of Salisbury Cathedral (Akande, 2016).
(b) Sketches of bat wings by Leonardo da Vinci (Zöllner and Nathan, 2014).

The understanding of morphology through biology can be transferred to architecture via the concept of structure in mathematics. In 1951, the Modulor, published by Le Corbusier, was developed from a theory of a sequence of harmonic numbers related to human proportions (Figure 3(a)), while in the late twentieth century, Richard Buckminster Fuller developed the geodesic dome following Thompson's statements on cell behavior and the geometry of Radiolaria[3] (Figure 3(b)). The broad range of studies of natural forms, force relationships and design processes were later used by many architects, including Frei Otto, Heinz Isler, Louis I. Kahn and Pier Luigi Nervi, as approaches for developing their methods in structural morphology.

Nowadays, the understanding of structural morphology through biology integrated with the development of computer technology has become a powerful tool for architects and structural engineers to explore new innovative structural shapes and geometries naturally and with increased energy efficiency.

2. Structural Morphology

Structural morphology is not a new discipline in the architectural design process. Gothic cathedrals obviously use structural dendriforms inspired from nature as their primary structures (Figure 4(a)). Several works of Leonardo da Vinci are based on bionic design and represent the strongest link to structural morphology (Figure 4(b)). In the twentieth century, there has been increasing concern regarding form, structure, morphology and organization (Hillier and Leaman, 1973), and this has resulted in a significant growth in the study of form and structure. The concepts of form and structure are defined from a structural viewpoint through structural morphology (Pultar, 1977).

The term ‘structural morphology’ was proposed by Michael Burt (Motro, 2011) but there is no clear definition from the SMG[4] due to its diversity of research approaches. Therefore, many researchers have provided their own definition for this term, for instance, Shen and Wu defined a discipline that studies the interaction between the structural form and its mechanical behavior from an integral perspective, with the aim of realizing the rationality and efficient of the structures (Shen and Wu, 2014). In contrast, Rene Motro described the meaning of structural morphology as the study of form and shape of a structure and the relations between form, forces and material, as shown in Figure 5.

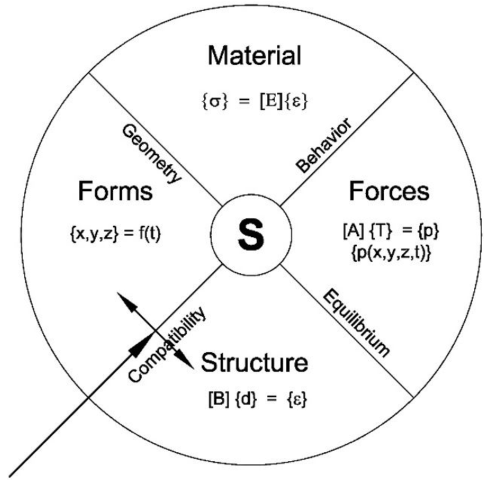


Figure 5. Motro’s conceptual scheme of structural morphology (Motro, 2009).

According to the conceptual scheme in Figure 5, Motro classified design parameters, based on the parametric analysis of the structural system, into four categories: structure, form, force and material. The position of structural morphology in this diagram is at the interface between form and structure. The direct relation between form and structure is affected by the behavior of the material and the need to ensure the static equilibrium of the system **S** being designed (Motro, 2009).

Although the conceptual scheme of structural morphology shows a strong relationship between form and structure but in some construction systems, the other two parameters play an important role in defining the form of equilibrium structure. Two equilibrium form systems, which the final form is the result of the forces applied, can be divided into two categories: funicular and self-stressing systems.

Funicular systems are systems where the equilibrium state of form is dependent on the magnitude and direction of the external forces, for example, the shape of a hanging chain. Gaudi applied this method to his notable design in the Church of Colònia Güell (Figure 6(a)) by constructing the model with ropes. The geometric shapes of ropes formed by concentrated loads represent the part of stresses. The compressed shapes were then created by inverting entirely tension systems to compression arches. *Self-stressing systems*, or form-active structures (Engel, 1997), are systems where the equilibrium state of form results from the internal stresses in static equilibrium (not effected by external actions). The systems in this category are cable nets and tension membranes (Figure 6(b)).

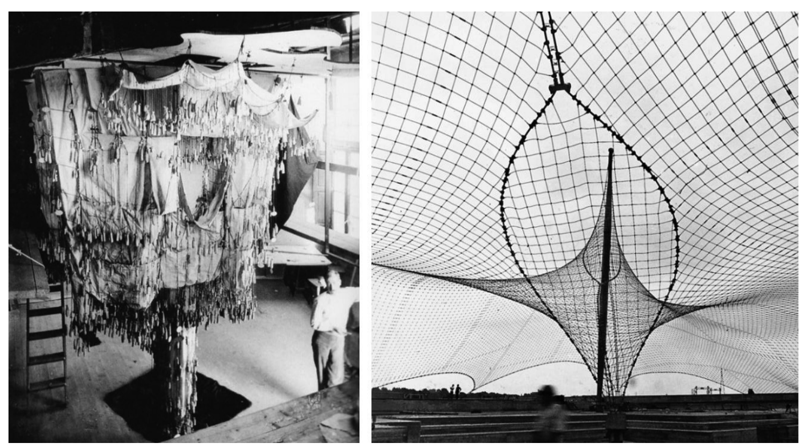


Figure 6. Funicular and self-stressing form systems: (a) hanging model for the Church of Colònia Güell made by Gaudi (Martin, 2015); (b) German Pavilion at Montreal in 1967 designed by Frei Otto (Detail 6, 2000).

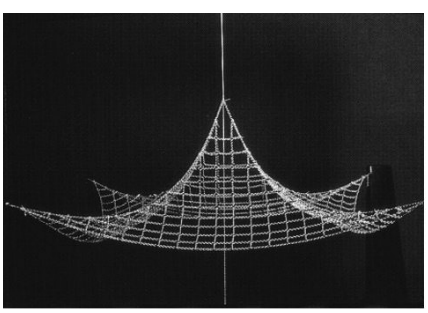
3. Form-Finding and Structural Optimization

For structural design, the geometric shape has an effect on its structural efficiency. Researchers and designers have developed many numerical methods for several decades to obtain the most efficient shape and concluded that its shape should depend on the flow of forces (Li et al., 2017). The two primary methods to generate the diverse structural forms based on the concept of structural morphology are form-finding and structural optimization (Li, 2018).

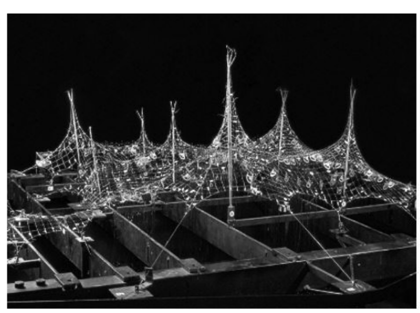
Form-finding is a process used to find the optimal geometry of a structure that is in static equilibrium with a design loading (Adriaenssens et al., 2014). It is usually used to determine the equilibrium shape of form- (e.g., cable nets and membranes) and surface-active structures (shells and grid shells). In the pre-computer age, architects and engineers conducted form-finding process through experiment using physical models (Chandana et al., 2005). These physical models can be divided into three groups: hanging (Figure 7(a)), tension (Figure 7(b)) and pneumatic models (Figure 7(c)).

These three groups of physical models are based on the "form follows force" principle and represent the equilibrium state of flexible materials under their self-weight and certain constraint conditions with the stress state in pure tension. The structural forms that are generated from physical models will perform with high structural efficiency.

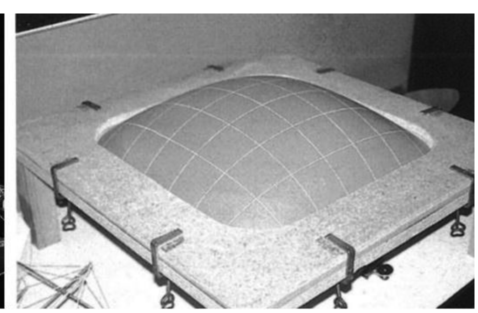
Since the mid-twentieth century, with the development of computer techniques, the new advanced in computational methods of form-finding have been developed to overcome the difficulties in the form-finding process. In 1970, the force density method was introduced by Linkwitz and Schek (Linkwitz and Schek, 1971) in response to the need for computational modelling of structures for the Munich Olympic complex (Lewis, 2003). The force density method is considered to be very effective, because the nonlinear equilibrium equations of the nodes are transformed to a linear system of equations and then be solved in the form-finding process (Zhang, and Ohsaki, 2006).



(a)
hanging model for the study
of suspended roof (Otto et al., 2006)



(b)
tension model for the German Pavilionin
Montreal (FAR Frohn and Rojas, 2016)



(c)
pneumatic model for bubble shell investigation
(Chilton, 2000)

Figure 7. Physical models for form-finding

Another method is the dynamic relaxation method, which was first applied to the static analysis of hanging roofs by Day and Bunce in 1970 (Day and Bunce, 1970) and further applied for the form-finding of cable nets and membrane structures by Lewis (Lewis et al., 1984) and Barnes (Barnes, 1994). The dynamic relaxation method is based on the iterative calculation of the movement of each node of the structure from its initial position until the structure comes to rest, due to artificial damping, in static equilibrium. According to the studies of the efficiency of numerical solutions by Barnes (Barnes, 1982) and Lewis (Lewis, 1989), it was revealed that the method of dynamic relaxation is very well suited to both the static analysis and form-finding of lightweight tensile structures.

Structural optimization refers to an optimization that seeks to find the best arrangement of structures or structural components to achieve certain objectives under specific conditions (Tsiptsis et al., 2019) or in other words, making an assemblage of materials sustain loads optimally (Christensen and Klarbring, 2009). The objectives of structural optimizations are to reduce the total cost by minimizing the total weight of the structure to improve the structural performance, such as mechanical behavior and aerodynamic performance. Another significant objective is to reduce the environmental impacts (Mei and Wang, 2021). Structural optimization can be divided into size, shape or topology optimization (Figures 8(a)-(c)). Size optimization is used to determine the effective cross-section size of frames and trusses, whereas the goal of shape optimization is to find the optimum shape of the design domain. Nevertheless, both size and shape optimization cannot change the structural topology during the optimization process. Among these three methods, the topology optimization is usually used as a tool to support the form-finding process in architectural design (Bialkowski, 2016) because it offers more freedom for a designer to create efficient conceptual designs (Huang and Xie, 2010).

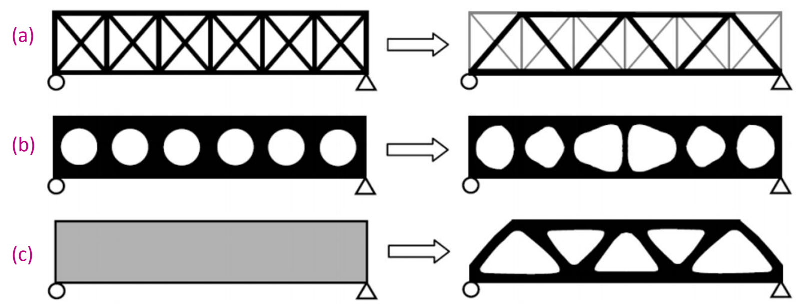
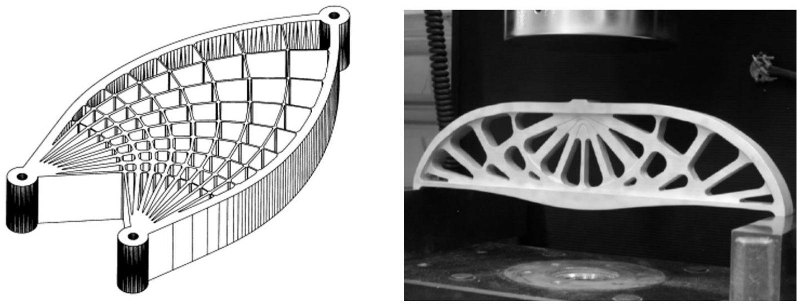


Figure 8. Three structural optimization categories: (a) sizing optimization; (b) shape optimization; (c) topology optimization. The initial problems are shown on the left and the optimal solutions are shown on the right (Bendsøe and Sigmund, 2003).

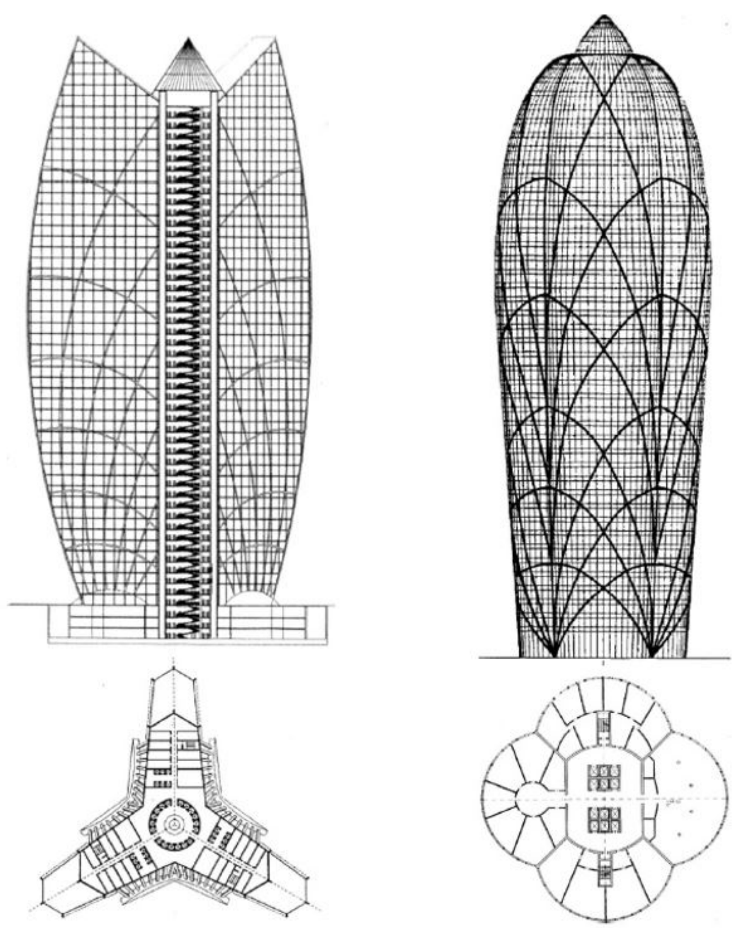
Topology optimization, the most general form of structural optimization, focuses on how nodes are connected and supported, with the aim of minimizing the stain energy of the structure by means of eliminating redundant structural members. The first approach of topology optimization concepts was found in the Maxwell theorem for frames, where he proposed optimal trusses with elements aligned in the directions of the principal stresses (Oliveira et al., 2018). In 1901, Anthony G.M. Michell, an Australian mechanical engineer, presented quasi-continuum truss structures as innovative solutions in the field of structural topology optimization by extending Maxwell's theory (Michell, 1904).

Michell studied the cantilever truss of optimal design to transmit the applied load to the given fixed point of support (Prager, 1977) and presented orthogonal curved trusses composed of tensile and compressive members intersecting at 90° angles to each other, with no shear stresses and maximum stiffness for the given structural mass. This optimal discrete truss structure is known as the *Michell Truss* (Figure 9(a)). Although Michell structures have shown agreement with mechanical properties, as shown by physical testing (Figure 9(b)), their applications are not practical due to the complex geometry.



(a) (b)

Figure 9. Michell structures: (a) Michell Truss (Dewhurst, 2001); (b) physically testing of Michell structure (Srithongchai, 2003).



(a) (b)

Figure 10. Skyscraper projects (Zalewski and Zablocki, 2002).

Michell's theory has been used by architects and engineers as an optimization technique for structural design applications. Wacław Zalewski and Wojciech Zabłocki, a Polish engineer and architect, respectively, proposed the general concepts of "tulip-like (bulbous) buildings" and "wingy buildings" based on Michell's theory as solutions for high-rise building design (Zabłocki, 2000). Wingy buildings are composed of three or four wings connected to the central core. The structure of each wing is analogous to the geometry of Michell structures and is used to transmit bending moments caused by wind loads (Figure 10(a)), while the shapes of tulip-like buildings are generated by the rotation of a plane Michell structure around the vertical axis (Graczykowski and Lewiński, 2020). The lateral forces caused by wind loads are transmitted by the double-curve of the steel structure on the building's façade (Figure 10(b)).

Topology optimization techniques can be utilized mathematically through the use of the finite element method (FEM), which was introduced by Richard Courant in 1940s (Williamson, F. Jr.,1980). With finite element analysis, the geometry of the design domain is subdivided into several small elements. Sigmund and Petersson (Sigmund and Petersson, 1998) used the FEM in a topology optimization solution as a number of black and white pixels of an image to represent and conceptualize the optimal structure. These pixels indicate the existence of a material. The structural layout is defined by the material distribution $\chi(x)$ as a function of location x in the design space, as shown in Figure 11 (Bletzinger and Ramm, 1998). In addition, Eschenauer and Olhoff classified the topology optimization techniques into two distinct categories of material or micro approaches based on the SIMP[5] method and geometry or macro approaches based on the ESO[6] method (Eschenauer and Olhoff, 2001).

$$\chi(x) = \begin{cases} 0 \rightarrow no\ material \\ 1 \rightarrow material \end{cases}$$

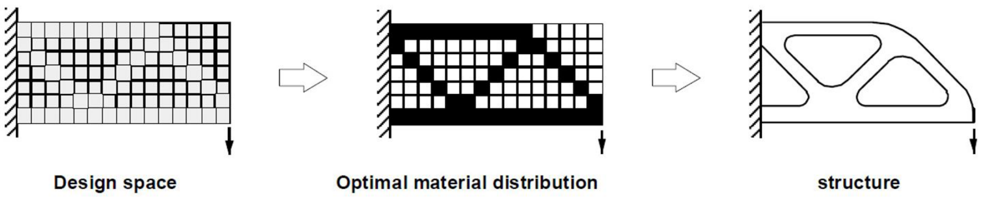
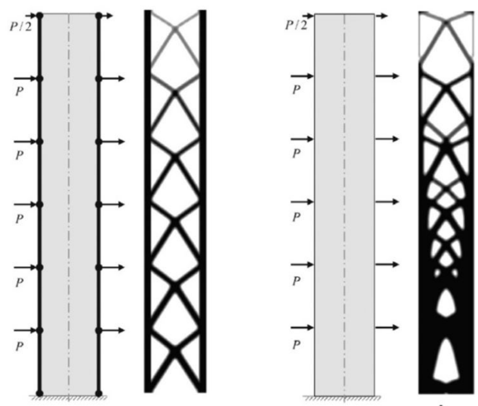


Figure 11. Topology optimization (Bletzinger and Ramm, 1998).

4. Applications of Structural Topology Optimization in Architecture

The applications of topology optimization in the field of structural design can be divided into discrete member optimization and continuum methods based on the type of structure (Stromberg et al., 2012a). The aim of discrete member optimization is to determine the optimal characteristics, such as number, shape, size or connectivity of elements for discrete structures, like trusses and frames. In contrast, the continuum optimization method is applied to solid shapes, such as shell structures, in order to find whether the topology of the structure should be a solid or void element (Figure 12).

The application of topology optimization techniques provides engineers with the ability to develop the optimal topology of lateral bracing systems with minimal material usage and rational cost (Stromberg et al., 2012b). Bracing systems have been used in several notable skyscrapers, such as the John Hancock Center in Chicago (Figure 13(a)) and the Bank of China Tower in Hong Kong (Figure 13(b)). All of these bracing systems are based on traditional designs (diagonal braces at 45° or 60°). However, there have been very few studies conducted to identify the optimal angle.



(a) (b)

Figure 12. Topology optimization techniques for structural systems: (a) problem statement for discrete member optimization approach:
(b) problem statement for continuum optimization method (Stromberg et al., 2012).

(a) (b)

Figure 13. Traditional diagonal bracing of high-rise buildings: (a) John Hancock Center in Chicago (Stoller, 2021); (b) Bank of China Tower in Hong Kong (Fu, 2019).

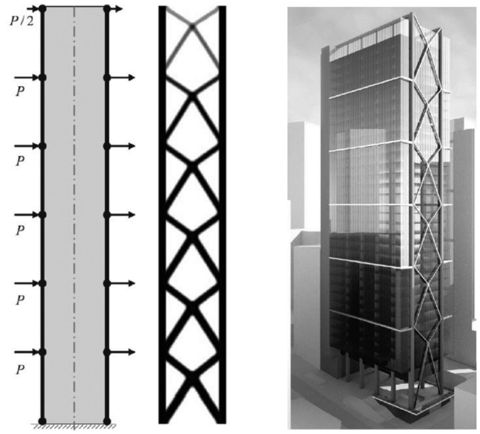
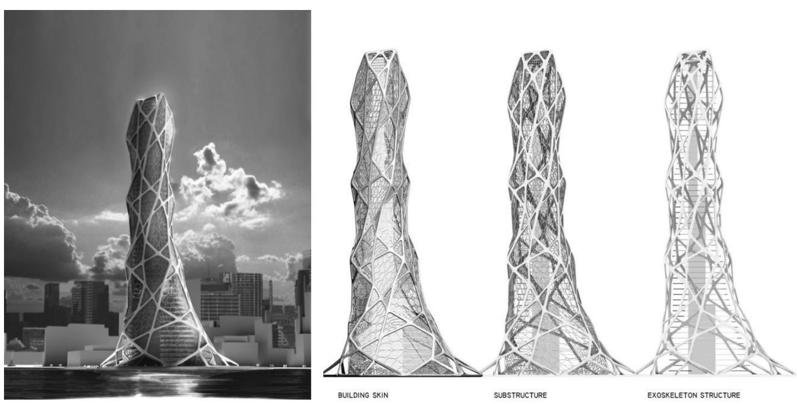


Figure 14. Topology optimization for diagonal bracing system of 100 Mount Street building (Beghini et al., 2014).



(a) (b)

Figure 15. Bionic Tower designed by LAVA Architecture Group:
(a) design proposal of tower (Poh, 2019); (b) Bionic Tower systems (Sadeghipour, 2015).

(a) (b)

Figure 16. Topology optimization applied in architectural form by Arata Izosaki:
(a) Illa de Blanes in Spain (Bialkowski, 2016);
(b) Qatar National Convention Centre in Doha (Sobek, 2021).

The conceptual design of the bracing system for the 100 Mount Street building, 288 m in height, in Australia, identified the optimal diagonal layout using topology optimization with a combination of continuum and discrete elements. The final result shows that the densities of the bracing elements increase as the load increases throughout the height of the structure (Figure 14). The application of the topology optimization technique enables engineers to identify the best bracing layout while satisfying the aesthetic expression of the architectural design.

Another example of using the topology optimization approach as a solution for complex high-rise structures is the architectural proposal of the Bionic Tower in Abu Dhabi (Figure 15(a)). The Bionic Tower is a biomorphic project inspired by nature. The building behaves like an organism or ecosystem, with a skin that controls external environments, including air pressure, temperature, humidity, air pollution and solar radiation. The main structures used for the tower are a braced outrigger and a concrete core, which is connected to the external bracing elements by a truss. The topology optimization technique has been applied to generate the optimal freeform of the entire exterior surface, which is a responsive skin and can be adjusted to the outside environment (Figure 15(b)).

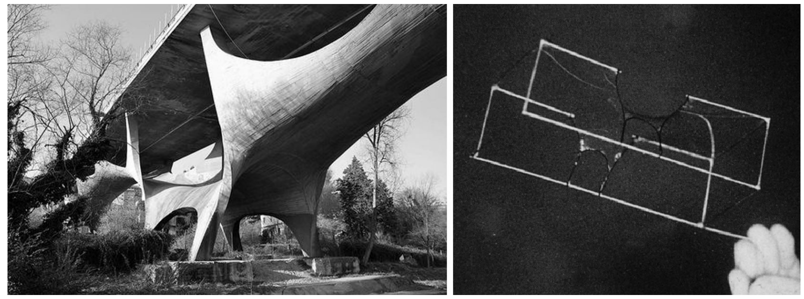
The previous examples are focused on the structural aspects resulting from the topology optimization approach. One of the first attempts to use topology optimization algorithms in design architectural form was found in the project *Illa de Blanes* in Spain (Figure 16(a)) designed by the Japanese architect Arata Izosaki in collaboration with Mutsuro Sasaki. Izosaki designed the tree-like columns to support the large double-curved roof, which covers 75,000 m^2 of usable space. These organic-shaped columns were generated by topology optimization based on the ESO algorithm (Januszkiewicz and Banachowicz, 2017). However, this project has never built due to budgetary constraints.

Izosaki also collaborated with Sasaki in the design of the structure for the Qatar National Convention Centre in Doha. The development of structural optimization technology, known as the extended evolutionary structure optimization (extended ESO), is applied to generate the forms of dendriform, tree-like structure, which was inspired by the Sidra[7] tree. The two enormous dendriform structures were designed to support the 250 m long by 30 m wide and 20 m high of building's exterior canopy (Figure 16(b)). The complex geometry of organic forms shaped were first generated from Izosaki's office by using the 3D extending ESO and then were rationalized through Rhino by Buro Happold before fabrication in Malaysia (Januszkiewicz and Banachowicz, 2017). By applying the ESO method to generate the structural shape, resulting in minimized bending stresses in the slab and optimized global behavior of the structural system (Larena, 2009).

Structural optimization in engineering usually takes natural constructions as an example in order to obtain both structural efficiency and architectural aesthetic value. The Basento Bridge (Figure 16(a)) is one of the very few shell-supported bridges, which achieve in the aesthetics form and optimized structure.

The shell-supported bridge over the Basento River in Potenza, Italy, designed by the Italian engineer Sergio Musmeci, is based on the conceptual idea of a harmonic relationship between structure and architectural form. Musmeci designed this bridge by inspiration from the concept of minimal surfaces, which minimize the material in creating structures. However, his main goal was not to save concrete, but to obtain a structure that was also an architectural object with aesthetical value (Fenu et al., 2020).

In the first step of the design process, Musmeci investigated the form of a bridge through the experiments made of soap-glycerin films (Figure 17(b)), which allowed him to achieve the optimal shape of the bridge. Based on these experiments, he



(a) (b)

Figure 17. (a) Basento Bridge in Potenza, Italy (Mirlostudio, 2021).
(b) Scherk surface: soap film model of Basento Bridge (Alessandra, 2012).

derived the minimal structures and created a physical model of two semi-arches using a sheet of rubber and then applied the compression forces on where the real structure would be subjected to. After working with several model tests and numerical calculations, Musmeci finally completed the bridge design by the form of the double-curved RC shell with a thickness of only 30 cm. The anticlastic shells of minimal area can efficiently transfer applied loads to the foundations. The structure of the Basento bridge is one of the very few shell-supported bridges, which reflects the complex procedure of the form-finding method in order to obtain the design and construction of optimized structures.

Nowadays, with the development of computer technologies, the complex design procedure through the use of physical models can be replaced by using numerical form-finding algorithms. The design principle of the shell-supported bridge developed by Musmeci was reinvestigated to improve the behavior of the bridge. According to the research of Bruno Briseghella (Briseghella et al., 2013), the unwished tensile stresses caused by unwished bending moments can be reduced by inserting cavities in the shell surface by means of topology optimization.

In order to investigate, a shell footbridge cross a deep canyon in Cagliari, Italy, was designed through a form-finding method in the same procedure as completed in the Basento bridge.

(a) (b)

Figure 18. (a) Model of shell-supported bridge (Briseghella et al., 2013).
(b) Model of shell-supported bridge with cavity insertion (Briseghella et al., 2013).

(a) (b)

Figure 19. (a) World Trade Center Transportation Hub, New York City, by Santiago Calatrava (Alan Karchmer, 2021). (b) Galaxy SOHO, Beijing, by Zaha Hadid (Hufton+Crow, 2017).

The computational model of the bridge, generated by using finite element analysis software (Figure 18(a)), indicated that there were unwanted bending moments occurred in some shell regions due to the second-order displacements and the bending stiffness of the RC shell.

To reduce these unwanted internal forces, some materials have to be eliminated from the shell regions. Therefore, cavities were inserted in the shell using topology optimization based on the SIMP method. The results show that this technique is very efficient in greatly reducing the area of the shell regions, which causes the unwanted tensile stresses. Furthermore, the use of the topology optimization technique also defines the suitable hole pattern on the structure (Figure 18(b)).

5. Potentials of Parametric Modeling for Structural Morphology

The advancements in of the computer-aided technology of last few decades had an impact on architecture and engineering. Computer-aided Design (CAD) as a tool of parametric design is not only used for 3D renderings and presentations but becomes a powerful design tool which allows architects to design free form with more complicated spaces and higher structural complexity as can be seen in the projects from Santiago Calatrava and Zaha Hadid, the pioneer architects who have carried out shape finding by using of parametric modeling (Figure 19).

Parametric modeling, first invented by Rhino, is the process of making a geometric representation of a design with components and attributes that have been parameterized (Barrios, 2005). The rise of parametric modeling tools has been further integrated to the design process, leading to the term 'parametric design' (Harding et al., 2012). Parametric design process consists of five distinct stages: determining the parameters, designing the relations between the parameters, determining the estimated geometry, creating variations and testing the resulting product. These stages are associated with each other and affect each other. The advantage of parametric modeling is the ability to change the shape of model geometry as soon as the dimension value is modified (Fu, 2018). This provides convenience for designers in terms of time and application. The use of parametric modeling for structural morphology and topology optimization offers a broader range of alternative design solutions and provides designers a faster way of rationalizing forms.

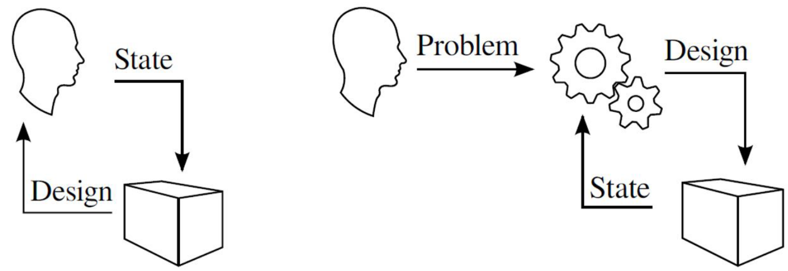
6. Conclusion

Architecture has long been inspired by natural forms. Many architects and engineers search for rational structures and form optimization using nature and biology as role models. In architectural design, the overall structural geometry, or structural form, has a significant influence, not only on the architectural image, but also the routing of the flow of internal forces. The study of the relations between form and structure is defined as structural morphology, which is a unique field of study in architecture.

The use of structural morphology and optimization algorithms has changed the way of design, as can be seen in the works from architects since the previous century, such as Antoni Gaudi, Frei Otto, Buckminster Fuller and Heinz Isler, who took inspirations from natural structures and carried out experiments using physical models to find efficient structural forms. This iterative process is known as form-finding and was later achieved by numerical methods, such as the force density method or dynamic relaxation. Both solution procedures are useful for determining the equilibrium position of a structural network in relation to the internal force (Kilian and Ochsendorf, 2005). The form-finding process is typically applied to form-active structure, such as cable-net structures and membrane structures.

However, finding the equilibrium shape using form-finding methods is inadequate because the shape is the result of the process without considering optimization problems that consist of many variables and constraints (Figure 20(a)). Hence, in order to obtain the optimal shape, the structural design problems need to consider the design process through the structural optimization method (Figure 20(b)).

The aim of structural optimization is to maximize the performance of a structural component under prescribed conditions (Stach, 2010). Structural optimization can be classified into sizing, shape and topology optimization. Topology optimization, an iterative computational process that works within confined space, is the most broadly applied to architecture in order to improve the structural performance and to reduce the total cost by minimizing the total weight of structure.



(a) Form-finding (b) Optimization

Figure 20. Two approaches of structural morphology in design process (Descamps, 2014).

To conclude, the use of structural morphology as an approach can lead to excellent integration between engineering and architectural design. The availability of advance computer technology for structural optimization and form-finding offers faster way for rationalizing form and provides opportunities for designing a new generation of buildings. Furthermore, the structures, which are designed through a structural optimization process require less materials, leading to the current trend towards economical and environmentally sustainable design.

Remarks

[1] Johann Wolfgang von Goethe (1749–1832): a German naturalist and philosopher.
[2] D'Arcy Wentworth Thompson (1860–1948): a Scottish biologist and mathematical scientist.
[3] Microscopic planktonic with a spherical shape that are mostly marine but with some freshwater variants.
[4] Structural Morphology Group (SMG) founded in 1991 by the 'gang of four', namely, Ture Western, Pieter Huybers, Jean-François Gabriel and René Motro.
[5] Solid isotropic material with penalization introduced by Bendose (1995).
[6] Evolutionary structural optimization proposed by Xie and Steven (1993).
[7] An iconic symbol tree in Qatari culture.

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