Calculating Leakage Inductances of Split-Windings in Dry-Type Traction Transformers

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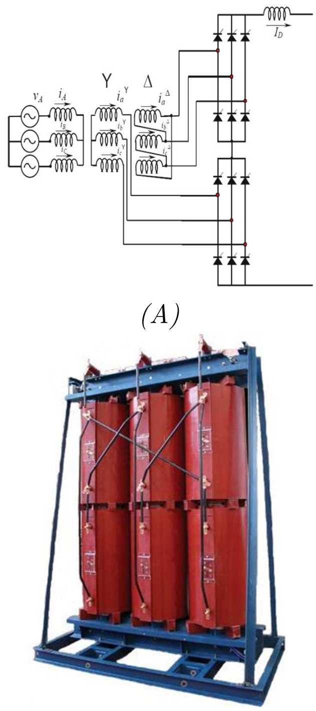
ABSTRACT

Calculating leakage inductances between windings of multi-winding traction transformers is one of the most important schemes in design procedure. In this paper, a new analytical method and finite element models have been introduced for electromagnetic modeling of traction transformers with four split windings. Using these methods, the leakage inductances between the split windings are calculated and the methods have been validated with the help of experimental data gathered from a typical 4000kVA cast-resin dry-type transformer. Some reformations are employed to improve the finite element results and effects of these reformations on accuracy of the models are studied. Employing the validated models, the effects of middle yoke on the leakage inductances of split-winding traction transformer are discussed in this paper.

Keywords: Traction, Dry-Type Transformer, Split Windings, Leakage Inductances, Analytic, Finite Element

1. INTRODUCTION

Along with the rapid development of subways and traction networks, the converter systems have come true and widely used in power systems. The main part of these converter systems is its transformer, called as “traction transformer”. In order to protect against flame and explosion, nowadays traction transformers are manufactured as dry-type transformers. Fig.1 shows a converter system and its related cast-resin dry-type transformer. As shown in Fig. 1, this type of transformer is usually manufactured from four split windings in which the high voltage windings are in parallel. As it is known, calculating leakage inductances of windings is an important factor in design procedure of multi-winding traction transformers [1-4]. Leakage inductances of a concentric three-winding transformer are discussed in [5]. Many different methods have been presented for inductance calculations in transformers with concentric and pancake windings [6-10]. Authors introduce an analytic and a semi-analytic method for electromagnetic modeling of traction transformer in [11]. But there is a need for a simple and fast method for leakage inductance calculations in the split-winding traction transformer with both concentric and pancake windings.



(A)

(B)

Fig.1: *A) Schematic diagram of a 12-pulse converter in a traction system, B) traction transformer with pancake windings*

Finite element is an accurate method but when a shorter computation time is essential it is not preferred and it is needed to introduce a fast and precise method. Thus in this paper, a new and simple analytic method is introduced for leakage inductance calculations in the split-winding traction transformer

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and using a typical 4000kVA dry-type traction transformer, the results are validated with the help of finite element and experimental results. In the following it is shown that the introduced analytic method is a powerful and accurate method that this is in addition of its small computation time. Afterwards, employing the presented methods, the most effective parameters on the leakage inductances of a traction transformer are studied. Finally a refinement is proposed in the geometry of the split-winding traction transformer and its effects on the leakage inductances are discussed. It is shown that how the geometry can be tailored to achieve the suitable leakage inductances between split windings by inserting a middle yoke between top and bottom windings.

2. ELECTROMAGNETIC MODELING IN A SPLIT-WINDING TRANSFORMER

Fig.2 shows a scheme of one leg in a four-winding cast-resin dry-type transformer.

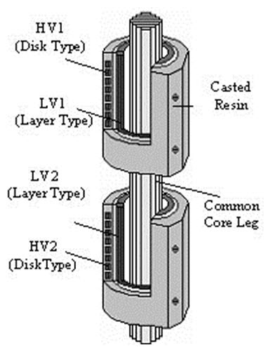


Fig.2: *Schematic view of one leg in a four-winding dry-type Traction transformer*

In order to measuring leakage inductance in a pair of windings in multi-winding transformer (Fig.2), it is convenient to use test circuit given in the Fig.3.

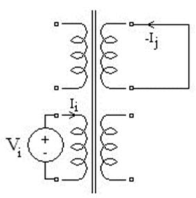


Fig.3: *Circuit for measuring leakage inductance of winding i and j*

If the nominal currents applied to the windings i and j (I_i for winding i, -I_j for winding j and zero for the other two windings), the magnetic vector potential is related to the leakage flux and can be calculated by solving the Poisson's equation as given in (1) [12].

$$\nabla^2 A - \mu\varepsilon\frac{\partial^2 A}{\partial t^2} = \mu \cdot J \tag{1}$$

where,

A: magnetic vector potential,
J: current density,
ε: permittivity, and
μ: permeability.

Consequently, leakage inductance (seen from side j) can be calculated, employing (2).

$$L_l^{ij} = \frac{2 \times W_m^{ij}}{I_j^2} \tag{2}$$

where, W_m^{ij} is the total energy stored in the magnetic field of Fig.3, and it can be determined as shown in (3).

$$W_m^{ij} = \int_{vol} A \cdot J dv \tag{3}$$

To solve the partial differential equation (1) is very intricate. Therefore, simplifying procedures should be employed. In the following, two methods are introduced for solving equation (1) and calculating the leakage inductances of the multi-winding transformer.

2.1 Analytical Method

Consider a pair of windings as shown in Fig.4. Calculating the leakage inductance of these windings directly, is very intricate. To simplifying the inductance calculation, the leakage inductance of windings shown in Fig.4, can be subdivided to two components (the axial and the radial) as presented in (4).

$$L_l^{ij} = \sqrt{L_{lr}^2 + L_{lz}^2} \tag{4}$$

where, L_{lr} and L_{lz} are the inductances related to leakage flux in radial and axial direction.

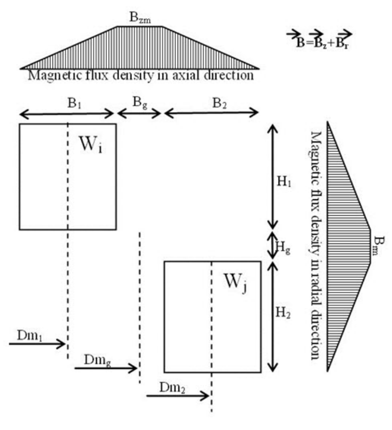


Fig.4: *Leakage flux density in axial and radial directions*

To calculate L_{lz}, the windings can be assumed as concentric windings with equal heights (Fig.5).

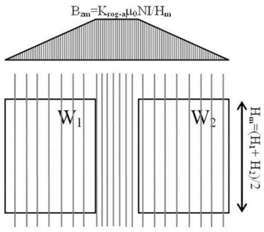


Fig. 5: *Concentric windings for inductance calculation due to the axial flux*

It can be assumed that the leakage flux in Fig.5 has only one component in the axial direction. Therefore using the relation in [13], the leakage inductance of those windings can be determined as follows:

$$L_{lz} = \frac{\mu_0 \pi N^2}{H_m / K_{rog-a}} \left(\frac{B_1 D_{m1}}{3} + \frac{B_2 D_{m2}}{3} + B_g D_{mg} \right) \quad (5)$$

where, $K_{rog\text{-}a}$ is the Rogofsky factor that is related to flux fringing in the top and bottom of the windings and it can be calculated as below [13]:

$$K_{rog-a} = \frac{1 - e^{-\pi H_m / (B_1 + B_g + B_2)}}{\pi H_m / (B_1 + B_g + B_2)} \quad (6)$$

Similarly, L_{lr} can be calculated by assuming the windings in the Fig.4 as two pancake windings with equal diameters as shown in Fig.6. It can be assumed that the leakage flux in the Fig.6 has only one component in the radial direction and the leakage inductance of presented windings can be described as:

$$L_{lr} = \frac{\mu_0 \pi N^2 D_m}{B_m / K_{rog-r}} \left(\frac{B_1}{3} + \frac{B_2}{3} + B_g \right) \quad (7)$$

Where similarly, K_{rog-r} can be calculated as

$$K_{rog-r} = \frac{1 - e^{-\pi B_m / (H_1 + H_g + H_2)}}{\pi B_m / (H_1 + H_g + H_2)} \quad (8)$$

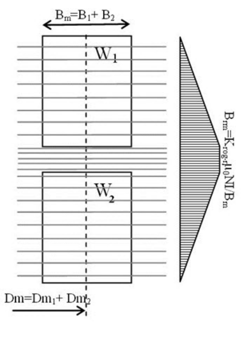


Fig. 6: *Pancake windings with same diameter for inductance calculation due to the radial flux*

2.2 Finite Element Method

Most convenient way to solve differential equations given in the previous section is to use a numerical method. In this paper, finite element (FE) method [12] employed for electromagnetic modeling and inductance calculation in multi-winding traction transformer (Fig.2). To simplify the model and to reduce the computation time of this modeling, it is suitable to model this three phase transformer as a single phase transformer. In this paper 2D and 3D finite element models are used for inductance calculation of a single phase traction transformer. Fig.7 shows simplest 2D finite element model in which the core is assumed as an open leg.

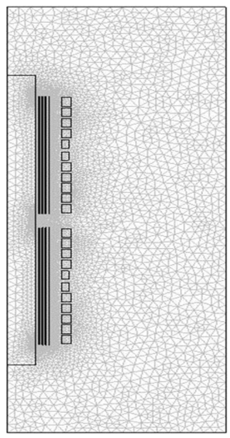


Fig. 7: *Simple Axi-symmetric 2D FE Model*

As the model shown in Fig.7 is not perfect for modeling the effect of core; the core model is modified as shown in the Fig.8(A).

Employing the axi-symmetric modeling, the effect of core in the 2D-F model is more than its effect in the actual core. To improve this modeling, it is better to use equivalent core model as shown in Fig.8(B) with given parameters in (9) and (10).

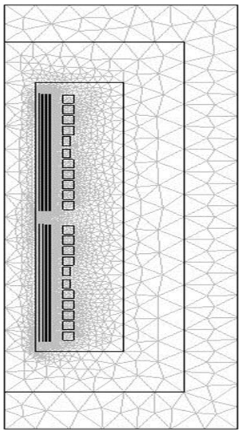
$$d_{yoke} = \frac{r_0^2}{2r} \quad (9)$$

$$d_{rel.leg} = \frac{r_0^2}{2E_s} \quad (10)$$

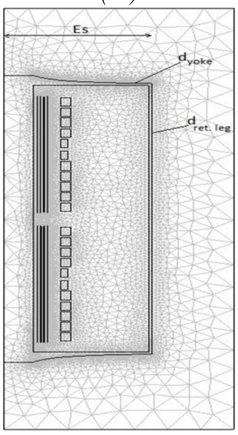
where,

- r_0: actual radius of the yoke
- r: radial distance
- E_s: width of the core window

Fig.9(A) shows the complete 3D finite element model for mentioned traction transformer. To solve the model shown in Fig.9(A), it would be very time consuming, however realizing the symmetry of geometry, it is sufficient to model only a quarter of this



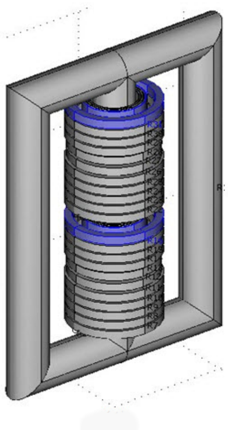
(A)

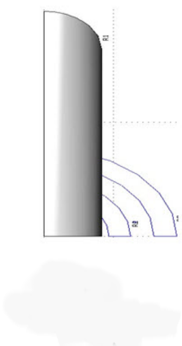


(B)

Fig.8: *Axi-symmetric 2D FE Model, B) full core: 2D-F, C) improved core: 2D-I*

transformer as it is shown in Fig.9(B). Using the finite element models, introduced in this section, leakage inductances of multi-winding traction transformer have been calculated in the next section.





(A) (B)

Fig.9: *3D FE models of the traction transformer- A) full model- B) quarter model*

3. ANALYSIS OF INDUCTANCES IN MULTI-WINDING DRY-TYPE TRANSFORMR

Consider a typical 4000kVA dry-type traction transformer given in the Fig.10 and table 1 [14]. Traction transformer explained in this table, had been modeled with the help of FEM and analytical methods mentioned in the previous section and test procedure given in the Fig.3 had been applied to all pair of the windings.

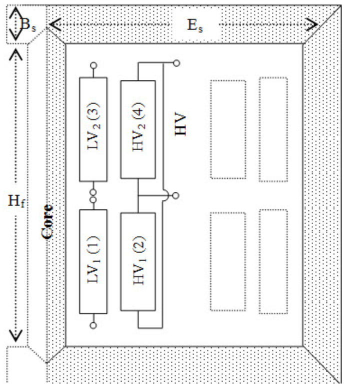


Fig.10: *Three-Phase four-winding traction transformer*

Table 1: *Conditions of test system under all type of faults*

Winding	HV	LV_1	LV_2
Rated Voltage	4 MVA	2 MVA	2 MVA
Rated Current	20000	750	750
Line Current	0.1155	1.54	1.54
Connection	D	y	D
Layer Number	1	3	3
Axial Channel	-	3×10 mm	1×12 mm
Disk Number	11	1	1
Radial Channel Between Disks	9×12 mm 1×26 mm	-	-
Height	1230 mm	1240 mm	1240 mm
Internal Diameter	654 mm	332 mm	929 mm
External Diameter	799 mm	454 mm	982 mm
E_s	1015		
H_f	1410		
B_s	320 mm		

Fig.11 and Fig.12 show the leakage flux density for HV_1-LV_1 and LV_1-LV_2 tests.

Consequently, Fig.13 and Fig.14 show the related magnetic energy for these leakage inductance tests.

Using the analytic method, leakage inductances have been calculated and the results are compared to the FE and experimental results in table 2. It is seen that both 3D-FE and analytic methods are accurate, but as a shorter computation time is essential in an automatic design procedure, the FE method is not preferred and the presented analytic method is a better choice.

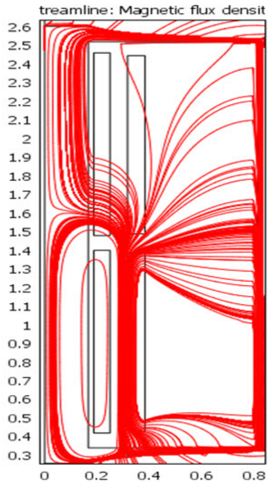


Fig.11: *Magnetic flux density due to LV_1 -HV_1 test*

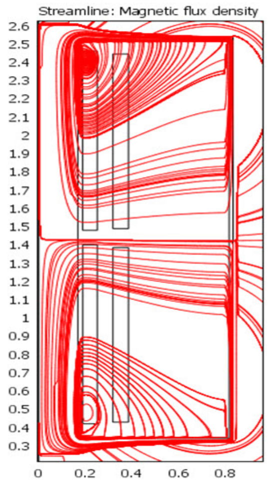


Fig.12: *Magnetic Flux due to LV_1 -LV_2 test*

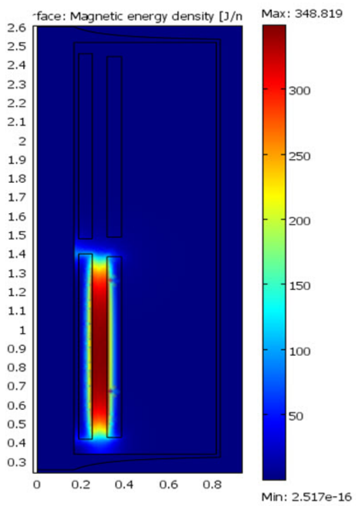


Fig.13: *Magnetic energy due to LV_1-HV_1 test*

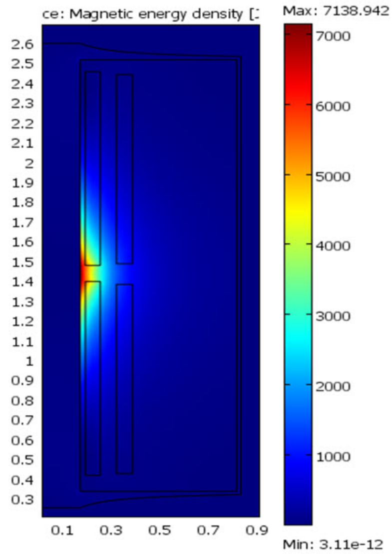


Fig.14: *Magnetic energy due to LV_1 -LV_2 test*

Table 2: *Perunited leakage reactances of the typical traction transformer*

		Leakage inductances (%)					
		LV_1-HV_1	LV_1-LV_2	LV_2-HV_1	LV_1-HV_2	LV_2-HV_2	HV_1-HV_2
Analytic		5.9	56	72	73	88	6.0
FE	2D-S	5.68	45	44	44	42	5.69
	2D-F	5.81	112	112	111	110	5.84
	2D-I	5.79	100	99	98	97	5.83
	3D	5.89	88	88	88	88	5.91
Exp.		5.80	73	72	74	72	5.94

As it can be seen from table 2, inductances between common axis windings (i.e. LV_1-LV_2) are much higher than the leakage inductances of concentric windings (i.e. LV_1-HV_1) of the same heights.

The inductances in the above table are calculated and measured in the nominal currents. In many cases it can be assumed that these leakage inductances are constant during the operation. But actually these values will be varied according to their current. This phenomena that is shown in Fig.15 and Fig.16, is related to nonlinearity of magnetic core. As it is clear, leakage inductance in a pair of concentric windings can be assumed as a constant value, but the effect of nonlinearity on leakage inductance of pancake windings is considerable and it cannot be neglected. In the converter transformers it is important to keep the leakage inductance of LV_1-LV_2 as high as it is possible. In order to improve and increase leakage inductances between the axial windings in the traction transformer, it might be better to use a middle yoke.

Fig.17 and Fig.18 show the effect of middle yoke on the magnetic field and magnetic energy generated in LV1-LV2 inductance test.

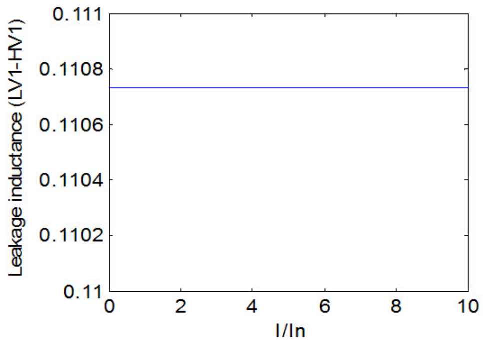


Fig.15: *: Leakage Inductance of concentric windings as a function of current*

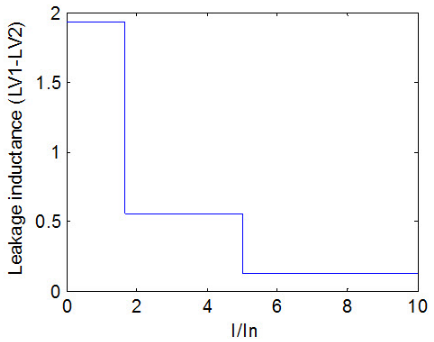


Fig.16: *Leakage Inductance of pancake windings as a function of current*

Table 3 compares the leakage inductances with and without middle yoke. As it is clear from table 3, the inductances of LV_1-LV_2, LV_1-HV_2 and LV_2-HV_1 increase rapidly with an increase in cross section of middle yoke. But the middle yoke has a small and negligible effect on inductances of LV_1-HV_1 and LV_2-HV_2.

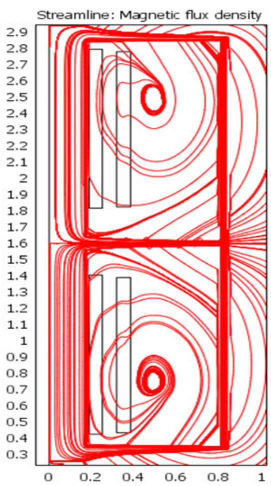


Fig.17: *Magnetic flux density due to* LV_1 *-*LV_2 *test in presence of middle yoke*

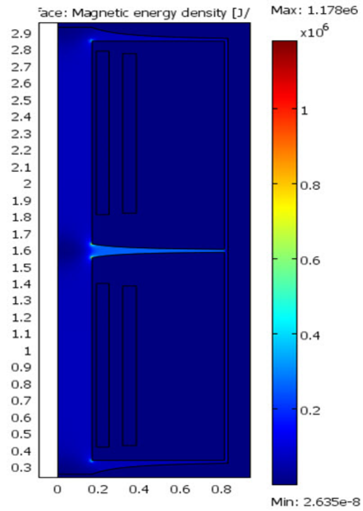


Fig.18: *Magnetic energy due to LV1 -LV2 test in presence of middle yoke*

Table 3: *Effect of middle yoke on leakage inductances*

Mid yoke / Main Yoke	Leakage inductances (%)					
	LV_1-HV_1	LV_1-LV_2	LV_2-HV_1	LV_1-HV_2	LV_2-HV_2	HV_1-HV_2
0%	5.79	100	99	98	97	5.83
1%	5.91	278	275	275	272	5.915
2%	5.90	434	431	430	428	5.913
4%	5.89	722	719	718	716	5.91
5%	5.89	857	854	853	851	5.91
9%	5.87	1337	1335	1333	1331	5.89
15%	5.86	1928	1926	1924	1922	5.88
22%	5.85	2473	2472	2470	2469	5.88
25%	5.85	2682	2681	2679	2677	5.88
30%	5.84	2963	2962	2958	2958	5.87
40%	5.84	3435	3431	3431	3431	5.87
50%	5.83	3798	3818	3815	3795	5.86
60%	5.83	4087	4087	4083	4083	5.86
80%	5.82	4508	4509	4505	4505	5.86
90%	5.82	4667	4667	4664	4664	5.85
100%	5.82	4802	4798	4798	4798	5.85

4. CONCLUSIONS

Analytical and FEM based methods had been introduced in this paper for inductance calculation of multi-winding traction transformers. It had been shown that FEM has better and most accurate results. The accuracy of 3D model is much higher than 2D models, but improved core 2D model (2D-I) has also a good result. Consequently, analytical method given in this paper is a fast and simple method with enough accuracy for leakage inductance calculation in multi-winding transformers and it can be used in an automatic design procedure.

Leakage inductances of axial windings are about ten times of leakage inductances in radial windings and it had been shown that, adding a middle yoke can improve and increase this ratio. It is clear that the effect of this middle yoke on the leakage inductances of radial windings is not considerable in spite of increase in leakage inductances of axial windings.

One can see that the nonlinearity has not considerable effects on leakage inductances of concentric windings, but this effect is not negligible when the leakage inductances between top and bottom windings are considered.

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Jawad Faiz received the Ph.D. degree in Electrical Engineering from the University of Newcastle upon Tyne, England in 1988. He is now a Professor at School of Electrical and Computer Engineering, University College of Engineering, University of Tehran. He has received a number of national and international awards. His teaching and research interests are switched reluctance and VR motors design, design and modeling of electrical machines and drives, transformer modeling and design and fault diagnosis in electrical machinery.

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