A Design Method for Simple Repetitive Controllers for Multiple-Input/Multiple-Output Time-Delay Plants

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ABSTRACT

The simple repetitive control system proposed by Yamada et al. is a type of servomechanism for the periodic reference input. That is, the simple repetitive control system follows the periodic reference input with small steady state error, even if a periodic disturbance or an uncertainty exists in the plant. In addition, simple repetitive control systems make transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. In addition, Yamada et al. clarified the parameterization of all stabilizing simple repetitive controllers. Recently, the parameterization of all stabilizing simple repetitive controllers for time-delay plants was clarified by Yamada et al. However their method cannot be applied to multiple-input/multiple-output time-delay plants. In this paper, we propose the parameterization of all stabilizing simple repetitive controllers for multiple-input/multiple-output time-delay plants.

Keywords: Multiple-Input/Multiple-Output Time-Delay Plant, Repetitive Control, Parameterization, Finite Number of Poles

1. INTRODUCTION

The repetitive control system is a type of servomechanism for the periodic reference input. That is, the repetitive control system follows the periodic reference input without steady state error, even if a periodic disturbance or an uncertainty exists in the plant [1–13]. It is difficult to design stabilizing controllers for the plant, because the repetitive control system follows any periodic reference input without steady state error is a neutral type of time-delay control system [11]. In order to design a repetitive control system that follows any periodic reference input without steady state error, the plant needs to be biproper [3–11]. In practice, the plant is strictly proper. Many design methods of repetitive control systems for the strictly proper plant are given [3–11]. These studies are divided into two types. One uses a low-pass filter [3–10] and the other uses an attenuator [11]. The latter is difficult to design because it uses a state variable time-delay in the repetitive controller [11]. The former has a simple structure and is easily designed. Therefore, the former type of repetitive control system is called the modified repetitive control system [3–10].

Using the modified repetitive controllers in [3–10], even if the plant does not include time-delay, transfer functions from the periodic reference input to the output and from the disturbance to the output generally have infinite numbers of poles. This makes it difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From the practical point of view, it is desirable that these characteristics are easily specified. To do this, transfer functions from the periodic reference input to the output and from the disturbance to the output are desirable to have finite numbers of poles. Yamada et al. proposed the concept of simple repetitive control systems such that the controller works as a modified repetitive controller and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles [14]. In addition, Yamada et al. clarified the parameterization of all stabilizing simple repetitive controllers. However, the method by Yamada et al. cannot be applied to time-delay plants. To solve this problem, the parameterization of all stabilizing simple repetitive controllers for time-delay plants was solved by Yamada et al. [15]. However, Yamada et al. [15] did not consider the parameterization of all stabilizing simple repetitive controllers for multiple-input/multiple-output time-delay plants. From the practical point of view, since many real plants are multiple-input/multiple-output systems and the parameterization is effective to design stabilizing controllers [16–24], the problem of obtaining the parameterization of all stabilizing simple repetitive controllers for multiple-input/multiple-output time-delay plants must be considered.

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In this paper, we expand the result in [15] and clarify the parameterization of all stabilizing simple repetitive controllers for multiple-input/multiple-output time-delay plants. This paper is organized as follows: In Section 2., the concept of the simple repetitive controllers for multiple-input/multiple-output time-delay plants is proposed. In addition, in Section 2., the problem considered in this paper is described. In Section 3., the structure of simple repetitive controllers for multiple-input/multiple-output time-delay plants is clarified. In Section 4.

and Section 5., using the result in Section 3., the parameterizations of all stabilizing simple repetitive controllers for both stable time-delay plants and for unstable time-delay plants are clarified. In Section 6., control characteristics of control system using simple repetitive controllers for time-delay plants are described. In Section 7., a design procedure of stabilizing simple repetitive controllers for multiple-input/multiple-output time-delay plants is presented. In Section 8., we show a numerical example to illustrate the effectiveness of the proposed method. Section 9. gives concluding remarks.

Notation

R	the set of real numbers.
R_+	$R \cup \{\infty\}$.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real rational functions.
A^T	transpose of A.
$\bar{\sigma}(\{\cdot\})$	largest singular value of $\{\cdot\}$.
diag $(a_1, \cdots, a_n)$	an $n \times n$ diagonal matrix with a_i as its i-th diagonal element.

2. PROBLEM FORMULATION

Consider the unity feedback control system in

$$\left\{ \begin{array}{rcl} y & = & G(s)e^{-sL}u + d \\ u & = & C(s)(r-y) \end{array} \right. , \tag{1}$$

where $G(s)e^{-sL}$ is the plant, $L > 0$ is the time-delay, $G(s) \in R^{p\times p}(s)$, $C(s)$ is the controller, $u \in R^p$ is the control input, $y \in R^p$ is the output, $d \in R^p$ is the disturbance and $r \in R^p$ is the periodic reference input with period $T > 0$ satisfying

$$r(t+T) = r(t) \ \ (\forall t \geq 0). \tag{2}$$

It is assumed that rank $G(s) = p$ and $T \geq L$. According to [3–10], the modified repetitive controller $C(s)$ in (1) is written by the form in

$$C(s) = \hat{C}(s) + \bar{C}(s)e^{-sT}\left(I - q(s)e^{-sT}\right)^{-1}, \tag{3}$$

where rank $\bar{C}(s) = p$ and $q(s) \in R^{p\times p}(s)$ is a proper low-pass filter satisfying $q(0) = I$ and rank $q(s) = p$. In the following, $e^{-sT}\left(I - q(s)e^{-sT}\right)^{-1}$ defines the internal model for the periodic signal with period T. According to [3–10], if the low-pass filter $q(s)$ satisfies

$$\bar{\sigma}\left\{I - q(j\omega_i)\right\} \simeq 0 \quad (i = 0, 1, \ldots, N_{max}), \tag{4}$$

where $\omega_i (i = 0, 1, \ldots, N_{max})$ are frequency components of the periodic reference input r written by

$$\omega_i = \frac{2\pi}{T} i \quad (i = 0, 1, \ldots, N_{max}) \tag{5}$$

and $\omega_{N_{max}}$ is the maximum frequency component of the periodic reference input r, then the output y in (1) follows the periodic reference input r with a small steady state error.

Using the modified repetitive controller $C(s)$ in (3), the transfer function from the periodic reference input r to the output y and that from the disturbance d to the output y in (1) are written as

$$\begin{aligned} y &= \left(I + G(s)C(s)e^{-sL}\right)^{-1} G(s)C(s)e^{-sL}r \\ &= G(s)\left\{\hat{C}(s)e^{-sL} + \left(\bar{C}(s) - \hat{C}(s)q(s)\right)\right. \\ &\quad \left. e^{-s(T+L)}\right\}\left[I - q(s)e^{-sT} + G(s)\hat{C}(s)e^{-sL}\right. \\ &\quad \left. + G(s)\left(\bar{C}(s) - \hat{C}(s)q(s)\right)e^{-s(T+L)}\right]^{-1} r \end{aligned} \tag{6}$$

and

$$\begin{aligned} y &= \left(I + G(s)C(s)e^{-sL}\right)^{-1} d \\ &= \left(I - q(s)e^{-sT}\right)\left[I - q(s)e^{-sT} + G(s)\hat{C}(s)e^{-sL}\right. \\ &\quad \left. + G(s)\left(\bar{C}(s) - \hat{C}(s)q(s)\right)e^{-s(T+L)}\right]^{-1} d, \end{aligned} \tag{7}$$

respectively. Generally, transfer functions from the periodic reference input r to the output y in (6) and from the disturbance d to the output y in (7) have infinite numbers of poles. This makes it difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From the practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic are easily specified. To do this, transfer functions from the periodic reference input to the output and from the disturbance to the output are desirable to have finite numbers of poles.

From above practical requirement, we define simple repetitive controllers for multiple-input/multiple-output time-delay plants as Definition 1 and clarify the parameterization of all stabilizing simple repetitive controllers for multiple-input/multiple-output time-delay plants.

Definition 1: (simple repetitive controller for multiple-input/multiple-output time-delay plants) We call the controller $C(s)$ a "simple repetitive controller for multiple-input/multiple-output time-delay plants", if following expressions hold true:

1. The controller $C(s)$ works as a modified repetitive controller. That is, the controller $C(s)$ is written by

(3), where rank $\bar{C}(s) = p$ and $q(s) \in R^{p\times p}(s)$ satisfies $q(0) = I$ and rank $q(s) = p$.
2. The controller $C(s)$ makes transfer functions from the periodic reference input r to the output y in (1) and from the disturbance d to the output y in (1) have finite numbers of poles.

3. STRUCTURE OF SIMPLE REPETITIVE CONTROLLERS FOR TIME-DELAY PLANTS

In this section, the structure of simple repetitive controllers for time-delay plants defined in Definition 1 is clarified.

From Definition 1, since the transfer function from the periodic reference input r to the output y has finite numbers of poles, the transfer function from the periodic reference input r to the output y in (1) is written by

$$y=\left(\bar{G}_1(s)e^{-sL}+\bar{G}_2(s)e^{-sT}+\bar{G}_3(s)e^{-s(T+L)}\right) r, \quad (8)$$

where $\bar{G}_i(s) \in RH_\infty^{p\times p}(i = 1,2,3)$. If the controller $C(s)$ in (3) is used to make the transfer function from the periodic reference input r to the output y conform to (8), the following theorem is satisfied.

Theorem 1: The controller $C(s)$ in (3) will make the transfer function from the periodic reference input r to the output y conform to (8) if and only if it has the structure in

$$\begin{aligned} C(s) &= \left(C_1(s) + C_2(s)e^{-s(T-L)} + C_3(s)e^{-sT}\right) \\ &\quad \left\{\left(I + C_4(s)e^{-sL}\right)\left(I - q(s)e^{-sT}\right)\right\}^{-1}, \quad (9) \end{aligned}$$

where $C_i(s) \in R^{p\times p}(s)(i = 1,\dots,4)$ and rank $C_2(s) = p$.

Proof: First the necessity is shown. That is, we show that if $C(s)$ is written by (3) and transfer functions from the periodic reference input r to the output y in (1) and from the disturbance d to the output y in (1) have finite numbers of poles, then $C(s)$ is written by (9). Since the controller $C(s)$ is written by (3), the transfer function from the periodic reference input r to the error $e = r - y$ in (1) is given by

$$\begin{aligned} e &= \left(I + G(s)C(s)e^{-sL}\right)^{-1} r \\ &= \left(I - q(s)e^{-sT}\right)\Big[I - q(s)e^{-sT} + G(s)\hat{C}(s)e^{-sL} \\ &\quad +G(s)\left(\bar{C}(s) - \hat{C}(s)q(s)\right)e^{-s(T+L)}\Big]^{-1} r. \quad (10) \end{aligned}$$

From the assumption that the transfer function from the periodic reference input r to the output y is written by (8), the transfer function in (10) is equal to

$$\begin{aligned} e &= \left(I - \bar{G}_1(s)e^{-sL} - \bar{G}_2(s)e^{-sT} \right. \\ &\quad \left. -\bar{G}_3(s)e^{-s(T+L)}\right) r. \quad (11) \end{aligned}$$

From (10) and (11), we find that (10) can be rewritten as

$$\begin{aligned} e &= \left(I + G(s)C(s)e^{-sL}\right)^{-1} r \\ &= \left(I + H(s)e^{-sL}\right)\left(I - q(s)e^{-sT}\right) r, \quad (12) \end{aligned}$$

where $H(s) \in R^{p\times p}(s)$. This equation implies that $C(s)$ is described by

$$\begin{aligned} C(s) &= \left(-G^{-1}(s)H(s) + G^{-1}(s)q(s)e^{-s(T-L)} \right. \\ &\quad \left. +G^{-1}(s)H(s)q(s)e^{-sT}\right) \\ &\quad \left\{\left(I + H(s)e^{-sL}\right)\left(I - q(s)e^{-sT}\right)\right\}^{-1} \\ &= \left(C_1(s) + C_2(s)e^{-s(T-L)} + C_3(s)e^{-sT}\right) \\ &\quad \left\{\left(I + C_4(s)e^{-sL}\right)\left(I - q(s)e^{-sT}\right)\right\}^{-1}, \quad (13) \end{aligned}$$

where

$$C_1(s) = -G^{-1}(s)H(s), \quad (14)$$

$$C_2(s) = G^{-1}(s)q(s), \quad (15)$$

$$C_3(s) = G^{-1}(s)H(s)q(s) \quad (16)$$

and

$$C_4(s) = H(s). \quad (17)$$

From the assumption of rank $q(s) = p$ and (15), rank $C_2(s) = p$ holds true. The necessity has been shown.

Next, the sufficiency is shown. That is, if $C(s)$ is written by (9), then $C(s)$ works as a modified repetitive controller, that is the controller $C(s)$ is written by (3), and makes transfer functions from the periodic reference input r to the output y in (1) and from the disturbance d to the output y in (1) have finite numbers of poles. After simple manipulations, we have

$$\begin{aligned} y &= \left(I + G(s)C(s)e^{-sL}\right)^{-1} G(s)C(s)e^{-sL} r \\ &= \left(-H(s)e^{-sL} + q(s)e^{-sT} \right. \\ &\quad \left. +H(s)q(s)e^{-s(T+L)}\right) r \quad (18) \end{aligned}$$

and

$$\begin{aligned} y &= \left(I + G(s)C(s)e^{-sL}\right)^{-1} d \\ &= \left(I + H(s)e^{-sL}\right)\left(I - q(s)e^{-sT}\right) d. \quad (19) \end{aligned}$$

Since $H(s) \in R^{p\times p}(s)$ and $q(s) \in R^{p\times p}(s)$, transfer functions from r to y in (18) and from d to y in (19) have finite numbers of poles.

Next we show that the controller $C(s)$ in (9) works as a modified repetitive controller. The controller $C(s)$ in (9) is rewritten by the form in (3), where

$$\hat{C}(s) = C_1(s)\left(I + C_4(s)e^{-sL}\right)^{-1} \quad (20)$$

and

$$\bar{C}(s) = \left(C_1(s)q(s) + C_2(s)e^{sL} + C_3(s)\right)\left(I + C_4(s)e^{-sL}\right)^{-1}. \quad (21)$$

From the assumption that rank $C_2(s) = p$ and (21), rank $\bar{C}(s) = p$ holds true. These expressions imply that the controller $C(s)$ in (9) works as a modified repetitive controller. Thus, the sufficiency has been shown.

We have thus proved Theorem 1. ■

4. THE PARAMETERIZATION OF ALL STABILIZING SIMPLE REPETITIVE CONTROLLERS FOR STABLE TIME-DELAY PLANTS

In this section, we propose the parameterization of all stabilizing simple repetitive controllers for time-delay plants when $G(s)e^{-sL}$ is stable.

This parameterization is summarized in the following theorem.

Theorem 2: It is assumed that $G(s)e^{-sL}$ is stable, $q(s) \in RH_\infty^{p\times p}$ and $G^{-1}(s)q(s) \in RH_\infty^{p\times p}$. The parameterization of all stabilizing simple repetitive controllers $C(s)$ is written by

$$C(s) = \left(Q(s) + G^{-1}(s)q(s)e^{-s(T-L)} - Q(s)q(s)e^{-sT}\right)\left\{\left(I - G(s)Q(s)e^{-sL}\right)\left(I - q(s)e^{-sT}\right)\right\}^{-1}, \quad (22)$$

where $Q(s) \in RH_\infty^{p\times p}$ is any function.

Proof: First, the necessity is shown. From Theorem 1, the structure of simple repetitive controller $C(s)$ is written by (9). Therefore, we show that if the controller $C(s)$ in (9) make the control system in (1) stable, then the controller $C(s)$ is written by (22). From the assumption that the controller $C(s)$ in (9) makes the transfer function from r to y of the control system in (1) have finite numbers of poles,

$$\begin{aligned}&\left(I + G(s)C(s)e^{-sL}\right)^{-1} G(s)C(s)e^{-sL}\\ &= G(s)\left(C_1(s) + C_2(s)e^{-s(T-L)} + C_3(s)e^{-sT}\right)\\ &\quad e^{-sL}\left\{I + \left(G(s)C_2(s) - q(s)\right)e^{-sT}\right.\\ &\quad + \left(G(s)C_1(s) + C_4(s)\right)e^{-sL} + \left(G(s)C_3(s)\right.\\ &\quad \left.\left. - C_4(s)q(s)\right)e^{-s(T+L)}\right\}^{-1}\end{aligned} \quad (23)$$

has finite numbers of poles. This implies that

$$C_2(s) = G^{-1}(s)q(s), \quad (24)$$

$$C_4(s) = -G(s)C_1(s) \quad (25)$$

and

$$C_3(s) = -C_1(s)q(s). \quad (26)$$

Substituting (24), (25) and (26) into (9), $C(s)$ must take the form

$$C(s) = \left(C_1(s) + G^{-1}(s)q(s)e^{-s(T-L)} - C_1(s)q(s)e^{-sT}\right)\left\{\left(I - G(s)C_1(s)e^{-sL}\right)\left(I - q(s)e^{-sT}\right)\right\}^{-1}. \quad (27)$$

From the assumption that $C(s)$ in (9) makes the control system in (1) stable, $\left(I+G(s)C(s)e^{-sL}\right)^{-1}G(s)C(s)e^{-sL}$, $\left(I+G(s)C(s)e^{-sL}\right)^{-1}$, $\left(I+C(s)G(s)e^{-sL}\right)^{-1}C(s)$ and $\left(I+G(s)C(s)e^{-sL}\right)^{-1}G(s)e^{-sL}$ are stable. From simple manipulations and (27), we have

$$\begin{aligned}&\left(I + G(s)C(s)e^{-sL}\right)^{-1} G(s)C(s)e^{-sL}\\ &= G(s)C_1(s)e^{-sL} + q(s)e^{-sT}\\ &\quad - G(s)C_1(s)q(s)e^{-s(T+L)},\end{aligned} \quad (28)$$

$$\left(I + C(s)G(s)e^{-sL}\right)^{-1} C(s) = C_1(s) + G^{-1}(s)q(s)e^{-s(T-L)} - C_1(s)q(s)e^{-sT}, \quad (29)$$

$$\left(I + G(s)C(s)e^{-sL}\right)^{-1} G(s)e^{-sL} = \left(I - G(s)C_1(s)e^{-sL}\right)\left(I - q(s)e^{-sT}\right)G(s)e^{-sL} \quad (30)$$

and

$$\left(I + G(s)C(s)e^{-sL}\right)^{-1} = \left(I - G(s)C_1(s)e^{-sL}\right)\left(I - q(s)e^{-sT}\right). \quad (31)$$

It is obvious that the necessary condition that transfer functions in (28), (29), (30) and (31) are stable is $C_1(s) \in RH_\infty^{p\times p}$. Let $C_1(s) = Q(s) \in RH_\infty^{p\times p}$, we have (22). Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, we show that if $C(s)$ is written by (22), then the controller $C(s)$ makes the control system in (1) stable and makes the transfer function from the periodic reference input r to the output y and that from the disturbance d to the output y of the control system in (1) have finite numbers of poles. After simple manipulations, we have

$$\begin{aligned}&\left(I + G(s)C(s)e^{-sL}\right)^{-1} G(s)C(s)e^{-sL}\\ &= G(s)Q(s)e^{-sL} + q(s)e^{-sT}\\ &\quad - G(s)Q(s)q(s)e^{-s(T+L)},\end{aligned} \quad (32)$$

$$\left(I + C(s)G(s)e^{-sL}\right)^{-1} C(s) = Q(s) + G^{-1}(s)q(s)e^{-s(T-L)} - Q(s)q(s)e^{-sT}, \quad (33)$$

$$\begin{aligned}&\left(I+G(s)C(s)e^{-sL}\right)^{-1}G(s)e^{-sL}\\&=\left(I-G(s)Q_1(s)e^{-sL}\right)\left(I-q(s)e^{-sT}\right)G(s)e^{-sL}\end{aligned}\tag{34}$$

and

$$\begin{aligned}&\left(I+G(s)C(s)e^{-sL}\right)^{-1}\\&=\left(I-G(s)Q(s)e^{-sL}\right)\left(I-q(s)e^{-sT}\right).\end{aligned}\tag{35}$$

From the assumption that $G(s)$ is stable, $q(s) \in RH_\infty^{p\times p}$, $G^{-1}(s)q(s) \in RH_\infty^{p\times p}$ and $Q(s) \in RH_\infty^{p\times p}$, transfer functions in (32), (33), (34) and (35) are stable. In addition, from the same reason, the transfer function from the periodic reference input r to the output y written by (32) and that from the disturbance d to the output y written by (35) of the control system in (1) have finite numbers of poles. Thus, the sufficiency has been shown.

We have thus proved Theorem 2. ■

5. THE PARAMETERIZATION OF ALL STABILIZING SIMPLE REPETITIVE CONTROLLERS FOR UNSTABLE TIME-DELAY PLANTS

In this section, we propose the parameterization of all stabilizing simple repetitive controllers for time-delay plants when $G(s)e^{-sL}$ is unstable.

This parameterization is summarized in the following theorem.

Theorem 3: It is assumed that $G(s)e^{-sL}$ is unstable, $q(s) \in RH_\infty^{p\times p}$ and $G^{-1}(s)q(s) \in RH_\infty^{p\times p}$. For simplicity, unstable poles of $G(s)e^{-sL}$ are assumed to be distinct. That is, when $s_i(i=1,\dots,n)$ denote unstable poles of $G(s)$, $s_i \neq s_j(i\neq j; i=1,\dots,n; j=1,\dots,n)$. Using these conditions, the parameterization of all stabilizing simple repetitive controllers is written as

$$\begin{aligned}&C(s)\\&=\left(Q(s)+G^{-1}(s)q(s)e^{-s(T-L)}-Q(s)q(s)e^{-sT}\right)\\&\quad\left\{\left(I-G(s)Q(s)e^{-sL}\right)\left(I-q(s)e^{-sT}\right)\right\}^{-1},\end{aligned}\tag{36}$$

where $Q(s) \in RH_\infty^{p\times p}$ is given by

$$Q(s)=D(s)\left(\bar{G}_u(s)+D(s)\tilde{Q}(s)\right),\tag{37}$$

$N(s) \in RH_\infty^{p\times p}$ and $D(s) \in RH_\infty^{p\times p}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s)=N(s)D^{-1}(s),\tag{38}$$

$\bar{G}_u(s) \in RH_\infty^{p\times p}$ is a function satisfying

$$N(s_i)\bar{G}_u(s_i)e^{-s_iL}=I\quad(i=1,\dots,n)\tag{39}$$

and $\tilde{Q}(s) \in RH_\infty^{p\times p}$ is any function.

Proof: First, the necessity is shown. From Theorem 1, the structure of simple repetitive controller $C(s)$ is written by (9). Therefore, we show that if the controller $C(s)$ in (9) make the control system in (1) stable, then the controller $C(s)$ is written by (36) and (37). From the assumption that the controller $C(s)$ in (9) makes the transfer function from r to y of the control system in (1) have finite numbers of poles,

$$\begin{aligned}&\left(I+G(s)C(s)e^{-sL}\right)^{-1}G(s)C(s)e^{-sL}\\&=G(s)\left(C_1(s)+C_2(s)e^{-s(T-L)}+C_3(s)e^{-sT}\right)\\&\quad e^{-sL}\left\{I+\left(G(s)C_2(s)-q(s)\right)e^{-sT}\right.\\&\quad+\left(G(s)C_1(s)+C_4(s)\right)e^{-sL}+\left(G(s)C_3(s)\right.\\&\quad\left.\left.-C_4(s)q(s)\right)e^{-s(T+L)}\right\}^{-1}\end{aligned}\tag{40}$$

has finite numbers of poles. This implies that

$$C_2(s)=G^{-1}(s)q(s),\tag{41}$$

$$C_4(s)=-G(s)C_1(s)\tag{42}$$

and

$$C_3(s)=-C_1(s)q(s)\tag{43}$$

are satisfied, that is, $C(s)$ is rewritten by

$$\begin{aligned}&C(s)\\&=\left(C_1(s)+G^{-1}(s)q(s)e^{-s(T-L)}-C_1(s)q(s)e^{-sT}\right)\\&\quad\left\{\left(I-G(s)C_1(s)e^{-sL}\right)\left(I-q(s)e^{-sT}\right)\right\}^{-1}.\end{aligned}\tag{44}$$

From the assumption that the controller $C(s)$ in (9) makes the control system in (1) stable, $\left(I+G(s)C(s)e^{-sL}\right)^{-1}G(s)C(s)e^{-sL}$, $\left(I+G(s)C(s)e^{-sL}\right)^{-1}$, $\left(I+C(s)G(s)e^{-sL}\right)^{-1}C(s)$ and $\left(I+G(s)C(s)e^{-sL}\right)^{-1}G(s)e^{-sL}$ are stable. From simple manipulations and (44), we have

$$\begin{aligned}&\left(I+G(s)C(s)e^{-sL}\right)^{-1}G(s)C(s)e^{-sL}\\&=G(s)C_1(s)e^{-sL}+q(s)e^{-sT}\\&\quad-G(s)C_1(s)q(s)e^{-s(T+L)},\end{aligned}\tag{45}$$

$$\begin{aligned}&\left(I+C(s)G(s)e^{-sL}\right)^{-1}C(s)\\&=C_1(s)+G^{-1}(s)q(s)e^{-s(T-L)}-C_1(s)q(s)e^{-sT},\end{aligned}\tag{46}$$

$$\begin{aligned}&\left(I+G(s)C(s)e^{-sL}\right)^{-1}G(s)e^{-sL}\\&=\left(I-G(s)C_1(s)e^{-sL}\right)\left(I-q(s)e^{-sT}\right)G(s)e^{-sL}\end{aligned}\tag{47}$$

and

$$\begin{aligned}&\left(I+G(s)C(s)e^{-sL}\right)^{-1}\\&=\left(I-G(s)C_1(s)e^{-sL}\right)\left(I-q(s)e^{-sT}\right).\end{aligned}\tag{48}$$

Since the transfer function in (45), (46) and (48) are stable, from $q(s) \in RH_\infty^{p\times p}$ and $G^{-1}(s)q(s) \in RH_\infty^{p\times p}$, we have $G(s)C_1(s) \in RH_\infty^{p\times p}$ and $C_1(s) \in RH_\infty^{p\times p}$. This implies that $C_1(s)$ is written by

$$C_1(s) = D(s)\hat{C}_1(s), \tag{49}$$

where $\hat{C}_1(s) \in RH_\infty^{p\times p}$. From the assumption that the transfer function in (47) is stable and from (49), for s_i $(i = 1, \ldots, n)$ which are unstable poles of $G(s)$,

$$\begin{aligned} I - G(s_i)C_1(s_i)e^{-s_iL} &= I - N(s_i)\hat{C}_1(s_i)e^{-s_iL} \\ &= 0 \quad (i = 1, \ldots n) \end{aligned} \tag{50}$$

must be satisfied. Because $\bar{G}_u(s) \in RH_\infty^{p\times p}$ and $\hat{C}_1(s) \in RH_\infty^{p\times p}$, $\hat{C}_1(s) - \bar{G}_u(s)$ is stable. From (39) and (50),

$$\hat{C}_1(s_i) - \bar{G}_u(s_i) = 0 \quad (i = 1, \ldots, n) \tag{51}$$

is satisfied. Equation (51) implies that s_i $(i = 1, \ldots, n)$, which are unstable poles of $G(s)$, are blocking zeros of $\hat{C}_1(s) - \bar{G}_u(s)$, because $\hat{C}_1(s) \in RH_\infty^{p\times p}$ and $\bar{G}_u(s) \in RH_\infty^{p\times p}$. When we rewrite $\hat{C}_1(s) - \bar{G}_u(s)$ as

$$\hat{C}_1(s) - \bar{G}_u(s) = D(s)\tilde{Q}(s), \tag{52}$$

then $\tilde{Q}(s) \in RH_\infty^{p\times p}$, because $\hat{C}_1(s) \in RH_\infty^{p\times p}$, $\bar{G}_u(s) \in RH_\infty^{p\times p}$ and $D(s) \in RH_\infty^{p\times p}$. Substituting (52) into (49), we have

$$C_1(s) = D(s)\left(\bar{G}_u(s) + D(s)\tilde{Q}(s)\right) \in RH_\infty^{p\times p}. \tag{53}$$

Using $Q(s) \in RH_\infty^{p\times p}$, let $C_1(s)$ in (53) be

$$\begin{aligned} C_1(s) &= Q(s) \\ &= D(s)\left(\bar{G}_u(s) + D(s)\tilde{Q}(s)\right) \in RH_\infty^{p\times p}. \end{aligned} \tag{54}$$

From (54) and (44), we have (36) and (37). In this way, it is shown that if the controller $C(s)$ in (9) makes the transfer function from r to y of the control system in (1) have finite numbers of poles and makes the control system in (1) stable, then $C(s)$ is written as (36) and (37).

Next, the sufficiency is shown. That is, we show that if $C(s)$ is written by (36) and (37), then the controller $C(s)$ makes the control system in (1) stable and makes the transfer function from r to y and that from the disturbance d to the output y of the control system in (1) have finite numbers of poles. After simple manipulations, we have

$$\begin{aligned} &\left(I + G(s)C(s)e^{-sL}\right)^{-1} G(s)C(s)e^{-sL} \\ &= N(s)\left(\bar{G}_u(s) + D(s)\tilde{Q}(s)\right)e^{-sL} + q(s)e^{-sT} \\ &\quad -N(s)\left(\bar{G}_u(s) + D(s)\tilde{Q}(s)\right)q(s)e^{-s(T+L)}, \end{aligned} \tag{55}$$

$$\begin{aligned} &\left(I + C(s)G(s)e^{-sL}\right)^{-1} C(s) \\ &= D(s)\left(\bar{G}_u(s) + D(s)\tilde{Q}(s)\right) \\ &\quad +G^{-1}(s)q(s)e^{-s(T-L)} \\ &\quad -D(s)\left(\bar{G}_u(s) + D(s)\tilde{Q}(s)\right)q(s)e^{-sT}, \end{aligned} \tag{56}$$

$$\begin{aligned} &\left(I + G(s)C(s)e^{-sL}\right)^{-1} G(s)e^{-sL} \\ &= \left\{I - N(s)\left(\bar{G}_u(s) + D(s)\tilde{Q}(s)\right)e^{-sL}\right\} \\ &\quad \left(I - q(s)e^{-sT}\right)G(s)e^{-sL} \end{aligned} \tag{57}$$

and

$$\begin{aligned} &\left(I + G(s)C(s)e^{-sL}\right)^{-1} \\ &= \left\{I - N(s)\left(\bar{G}_u(s) + D(s)\tilde{Q}(s)\right)e^{-sL}\right\} \\ &\quad \left(I - q(s)e^{-sT}\right). \end{aligned} \tag{58}$$

Since $\bar{G}_u(s) \in RH_\infty^{p\times p}$, $\tilde{Q}(s) \in RH_\infty^{p\times p}$, $G^{-1}(s)q(s) \in RH_\infty^{p\times p}$, $q(s) \in RH_\infty^{p\times p}$, $N(s) \in RH_\infty^{p\times p}$ and $D(s) \in RH_\infty^{p\times p}$, the transfer functions in (55), (56), (58) are stable. If the transfer function in (57) is unstable, unstable poles of the transfer function in (57) are unstable poles of $G(s)$. From the assumption that $\bar{G}_u(s)$ satisfies (39), unstable poles of $G(s)$ are not poles of $\{I - N(s)(\bar{G}_u(s) + D(s)\tilde{Q}(s))e^{-sL}\}(I - q(s)e^{-sT})G(s)e^{-sL}$. Therefore, the transfer function in (57) is stable. In addition, from the same reason, the transfer function from the periodic reference input r to the output y written by (55) and that from the disturbance d to the output y written by (58) of the control system in (1) have finite numbers of poles. Thus, the sufficiency has been shown.

We have thus proved Theorem 3. ■

6. CONTROL CHARACTERISTICS

In this section, we present control characteristics of simple repetitive control system using the parameterizations of all stabilizing simple repetitive controllers for stable time-delay plants in Theorem 2 and for unstable time-delay plants in Theorem 3.

First, the input-output characteristic is shown. Using the parameterization of all stabilizing simple repetitive controllers for time-delay plants in Theorem 2 and Theorem 3, the transfer function $S(s)$ from the periodic reference input r to the error $e = r - y$ in (1) is written by

$$S(s) = \left(I - G(s)Q(s)e^{-sL}\right)\left(I - q(s)e^{-sT}\right). \tag{59}$$

From (59), for frequency components ω_i $(i = 0, 1, \ldots, N_{max})$ in (5) of the periodic reference input r, if $q(s) \in RH_\infty^{p\times p}$ is settled to satisfy (4), then the output y follows the periodic reference input r with small steady state error.

Next the disturbance attenuation characteristic is shown. Using the parameterization of all stabilizing simple repetitive controllers for time-delay plants in Theorem 2 and Theorem 3, the transfer function $S(s)$ from the disturbance d to the output y in (1) is written by (59). From (59), for the frequency components $\omega_i(i=0,1,\dots,N_{max})$ in (5) of the disturbance d those are same to those of the periodic reference input r, since $S(s)$ satisfies $\bar{\sigma}\{S(j\omega_i)\}\simeq 0$ $(\forall i=0,1,\dots,N_{max})$, the disturbance d is attenuated effectively. For the frequency component ω_d of the disturbance d that is different from that of the periodic reference input r, that is $\omega_d\neq\omega_i(\forall i=0,1,\dots,N_{max})$, even if

$$\bar{\sigma}\{I-q(j\omega_d)\}\simeq 0, \tag{60}$$

the disturbance d cannot be attenuated, because

$$e^{-j\omega_d T}\neq 1 \tag{61}$$

and

$$\bar{\sigma}\left\{I-q(j\omega_d)e^{-j\omega_d T}\right\}\neq 0. \tag{62}$$

In order to attenuate this frequency component, we must find $Q(s)$ that satisfies

$$\bar{\sigma}\left\{I-G(j\omega_d)Q(j\omega_d)e^{-j\omega_d L}\right\}\simeq 0. \tag{63}$$

From above discussion, the role of $q(s)$ is to specify the input-output characteristic for the periodic reference input r and to specify the disturbance attenuation characteristic for the disturbance d with same frequency components ω_i $(i=0,1,\dots,N_{max})$ of the periodic reference input r. The role of $Q(s)$ is to specify the disturbance attenuation characteristic for the disturbance d with frequency components $\omega_d\neq\omega_i$ $(\forall i=0,1,\dots,N_{max})$.

7. DESIGN PROCEDURE

In this section, a design procedure of stabilizing simple repetitive controller satisfying Theorem 2 and Theorem 3 is presented.

When $G(s)e^{-sL}$ is stable, a procedure is summarized as follows.

Procedure 1

Step 1) $q(s)\in RH_\infty^{p\times p}$ is settled so that for the frequency components $\omega_i(i=0,1,\dots,N_{max})$ of the periodic reference input $r(s)$,

$$\bar{\sigma}\{I-q(j\omega_i)\}\simeq 0\ \ (\forall i=0,1,\dots,N_{max}) \tag{64}$$

and

$$G^{-1}(s)q(s)\in RH_\infty^{p\times p} \tag{65}$$

are satisfied. In order to satisfy both (64) and (65), for example, $q(s)$ is designed by

$$q(s) = G_i(s)q_r(s), \tag{66}$$

where $G_i(s)\in RH_\infty^{p\times p}$ is an inner function of $G(s)$ satisfying $G_i(0)=I$ and

$$G(s)=G_i(s)G_o(s), \tag{67}$$

$G_o(s)\in RH_\infty^{p\times p}$ is an outer function, $q_r(s)$ is a low-pass filter satisfying $q_r(0)=I$, as

$$q_r(s) = \mathrm{diag}\left\{\frac{1}{(1+s\tau_{r1})^{\alpha_{r1}}},\cdots,\frac{1}{(1+s\tau_{rp})^{\alpha_{rp}}}\right\} \tag{68}$$

is valid, α_{ri} $(i=1,\dots,p)$ are arbitrary positive integers to make $G_o^{-1}(s)q_r(s)$ proper and $\tau_{ri}\in R$ $(i=1,\dots,p)$ are any positive real numbers satisfying

$$\bar{\sigma}\left[I-G_i(j\omega_i)\mathrm{diag}\left\{\frac{1}{(1+j\omega_r\tau_{r1})^{\alpha_{r1}}},\cdots,\frac{1}{(1+j\omega_i\tau_{rp})^{\alpha_{rp}}}\right\}\right]\simeq 0. \tag{69}$$

Step 2) $Q(s)\in RH_\infty^{p\times p}$ is designed so that for the frequency component ω_d of the disturbance d,

$$\bar{\sigma}\left\{I-Q(j\omega_d)G(j\omega_d)e^{-j\omega_d L}\right\}\simeq 0 \tag{70}$$

is satisfied.

Step 3) Substituting above $Q(s)$ and $q(s)$ for (22), we obtain a stabilizing simple repetitive controller for stable time-delay plant.

When $G(s)e^{-sL}$ is unstable, a procedure is summarized as follows.

Procedure 2

Step 1) Obtain the coprime factors $N(s)\in RH_\infty^{p\times p}$ and $D(s)\in RH_\infty^{p\times p}$ of $G(s)$ satisfying (38).

Step 2) $\bar{G}_u(s)\in RH_\infty^{p\times p}$ is settled satisfying (39).

Step 3) $q(s)\in RH_\infty^{p\times p}$ is settled so that for the frequency components $\omega_i(i=0,1,\dots,N_{max})$ of the periodic reference input $r(s)$,

$$\bar{\sigma}\{I-q(j\omega_i)\}\simeq 0\ \ (\forall i=0,1,\dots,N_{max}) \tag{71}$$

and $G^{-1}(s)q(s)\in RH_\infty^{p\times p}$ are satisfied. Such $q(s)$ can be designed using the method described in Step 1) in Procedure 1.

Step 4) $\tilde{Q}(s)\in RH_\infty^{p\times p}$ is designed so that for the frequency component ω_d of the disturbance d,

$$\begin{aligned}&\bar{\sigma}\left\{I-Q(j\omega_d)G(j\omega_d)e^{-j\omega_d L}\right\}\\ =\ &\bar{\sigma}\left\{I-N(j\omega_d)\left(\bar{G}_u(j\omega_d)+D(j\omega_d)\tilde{Q}(j\omega_d)\right)e^{-j\omega_d L}\right\}\\ \simeq\ &0\end{aligned} \tag{72}$$

is satisfied.

Step 5) Substituting above $N(s)$, $D(s)$, $\bar{G}_u(s)$, $\tilde{Q}(s)$ and $q(s)$ for (36) and (37), we obtain a stabilizing simple repetitive controller for unstable time-delay plant.

8. NUMERICAL EXAMPLE

In this section, a numerical example is shown to illustrate the effectiveness of the proposed method.

Consider the problem to obtain the parameterization of all stabilizing simple repetitive controllers for the stable time-delay plant $G(s)e^{-sL}$ written by

$$G(s)e^{-sL} = \begin{bmatrix} \frac{s-256}{(s+1)(s+2)} & \frac{-128}{(s+1)(s+2)} \\ \frac{128}{(s+1)(s+2)} & \frac{s-512}{(s+1)(s+2)} \end{bmatrix} e^{-4s}, \quad (73)$$

that follows the periodic reference input $r(t)$ with period $T = 4$[sec] with small steady state error, where $G(s)$ and L are

$$G(s) = \begin{bmatrix} \frac{s-256}{(s+1)(s+2)} & \frac{-128}{(s+1)(s+2)} \\ \frac{128}{(s+1)(s+2)} & \frac{s-512}{(s+1)(s+2)} \end{bmatrix} \quad (74)$$

and

$$L = 4[\mathrm{sec}], \quad (75)$$

respectively.

From Theorem 2, the parameterization of all stabilizing simple repetitive controllers for stable time-delay plants $G(s)e^{-sL}$ in (73) is given by (22), where $Q(s) \in RH_\infty^{p\times p}$ is any function.

In order for the output $y(t) = [y_1(t), y_2(t)]^T$ to follow the periodic reference input

$$r(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} = \begin{bmatrix} \sin\left(\frac{\pi t}{2}\right) \\ 2\sin\left(\frac{\pi t}{2}\right) \end{bmatrix} \quad (76)$$

with small steady state error and for the disturbance

$$d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} = \begin{bmatrix} \sin(\pi t) \\ 2\sin(\pi t) \end{bmatrix} \quad (77)$$

for which the frequency component is same to that of the periodic reference input $r(t)$, to be attenuated effectively, $q(s)$ is settled by

$$q(s) = G_i(s)q_r(s), \quad (78)$$

where

$$G_i(s) = \frac{s^2 - 768s + 1.48\cdot 10^5}{s^2 + 768s + 1.48\cdot 10^5}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (79)$$

and

$$q_r(s) = \begin{bmatrix} \frac{1}{0.002s+1} & 0 \\ 0 & \frac{1}{0.003s+1} \end{bmatrix}. \quad (80)$$

In order for the disturbance

$$d(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} = \begin{bmatrix} \sin\left(\frac{\pi t}{4}\right) \\ 2\sin\left(\frac{\pi t}{4}\right) \end{bmatrix} \quad (81)$$

for which the frequency component is different from that of the periodic reference input $r(t)$, to be attenuated effectively, $Q(s)$ is selected satisfying (63) as

$$Q(s) = \begin{bmatrix} \frac{52.87s+22.94}{(s+50)(s+100)} & \frac{13.23s+5.719}{(s+50)(s+100)} \\ \frac{-13.23s-5.719}{(s+50)(s+100)} & \frac{26.41s+11.50}{(s+50)(s+100)} \end{bmatrix}. \quad (82)$$

Using above-mentioned parameters, we have a stabilizing simple repetitive controller. When the designed stabilizing simple repetitive controller $C(s)$ is used, the response of the error $e(t) = r(t) - y(t) = [e_1(t), e_2(t)]^T$ in (1) for the periodic reference input $r(t)$ in (76) is shown in Fig. 1. Here, the broken line shows the response of the periodic reference input $r_1(t)$, the dotted line shows that of the periodic reference input $r_2(t)$, the solid line shows that of the error $e_1(t)$ and the dotted and broken line shows that of the error $e_2(t)$. Figure 1 shows that the output $y(t)$ follows the periodic reference input $r(t)$ in (76) with small steady state error.

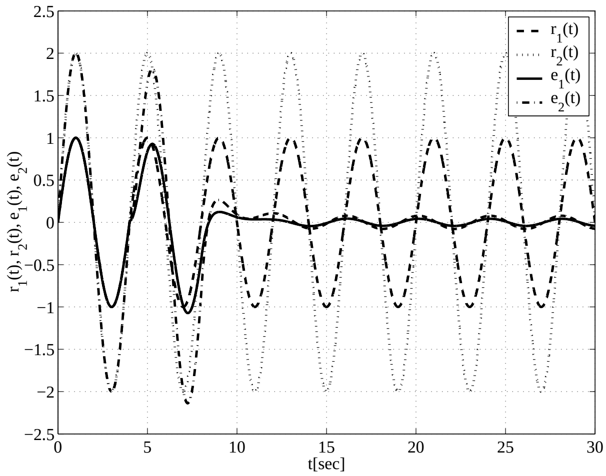

Fig.1: *The response of the error $e(t)$ for the periodic reference input $r(t)$ in (76)*

Next, using the designed stabilizing simple repetitive controller $C(s)$, the disturbance attenuation characteristic is shown. The response of the output $y(t)$ for the disturbance $d(t)$ in (77), for which the frequency component is equivalent to that of the periodic reference input $r(t)$ is shown in Fig. 2. Here, the broken line shows the response of the disturbance $d_1(t)$, the dotted line shows that of the disturbance $d_2(t)$, the solid line shows that of the output $y_1(t)$ and the dotted and broken line shows that of the output $y_2(t)$. Figure 2 shows that the disturbance $d(t)$ in (77) is attenuated effectively.

Finally, the response of the output $y(t)$ for the disturbance $d(t)$ in (81), for which the frequency component is different from that of the periodic reference input $r(t)$ is shown in Fig. 3. Here, the broken line shows the response of the disturbance $d_1(t)$, the dotted line shows that of the disturbance $d_2(t)$, the solid

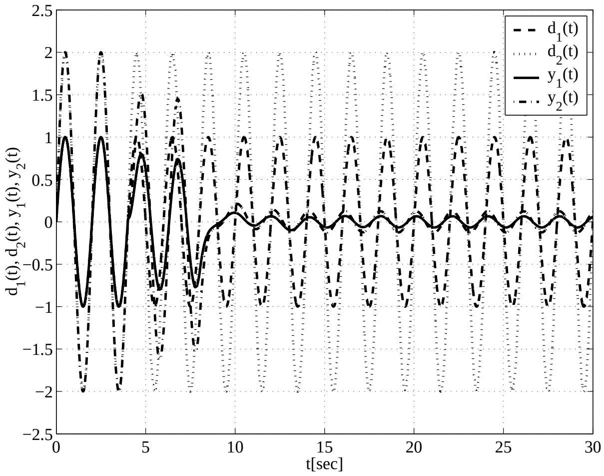

Fig.2: *The response of the output $y(t)$ for the disturbance $d(t)$ in (77)*

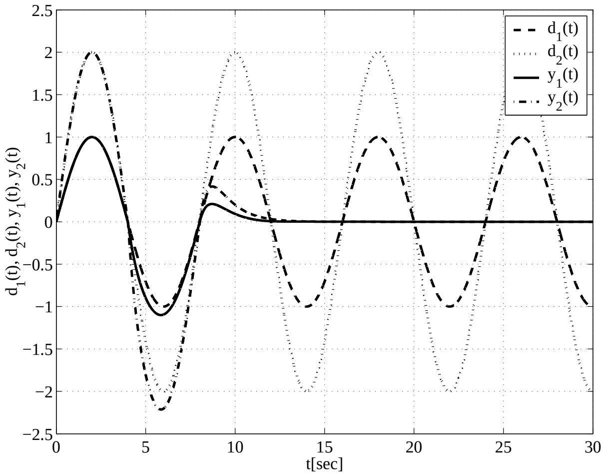

Fig.3: *The response of the output $y(t)$ for the disturbance $d(t)$ in (81)*

line shows that of the output $y_1(t)$ and the dotted and broken line shows that of the output $y_2(t)$. Figure 3 shows that the disturbance $d(t)$ in (81) is attenuated effectively.

A stabilizing simple repetitive controller for time-delay plant can be easily designed in the way shown here.

9. CONCLUSIONS

In this paper, we have proposed the concept of simple repetitive controllers for multiple-input/multiple-output time-delay plants and clarified the parameterization of all stabilizing simple repetitive controllers for both stable time-delay plants and unstable time-delay plants. That is, we found out all controllers $C(s)$ written as the form in (9) such that the control system in (1) is stable, the output y follows the periodic reference input r with small steady state error even in the presence of time-delay and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. Control characteristics of simple repetitive control systems for time-delay plants were presented, as well as a design procedure of stabilizing simple repetitive controllers for time-delay plants was presented. Finally, a numerical example was shown to illustrate the effectiveness of the proposed method.

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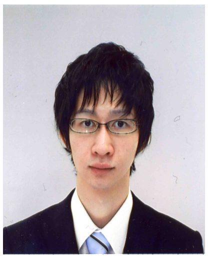

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