Robust Digital Control for Boost DC-DC Converter

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ABSTRACT

If the duty ratio, load resistance and input voltage in boost DC-DC converter are changed, the dynamic characteristics is varied greatly, that is, boost DC-DC converter has non-linear characteristics. In many applications of DC-DC converters, the load cannot be specified in advance, and it will be changed suddenly from no load to full load. In the boost DC-DC converter system used a conventional single controller cannot be adapted to change dynamics and it occurs large output voltage variation. In this paper, an approximate 2-Degree-of-Freedom (2DOF) digital controller for suppressing the change of step response characteristics and variation of output voltage in the load sudden changes is proposed. Experimental studies using micro-processor for controller demonstrate that this type of digital controller is effective to suppress variations.

Keywords: Boost DC-DC Converter, Approximate 2DOF, Robust Digital Control, Micro-Processor

1. INTRODUCTION

In many applications of DC-DC converters, load cannot be specified in advance, i.e., their amplitudes are suddenly changed from the zero to the maximum rating. In a boost DC-DC converter, if the duty ratio, load resistance and input voltage are changed, the dynamic characteristics are varied greatly, that is, the boost DC-DC converter has non-linear characteristics. Usually, the controller of the boost DC-DC converter is designed to the approximated linear controlled object at one operating point. In a non-linear boost DC-DC converter system, it is not enough for the design of controller considering only one operating point. As a technique to improve dynamic performance, the gain-scheduled control is proposed. This method switch many controllers designed to many operating points. However, it requires complicated control routine when controllers are implemented to micro-processor. Then, the controller which can cover sudden load changes and dynamic characteristics changes with only one controller is needed. PI control etc.[1-6] are conventionally used for the boost DC-DC converter, but robustness is not enough. In order to improve robustness, various methods[7-11] are proposed, but these have not improved so much, either.

In this paper, an approximate 2-degree-of-freedom (2DOF) design method robust enough which suppresses very small the output voltage regulation at load sudden change is proposed.
Robust control method using an approximate 2DOF for improving start-up characteristics and load sudden change characteristics of DC-DC converters has been proposed [12-13]. However, it was applied to buck DC-DC converter. The Boost DC-DC converter is non-linear system and the dynamic characteristics are changed at each operating point. The design method of the approximate 2DOF digital controller which considering much operating points with one controller is proposed. By this method, even if the dynamic characteristics of converter are change, the variation of the output voltage and dynamic characteristic can be suppressed enough. This controller is actually implemented on a micro-processor and is connected to the boost DC-DC converter. Experimental studies demonstrate that the digital controller designed by proposed method satisfies the given specifications and is useful.

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2. BOOST DC-DC CONVERTER

2.1 State-space model of boost DC-DC converter

The boost DC-DC converter is shown in Fig. 1. In Fig.1, v_i is input AC voltage, C_{in} is smoothing capacitor, Q_0 is main switch, L_0 and D_0 are boost inductance and diode, C_0 is output capacitor, R_L is output load resistance, i_L is inductor current, i_o is output current, and v_o is output voltage. Here C_{in} is 1[μF], L_0 is 150[μH] and C_0 is 940[μF]. Using averaging method, the state equation of the controlled object in Fig.1 becomes as follows [14]:

$$\frac{d}{dt}\begin{bmatrix} v_o \\ i_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_L C_0} & \frac{1-\mu}{C_0} \\ \frac{1-\mu}{L_0} & \frac{R_0}{L_0} \end{bmatrix}\begin{bmatrix} v_o \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{V_i}{L_0} \end{bmatrix} + \left\{ v_o \begin{bmatrix} 0 \\ \frac{1}{L_0} \end{bmatrix} + i_L \begin{bmatrix} -\frac{1}{C_0} \\ 0 \end{bmatrix} \right\} \tag{1}$$

Here equivalent resistance of inductor R_0 is 1.8[Ω] and rectified input voltage V_i is 141[VDC]. μ is duty ratio. $\mu = \mu_s + \tilde{\mu}, \mu_s$ is the value at each operating point and $\tilde{\mu}$ is the small variation at operating points. The boost DC-DC converter has non-liner characteristics because this equation has the product of state variable and duty ratio.

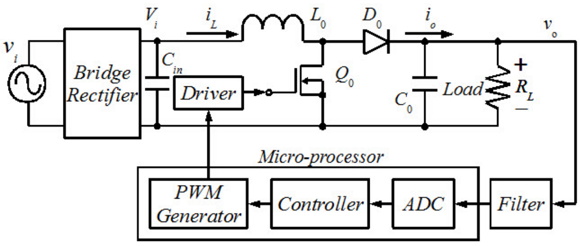


Fig.1: *Boost DC-DC Converter*

2.2 Static characteristics of boost converter

In the static characteristics, the differential values of state variables and $\tilde{\mu}$ are zero. Then average of output voltage V_s and inductor current I_s at operating points becomes as follows:

$$\begin{aligned} V_s &= \frac{1}{1 - \dfrac{1}{(1-\mu_s)^2}\dfrac{R_0}{R_L}} \frac{1}{1-\mu_s} V_i \\ I_s &= \frac{1}{R_L}\frac{V_s}{1-\mu_s} \end{aligned} \tag{2}$$

Eq. (2) turns out that the boost DC-DC converter is non-linear system. The static characteristics of the boost DC-DC converter is changed greatly with load resistances, and it influences the dynamic characteristics of converter. In addition, the static characteristics will be changed with input voltage variations [9].

$$\begin{aligned} \dot{x}(t) &= A_c x(t) + B_c u(t) \\ y(t) &= C_c x(t) \end{aligned} \tag{3}$$

Where

$$A_c = \begin{bmatrix} -\dfrac{R_0}{L_0} & -\dfrac{1-\mu_s}{L_0} \\ \dfrac{1-\mu_s}{C_0} & \dfrac{1}{R_L C_0} \end{bmatrix}, B_c = \begin{bmatrix} \dfrac{V_s}{L_0} \\ -\dfrac{I_s}{C_0} \end{bmatrix}$$
$$x(t) = \begin{bmatrix} i_L(t) \\ v_o(t) \end{bmatrix}, u(t) = \tilde{\mu}(t), C_c = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

From this equation, each matrix of the boost converter depends on duty ratio μ_s. Therefore, the initial value and converter response at the operating point will be changed depending on duty ratio and operating point variations. The load changes of the controlled object and output voltage change are considered as parameter changes in eq. (3). Such parameter changes can be transformed to equivalent disturbances q_u and q_y as shown in Fig.5. Therefore, what is necessary is just to constitute the control systems whose pulse transfer functions from equivalent disturbances q_u and q_y to the output y become as small as possible in their amplitudes, in order to robustize or suppress the influence of these parameter changes.

The controller of the DC-DC boost converter used as inverter power supplies for EV (electric vehicle) and Air Conditioner, etc. is designed. Then the controller which satisfies the following specifications will be designed.

1. Input voltage vi is 100[VAC],
   And output voltage vo changes from 240[VDC] to 385[VDC].
2. Step responses are almost the same at resistive loads where $300 \leq R_L$¡5k [Ω],
   And an over-shoot is less than 10[%] in the step response.
3. The dynamic load response is smaller than 20[VDC] against change of load between 30∼500[W].

Under these specifications, three operating points for deciding the controller was determined as follows:

Point 1: Output voltage is 385[VDC],
Resistive load is 5[kΩ]
Point 2: Output voltage is 385[VDC],
Resistive load is 300[Ω]
Point 3: Output voltage is 240[VDC],
Resistive load is 300[Ω]

The gains and phases characteristics of the boost DC-DC converter are different at each operating point. One approximate 2DOF controller is designed by considering these operating points.

3. DIGITAL ROBUST CONTROLLER

3.1 Discrization of controlled object

The continuous system of eq. (3) is transformed into the discrete system as follows:

$$\begin{aligned} x_d(k+1) &= A_d x_d(k) + B_d u(k) + q_u(k) \\ y(k) &= C_d x_d(k) + q_y(k) \end{aligned} \tag{4}$$

Where

$$A_d = \left[e^{A_c T}\right], B_d = \left[\int_0^T e^{A_c \tau} B_c d\tau\right], C_d = C_c$$

And sampling period $T_s = 10[\mu s]$. Here, in order to compensate the delay $L_d = 0.99T_s$ by ADC conversion time and micro-processor operation time etc., one delay (state ξ_1) is introduced to input of the controlled object. And, more one delay (state ξ_2) is also introduced to input of the controlled object for the current feedback equivalent conversion. Then, a new controlled object with two delays is shown in Fig.2. The state-space equation is described as follows:

$$\begin{aligned} x_{dw}(k+1) &= A_{dw}x_{dw}(k) + B_{dw}v(k) \\ y(k) &= C_{dw}x_{dw}(k) \end{aligned} \tag{5}$$

Where

$$\begin{aligned}
A_{dw} &= \begin{bmatrix} A_w & B_w \\ 0 & 0 \end{bmatrix}, B_{dw} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
C_{dw} &= \begin{bmatrix} C_c & 0 & 0 \end{bmatrix}, \xi_2(k-1) = \xi_1(k) \\
A_w &= \begin{bmatrix} e^{A_cT} & e^{A_c(T-L_d)}\int_0^{L_d} e^{A_c\tau}B_c d\tau \\ 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 \end{bmatrix} \\
B_w &= \begin{bmatrix} \int_0^{T-L_d} e^{A_c\tau}B_c d\tau \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} \\ b_{21} \\ 1 \end{bmatrix} \\
x_{dw}(k) &= \begin{bmatrix} x_d(k) \\ \xi_1(k) \\ \xi_2(k) \end{bmatrix} = \begin{bmatrix} v_o(k) \\ i_L(k) \\ uk \\ v(k-1) \end{bmatrix}
\end{aligned}$$

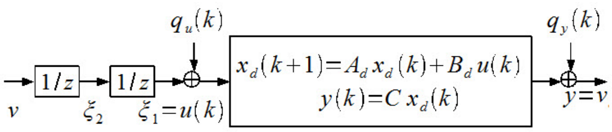


Fig.2: *New controlled object with two input delays*

3.2 Design method for approximate 2DOF digital controller

To the new controlled object of eq. (5), the model matching control system is constituted using the state feedback as shown in Fig.3.

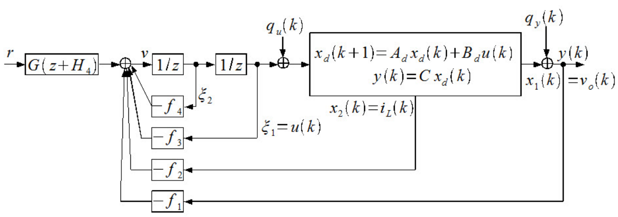


Fig.3: *Model matching system using state feedback*

The system of Fig.3, transfer function from reference input r to output y is described as follows:

$$\begin{aligned} W_{ry}(z) &= \frac{(1+H_1)(1+H_2)(1+H_3)}{(z+H_1)(z+H_2)(z+H_3)} \\ &\times \frac{(z-n_1)(z-n_2)(1+H_4)}{(1-n_1)(1-n_2)(z+H_4)} \end{aligned} \tag{6}$$

Here n_1 and n_2 are the zeros of the discrete-time controlled object. It shall be specified that the relation of H_1 and H_2, H_3 become $|H_1| \gg |H_2|, |H_3|$. Then W_{ry} can be approximated to the following first-order discrete model:

$$W_{ry}(z) \approx W_m(z) = \frac{1+H_1}{z+H_1} \tag{7}$$

This target characteristic W_m is realizable by applying a state feedback shown in Fig.3:

$$v = -Fx_{dw} - G(z-H_4)r \tag{8}$$

Where

$$\begin{aligned} F &= \begin{bmatrix} f_1 & f_2 & f_3 & f_4 \end{bmatrix} \\ G &= \frac{1}{C_{dw}\dfrac{I}{I-(A_{dw}-B_{dw}F)B_{dw}}}\frac{1}{1+H_4} \end{aligned}$$

The current feedback is used in Fig.3. This is transformed into voltage and control input feedbacks without changing the pulse transfer function W_{ry} by an equivalent transform using following equation:

$$\begin{aligned} -f_2x_2(k) = -\frac{f_2}{a_{12}}(&-x_1(k+1) \\ &- a_{11}x_1(k) - a_{13}\xi_1(k) - b_{11}\xi_2(k)) \end{aligned} \tag{9}$$

Using this equation, Fig.3 is transformed to Fig.4, and then to Fig.5.

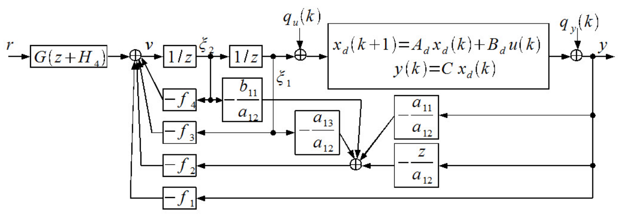


Fig.4: *Equivalent transform the current feedback*

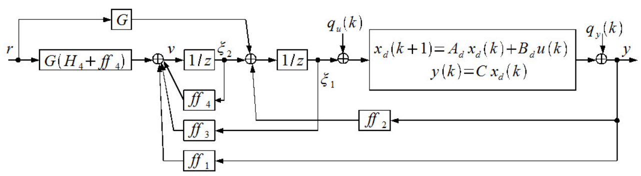


Fig.5: *Model matching system using only voltage feedback*

In Fig.5, the parameters are as follows:

$$
\begin{aligned}
&ff_1 = -f_1 + \frac{f_2}{a_{12}}\left(a_{11} - f_4 + \frac{f_2 b_{11}}{a_{12}}\right), f_2 = -\frac{f_2}{a_{12}} \\
&ff_3 = -f_3 + \frac{f_2 a_{13}}{a_{12}}, \quad ff_4 = -f_4 + \frac{f_2 b_{11}}{q_{112}}
\end{aligned} \tag{10}
$$

Where $f_i, i = 1, \ldots, 4$ are state feedback gain $F = [f_1 \;\; f_2 \;\; f_3 \;\; f_4]$ that obtained from state feedback method for constituting the model matching system. The system added the inverse system and the filter to the system in Fig.5 is constituted as shown in Fig.6.

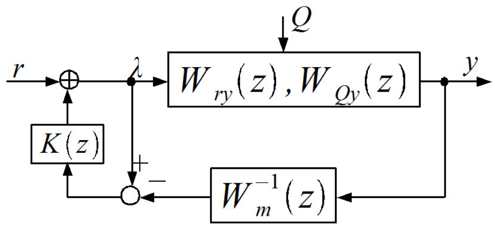


Fig.6: *System reconstituted with inverse system and filter*

In Fig.6, transfer function K(z) is following equation:

$$K(z) = \frac{k_z}{z - 1 + k_z} \tag{11}$$

And, the transfer function $W_{Qy}(z)$ between this equivalent disturbance $Q = [q_u \; q_y]^T$ to y of the system in Fig.6 is defined as follows:

$$W_{Qy}(z) = \lfloor \; W_{q_u y}(z) \quad W_{q_y y}(z) \; \rfloor \tag{12}$$

The transfer functions between $r - y, q_u - y$ and $q_y - y$ of the system in Fig.6 are given by following equation:

$$y = \frac{1 + H_1}{z + H_1}\frac{z - 1 + k_z}{z - 1 + k_z W_s(z)} W_s(z) r \tag{13}$$

$$y = \frac{z - 1 + k_z}{z - 1 + k_z}\frac{z - 1 + k_z}{z - 1 + k_z W_s(z)} W_{Qy}(z) Q \tag{14}$$

Where

$$W_s(z) = \frac{(1 + H_2)}{(z + H_2)}\frac{(1 + H_3)}{(z + H_3)}\frac{(z - n_1)}{1 - n_1}\frac{(z - n_2)}{1 - n_2} \tag{15}$$

Here, if $W_s(z) \approx 1$, then eq. (13) and eq. (14) become respectively as follows:

$$y = \frac{1 + H_1}{z + H_1} r \tag{16}$$

$$y = \frac{z - 1}{z - 1 + k_z} W_{Qy}(z) Q \tag{17}$$

From eq. (15), (16), it turns out that the characteristics from r to y can be specified with H_1 and the characteristics from q_u and q_y to y can be independently specified with k_z. That is the system in Fig.6 is an approximate 2DOF system, and its sensitivity against disturbances becomes lower with the increase of k_z. If k_z is sufficiently large, the system becomes robust for parameter changes and uncertainness. Equivalent conversion of the controller in Fig.6, we obtain Fig.7.

Then, substituting a system of Fig.5 to Fig.7, approximate 2DOF digital integral type control system will be obtained as shown in Fig.8.

In Fig.8, the parameters of the controller are as follows:

$$
\begin{aligned}
&k_1 = ff_1 - \frac{G(H_4 + ff_4)k_2}{1 + H_2}, k_2 = ff_2 - \frac{Gk_z}{1 + H_2} \\
&k_3 = ff_3, k_4 = ff_4, k_i = G(H_4 + ff_4)k_z \\
&k_{iz} = Gk_2, k_{1r} = G, k_{2r} = G(H_4 + ff_4)
\end{aligned} \tag{18}
$$

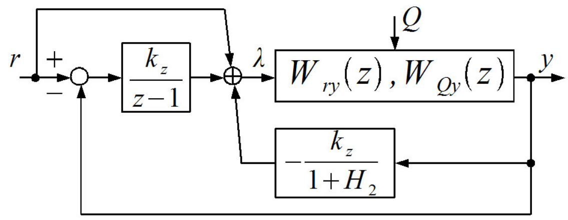


Fig.7: *Equivalent conversion of the robust digital controller*

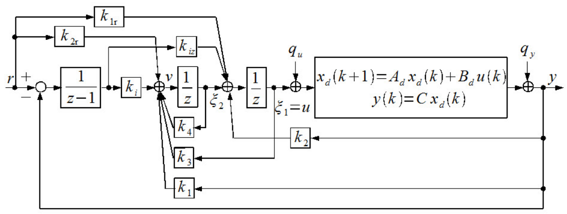


Fig.8: *Approximate 2DOF digital integral type control system*

3.3 Design of control system parameters

The control system is designed for eq. (3) at operating point 1 as a nominal controlled object. First of all, determine the parameter H_1 to fill specification (2). And the parameters $H_i, i = 2, 3, 4$ are specified to keep transfer characteristic as follows:

$$
\begin{aligned}
&H_1 = -0.99973, H_2 = -0.4 + 0.01i \\
&H_3 = -0.4 - 0.01i, H_4 = -0.99973
\end{aligned} \tag{19}
$$

Next, how to determine k_z using the root locus characteristics will be shown. The root locus of this system is derived from the closed loop characteristics of Fig.8. The root locus characteristics are shown in Fig.9.

In this figure, poles p_1, p_2 and p_3 of this system should be inside of unit circle in z-place. In this situation, if k_z is increase from 0 to 0.5, poles are move along these allows. It shows the k_z should be under 0.3 for system stability. From Fig.9, k_z is set at 0.3. Then the parameters of controller become as follows using eq. (8), (10), (17):

$$
\begin{aligned}
k_1 &= 19.89597, k_2 = -36.74662 \\
k_3 &= -0.355444, k_4 = 0.1595481 \\
k_i &= 0.0001961167, k_{iz} = 0.003293415
\end{aligned} \tag{20}
$$

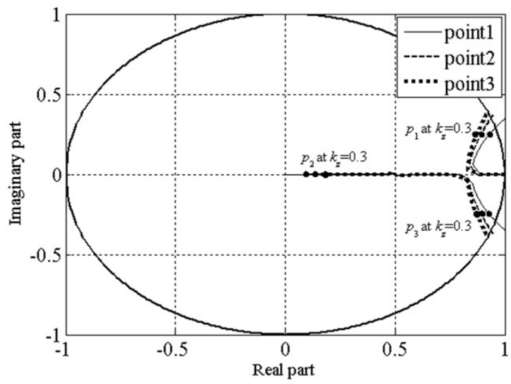


Fig.9: *Root locus*

4. EXPERIMENTAL RESULTS

Experimental setup system is shown in Fig.10. In this experiment, the micro-processor SH7216 by Renesas Electronics Corp. is used. The experiment results at step responses and load sudden change is shown in Fig.11 and Fig.12. In Fig.11, rising time in step responses is about 80[ms], and it turns out that even if the operating point changes, the step responses are not change and it satisfy specification without over-shoot. In Fig.12, the output voltage variation in sudden load change is about 4.5[V] (1.17%). The experimental result at load sudden change used usual PI controller is shown in Fig.13. In this figure, the output voltage variation in sudden load change is over 20[V] (5.19%), and recovery time are over 100[ms]. From these results, the control system using PI controller cannot satisfy specification. As a result, it turns out that proposed one is effective practically.

5. CONCLUSIONS

In this paper, the concept of controller for nonlinear boost DC-DC converter to attain good robust-

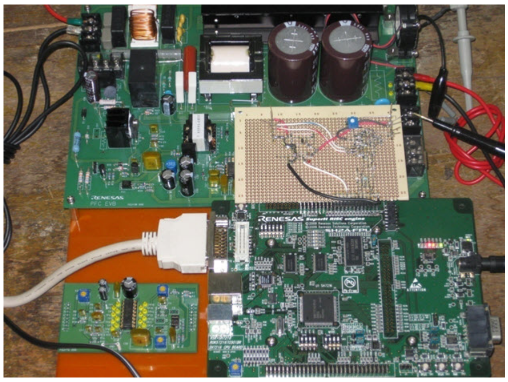
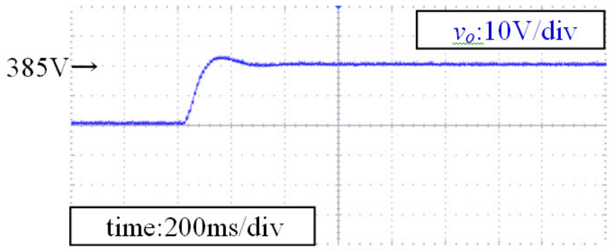
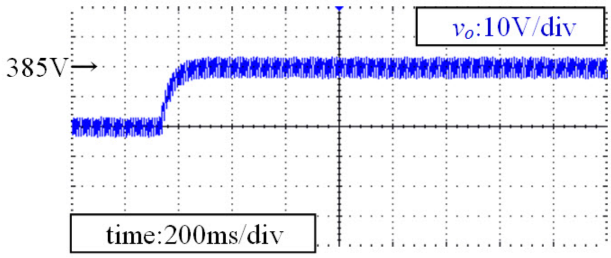


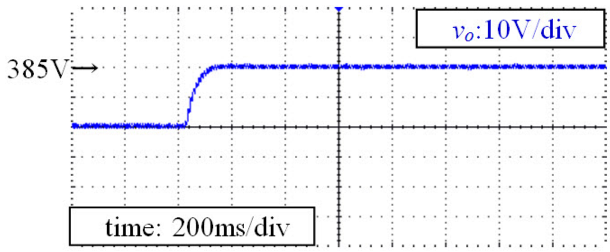
Fig.10: *Experimental setup system*



(a) At operating point 1



(b) At operating point 2



(c) At operating point 3

Fig.11: *Experimental results of step responses*

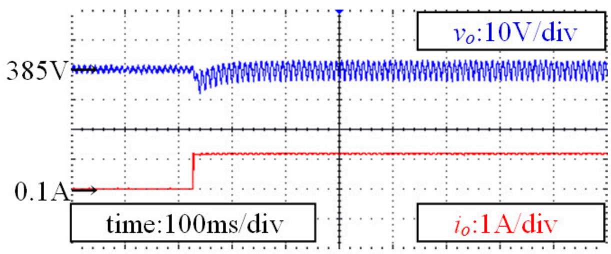


Fig.12: *Experimental results of sudden load change*

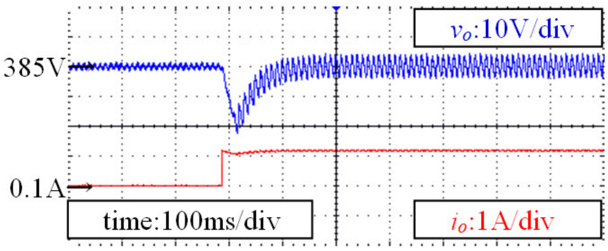


Fig.13: *Experimental results of sudden load change, where using digital PI controller*

ness was given. The proposed digital controller was implemented on the micro-processor. The DC-DC converter built-in this micro-processor was manufactured. It was shown from experiments that the proposed approximate 2DOF digital controller can suppress the variation of step responses in each operating point. And proposed method can reduce output voltage variation in sudden load change better than PI controller. This fact demonstrates the usefulness and practicality of our proposed method. As a result, the proposing method is an innovative method applicable to many boost converters of which capacities differ.

The output voltage regulation at input voltage change is not checked. It is a future subject to check this.

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