A Design Method for Control System to Attenuate Periodic Input Disturbances Using Disturbance Observers for Time-Delay Plants

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ABSTRACT

In this paper, we examine a design method for control system to attenuate periodic input disturbances using disturbance observers for time-delay plants. The disturbance observers have been used to estimate the disturbance in the plant. Several papers on design methods of disturbance observers have been published. Recently, parameterizations of all disturbance observers and all linear functional disturbance observers for time-delay plants with any input disturbance were clarified. If parameterizations of all such observers for time-delay plants with any input disturbance are used, there is a possibility that we can design control systems to attenuate input disturbances effectively. However, no paper examines a design method for control system using parameterizations of all disturbance observers and all linear functional disturbance observers for time-delay plants with any input disturbance. In this paper, to attenuate periodic input disturbances effectively, we propose a design method for control system using these parameterizations.

Keywords: Time-Delay Plant, Periodic Disturbance, Disturbance Observer, Parameterization

1. INTRODUCTION

In this paper, we examine a design method for control system to attenuate periodic input disturbances using the parameterization of all disturbance observers for time-delay plants with any input disturbance. A disturbance observer is used in the motion-control field to cancel the disturbance or to make the closed-loop system robustly stable [1–8]. Generally, the disturbance observer consists of the disturbance signal generator and observer. And then, the disturbance, which is usually assumed to be step disturbance, is estimated using observer. Since the disturbance observer has simple structure and is easy to understand, the disturbance observer is applied to many applications [1–6, 8]. However, Mita et al. point out that the disturbance observer is nothing more than an alternative design of an integral controller [7]. That is, the control system with the disturbance observer does not guarantee the robust stability. In addition, in [7], an extended H_∞ control is proposed as a robust motion control method which achieves the disturbance cancellation ability. This implies that using the method in [7], a control system with a disturbance observer can be designed to guarantee the robust stability. From other viewpoint, Kobayashi et al. consider the robust stability of the control system with a disturbance observer and examine an analysis of parameter variations of disturbance observer [8]. In this way, robustness analysis of control system with a disturbance observer has been considered.

On the other hand, another important control problem is the parameterization problem, the problem of finding all stabilizing controllers for a plant [9–14]. If the parameterization of all disturbance observers for any disturbance could be obtained, we could express previous studies of disturbance observer in a uniform manner. In addition, disturbance observer for any disturbance could be designed systematically. From this viewpoint, Yamada et al. examine parameterizations of all disturbance observers and all linear functional disturbance observers for plants with any input disturbance [15]. Yamada et al. expand the result in [15] and propose parameterizations of all disturbance observers and all linear functional disturbance observers for time-delay plants with any input disturbance [16]. If parameterizations of all disturbance observers and all linear functional disturbance observers for time-delay plants with any input disturbance in [16] are used, there is a possibility that we can design control systems to attenuate input disturbances effectively. However, no paper examines a design method for control system using parameterizations of all disturbance observers and all linear functional disturbance observers for time-delay plants with any input disturbance.

In this paper, in order to show the effectiveness of these parameterizations, we propose a design method for control system to attenuate periodic input disturbances effectively using these parameterizations for time-delay plants with any input disturbance. First,

Manuscript received on July 12, 2012 ; revised on September 12, 2012.
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the disturbance observer and the linear functional disturbance observer for time-delay plants with any input disturbance are introduced. Next, to attenuate periodic input disturbances effectively, a design method for control system using these parameterizations of all disturbance observers and of all linear functional disturbance observers for time-delay plants with any input disturbance is proposed. In addition, control characteristics of control system using these parameterizations are clarified. Note that the repetitive control system [17] is well known as an effective control system to attenuate periodic disturbances. It is shown that the proposed method can attenuate periodic disturbances effectively without using repetitive controllers. A design procedure is also given. Finally, a numerical example is illustrated to show the effectiveness of the proposed method.

Notation

R	the set of real numbers.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real rational functions.
$\mathcal{U}$	the unimodular procession in RH_∞. That is, $P(s) \in \mathcal{U}$ means that $P(s) \in RH_\infty$ and $P^{-1}(s) \in RH_\infty$.
A^T	transpose of A.
$\bar{\sigma}(\{\cdot\})$	largest singular value of $\{\cdot\}$.
diag $(a_1, \cdots, a_n)$	an $n \times n$ diagonal matrix with a_i as its i-th diagonal element.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	represents the state space description $C(sI-A)^{-1}B+D$.
$\mathcal{L}\{\cdot\}$	the Laplace transformation of $\{\cdot\}$.

2. DISTURBANCE OBSERVER AND LINEAR FUNCTIONAL DISTURBANCE OBSERVER

In this section, we briefly introduce a disturbance observer and a linear functional disturbance observer for time-delay plants with any input disturbance and explain the problem considered in this paper.

Consider the plant written by

$$\left\{\begin{array}{rcl} \dot{x}(t) & = & Ax(t) + B\left(u(t-L) + d(t-L)\right) \\ y(t) & = & Cx(t) + D\left(u(t-L) + d(t-L)\right) \end{array}\right. , \quad (1)$$

where $x \in R^n$ is the state variable, $u \in R^p$ is the control input, $y \in R^m$ is the output, $d \in R^p$ is the periodic disturbance with period $T > 0$ ($T \in R$) satisfying

$$d(t+T) = d(t) \ (\forall t \geq 0), \quad (2)$$

$A \in R^{n\times n}$, $B \in R^{n\times p}$, $C \in R^{m\times n}$ and $D \in R^{m\times p}$. It is assumed that (A, B) is stabilizable, (C, A) is detectable, $u(t)$ and $y(t)$ are available, but $d(t)$ is unavailable. The transfer function $y(s)$ in (1) is denoted by

$$y(s) = G(s)e^{-sL}u(s) + G(s)e^{-sL}d(s), \quad (3)$$

where

$$G(s) = C\left(sI - A\right)^{-1}B + D \quad \in R^{m\times p}(s). \quad (4)$$

When the disturbance $d(t)$ is not available, in many cases, the disturbance estimator named the disturbance observer is used. The disturbance observer estimates the disturbances $d(t)$ in (1) using available measurements. Available measurements of the time-delay plant in (1) are the control input $u(t-\delta)(L \geq \delta \geq 0)$ and the output $y(t)$. For simplicity, we select $\delta = L$, and then the general form of the disturbance observer $\tilde{d}(s)$ for (1) is written by

$$\tilde{d}(s) = F_1(s)y(s) + F_2(s)e^{-sL}u(s), \quad (5)$$

where $F_1(s) \in R^{p\times m}(s)$, $F_2(s) \in R^{p\times p}(s)$, $\tilde{d}(s) = \mathcal{L}\{\tilde{d}(t)\}$ and $\tilde{d}(t) \in R^p(t)$. That is, the general form of the disturbance observer $\tilde{d}(s)$ is shown in Fig. 1. In

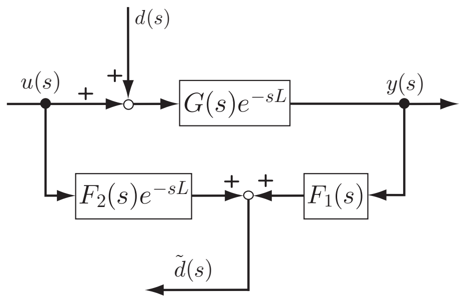

Fig.1: *Structure of a disturbance observer and that of a linear functional disturbance observer*

the following, we call the system $\tilde{d}(s)$ in (5) a disturbance observer for time-delay plants with any input disturbance, if

$$\lim_{t\to\infty}\left(d(t-L) - \tilde{d}(t)\right) = 0 \quad (6)$$

is satisfied for any initial state $x(0)$, control input $u(t)$ and disturbance $d(t)$.

According to [16], there exists a disturbance observer $\tilde{d}(s)$ satisfying (6) if and only if $m \geq p$ and $G(s)$ is biproper and of minimum phase, that is, D is of full rank and

$$\text{rank}\left[\begin{array}{cc} A - sI & B \\ C & D \end{array}\right] = n + \min(m, p)$$
$$(\forall \Re\{s\} \geq 0). \quad (7)$$

In addition, when above-mentioned expressions hold, the parameterization of all disturbance observers for time-delay plants $G(s)e^{-sL}$ with any input disturbance is written by (5), where

$$F_1(s) = D(s)N^*(s) + Q(s)N^\perp(s) \in RH_\infty^{p\times m} \quad (8)$$

and

$$F_2(s) = -I \in RH_\infty^{p\times p}, \tag{9}$$

respectively, where $N(s) \in RH_\infty^{m\times p}$ and $D(s) \in RH_\infty^{p\times p}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = N(s)D^{-1}(s), \tag{10}$$

$N^*(s)$ is a pseudo inverse of $N(s)$ satisfying

$$N^*(s)N(s) = I \tag{11}$$

and $Q(s) \in RH_\infty^{p\times(m-p)}$ is any function.

When one of following expressions:
1. $m \geq p$.
2. $G(s)$ is of minimum phase.
3. $G(s)$ is biproper.

does not hold, there exists no disturbance observer for time-delay plants with any input disturbance satisfying (6) [16]. Since many plants in the motion-control field are strictly proper and of non-minimum phase, this is a problem for the disturbance observer for time-delay plants with any input disturbance to be solved. When a disturbance observer for time-delay plants with any input disturbance is used to attenuate disturbances such as in [1–6], even if $\tilde{d}(s)$ satisfying (6) cannot be designed, the control system can be designed to attenuate disturbance effectively. That is, in order to attenuate disturbances, it is enough to estimate $(I - F(s))e^{-sL}d(s)$, where $F(s) \in RH_\infty^{p\times p}$. From this point of view, when $G(s)$ is strictly proper and of non-minimum phase, Yamada et al. defined a linear functional disturbance observer for time-delay plants with any input disturbance [16].

For any initial state $x(0)$, control input $u(t)$ and disturbance $d(s)$, we call $\tilde{d}(s)$ the linear functional disturbance observer for time-delay plants with any input disturbance if

$$e^{-sL}d(s) - \tilde{d}(s) = F(s)e^{-sL}d(s) \tag{12}$$

is satisfied, where $F(s) \in RH_\infty^{p\times p}$ [16]. Available measurements of the time-delay plant in (1) are the control input $u(t-\delta)(L \geq \delta \geq 0)$ and the output $y(t)$. For simplicity, we select $\delta = L$, and then the general form of the linear functional disturbance observer for time-delay plants with any input disturbance is written by (5), where $F_1(s) \in R^{p\times m}(s)$ and $F_2(s) \in R^{p\times p}(s)$. That is, the general form of the linear functional disturbance observer $\tilde{d}(s)$ is shown in Fig. 1. When $m = p$ holds and the time-delay plant $G(s)e^{-sL}$ is stable, the system $\tilde{d}(s)$ in (5) is a linear functional disturbance observer for stable time-delay plants with any input disturbance if and only if $F_1(s)$, $F_2(s)$ and $F(s)$ are written by

$$F_1(s) = I + Q(s), \tag{13}$$

$$F_2(s) = -G(s) - Q(s)G(s) \tag{14}$$

and

$$F(s) = I - F_1(s)G(s), \tag{15}$$

respectively, where $Q(s) \in RH_\infty^{p\times p}$ is any function. On the other hand, when $m = p$ holds and the time-delay plant $G(s)e^{-sL}$ is unstable, the system $\tilde{d}(s)$ in (5) is a linear functional disturbance observer for unstable time-delay plants with any input disturbance if and only if $F_1(s)$, $F_2(s)$ and $F(s)$ are written by

$$F_1(s) = \tilde{D}(s) + Q(s)\tilde{D}(s), \tag{16}$$

$$F_2(s) = -\tilde{N}(s) - Q(s)\tilde{N}(s) \tag{17}$$

and

$$F(s) = I - F_1(s)G(s), \tag{18}$$

respectively, where $\tilde{N}(s) \in RH_\infty^{m\times p}$ and $\tilde{D}(s) \in RH_\infty^{m\times m}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \tilde{D}^{-1}(s)\tilde{N}(s) \tag{19}$$

and

$$\tilde{D}(s)N(s) - \tilde{N}(s)D(s) = 0 \tag{20}$$

and $Q(s) \in RH_\infty^{p\times p}$ is any function.

The problem considered in this paper is to propose a design method for control system to attenuate periodic input disturbances effectively using the parameterizations of all disturbance observers and all linear functional disturbance observers for time-delay plants with any input disturbance.

3. DESIGN METHOD FOR CONTROL SYSTEM FOR MINIMUM-PHASE AND BIPROPER PLANTS

In this section, we propose a design method for control system to attenuate periodic input disturbances effectively using the parameterization of all disturbance observers for time-delay plants with any input disturbance.

When $G(s)$ is of minimum-phase and biproper, we propose a control system using the parameterization of all disturbance observers for time-delay plants with any input disturbance as shown in Fig. 2. Here, $C(s) \in R^{p\times m}(s)$ and $\hat{C}(s) \in R^{p\times p}(s)$ are controllers, $\bar{L} \geq 0$ is chosen as

$$\bar{L} = nT - L, \tag{21}$$

n is the smallest positive integer that makes $\bar{L}$ in (21) nonnegative, and $F_1(s)$ and $F_2(s)$ are given by (8) and (9), respectively.

Next, we clarify control characteristics of the control system in Fig. 2. First, the input-output characteristic of control system in Fig. 2 is shown. The

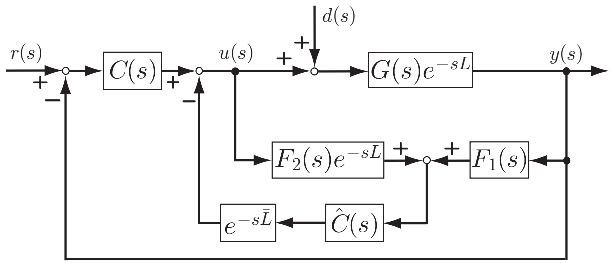

Fig.2: *Control system using the parameterization of all disturbance observers*

transfer function from the reference input $r(s)$ to the output $y(s)$ and that from the reference input $r(s)$ to the error $e(s) = r(s) - y(s)$ are written by

$$y(s) = \left(I + G(s)e^{-sL}C(s)\right)^{-1} G(s)e^{-sL}C(s)r(s) \quad (22)$$

and

$$e(s) = r(s) - y(s) = \left(I + G(s)e^{-sL}C(s)\right)^{-1} r(s), \quad (23)$$

respectively. Therefore, the input-output characteristic in Fig. 2 is specified using $C(s)$.

Next, the disturbance attenuation characteristic of control system in Fig. 2 is shown. The transfer function from the disturbance $d(s)$ to the output $y(s)$ is given by

$$\begin{aligned} y(s) &= \left(I + G(s)e^{-sL}C(s)\right)^{-1} G(s)e^{-sL} \\ &\quad \left(I - \hat{C}(s)e^{-snT}\right) d(s). \end{aligned} \quad (24)$$

From (24), if $\hat{C}(s)$ is set satisfying

$$\bar{\sigma}\left\{I - \hat{C}(j\omega_i)\right\} \simeq 0 \ \ (i = 1, \ldots, n_d), \quad (25)$$

then the periodic disturbance $d(s)$ with frequency component ω_i $(i = 1, \ldots, n_d)$ written by

$$\omega_i = \frac{2\pi}{T} i \ \ (i = 1, \ldots, n_d), \quad (26)$$

is attenuated effectively, where ω_{n_d} is the maximum frequency component of the periodic disturbance $d(s)$ with period T. To satisfy (25), $\hat{C}(s)$ is set according to

$$\begin{aligned} &\hat{C}(s) \\ &= \ \mathrm{diag}\left\{ \frac{1}{(1+s\tau_1)^{\alpha_1}} \ \cdots \ \frac{1}{(1+s\tau_p)^{\alpha_p}} \right\}, \end{aligned} \quad (27)$$

where $\tau_i \in R$ $(i = 1, \ldots, p)$, and α_i $(i = 1, \ldots, p)$ are arbitrary positive integers satisfying

$$\begin{aligned} &\bar{\sigma}\left[I - \mathrm{diag}\left\{\frac{1}{(1+j\omega_i\tau_1)^{\alpha_1}} \cdots \frac{1}{(1+j\omega_i\tau_p)^{\alpha_p}}\right\}\right] \\ &\qquad \simeq 0 \ \ (i = 1, \ldots, n_d). \end{aligned} \quad (28)$$

When $\hat{C}(s)$ is settled as (27), we have

$$\begin{aligned} &y(j\omega_i) \\ &= \left(I + G(j\omega_i)e^{-j\omega_i L}C(j\omega_i)\right)^{-1} G(j\omega_i)e^{-j\omega_i L} \\ &\quad \left[I - \mathrm{diag}\left\{\frac{1}{(1+j\omega_i\tau_1)^{\alpha_1}} \cdots \frac{1}{(1+j\omega_i\tau_p)^{\alpha_p}}\right\}\right] \\ &\quad d(j\omega_i) \ \ (i = 1, \ldots, n_d). \end{aligned} \quad (29)$$

Under the condition in (28),

$$\bar{\sigma}\left\{y(j\omega_i)\right\} \simeq 0 \ \ (i = 1, \ldots, n_d) \quad (30)$$

hold true. This implies that when $\hat{C}(s)$ is chosen as (27), the control system in Fig. 2 can attenuate periodic input disturbances effectively without using repetitive controllers in [17].

From (22) and (24), the role of $C(s)$ is different from that of $\hat{C}(s)$. $C(s)$ specifies the input-output characteristic, while $\hat{C}(s)$ specifies the disturbance attenuation characteristic. That is, the control system in Fig. 2 has one of two-degree-of-freedom structures.

Finally, the condition that the control system in Fig. 2 is stable is clarified. From (22) and (24), it is obvious that the control system in Fig. 2 is stable if and only if following expressions hold.

1. $C(s)$ makes the unity feedback control system in

$$\left\{ \begin{array}{rcl} y(s) &=& G(s)e^{-sL}u(s) \\ u(s) &=& -C(s) \end{array} \right. \quad (31)$$

stable.

2. $Q(s) \in RH_\infty^{p\times(m-p)}$.
3. $\hat{C}(s) \in RH_\infty^{p\times p}$.

4. DESIGN METHOD FOR CONTROL SYSTEM FOR NON-MINIMUM-PHASE AND/OR STRICTLY PROPER PLANTS

In this section, we propose a design method for control system to attenuate periodic input disturbances effectively using the parameterization of all linear functional disturbance observers for non-minimum-phase and/or strictly proper time-delay plants with any input disturbance.

Even if $G(s)$ is of non-minimum-phase and/or strictly proper, we can design a control system using the parameterization of all linear functional disturbance observers for time-delay plants with any input disturbance in Fig. 2. Here, $C(s) \in R^{p\times m}(s)$ and $\hat{C}(s) \in R^{p\times p}(s)$ are controllers, $\bar{L} \geq 0$ is chosen as (21), n is the smallest positive integer that makes $\bar{L}$ in (21) nonnegative. When $G(s)$ is stable, $F_1(s)$ and $F_2(s)$ are given by (13) and (14), respectively. When $G(s)$ is unstable, $F_1(s)$ and $F_2(s)$ are given by (16) and (17), respectively.

Next, we clarify control characteristics of the control system in Fig. 2. First, the input-output characteristic of control system in Fig. 2 is shown. The transfer function from the reference input $r(s)$ to the output $y(s)$ and that from the reference input $r(s)$ to the error $e(s) = r(s) - y(s)$ are written by

$$y(s) = \left(I + G(s)e^{-sL}C(s)\right)^{-1} G(s)e^{-sL}C(s)r(s) \quad (32)$$

and

$$e(s) = r(s) - y(s) = \left(I + G(s)e^{-sL}C(s)\right)^{-1} r(s), \quad (33)$$

respectively. Therefore, the input-output characteristic in Fig. 2 is specified using $C(s)$. For example, the output $y(t)$ follows the step reference input $r(t)$ without steady state error if

$$(I + G(0)C(0))^{-1} = 0. \quad (34)$$

Next, the disturbance attenuation characteristic of control system in Fig. 2 is shown. The transfer function from the disturbance $d(s)$ to the output $y(s)$ is given by

$$\begin{aligned} y(s) &= \left(I + G(s)e^{-sL}C(s)\right)^{-1} G(s)e^{-sL} \\ &\quad \left\{I - \hat{C}(s)\left(I + Q(s)\right)\tilde{N}(s)e^{-snT}\right\} d(s) \\ &= \left(I + G(s)e^{-sL}C(s)\right)^{-1} \tilde{D}^{-1}(s)e^{-sL} \\ &\quad \left\{I - \tilde{N}(s)\hat{C}(s)\left(I + Q(s)\right)e^{-snT}\right\} \tilde{N}(s)d(s). \end{aligned} \quad (35)$$

From (35), if $\hat{C}(s)$ is set satisfying

$$\bar{\sigma}\left\{I - \tilde{N}(j\omega_i)\hat{C}(j\omega_i)\left(I + Q(j\omega_i)\right)\right\} \simeq 0 \quad (i = 1, \ldots, n_d), \quad (36)$$

then the periodic disturbance $d(s)$ with frequency component ω_i $(i = 1, \ldots, n_d)$ written by

$$\omega_i = \frac{2\pi}{T} i \quad (i = 1, \ldots, n_d), \quad (37)$$

is attenuated effectively. When $G(s)$ is of minimum phase, that is $\tilde{N}(s)$ is of minimum phase, there exists $\tilde{N}_r(s) \in RH_\infty^{m\times m}$ satisfying

$$\begin{aligned} &\tilde{N}(s)\tilde{N}_r(s) \\ &= \bar{Q}(s) \\ &= \text{diag}\left\{ \frac{1}{(1+\tau_1 s)^{\alpha_1}} \quad \cdots \quad \frac{1}{(1+\tau_m s)^{\alpha_m}} \right\}, \end{aligned} \quad (38)$$

where α_i $(i = 1, \ldots, m)$ are arbitrary positive integers to make $\tilde{N}_r(s)$ proper and $\tau_i \in R$ $(i = 1, \ldots, m)$ are any positive real numbers satisfying

$$\begin{aligned} \bar{\sigma}\left\{I - \bar{Q}(j\omega_i)\right\} &= \bar{\sigma}\left[I - \text{diag}\left\{\frac{1}{(1+j\omega_i\tau_1)^{\alpha_1}} \cdots \frac{1}{(1+j\omega_i\tau_m)^{\alpha_m}}\right\}\right] \\ &\simeq 0 \quad (i = 1, \ldots, n_d). \end{aligned} \quad (39)$$

Using $\tilde{N}_r(s)$, if $\hat{C}(s)$ is selected as

$$\hat{C}(s) = \tilde{N}_r(s)\left(I + Q(s)\right)^{-1}, \quad (40)$$

where $Q(s)$ is selected so that $(I + Q(s)) \in \mathcal{U}$, then we have

$$\begin{aligned} &y(j\omega_i) \\ &= \left(I + G(j\omega_i)e^{-j\omega_i L}C(j\omega_i)\right)^{-1} \tilde{D}^{-1}(j\omega_i)e^{-j\omega_i L} \\ &\quad \left\{I - \bar{Q}(j\omega_i)\right\} \tilde{N}(j\omega_i)d(j\omega_i) \quad (i = 1, \ldots, n_d). \end{aligned} \quad (41)$$

Under the condition in (39),

$$\bar{\sigma}\left\{y(j\omega_i)\right\} \simeq 0 \quad (i = 1, \ldots, n_d) \quad (42)$$

hold true. That is, the periodic input disturbances with frequency components ω_i $(i = 1, \ldots, n_d)$ is attenuate effectively without using repetitive controllers in [17]. On the other hand, when $G(s)$ is of non-minimum phase, that is $\tilde{N}(s)$ is of non-minimum phase, there exists $\hat{\tilde{N}}_r(s) \in RH_\infty^{m\times m}$ satisfying

$$\begin{aligned} &\tilde{N}(s)\hat{\tilde{N}}_r(s) \\ &= \bar{Q}(s)\tilde{N}_i(s) \\ &= \text{diag}\left\{ \frac{1}{(1+\tau_1 s)^{\alpha_1}} \quad \cdots \quad \frac{1}{(1+\tau_m s)^{\alpha_m}} \right\} \\ &\quad \tilde{N}_i(s), \end{aligned} \quad (43)$$

where $\tilde{N}_i(s) \in RH_\infty$ is an inner function satisfying $\tilde{N}_i(0) = I$, α_i $(i = 1, \ldots, m)$ are arbitrary positive integers to make $\hat{\tilde{N}}_r(s)$ proper and $\tau_i \in R$ $(i = 1, \ldots, m)$ are any positive real numbers satisfying

$$\begin{aligned} &\bar{\sigma}\left\{I - \bar{Q}(j\omega_i)\tilde{N}_i(j\omega_i)\right\} \\ &= \bar{\sigma}\left[I - \text{diag}\left\{\frac{1}{(1+j\omega_i\tau_1)^{\alpha_1}} \cdots \frac{1}{(1+j\omega_i\tau_m)^{\alpha_m}}\right\}\tilde{N}_i(j\omega_i)\right] \\ &\simeq 0 \quad (i = 1, \ldots, n_d). \end{aligned} \quad (44)$$

Using $\hat{\tilde{N}}_r(s)$, if $\hat{C}(s)$ is selected as

$$\hat{C}(s) = \hat{\tilde{N}}_r(s)\left(I + Q(s)\right)^{-1}, \quad (45)$$

where $Q(s)$ is selected so that $(I + Q(s)) \in \mathcal{U}$, then we have

$$\begin{aligned} &y(j\omega_i) \\ &= \left(I + G(j\omega_i)e^{-j\omega_i L}C(j\omega_i)\right)^{-1} \tilde{D}^{-1}(j\omega_i)e^{-j\omega_i L} \\ &\quad \left\{I - \bar{Q}(j\omega_i)\tilde{N}_i(j\omega_i)\right\} \tilde{N}(j\omega_i)d(j\omega_i) \\ &\quad (i = 1, \ldots, n_d). \end{aligned} \quad (46)$$

Under the condition in (44),

$$\bar{\sigma}\left\{y(j\omega_i)\right\} \simeq 0 \quad (i = 1, \ldots, n_d) \quad (47)$$

hold true. That is, the periodic input disturbances with frequency components ω_i $(i = 1, \ldots, n_d)$ is attenuate effectively without using repetitive controllers in [17].

Note that $\tilde{N}_r(s) \in RH_\infty^{m\times m}$ satisfying (38) can be designed using the method in [18]. $\hat{\tilde{N}}_r(s) \in RH_\infty^{m\times m}$ satisfying (43) can be designed using the method in [19–22]. The method in [19] makes $\tilde{N}_i(s)$ in (43) be not necessarily a diagonal functional matrix, but the method in [21, 22] makes $\tilde{N}_i(s)$ in (43) by a diagonal functional matrix. Therefore, in order to attenuate the periodic disturbance $d(s)$ effectively, we had better design $\hat{\tilde{N}}_r(s)$ using the method in [21, 22].

From (32) and (35), the role of $C(s)$ is different from that of $\hat{C}(s)$. $C(s)$ specifies the input-output characteristic, while $\hat{C}(s)$ specifies the disturbance attenuation characteristic. That is, the control system in Fig. 2 has one of two-degree-of-freedom structures.

Finally, the condition that the control system in Fig. 2 is stable is clarified. From (32) and (35), it is obvious that the control system in Fig. 2 is stable if and only if following expressions hold.

1. $C(s)$ makes the unity feedback control system in

$$\left\{ \begin{array}{rcl} y(s) & = & G(s)e^{-sL}u(s) \\ u(s) & = & -C(s) \end{array} \right. \tag{48}$$

stable.

2. $Q(s) \in RH_\infty^{p\times p}$.
3. $\hat{C}(s) \in RH_\infty^{p\times p}$.

5. DESIGN PROCEDURE

In this section, a design procedure of control system using the linear functional disturbance observer in Fig. 2 is presented.

A simple design procedure of control system using the linear functional disturbance observer in Fig. 2 is summarized as follows:

1. Obtain coprime factors $N(s)$, $D(s)$, $\tilde{N}(s)$ and $\tilde{D}(s)$ of the plant $G(s)$ on RH_∞ satisfying (10), (19) and (20). A state space description of $N(s)$, $D(s)$, $\tilde{N}(s)$ and $\tilde{D}(s)$ of the plant $G(s)$ satisfying (10), (19) and (20) are obtained using the method in [23].
2. Design $\hat{C}(s)$ to attenuate periodic input disturbance $d(s)$ effectively using the method described in Section 4.
3. Design $C(s)$ to stabilize the unity feedback control system in (48). Such controller $C(s)$ can be designed using the parameterization of all stabilizing modified Smith predictors for multiple-input/multiple-output plants in [24].

According to [24], when $G(s)e^{-sL}$ is stable, the parameterization of all stabilizing modified Smith predictors $C(s)$ for stable plants $G(s)e^{-sL}$ is written as

$$C(s) = Q_c(s)\left(I - G(s)Q_c(s)e^{-sT}\right)^{-1}, \tag{49}$$

where $Q_c(s) \in RH_\infty^{m\times m}$ is any function and settled to specify the input-output characteristic. For example, in order for the output $y(t)$ to follow the step reference input $r(t)$ without steady state error, $Q_c(s)$ must be settled to satisfy

$$G(0)Q_c(0) = I. \tag{50}$$

On the other hand, when $G(s)e^{-sL}$ is unstable and unstable poles $s_i (i = 1, \ldots, n)$ of $G(s)$ satisfies $s_i \neq s_j (i \neq j; i = 1, \ldots, n; j = 1, \ldots, n)$, the parameterization of all stabilizing modified Smith predictors $C(s)$ for unstable plants $G(s)e^{-sL}$ is written as

$$C(s) = C_1(s)\left(I - G(s)C_1(s)e^{-sT}\right)^{-1}, \tag{51}$$

where $C_1(s)$ is given by

$$C_1(s) = D(s)\left(\bar{G}_u(s) + \hat{G}_u^{-1}(s)Q_c(s)\right), \tag{52}$$

$\bar{G}_u(s) \in RH_\infty^{m\times m}$ is a function satisfying

$$N(s_i)\bar{G}_u(s_i)e^{-s_iL} = I \quad (\forall i = 1, \ldots, n), \tag{53}$$

$$\hat{G}_u(s) = \frac{f(s)}{\prod_{i=1}^{n}(s - s_i)} I \in R^{m\times m}(s), \tag{54}$$

$f(s)$ is a Hurwitz polynomial with n-th degree and $Q_c(s) \in RH_\infty^{m\times m}$ is any function and settled to specify the input-output characteristic. For example, when $G(s)$ has a pole at the origin, the output $y(s)$ follows the step reference input $r(s)$ without steady state error, independent from $Q_c(s) \in RH_\infty^{m\times m}$ in (52). On the other hand, when $G(s)$ has no pole at the origin, for the output $y(t)$ to follow the step reference input $r(t)$ without steady state error, $Q_c(s)$ must be settled to satisfy

$$N(0)\left(\bar{G}_u(0) + \hat{G}_u^{-1}(0)Q_c(0)\right) = I. \tag{55}$$

6. NUMERICAL EXAMPLE

In this section, we show a numerical example to illustrate the effectiveness of the proposed method.

We consider the problem to design a control system in Fig. 2 for the output $y(t)$ to follow the step reference input $r(t)$ written by

$$r(t) = \left[\begin{array}{c} r_1(t) \\ r_2(t) \end{array}\right] = \left[\begin{array}{c} 1 \\ 2 \end{array}\right] \tag{56}$$

and to attenuate periodic input disturbance $d(t)$ with period $T = 2$[sec] written by

$$d(t) = \left[\begin{array}{c} d_1(t) \\ d_2(t) \end{array}\right] = \left[\begin{array}{c} \sin(\pi t) \\ 2\sin(\pi t) \end{array}\right] \tag{57}$$

effectively for the time-delay plant $G(s)e^{-sL}$ written by

$$G(s)e^{-sL} = \begin{bmatrix} \frac{s-1}{s(s+25)} & \frac{-1}{s(s+25)} \\ \frac{1}{s(s+25)} & \frac{s-3}{s(s+25)} \end{bmatrix} e^{-1.2s} = \left[\begin{array}{cccc|cc} 0 & 0 & 0 & 0 & -0.04 & -0.04 \\ 0 & 0 & 0 & 0 & 0.04 & -0.12 \\ 0 & 0 & -25 & 0 & 1.04 & 0.04 \\ 0 & 0 & 0 & -25 & -0.04 & 1.12 \\ \hline 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right] e^{-1.2s}, \tag{58}$$

where

$$G(s) = \begin{bmatrix} \frac{s-1}{s(s+25)} & \frac{-1}{s(s+25)} \\ \frac{1}{s(s+25)} & \frac{s-3}{s(s+25)} \end{bmatrix} = \left[\begin{array}{cccc|cc} 0 & 0 & 0 & 0 & -0.04 & -0.04 \\ 0 & 0 & 0 & 0 & 0.04 & -0.12 \\ 0 & 0 & -25 & 0 & 1.04 & 0.04 \\ 0 & 0 & 0 & -25 & -0.04 & 1.12 \\ \hline 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right] \tag{59}$$

and

$$L = 1.2[\text{sec}]. \tag{60}$$

$G(s)$ in (59) is unstable and of non-minimum phase, since poles of $G(s)$ in (59) are in $(0,0)$ and $(-25,0)$ and invariant zeroes of $G(s)$ are in $(2,0)$ and $(2,0)$. The smallest positive integer n that makes $\bar{L}$ in (21) nonnegative is $n = 1$, and then $\bar{L}$ in (21) is obtained as

$$\bar{L} = 0.8[\text{sec}]. \tag{61}$$

$C(s)$, $\hat{C}(s)$, $F_1(s)$ and $F_2(s)$ in Fig. 2 are designed using the method described in Section 5. Coprime factors $N(s) \in RH_\infty$, $D(s) \in RH_\infty$, $\tilde{N}(s) \in RH_\infty$ and $\tilde{D}(s) \in RH_\infty$ of the plant $G(s)$ in (59) on RH_∞ satisfying (10), (19) and (20) are given by

$$N(s) = \tilde{N}(s) = \begin{bmatrix} \frac{s-1}{(s+2)(s+25)} & \frac{-1}{(s+2)(s+25)} \\ \frac{1}{(s+2)(s+25)} & \frac{s-3}{(s+2)(s+25)} \end{bmatrix} \tag{62}$$

and

$$D(s) = \tilde{D}(s) = \begin{bmatrix} \frac{s}{s+2} & 0 \\ 0 & \frac{s}{s+2} \end{bmatrix}. \tag{63}$$

$Q(s)$ is selected as $Q(s) = 0$. $\hat{C}(s)$ in Fig. 2 is given by (45), where $\hat{\tilde{N}}_r(s)$ is designed satisfying (43), $\tau_1 = 0.001$, $\tau_2 = 0.001$, $\alpha_1 = 1$, $\alpha_2 = 1$ and

$$\tilde{N}_i(s) = \begin{bmatrix} \frac{s^2-4s+4}{s^2+4s+4} & 0 \\ 0 & \frac{s^2-4s+4}{s^2+4s+4} \end{bmatrix}. \tag{64}$$

Next, we design $C(s)$ as (51) in order for the output $y(t)$ to follow the step reference input $r(t)$ in (56) without steady state error. One of $\bar{G}_u(s) \in RH_\infty$ in (52) satisfying (53) is given by

$$\bar{G}_u(s) = \begin{bmatrix} \frac{-3750}{s+100} & \frac{1250}{s+100} \\ \frac{-1250}{s+100} & \frac{-1250}{s+100} \end{bmatrix}. \tag{65}$$

$\hat{G}_u(s) \in R(s)$ in (54) is set as

$$\hat{G}_u(s) = \begin{bmatrix} \frac{s+2}{s} & 0 \\ 0 & \frac{s+2}{s} \end{bmatrix}. \tag{66}$$

In order for the output $y(t)$ to follow the step reference input $r(t)$ in (56), $Q_c(s) \in RH_\infty$ is set as

$$Q_c(s) = \begin{bmatrix} \frac{100}{s+100} & 0 \\ 0 & \frac{100}{s+100} \end{bmatrix}. \tag{67}$$

Note that $G(s)$ in (59) has a pole at the origin, the output $y(t)$ follows the step reference input $r(t)$ in (56) without steady state error, independent from $Q_c(s)$ in (67). Substitution of above mentioned parameters for (51) gives $C(s)$.

Using the designed control system in Fig. 2, when the initial state $x(0)$ and the disturbance $d(t)$ are given by

$$x(0) = \begin{bmatrix} 0.1, & 0.1, & -0.1, & -0.1 \end{bmatrix}^T \tag{68}$$

and $d(t) = [\ 0,\ \ 0\ ]^T$, respectively, the response of the output $y(t) = [\ y_1(t),\ \ y_2(t)\ ]^T$ for the step reference input $r(t)$ in (56) is shown in Fig. 3. Here, the broken line shows the response of the reference input $r_1(t)$, the dotted line shows that of the reference input $r_2(t)$, the solid line shows that of the output $y_1(t)$, and the dotted and broken line shows that of the output $y_2(t)$. Figure 3 shows that the output $y(t)$ follows the reference input $r(t)$ without steady state error.

Next, the disturbance attenuation characteristic is shown. When the initial state $x(0)$ and the reference input $r(t)$ are given by

$$x(0) = \begin{bmatrix} 0.1 & 0.1 & -0.1 & -0.1 \end{bmatrix}^T \tag{69}$$

and $r(t) = [\ 0,\ \ 0\ ]^T$, respectively, the response of the output $y(t) = [\ y_1(t),\ \ y_2(t)\ ]^T$ for the periodic input disturbance in (57) is shown in Fig. 4. Here, the broken line shows the response of the periodic input disturbance $d_1(t)$, the dotted line shows that of the periodic input disturbance $d_2(t)$, the solid line

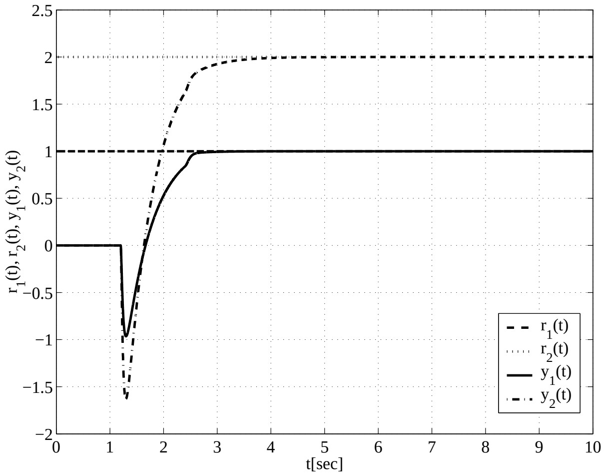

Fig.3: *Response of the output $y(t) = [y_1(t), y_2(t)]^T$ for the step reference input $r(t)$ in (56)*

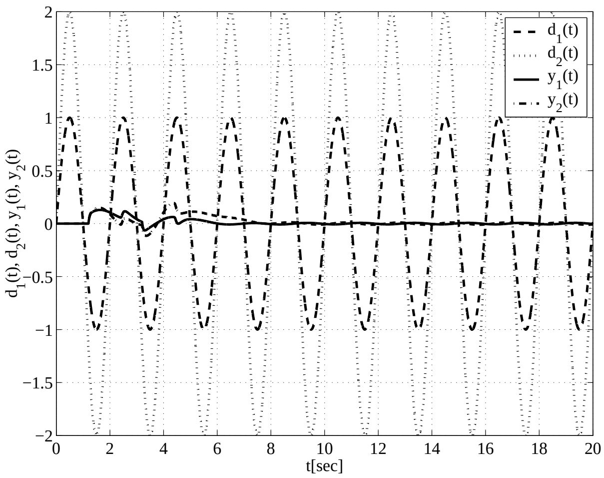

Fig.4: *Response of the output $y(t) = [y_1(t), y_2(t)]^T$ for the periodic input disturbance $d(t)$ in (57)*

shows that of the output $y_1(t)$ and the dotted and broken line shows that of the output $y_2(t)$. Figure 4 shows that the periodic input disturbance $d(t)$ is attenuated effectively.

In this way, it is shown that using the proposed method, we can easily design control system to follow the reference input $r(t)$ and to attenuate periodic input disturbance $d(t)$ effectively.

7. CONCLUSIONS

In this paper, we proposed a design method for control system to attenuate periodic input disturbances effectively using the parameterizations of all disturbance observers and all linear functional disturbance observers for time-delay plants with any input disturbance. Results obtained in this paper are as follows:

1. For minimum-phase and biproper time-delay plants with periodic input disturbance, a design method for control system to attenuate periodic input disturbance effectively using the parameterization of all disturbance observers is proposed. It is shown that the disturbance observer works in only the case that disturbances exist.
2. Control characteristics of proposed control system for minimum-phase and biproper time-delay plants are clarified. We have shown that the proposed control system can attenuate periodic disturbances effectively without using repetitive controllers. A design method of controller to attenuate periodic input disturbance effectively is given.
3. For non-minimum-phase and/or strictly proper time-delay plants with periodic input disturbance, a design method for control system to attenuate periodic input disturbance effectively using the parameterization of all linear functional disturbance observers is proposed.
4. Control characteristics of proposed control system for non-minimum-phase and/or strictly proper time-delay plants are clarified. A design method of controller to attenuate periodic input disturbance effectively is given.
5. Proposed control system includes three parameters to be designed, $C(s)$, $\hat{C}(s)$ and $Q(s)$. It is shown that the role of $C(s)$ is different from that of $\hat{C}(s)$ and $Q(s)$. The role of $C(s)$ is to specify the input-output characteristic. The role of $\hat{C}(s)$ and $Q(s)$ is to specify the disturbance attenuation characteristic.
6. A design procedure of control system using linear functional disturbance observer is presented.
7. A numerical example is shown to illustrate the effectiveness of the proposed method.

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