Multi-Objective Design for Switched Reluctance Machines Using Genetic and Fuzzy Algorithms

Satit Owatchaiphong[*1] and **Nisai Fuengwarodsakul**[*2], Non-members

ABSTRACT

This paper generalizes an automatic design process for dimensioning switched reluctance machines using a combination between genetic and fuzzy algorithms. The process considers all desired performance, dimensions and design guidelines as optimization objectives for determining fitness score of genetic algorithms whereas most automatic processes neither consider the restricted dimensions as objectives nor use them during adjusting machine dimensions. The processes are immediately interrupted and reject adjusted models in case that any dimension of these models does not comply with the restrictions regardless of their performances. In this paper, all interrupts are eliminated from design process. Besides, optimization objectives from design guidelines based on empirical experience improve the calculation accuracy of analytical analysis while still satisfying the requirements. Despite the increased number of objectives, the fitness of each objective is still normally determined since fuzzy algorithms connect all objectives together to convert model goodness into a scale factor used for scaling the objectives before summing into the fitness score. This formulates imprecise decision of designers into fuzzy rules and compensates the lacks of design experiences. The proposed method demonstrates the promising results verified by statistical records of model optimization from a mass simulation.

Keywords: Switched Reluctance Machine, Multi-Objective Optimization Design, Genetic Algorithms, Fuzzy Algorithms.

1. INTRODUCTION

In most conventional design processes, preliminary models of switched reluctance machines (SRM) are approximated by design guidelines and suggestions published in [1-5]. Characteristics and performances of the generated models are evaluated and compared by design objectives. Afterwards, the dimensions of the models are adjusted based on these results. By repeating this routine, an optimal model is developed from time to time. The process consumes effort and time. It also requires the great experience of designers to develop a well-optimized model. In addition, it is difficult to find globally-optimized solutions for multi-objective problems when many design parameters and design requirements are concerned. Mass calculation for all models created in design boundaries and deterministic search algorithms are practically impossible due to extensively long computing time. Therefore, an automatic design utilizing a stochastic optimization algorithm is an appropriate solution for this kind of problems.

Evolutionary computation is a search technique that can be considered as global optimization with a stochastic optimization algorithm. The evolutionary computation is mostly applied for underivable problems. It uses iterative progress of population development. Then, the population is selected by the stochastic algorithms to achieve desired solution. Such processes are often inspired by biological mechanisms of evolution [6]. Evolutionary computation mostly includes evolutionary algorithms such as genetic algorithm (GA), evolutionary programming, evolution strategies, genetic programming and differential evolution and swarm intelligences such as ant colony optimization and particle swarm optimization (PSO). To develop new individuals, GA uses three operators; parent selection, crossover and mutation while the operator of PSO adjusts its particles or individuals by their flying experiences in group and flight conditions. Regardless of the complexity of implementation, the comparison of search performance among algorithms is still a controversial topic, especially for GA and PSO.

Some papers proposed that characteristics of PSO are somewhat between genetic algorithms and evolutionary programming. Three operators of GA can be described by the behaviors of the operator of PSO. Crossover of GA is midway of the operator of PSO. The selected individuals exchange their information by the crossover to generate new individuals while new individuals of PSO are the swarms of particles gathering around the local optimal particles and occasionally the global optimal particles. In mutation of GA, new individuals can be created at every point by flipping a bit of selected individuals. Similar to the mutation, the particles of PSO can reach any point by providing sufficient velocity. Since the particles are intact from one generation into the next generation, the particles can go anywhere if the enough

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[1] The authors are with Department of Sirindhorn International Thai-German Graduate School of Engineering, King's Mongkut University of Technology North Bangkok , E-mail: satito@kmutnb.ac.th[1] and nisai.f.epe@tggs-bangkok.org[2].

number of generations is given.

In the early generations, the crossover has significant effects. The individuals are created over a relatively large distance since the information of parameters is exchanged between randomly selected individuals over the design universe while the swarms of particles move toward the best particle in PSO. It results in good local search ability for PSO but poor local search ability for GA. In the late generations, most individuals have converged and have similar information. It means that the crossover has less effect and then the mutation plays a role instead. The variation caused by mutation can theoretically reach any point in the design universe. However, by flipping a bit, individuals can be changed dramatically. It can be the destruction of invaluable individuals or the important variation. Therefore, the mutation rate must be carefully selected. Similar to the mutation rate, the velocity of PSO must be defined carefully since new particles can fly far away from the direction toward the best particle.

The complication of the comparison is explained by the fact that the effects of the different algorithms vary over the total number of generations prior to termination. There is not any distinct proof which method is better. However, there are strong points for selecting GA as the search technique for this work. GA has proven its robustness and reliability by many previous works in machine design applications collected in [7]. In addition, GA is frequently used as the reference method to evaluate the performances of other search techniques.

In most automatic design process presented in Figure 1, after specifying objectives, a group of individuals or machine models are randomly generated. Afterwards, machine capabilities are calculated. Objective scores are evaluated and then added up into a fitness score used for measuring their goodness. Regarding their fitness, some models are chosen to generate new individuals. The reproduction process of GA consists of three steps; parent selection, crossover and mutation. Their functionalities are introduced in the next section. With the reproduction process, the evolution of GA develops the models from generation to generation.

However, the new individuals are often invalid since the exchanged information between two corresponding individuals is inconsistent with each other during the reproduction process. In addition, they are not preferred by requirement of specified geometries. The inconsistent and unpleasant geometries are inevitable for all automatic design processes. For this reason, most automatic design processes add a checking step into the design cycle to assure the consistence of all models and the existence of preferred dimensions after the reproduction step as shown in Figure 1. In case that a model is considered as invalid, it will be discarded and a new model is produced until

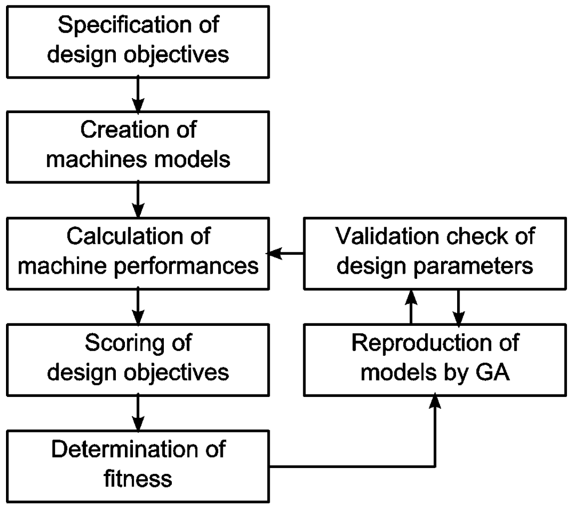

Fig.1: *Automatic design cycle with validation checking step after reproduction step.*

the required number of individuals is fulfilled.

The evolution caused by new individuals that inherit useful information of the chosen individuals in previous generations through crossover process is considered as the most powerful method of GA. Instead, the checking process denies some models formed by the reproduction process when they merely inherit an unpleasant geometry without regarding their fitness. In parent selection, the probability for being selected is given to every model and is directly proportional to its fitness from an assumption that the model may inherit a valuable geometry. This conflict between the checking process and the parent selection retards performances of the design process.

Therefore, the proposed design cycle removes this checking step and includes the preferred dimensions in fitness determination as the design objectives to improve the searching performance and to decrease the computing time of the design process. As results proposed in [8-9], the optimized models possess the preferred dimensions whereas models inheriting unpleasant dimensions cannot survive by means of natural selection. It can generalize automatic design processes of SRMs by congregating every consideration into one step. However, the number of objectives is readily increased and makes the fitness determination much complicated.

In the following section, after a brief introduction of genetic algorithms and multi-objective genetic algorithms (MOGA), fitness determination of the proposed design method is described in details. Selection of machine geometry parameters to form the machine model and design objectives applied in this paper are explained. Simulation results and conclusion are finally given to verify functionalities of the proposed method.

2. GENETIC ALGOTRITHMS AND MULTI-OBJECTIVE OPTIMIZATION

Genetic algorithms are stochastic search method, based on fundamental principles of natural selection and genetic evolution. The method for creating an individual vector, biologically called chromosome, makes it different from other stochastic search techniques. GA performs search technique from generation to generation. Each generation is composed by a group of individual vectors. A chain of design parameters referred as genes forms the individual vector or chromosome. Every chromosome represents a model in a design universe while its dimension is equal to the number of design parameters. Similar to other design processes, every model is evaluated into a score called fitness. It is an important data since it is only one measure to present model capability. The fitness is used for selecting some individuals to reproduce new individuals for the next generation. The reproduction process of GA consists 3 steps; parent selection, crossover and mutation.

2.1 Parent selection

To develop new models, some models will be chosen as survivals in this process called parent selection. Unlike selection processes of deterministic search methods that practically select the most fits, a selection decision of GA is made by probability. A higher reproductive chance is naturally given to individuals having higher fitness scores but this chance still remains for individuals having lower fitness scores. It can simply explain by a roulette game which each slot belongs to each individual but the slot opening width is proportional to its fitness. Therefore, the most fitness individual has the widest slot which results in the highest chance for being selected whereas the least fit having the narrowest slot still has a chance for being selected as parent by this method.

It prevents this search technique from being trapped by local optimal solutions since the design parameters are not only copied from the most fit parent but also from the least fit to their children in the next generation.

2.2 Crossover

Crossover imitates a recombination process of natural evolution. From the parent selection, two corresponding parents exchange their information or genes to create two new individuals called children. Some say that crossover is the most important process in GA. Unless the operation does exist, the results are no longer a genetic algorithm.

2.3 Mutation

Creation of a new individual from one corresponding chromosome is the mutation process. There are also many methods of mutation purposed. The most obvious one is alternation of characteristic of the concerning genes. The mutation rate is naturally rare and random. However, it still provides sufficient variation of genes.

Unlike single-objective optimization that its solution is an individual having the highest score of that objective, multi-objective optimization problems involve with trade-offs and competition among objectives. Therefore, there is no perfect solution for such problems but rather there is a set of solutions called Pareto-set. Aims to improve any objective of the solutions in Pareto-set further will result in degrading other objectives. Practically, they receive an identical fitness score. Shown in Figure 2, points M1-M5 are members of Pareto-set coordinated by fitness scores of objective 1 and 2. The sum of objective scores results in an identical fitness score presented by points F1-F5. Since the fitness is a scalar quantity, it cannot distinguish the capability of each individual. In fact, a fitness score is the most important value since it is the only indicator of model ability directly affecting parent selection of GA. Therefore, to differentiate the fitness for each model, the objectives must be involved with fitness determination. This complicates fitness function description.

One simple approach is made by assigning weights to the score of each objective before determining the fitness. It can discriminate all individuals including the members of Pareto-set. However, it is very difficult to accurately specify the weight even having comprehension of relative importance of each objective or knowledge of problems. Small difference in weighting can occasionally lead to quite different solutions. Besides, if the objectives are equal in term of importance, it cannot figure out the only individual having a middle score of both objectives to be the best fit by specifying an identical weight for both objectives.

Regardless of relative importance, many have proposed different algorithms in fitness determination without assigning the weights of each objectives, but by involving all objectives; for examples, the first MOGA applied by Schaffer's VEGA [10], Fonceca and Fleming's MOGA [11], Horn, Nafploitis and Goldberg's NPGA [12] and a survey presented in [13]. These algorithms determine fitness of an individual by comparing its objective scores with those of other individuals in generation to classify the individuals into different fitness ranks and then into different fitness. However, the ranking procedures become ineffective for deciding good quality solutions when many objectives are concerned [14-15]. Furthermore, the algorithms do not take any design knowledge and designer's reasoning into consideration. It results in very stiff decision for solutions of MOGA as the members of Pareto-set. After all, few individuals have to be selected from the set as the optimized solution by designer's reasoning at the end.

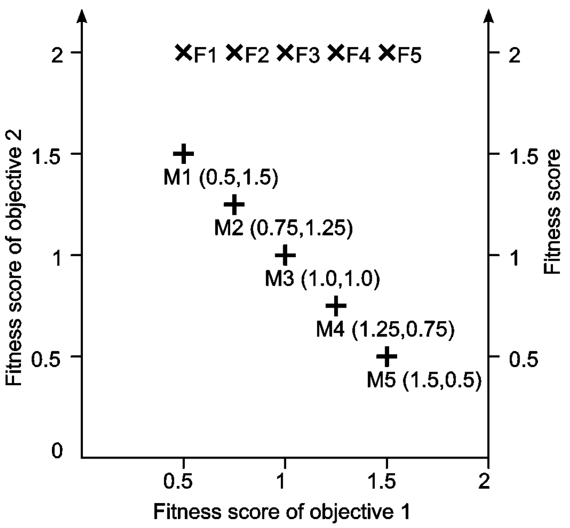

Fig.2: *Objective scores and fitness scores of solutions in Pareto-set.*

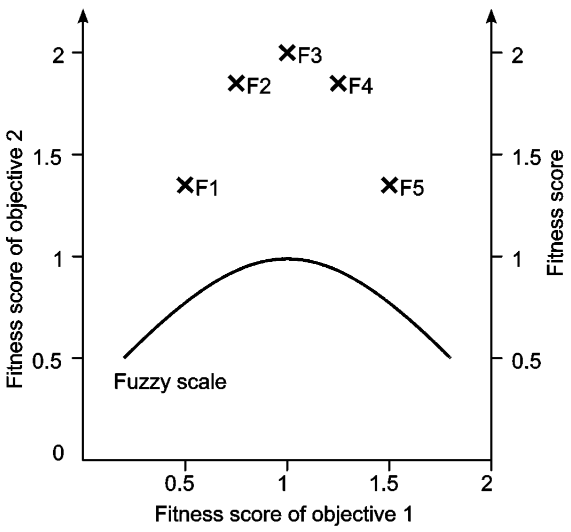

Fig.3: *Fuzzy scaling factor and fitness scores after multiplying with the scaling factors.*

3. PROPOSED DESIGN PROCESS

On the other hand, cooperation use of objectives and designer's reasoning for determining different fitness is an interesting option. Many have combined decision strategies of designers in fitness assignment by utilizing neural computing, machine learning, fuzzy algorithms, evolutionary algorithms and agent-based methods. These methods give an opportunity to designers to employ both expert knowledge and raw data of desired objectives for fitness determination, for examples, see [16-18].

In this paper, fuzzy algorithms (FA) is employed for formulating expert knowledge of design guidelines and decision strategies of designers for all objectives in terms of fuzzy rule bases, IF-THEN rules. Since human experts know which individual is good or bad, but cannot exactly express in form of the fitness function, expert knowledge and designer's reasoning are therefore transformed into a set of relationships between objectives and fitness known as fuzzy rules. In this paper, the input is objectives while the output is not fitness score, but a factor use for scaling the fitness named fuzzy scaling factor, kf. After scoring design objectives, the scores are used for inferring the fuzzy scaling factor. For example, two fuzzy rules are exploited for inferring the scaling factor for M1-M5 in Figure 2. The rule bases are defined as:

i. IF all objectives are middle THEN scale is good.
ii. IF any objective is lower THEN scale is bad.

As result, the scaling factor is highest and equal to one in case that both objective scores are one. Otherwise, the factor becomes smaller than one as shown in Figure 3. By this method, the scaling factor directly indicates model's goodness although it does not calculate the fitness itself. After multiplying the objectives with the scaling factors, the fitness scores are different as shown by F1-F5 and M3 has the highest score as expected from the assigned fuzzy rules.

Figure 4 presents the design procedure of the proposed method generalizing automatic design process of SRMs. Unlike almost of classical automated design cycles, the checking condition from preferred geometries and design suggestions is removed and congregated into the fitness assignment as optimization objectives.

The proposed algorithm simplifies the complexity of fitness determination by separating one complicated task into two simple steps. The score of objectives is normally described by linear relationships of the performance and requirement without any boundaries or conditions that are normally set to prohibit fitness compensation. After that, the scores from these objectives are used for inferring the fuzzy scaling factor according to the fuzzy rule base. The objective scores are scaled and then summed up together to determine the fitness. As results, all individuals are discriminated by their own fitness.

In this paper, it presents the uses of FA to determine the scaling factor, not the fitness directly. Then, the factor differentiates the fitness. The proposed method is focused on simplification of design process. Even the fuzzy rules are neither complicated nor specified for design of SRMs. Only three fuzzy rules are established by basic reasoning to verify that expert knowledge and design experiences are not necessary for construction of fuzzy rule base. The knowledge and experiences can be used as optimization objectives. Hence, the proposed algorithm can be simply adapted for optimizing other electrical machines.

4. DESIGN PARAMETERS

A model of SRMs is presented by geometries as shown in Figure 5. For standard types of multi-phase SRMs, a cross-section is composed of 9 parameters,

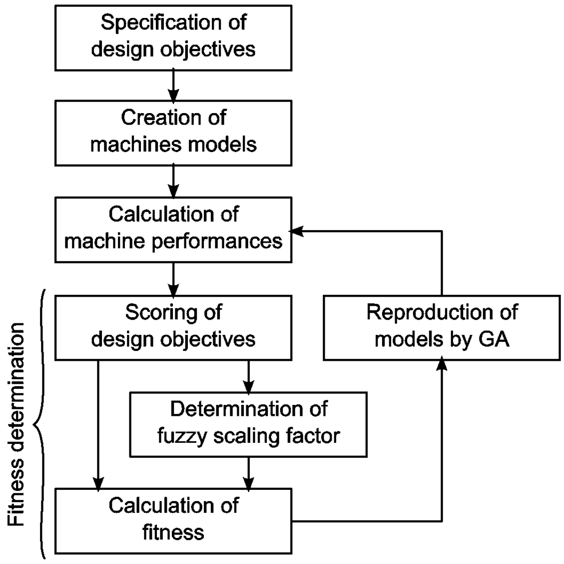

Fig.4: *Proposed design cycle without interrupting from validation check.*

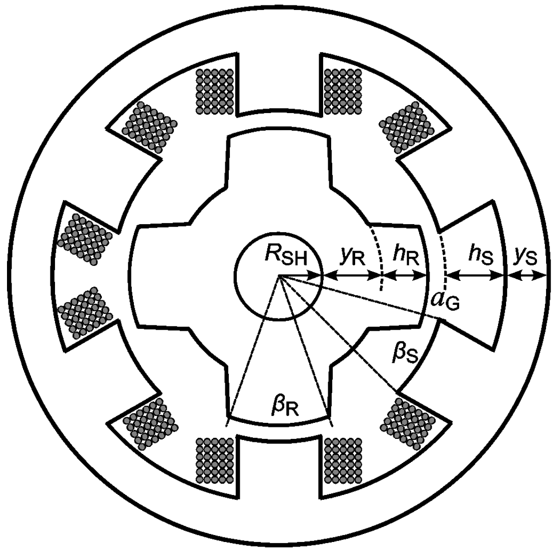

Fig.5: *Cross-section and geometry parameters of SRMs.*

shaft radius, rotor inner radius and outer radius, air gap, stator inner radius and outer radius, stator pole arc and rotor pole arc $R_{SH}, R_0, R_1, a_G, R_2, R_3, \beta_S$ and β_R respectively, including stack length, L_{STK}.

In automatic design processes, the design parameters are adjusted in a boundary of each parameter. The most advantage of this format is that desired geometries can be prescribed from the beginning and are kept constant during the design process. However, a situation that inner radii are larger than outer radii frequently occurs after reproducing the design parameters since the boundaries are usually overlapped. In this case, the design cycle will be interrupted as mentioned in the previous section.

Figure 5 also presents an additive chromosome format to avoid the mentioned problem. Geometries y_R, h_R, h_s and y_S are rotor yoke thickness, stator yoke thickness, rotor pole depth and stator pole depth. This additive format is utilized for the highest degree of freedom for generating models. It assures the consistence of every individual. Including phase number, N_{PH} and pole number, N_P , the chromosome is described as:

$$x = [R_{SH}, y_R, h_R, a_G, h_S, y_S, \beta_S, \beta_R, L_{STK}, N_{PH}, N_P] \quad (1)$$

5. IMPLEMENTATION OF THE PROPOSED METHOD

To determine a fitness score for each model, machine performances are scored by objective functions, f_1 to f_9 and then weighted by weights of objective importance, w_1 to w_9. In this paper, all weights are equal to one. The difference of objective importance can be interpreted into fuzzy rules. Finally, the sum of the objective scores is scaled by a fuzzy scaling factor, k_f to determine the fitness score of a model as expressed in (2).

$$f = k_f \cdot (f_1 \cdot w_1 + f_2 \cdot w_2 + f_3 \cdot w_3 + ... + f_9 \cdot w_9) \quad (2)$$

5.1 Objectives assigned by required performances

Torque is usually the most important characteristic of a machine. A desired value of torque, T_{REF} is typically specified by the relationship of torque and speed. For SRMs, machine torque, T_{CO} is calculated by a flux-linkage curve as shown in Figure 6. The fitness function of torque objective is described as:

$$f_1 = \frac{T_{CO}}{T_{REF}} \quad (3)$$

The enclosed area in the flux-linkage curve is electrical energy called co-energy, W_{CO} which can be converted into electrical torque in one stroke. In one revolution, the number of strokes is the product of phase number, N_{PH} and rotor pole number, N_R. An average torque in one revolution is determined as:

$$T_{CO} = \frac{N_{PH} \cdot N_R}{2\pi} \cdot W_{CO} \quad (4)$$

If the phase current supplied to a machine is kept constant from unaligned to aligned position, the co-energy and average torque are at their maximum for that current. By calculating the maximum torque for the certain current, torque capacity of a machine can be specified as presented by the dashed line of Figure 7. Subsequently, the current value is converted into current density of a stator winding which must be suitable for power ranges, applications and especially cooling methods described in [4]. For

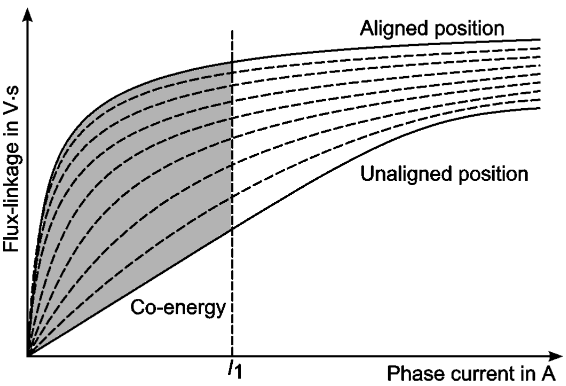

Fig. 6: *Flux-linkage characteristic and co-energy.*

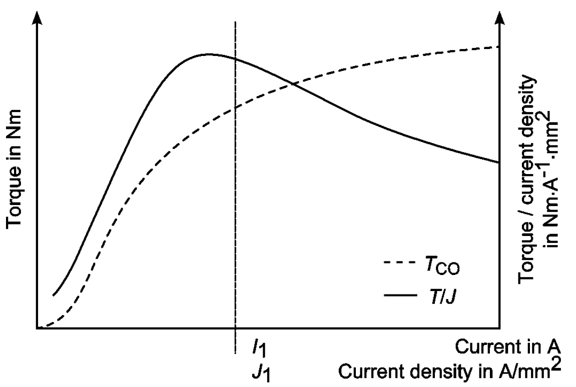

Fig. 7: *Static torque and ratio of torque and current density in function of current and current density.*

the rectangular-shaped pulse current having the peak current, I_{PEAK}, the current density, J_{RMS} is calculated as:

$$J_{RMS} = \frac{0.707 \cdot I_{PEAK}}{0.5 \cdot A_{SLOT} \cdot k_{FILL}} \tag{5}$$

where A_{SLOT} is the area of stator slot and k_{FILL} is fill factor of conductors in the stator slot. The second objective is described by current density, J_{RMS} at the required torque. The fitness is determined by:

$$f_2 = 2 - \frac{J_{RMS}}{J_{REF}} \tag{6}$$

where J_{REF} is current density specified by the cooling methods and machine applications. In contrast to the torque objective, this objective gives higher scores for machines with a lower current density at the required torque.

It should not be confused that the first and the second objective functions can be included as one objective. The aim of this optimization is to find a machine satisfying the torque constraint, but not to find a machine generating the greatest possible torque. The second objective is to indicate the margin between the specified current density and the actual one at the required torque.

Besides torque and current density, a ratio between torque and current density is also a very important objective which represents the machine utilization. It presents how good the winding slot area and the flux distribution in a machine are managed. The slot area is directly related to the current density whereas the flux is distributed in lamination. It is the trade-off between current density and torque producing flux. The solid line in Figure 7 presents the ratio of torque and current density of well-designed machines typically having their nominal operating point at the maximal ratio or slightly lower ratios since many machines are usually operated at 30-80% of their power rating in most applications. The fitness is determined by:

$$f_3 = \frac{\frac{T_{CO}}{J_{REF}}}{max\left(\frac{T}{J_{RMS}}\right)} \tag{7}$$

5.2 Objectives assigned by dimension restrictions

As mentioned in the design process, dimension restrictions and preferred dimensions are treated as objectives. The most restricted dimensions are the outer diameter and axial length. Therefore, the fitness function is used for selecting machines with the specified diameter, not for minimizing the diameter. The fitness function is described by:

$$f_4 = 1 - |\frac{R_{REF} - R_3}{R_{REF}}| \tag{8}$$

The triangle-shaped function gives the highest score only to machines having the radius, R_3 equal to the specified radius, R_{REF}. Unlike the outer diameter, the axial length of lamination steel or stack length is usually set as a limitation. As the length of machines is fixed, the stack length is limited by types of winding-end, position sensors and a propeller on one side of air-cooled type machines. It can be shorter, but not longer. The fitness function is described by:

$$f_5 = \begin{cases} 1 & ; L_{STK} \le L_{REF} \\ 0 & ; L_{STK} > L_{REF} \end{cases} \tag{9}$$

The fitness will be zero in case that stack length, L_STK is longer than limitation, L_{REF}. For machines with shorter length than L_{REF} , the fitness is not increased since the effect of axial magnetic flux at both ends or end-effect has much influence on shorter machines. In practice, the end-effect causes the reduction of unsaturated inductance ratio between aligned and unaligned positions. It directly reduces machine torque capacity and in addition, the accuracy of the calculated flux-linkage curves. The unaligned inductance calculated by 2-dimension finite element analysis (FEA) is about 15% different from the existing machine when $L_{STK} = 4 \times R_3$ [19]. To minimize the end effect, the objective is not obliged to shorten machine length, but to stay with its limit.

5.3 Objectives assigned by design guidelines

In automatic offered design process, plenty of models are created and then evaluated for developing an optimized one. The essence of such process is short computing time taken to calculate each model. Finite element analysis is accepted for its accuracy, but extremely long computing time is its most drawbacks. Analytical analysis is very fast in computing. Its accuracy is still acceptable but much dependent on shapes of machines. Some assumptions are made to simplify calculating methods such that magnetic saturation occurs in poles before in yokes or a mutual flux between simultaneously conducting phases is negligible [20].

Stator and rotor poles are assumed to be saturated before stator and rotor yokes. Therefore, most of analytical analysis is focused on magnetic parts in the poles. For 3-phase SRMs stator yoke and rotor yoke thickness, y_S and y_R must be at least half of stator pole and rotor pole width b_S and b_R since the magnetic flux at pole is divided by half into yoke. Because of this, some suggestions for ratios of stator and rotor yoke thickness and pole width $a_{S,REF}$ and $a_{R,REF}$ are made. Since the overlap of excitation between two phases is only 5% for 3-phase SRMs, the ratio is about 0.7-1. For 4 or higher phase numbers, the excitation overlap is about 30%. The suggested ratio is about 1-1.2. The fitness function is described as:

$$f_6 = \frac{\frac{y_S}{b_S}}{a_{S,REF}} \tag{10}$$

$$f_7 = \frac{\frac{y_R}{b_R}}{a_{R,REF}} \tag{11}$$

The other objective is provided by unsaturated inductance ratio between aligned and unaligned position, a_{LaLu}. The ratio directly affects volt-ampere rating required for selecting a converter [21]. For well-designed models, the inductance ratio reference, a_{REF} is about 8-10.

$$f_8 = \frac{a_{LauLuu}}{a_{REF}} \tag{12}$$

The last objective is about acoustic noises and vibration of SRMs since one of their major drawbacks is high acoustic noise emission. For preliminary model design, the stator is considered as a cylindrical shell with an outer radius, stator yoke thickness and stack length. For axial vibration nodal, m = 0, natural frequencies of radial vibration, f_n are obtained by [22]:

$$f_n = \frac{1}{2\pi}\underbrace{\sqrt{\frac{E}{\rho(1-\mu^2)}}}_{\text{Material property}} \cdot \underbrace{\frac{y_S}{\sqrt{12}R_3^2}}_{\text{Geometry}} \cdot \underbrace{\frac{n(n^2-1)}{\sqrt{n^2+1}}}_{\text{Vibration modw}} \qquad n = 2, 3, 4, \ldots \tag{13}$$

Where n is mode number. The equation illustrates distinct relationship among material property, geometry and vibration mode. The most interest belongs to yoke thickness since outer radius is predefined. The natural frequency will increase if the yoke becomes thicker. Consequently, acoustic noise emission will be reduced. Thus, the yoke thickness plays an important role for this constraint. This objective is described by:

$$f_9 = \frac{y_S}{y_{REF}} \tag{14}$$

where y_{REF} is yoke thickness reference calculated for a certain frequency related with the switching frequency of converter and the rotational speed of rotor.

5.4 Fuzzy scaling factor

This section explains the implementation of the fuzzy algorithm for determining the scaling factor k_f presented in (2). Two membership functions (MF) are applied for this work. The first MF is composed of three fuzzy sets; low, medium and high. It is assigned to the objectives of machine performances for torque f_1, current density f_2 and inductance ratio f_3 and geometries related with flux distribution f_6 and f_7 and acoustic noises f_9 as shown in Figure 8 (left). The machine performances and geometries are expected for the better values than their reference values based on the requirements and empirical values. The second MF is composed of two fuzzy sets; bad and good. It is assigned to the constraint of the maximum point f_3 and the limitation of geometries f_4 and f_5 in Figure 8 (right). The fitness for the ratio of torque and current density cannot be higher than one since it is compared with its maximum value, and the constraints of geometries are aimed to limit the stack length and to have the preferred outer radius. Therefore, the evaluation of the fitness gives only two results which are bad or good.

Since the fuzzy algorithm is applied for minimizing the compensation of fitness among objectives, not directly for determining the fitness value, the lower numbers of fuzzy sets are appropriate for this work. In addition, the lower numbers of fuzzy sets minimize the number of fuzzy rules. Consequently, the fuzzy rules are simple. For this reason, any intelligence algorithm for defining the MFs and constructing the fuzzy rules is not necessary.

For defining the fuzzy set, a trapezoidal shape function is selected due to its flat top. It represents no degree difference of membership for a certain range of fitness scores. Within this range, the fuzzy scale will be identical and the fitness itself differentiates the model ability. For this reason, the trapezoidal shape is well matched for this purpose. To construct the trapezoidal MF, it means to set values of bottom points called feet and top points called shoulders.

Many publications proposed methods for automatically defining MFs and constructing fuzzy rules. The

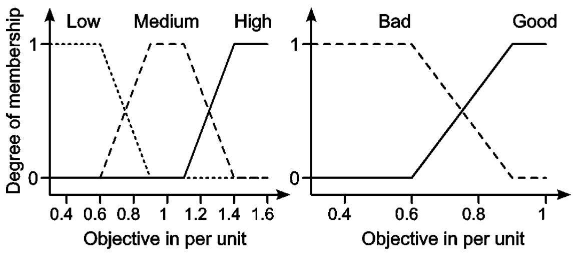

Fig.8: *Membership function of 3 and 2 fuzzy sets.*

methods can be applied for other forms as well. However, it is the fact that the automated adjustment methods require training which makes the created MF much dependent on the limited applications even design for the same type of machine. Conventionally, the feet and shoulders are set by trial and error based on experiences and empirical values. Although it consumes time and effort but for this level of work, it is practically suitable. The shoulders are defined by ±10% of the reference or requirements (0.9-1.1) while the feet are defined by 10% of the reference over half of reference (0.6) and 10% below one and half of reference (1.4). As a rule of thumb, ±10% of reference should be the minimum acceptable limit. This will also demonstrate the simplicity of utilizing fuzzy algorithms for managing the fitness function description that results in the promising performances for designing SRMs.

Afterward, the fuzzy sets are aggregated by Mamdani's fuzzy inference method according to rule bases. In this paper, the rule bases are constructed by basic reasons for prohibiting high fitness scores of an individual due to the compensation. Therefore, the construction of fuzzy rule bases does not require any knowledge of machine design or designer's experiences. The three rules are created as follows:

i. IF any objective is low or bad THEN k_f is low.
ii. IF all objectives are medium or good THEN k_f is medium.
iii. IF all objectives are high or good THEN k_f is high.

The membership functions of the output also consist of three fuzzy sets; low, medium and high as shown in Figure 8 (left). The scaling factor is normally determined by weighted average method of the output areas.

6. VALIDATION OF THE PROPOSED METHOD

To verify the functionalities of the proposed method, optimization of SRM was conducted to assure whether it can find the optimized model out of a design universe. After presenting the construction of the design universe and its boundaries, the simulation results show the quality of the proposed method.

6.1 Design universe and boundaries

Consequently, the design universe of SRM models was created within boundaries of design parameters as shown in Table 1. A mass simulation was made for every model created by varying design geometry against each others. Calculation of machine performances and characteristics are recorded as machine database. Every model is scored by the objective functions and then the fitness function. The computing time for collecting the machine data took almost 2 months for all 2,097,152 models (approx. 2.5s for each model).

For boundaries, the first design parameter, shaft radius, is practically predefined according to required stiffness based on material's strength and standard sizes of shaft coupler. Similar to shaft radius, air gap is also practically prescribed. Some said that smaller air gap produce better performances but it requires more precision for machine production. Hence, the air gap is also fixed by a standard value for the required power class. Finally, for the other parameters, the boundaries are defined as follows:

i. Phase and pole numbers are selected from the most frequently used ones for standard applications.
ii. Pole arc angles are determined by feasible triangle for each combination of phase and pole numbers.
iii. Pole depth and yoke thickness are varied in a proper range based on the pole arc angles.
iv. Stack length is varied by the relationship with a range of outer diameter.

Table 2 presents the reference values of each objective specified as the requirements of optimization. The values are based on the design guideline in [1-4]. Alongside, the performances of the best model in the design universe are presented. The comparison presents that the requirements are challenging and are not easy for finding the optimal models of the design problem since the requirements and the best model's performances are closed. Otherwise, the design problem can be considered as very easy in case that the references are much lower than those of the best fit since there is not much trade-offs among objectives.

6.2 Calculation results

Out of the design universe, there are 280 models that satisfy all constraint references, excepting the ratio of torque and current density. The best fit is a 3-phase/1-pole SRM and its characteristics are presented in Table 2. To verify its performances by the statistical record, the proposed process was performed 1,000 times with the GA-parameters as shown in Table 3.

The most fit from each optimization was compared by those 280 models of the design universe. The number of optimal model for each rank is counted and pre-

Table 1: Design parameter and boundaries

Parameter	Boundary			
Phase	3	3	4	4
pole	1	2	1	2
R_{SH} in mm	10			
y_R in mm	15-22	34-41	30-37	41-48
h_R in mm	12-19	9-12.5	10-17	7-10.5
a_G in mm	0.5			
h_S in mm	12-19	9-12.5	10-17	7-10.5
y_S in mm	12-19	9-12.5	10-17	7-10.5
β_S in °	16-31	10-17.5	12-19.5	8-15.5
β_R in °	16-31	10-17.5	12-19.5	8-15.5
L_{STK} in mm	85-105			

Table 2: References of optimization objectives compared with the best fit

Constraint and guide line	Reference	Best fit
Torque at 10 A/mm^2	15 Nm	15.83 Nm
Current density at 15 Nm	10 A/mm^2	9.6 A/mm^2
Outer radius	80 mm	80 mm
Stack length	≤105 mm	105 mm
Torque per current density at 10 A/mm^2	1	0.90
Ratio of aligned and unaligned inductance	9	9.26
Ratio of stator yoke thickness and pole width	0.85/3ph. 1.0/4ph.	0.9
Ratio of rotor yoke thickness and pole width	1.2	1.54
Stator yoke thickness	≥ 10 mm	13 mm

Table 3: References of optimization objectives compared with the best fit

GA-parameter	
Number of individuals	500
Number of generations	100
Crossover rate	95%
Mutation rate	5%

sented by the solid line as shown in Figure 9 while the accumulated number of optimal model is presented by the dashed line. The most fit was found 28 times out of 1000 or 2.8% while the 10 best models and 100 best models were found by 23.2% and 94.3%, respectively. It confirms that the probability to obtain the model in 100 best designs out of the extremely large design universe is higher than 90%. This can verify promising performance of the proposed method.

To illustrate the searching performances, development of some objectives is pointed out. Figure 10 presents the average number of model in each generation that satisfies the crucial constraints of torque, current density and outer dimensions. In the first

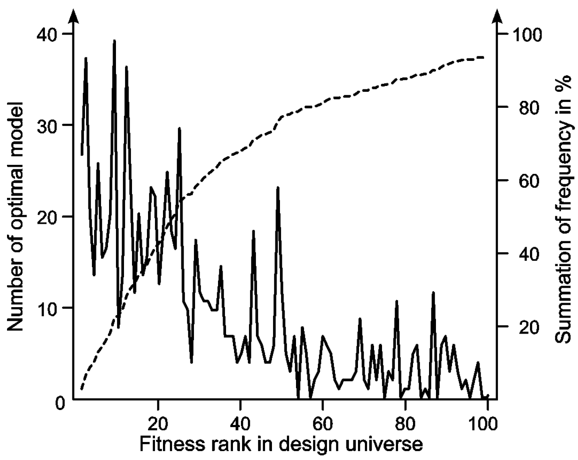

Fig.9: Statistic record of the best fit from simulation within the best 100 rank of design universe and summation of frequency.

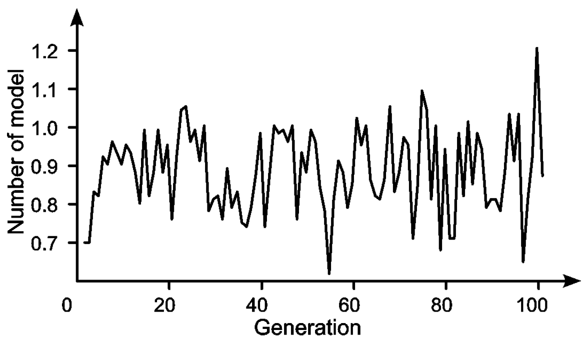

Fig.10: Number of model that fulfils torque, current density, outer radius and stack length requirement in each generation.

generation, there was less than one from 500 models that satisfy the objectives. After the 20th generation, it frequently presents the number higher than one. Figure 11 and 12 present the increasing number of model that satisfies for each objective separately. The number is around tens for the torque and current density objective but it is around hundreds for the stack length, ratio of stator yoke thickness and pole width and ration of the unsaturated aligned and unaligned inductance. It can present the speed of the process for optimizing each objective through the difficulty in finding one globally-optimized model.

To verify the accuracy of machine characteristics of the optimized model, comparison of flux-linkage curve calculated by finite element analysis (FEA) was made. The difference of flux-linkage curves are less than 1% at aligned position and less than 3% at unaligned position as shown in Figure 13. The comparison results can refer to an acceptable accuracy of the other performances of the optimized model. To emphasize the benefit gaining from the proper ratios between yoke thickness and pole width, another model having low scores of these objectives was examined.

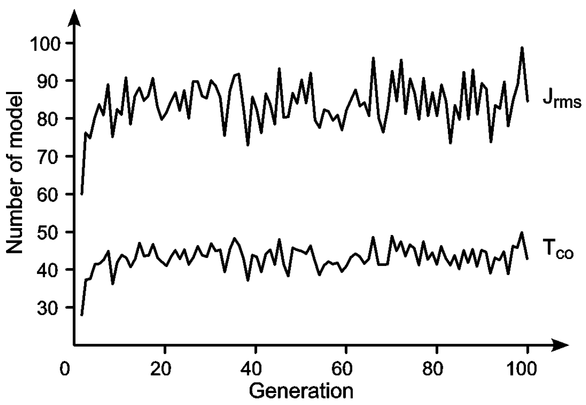

Fig.11: *Number of model that fulfils the torque, current density requirements in each generation.*

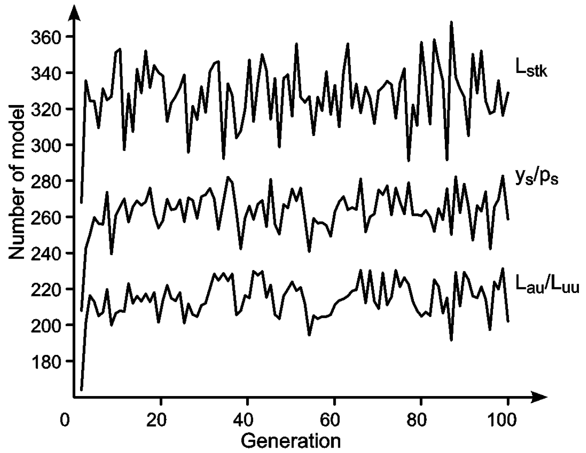

Fig.12: *Number of model that fulfils stack length requirement and design guidelines of stator yoke and pole ratio and inductance ratio in each generation .*

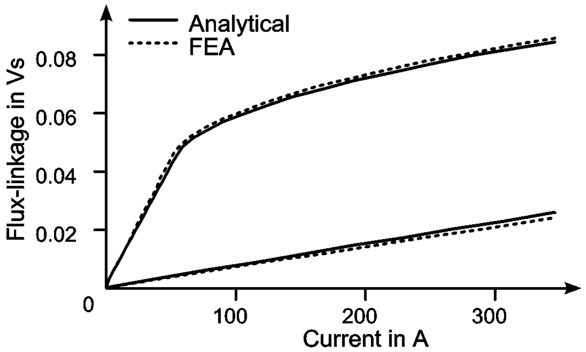

Fig.13: *Comparison of flux-linkage curves between analytical and finite element calculation of the best fit.*

The difference of the flux-linkage curves is about 5% at both positions as shown in Figure 14. It can be concluded that the objectives of the design guidelines directly improve machine performances and indirectly support the accuracy of analytical calculation.

Figure15 illustrates the ratio of torque and current

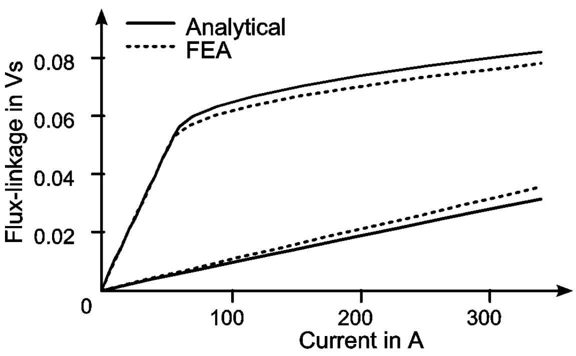

Fig.14: *Comparison of flux-linkage curves between analytical and finite element calculation of the model unsatisfied by design guidelines.*

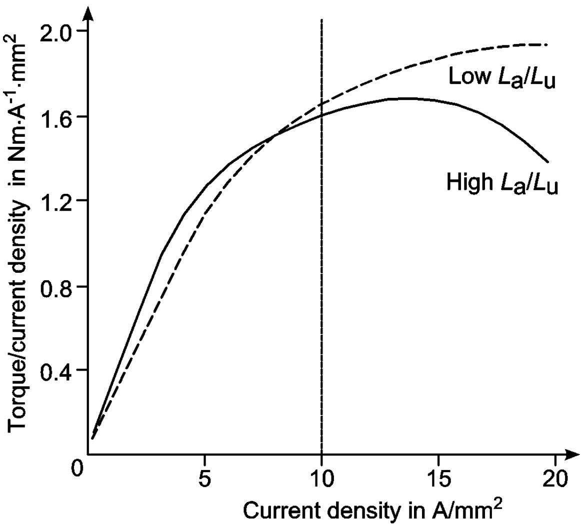

Fig.15: *Torque per current density for different inductance ratios.*

density in relationship with current density. The best model gives a good result of machine utilization since the ratio at the nominal current density ($10\mathrm{A/mm}^2$) is near at its maximum as shown by the solid line. In comparison to the best fit, another model has a slightly higher ratio of torque and current density at $10\mathrm{A/mm}^2$, but it is far from its maximum as presented by the dashed line. The higher ratio is traded by the reduction of the inductance ratio. As a result, this model generates the slightly higher torque at nominal current density, but it takes higher current density over the lower torque range which is normally used in most applications.

Besides the machine performances, the computing time of the design cycle is of essence. 50,000 models (100 generations$\times$500 individuals) were simulated for the design process. It takes about 36 hours or 40 times faster than those of the mass simulation of 2 months. Since the most fit models were frequently found out by generation 70 as shown in Figure 16, it can be pointed out that the computing time can

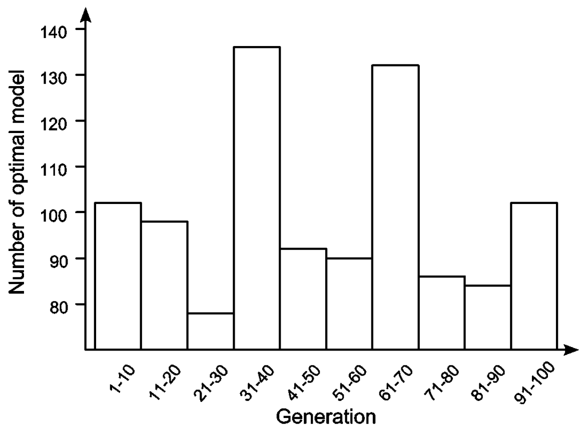

Fig.16: *Statistical record of the number of generations to find the optimal models.*

be shortened by reducing the number of generations or by terminating the design process in case that all constraints are already satisfied.

7. CONCLUSION

The proposed method simplifies design procedures for sizing preliminary models of SRMs by combining the uses of genetic and fuzzy algorithms. All requirements and design guidelines are treated as optimization objectives. It eliminates all undesired interrupts violating the reproduction process and then maintains the continuous flow of genetic algorithms. The design process was performed by 1,000 times and the results validate its functionalities. 94.3% of the best fits from the proposed method stay in the 100 best models of over 2 million models in the design universe. It assures the promising search performance while the statistical record presents its reliability. In addition, the objectives based on design guidelines profit to higher accuracy of the flux-linkage curves calculated by analytical analysis. This directly improves the accuracy of machine performances which are verified by the calculation results from finite element analysis.

The difficulty for defining the fitness function is simplified by fuzzy algorithms. Compensation among objective scores is prohibited by the fuzzy rule base. The results demonstrate the effectiveness of the most standard membership functions and the simple fuzzy rules which can be quickly and comfortably adapted for any special optimization purposes.

However, the operators used by genetic algorithms are conventional. The adaptation of crossover and mutation can be improved for shortening the computing time and increasing the resolution of the design parameters. For the crossover, it does not have much of action near the end of optimization and vice versa for the mutation. For the improvement, the crossover and mutation rates can be self-modified by considering variances and convergence of individuals so that the optimal solution will be found out sooner. In addition to the modification of operators, the resolution of design parameters can be modified by reducing the bit-resolution of the converged parameters and increasing the resolution of the others. In this case, if the termination is specified by the number of generation, the optimized parameters will result in highly-resolute quantities.

Regards of search methods, the uses of other stochastic search algorithms such as adapted GAs, PSO and ant colony optimization can be implemented for the future work as options for specified design applications.

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Satit Owatchaiphong received the B.Eng. (Hons) degree of Electrical Engineering in 2000 from King Mongkut's University of Technology North Bangkok, Thailand, and M.Sc. degree in Electrical Power Engineering in 2004 from RWTH-Aachen University, Germany. He is conducting his Ph.D. in fields of electrical machines and drives at King Mongkut's University of Technology North Bangkok. His research interests include machine design, modelling of machines, and drives.

Nisai H. Fuengwarodsakul received his Bachelor of Electrical Engineering in 1998 from KingÂt's Mongkut University of Technology North Bangkok and his Master of Science and Doctoral Degree in Electrical Power Engineering in 2001 and 2007 from RWTH-Aachen University, respectively. Since July 2007, he is with The Sirindhorn International Thai-German Graduate School of Engineering, King's Mongkut University of Technology North Bangkok as a lecturer/researcher. His research activities are in the area of electrical drives, battery and electric vehicles.