Performance Analysis of MU-MIMO Systems Using HMRS Technique for Various Transmission Modes

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ABSTRACT

It is the fact that the bandwidth of wireless communication system is such a limited resource that several techniques are selectively applied to increase the bandwidth efficiency. The highest bandwidth efficiency can be taken by applying Multi-User Multiple-Input Multiple-Output (MU-MIMO) technique. For this technique, the complexity of detection is rapidly increased by increasing the number of users. Thus the lower complex detection is necessarily required for MU-MIMO system. Recently, the simple detection technique called hybrid-MIMO receiver scheme (HMRS) has been proposed by the authors. However, that study neglected the demands of multiple users for transmitting MIMO modes which are crucially unpredictable in practice. In this paper, the performance analysis of MU-MIMO system using HMRS technique to support various types of user transmission modes is presented. Moreover, the nearly exact symbol error rate (SER) analysis of HMRS with the nonlinear error propagation effect over Rayleigh channels is originally presented. The recursive procedure is adopted to derive the nearly closed-form expressions of the error probability of each user. The results indicate that HMRS technique can improve the error rate more than the existing hybrid-MIMO about 8 dB at 10^{-4} SER, increasing the total number of user and number of SM user introduce the diversity gain loss. The simulation results illustrate the performance accuracy of the proposed analysis.

Keywords: Multiple-Input Multiple-Output (MIMO), Maximum Likelihood Detection (MLD), Successive Interference Cancellation (SIC), Space-time Block Code (STBC), Symbol Error Rate (SER).

1. INTRODUCTION

Wireless communications are widely used around the world. The new applications have been frequently established such as WLAN, 3GB, LTE, etc. Many customers need to access the limited spectrum in the same time. Therefore the available bandwidth cannot be enough for every user [1]. The saving bandwidth techniques for wireless communication have been researched and already applied such as TDMA, CDMA, MIMO [2-4], hybrid-MIMO [5-7], MU-MIMO [8], etc. Fig. 1 illustrates the basic structure of MU-MIMO system. The transmitted signals from N users are sent through MIMO channel and reached to the receiver in the same time and frequency to achieve a high bandwidth efficiency, to increase a capacity gain and to keep a benefit of diversity gain. The receiver must have capability to suppress interference and detect all symbols from all users by using multi-user detections such as QR decomposition [9], ZF detection, MMSE detection [10], ML detection [11], Sphere detection [12], IC [13], etc. In 2005, the simple hybrid-MIMO technique called Hybrid-MIMO Transceiver Scheme (HMTS) [14] has been proposed. The two users at transmitting side can select either Space-Time Block Code (STBC) or Spatial Multiplexing (SM) to encode the transmitted signals. It has a simple structure when a few users are operated in the system because MMSE detection and Successive Interference Cancellation (SIC) [15] are jointly applied. However, the complexity of detection at receiver is increased when a lot of users are operated because the MMSE detection and SIC are operated every time of detecting SM layer. Recently, the authors proposed the novel hybrid-MIMO technique called Hybrid-MIMO Receiver Scheme (HMRS) [16] that the system applies ML detection and SIC jointly to detect all symbols in entire layers. The ML detection is operated only one time for every case of N users, thus this scheme has the number of detecting procedures less than HMTS. The comparison of SER performance between HMRS and HMTS is presented in Section IV. However, the work in [16] did not study on the various types of user demand on MIMO transmission modes. In fact, the user demand cannot be predicted and this deviate the system performance from results presented in [16]. Hence, this paper has concerned this issue and proposes the performance analysis to investigate the effect of various MU-MIMO transmission modes.

In literature, the performance analysis of MLD over fading channel is presented in [17-21]. In [22-26],

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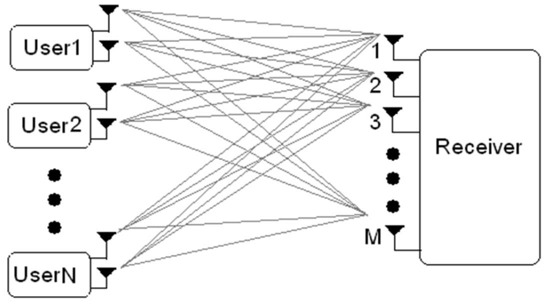

Fig.1: *MU-MIMO system.*

the closed-form expression of symbol error probability is derived for orthogonal space-time block coded (OSTBC) MIMO Rayleigh fading channels with arbitrary number of transmit and receive antennas. The successive procedure of error rate analysis for zero-forcing successive interference cancellation (ZF-SIC) is obviously described in [27] and [28]. However, so far in literature the performance analysis of hybrid-MIMO scheme has never been proposed.

In this paper, the authors investigate the SER performances of HMRS in case of N users, where N can be assigned as 2, 3 and 4, in order to study the effect of the various transmission modes. Because the several users may be randomly operated in practice, thus the evaluation of SER performance under this situation is an important task. The performance analysis of symbol error rate for HMRS hybrid-MIMO systems is also derived. From the above analyzing results, the readers can utilize it on the design of the proper transmitting parameters keenly including the transmitting power, antenna gain, modulation scheme, channel coding scheme and the number of antennas. Moreover, the transmitted signal, the received signal and the structure of N users using HMRS are presented in this work in order to reveal the procedure of the HMRS system.

The remainder of this paper is structured as follows. In section 2, the authors present the system model. Then the analysis of symbol error rate for MLD and STBC systems is described in section 3. The performance analysis of HMRS is explained in section 4. The numerical results and discussion are presented in section 5 and followed by the conclusions in section 6.

2. SYSTEM MODEL

The structure of MU-MIMO system is shown in Fig. 1. All MIMO streams are simultaneously transmitted by N users. Each user equipped with a 2-element antenna array and applies either 2×M STBC or 2×M SM system. The receiver equipped with an M-element antenna array. The transmitted signal vector s is sent through a random channel matrix **H** in uplink channel. Flat and slow Rayleigh fading is assumed to evaluate the SER performance. The receiver can detect all symbols from all users by using MU detection. The received signals can be expressed as

$$\mathbf{y}=\mathbf{Hs}+\mathbf{w} \tag{1}$$

where **y** is the vector of received signals, **H** is $M\times 2N$ MIMO channel matrix, s represents the $2N\times 1$ transmitted signal vector consisted of $2N$ BPSK symbols with a constellation size of C and average symbol energy E_0 ($E_0=E_s/2N$) and the M elements of vector **w** are samples of independent complex additive white Gaussian noise (AWGN) processes with single-sided power spectral density σ_ω^2. The channel matrix **H** is assumed to have a unit variance that can be described as

$$E[\|\mathbf{H}\|_F^2] = 2NM \tag{2}$$

where $\|\mathbf{A}\|_F$ denotes the Frobenius norm of matrix **A**, it can be expressed as $\|\mathbf{A}\|_F = \sqrt{\sum_{i=1}^{a}\sum_{j=1}^{b}|A_{ij}|^2}$ and $E[\cdot]$ represents the expectation operator.

3. SYMBOL ERROR RATE ANALYSIS OF MLD AND STBC DETECTION

The transmitted signals of HMRS can be simultaneously sent by multi-user including SM users and STBC users. The detection of each scheme can be done by using the different algorithm. Therefore, SER of each technique can be derived by applying the different methods and various factors. In this section, the SER analysis of both SM and STBC techniques are separately explained. Then in section 4, the SER expression of both techniques are jointly combined to get the average SER of each user by considering the error propagation effect in SIC process.

3.1 Union Bound on SER for MLD Systems

At the transmitting side of HMRS, any users can apply MLD technique to meet the high speed data rate. The tight union bound on the SER of the nth ($n = 1,,2N$) transmitted signal stream can be derived by applying the expression in [17] and [18]. It is assumed that all the possible symbols in constellation are equally probable. The authors define sc as the set of all C possible symbols transmitted at an especial antenna, and s represents the set of C^{2N} possible symbol vectors form the $2N$ transmit antennas. The authors define the C^{2N-1} vectors s_j as the subset of s for comparing with s_i that differs in their nth position form s_j. The number of s_j vectors can be defined by $C^{2N}-(C^{2N-1})$. The distance metrics of s_i and s_j are denoted by d_i and d_j, respectively. A pairwise error occurs when $D_{ij} = d_i - d_j < 0$. Thus, the union bound on the SER of the signal stream transmitted by the nth antenna is given by

$$P_{ML} \leq C^{-2N} \sum_{j} \sum_{i,i\neq j} P_{s_c,ij} \tag{3}$$

where $P_{sc,ij} = P(D_{ij} < 0|s_c, s_j = \int_{-\infty}^{0} p(D_{ij})dD_{ij}$ represents the pairwise error probability (PEP) between s_i and s_j and $p(D_{ij})$ is the pdf of D_{ij}. The closed-form expression of $P_{s_c,ij}$ has been presented as

$$P_{s_c,ij} = \frac{1}{(1+r_{c,ij})^{2M-1}} \sum_{a}^{M-1} \begin{pmatrix} 2M-1 \\ a \end{pmatrix} r_{c_ij}^{a} \tag{4}$$

where

$$r_{c,ij} = A_{c,ij}\Gamma_{c,j} + \sqrt{(A_{c,ij}\Gamma_{c,j})^2 + 2(A_{c,ij}\Gamma_{c,j})} + 1 \tag{5}$$

and

$$\Gamma_{c,j} = \gamma_c = E_s/\sigma_\omega^2 = \bar{\gamma} \tag{6}$$

and

$$A_{c,ij} = \|\mathbf{s}_i - \mathbf{s}_j\|^2/2E_0 \quad ,i \neq j \tag{7}$$

The BPSK codebook of each transmitting antenna is $\left\{+\sqrt{E_s/2N}, -\sqrt{E_s/2N}\right\}$. Therefore, s_i - s_j in (7) can only be chosen from $\left\{+2\sqrt{E_s/2N}, -2\sqrt{E_s/2N}\right\}$.

3.2 SER Analysis of STBC Systems

After STBC streams pass through the MIMO channel, the received signals are combined according to the STBC decoding algorithm. The combined signal y_c at the receiver can be written as

$$\begin{aligned} y_c &= \left[\sum_{i=1}^{2N} \sum_{j=1}^{M} |h_{ij}|^2\right] s + \widetilde{w} \\ &= \|\mathbf{H}\|_F^2 s_l + \widetilde{w}_l, \qquad l = 1, 2, ..., L \end{aligned} \tag{8}$$

where L denotes the symbols transmitted over T time slots, the code rate of OSTBC is R=L/T, $\widetilde{w}_l$ is the noise term after combining with a distribution $CN(0, \|\mathbf{H}\|_F^2\sigma_\omega^2)$. The transmitted symbol can be decoded by $\hat{s}_c = argmin_{s\in C}|y_c - \|\mathbf{H}\|_F^2 s|^2$. Thus, the effective signal-to-noise ratio (SNR) per symbol after STBC decoding can be determined by

$$\begin{aligned} \gamma_{STBC} &= \frac{\|\mathbf{H}\|_F^4 E_0}{\|\mathbf{H}\|_F^2 \sigma_\omega^2} = \frac{E_s}{\sigma_\omega^2} \frac{1}{2N} \|\mathbf{H}\|_F^2 \\ &= \frac{\bar{\gamma}}{2N} \|\mathbf{H}\|_F^2 \end{aligned} \tag{9}$$

where $\bar{\gamma} = \frac{E_s}{\sigma_\omega^2}$ is the average SNR per receiving antenna, $E_0 = E[|s_l|^2] = E_s/2N$. For the case of perfect CSI in [25], the authors can get the following SER for BPSK modulation case.

$$\begin{aligned} P_{STBC} &= 2\Im\left(gPSK, \frac{2N}{\bar{\gamma}}, G\right) \\ &= \frac{1}{\sqrt{\pi}(1+b)^G} \frac{\Gamma(G+\frac{1}{2})}{\Gamma(G+1)} \, {}_2F_1\left(G, \frac{1}{2}; G+\frac{1}{2}; \frac{1}{1+b}\right) \\ &= 1 - \zeta \sum_{k=0}^{G-1} \begin{pmatrix} 2k \\ k \end{pmatrix} \left(\frac{1-\zeta^2}{4}\right)^k \end{aligned} \tag{10}$$

where $\Im(p, q, m) = \frac{q^m}{\Gamma(m)} \int_0^\infty Q(px)e^{-qx}x^{m-1}dx$, $Q(x) = \frac{1}{\pi}\int_0^{\pi/2} exp\left(-\frac{x^2}{2sin^2\theta}\right)d\theta$, the Gaussian hypergeometric function defined as ${}_2F_1(e, f; g; r) = \sum_{k=0}^{\infty} \frac{(e)_k(f)_k}{(g)_k}\frac{r_k}{k!}$, gPSK $= 2\sin^2(\pi/\mathrm{C})$, $G = 2NM$, $\zeta = \sqrt{\frac{gPSK}{2q+gPSK}}$, $q = 2N/\bar{\gamma} = \|\mathbf{H}\|_F^2/\gamma_{STBC}$ and $b = gPSK/2(2N/\bar{\gamma})$.

4. PERFORMANCE ANALYSIS OF HMRS

In this section, the expression of symbol error rate for HMRS systems is derived by considering both SM and STBC users. By considering section 3, the expression of SER analysis of MLD and STBC are clearly explained and it is ready to be applied in this section. In order to understand the method to derive the SER expression of HMRS, the HMRS algorithm has to be firstly discussed in section 4.1. Then, the encoding process at transmitting side and the detection at receiving side for 2 users are explained later.

4.1 HMRS Systems

The hybrid-MIMO system applies both SM and STBC to encode the transmitted signals for each user at the transmitting side. The structure of multi-user HMRS is illustrated in Fig. 2. The N users at transmitting side are encoded by using SM and STBC code where J users and $N-J$ users are applied by encoding SM and STBC code, respectively. The transmitted signals are sent through the MIMO channel. At the receiver, the ML detection is used to get all symbols of SM users. All symbols of STBC users can be successively taken by operating the $N-J$ modules of the sub-received signals generator (GEN Sub-RX), SIC and STBC decoding. In this section, the signals and detecting procedure of the HMRS are demonstrated (where N=3, J=2 and M=4). The equivalent Space-Time Coding matrix for three users HMRS can be given by

$$\mathbf{s} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ s_7 & s_8 & s_9 & s_{10} & -s_6^* & s_5^* \end{bmatrix}^T \tag{11}$$

the $M \times 2N$ MIMO channel can be described as

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{21} & h_{31} & h_{41} & h_{51} & h_{61} \\ h_{12} & h_{22} & h_{32} & h_{42} & h_{52} & h_{62} \\ h_{13} & h_{23} & h_{33} & h_{43} & h_{53} & h_{63} \\ h_{14} & h_{24} & h_{34} & h_{44} & h_{54} & h_{64} \end{bmatrix} \tag{12}$$

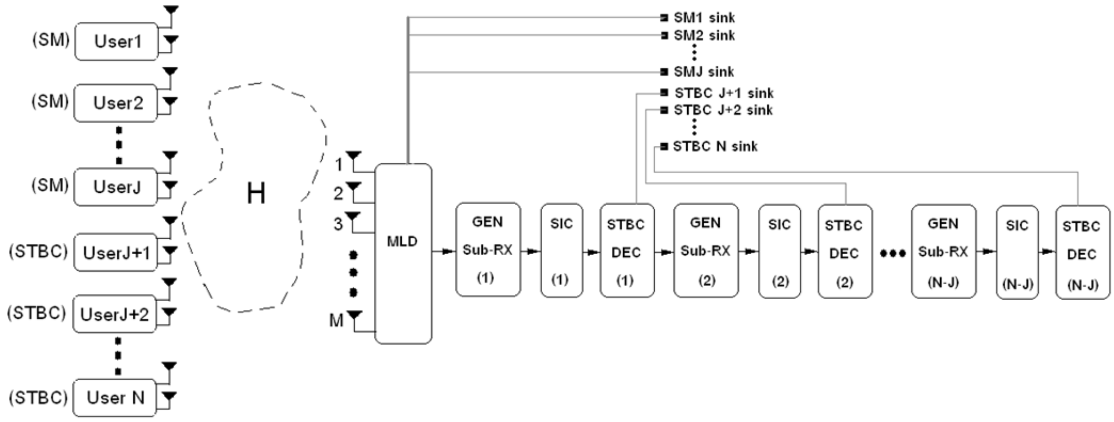

Fig.2: *HMRS technique for MU-MIMO system.*

where h_{mn} is the complex channel coefficient of the m^{th} transmitting antenna to the nth receiving antenna. The corresponding received signals can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w} = \begin{bmatrix} y_1 & y_5 \\ y_2 & y_6 \\ y_3 & y_7 \\ y_4 & y_8 \end{bmatrix} \tag{13}$$

then, all symbols of SM user can be calculated by

$$\hat{\mathbf{s}} = \underset{\tilde{s} \in C^{2N}}{arg\ min} \ \|\mathbf{y} - \mathbf{H}\tilde{\mathbf{s}}\|_F^2 \tag{14}$$

$$\hat{\mathbf{s}}_{SM} = \begin{bmatrix} \hat{s}_1 & \hat{s}_2 & \hat{s}_3 & \hat{s}_4 \\ \hat{s}_7 & \hat{s}_8 & \hat{s}_9 & \hat{s}_{10} \end{bmatrix}^T \tag{15}$$

where $\hat{\mathbf{s}}_{SM}$ denotes the 1^{st} row to the 4^{th} row in (14). The equivalent received signals of SM users can be generated as

$$\hat{\mathbf{y}}_{SM} = \mathbf{H}_{SM}\hat{\mathbf{s}}_{SM} \tag{16}$$

where $\mathbf{H}_{SM}$ is the 1st column to the 4^{th} column of MIMO channel $\mathbf{H}$. The equivalent received signals of STBC users can be calculated as

$$\hat{\mathbf{y}}_{STBC} = \mathbf{y} - \hat{\mathbf{y}}_{SM} = \begin{bmatrix} y_a & y_b \\ y_c & y_d \\ y_e & y_f \\ y_g & y_h \end{bmatrix} \tag{17}$$

$$\mathbf{y}^{\nabla}_{STBC} = [y_a \ y_b^* \ y_c \ y_d^* \ y_e \ y_f^* \ y_g \ y_h^*]^T \tag{18}$$

where $\mathbf{H}_{STBC}$ is the 5^{th} column to the 6^{th} column of MIMO channel $\mathbf{H}$. The modified $\mathbf{H}_{STBC}$ can be reformed by

$$\mathbf{H}^{\nabla}_{STBC} = \begin{bmatrix} h_{51} & h_{61}^* & h_{52} & h_{62}^* & h_{53} & h_{63}^* & h_{54} & h_{64}^* \\ h_{61} & -h_{51}^* & h_{62} & -h_{52}^* & h_{63} & -h_{53}^* & h_{64} & -h_{54}^* \end{bmatrix} \tag{19}$$

$$\hat{\mathbf{s}}_{STBC} = \mathbf{H}^{\nabla}_{STBC}\mathbf{y}^{\nabla}_{STBC} = \begin{bmatrix} \hat{s}_5 \\ \hat{s}_6 \end{bmatrix} \tag{20}$$

Finally, the decoding symbols of SM and STBC users can be taken from (15) and (20), respectively. As indicated in the HMRS procedure, all symbols of SM users can easily be obtained by using ML detection in only one time but HMTS technique in [14] applies MMSE filter many times to take all symbols of SM users. Hence it can be indicated that the HMRS can offer the benefit of simplification more than the HMTS.

4.2 Performance Analysis of 1SM+1STBC HMRS Systems

In this mode, each user applies MLD and STBC, respectively. The transmitted symbol can be detected by using MLD, SIC and STBC decoding. By considering from (14), the SER expression after MLD detecting as well as SER of SM user ($P_{e,SM}$) can be determined by (3) where the authors define N=2 and M=4. P_{ML} represents the probability of symbol error equally for each transmitted stream from all transmitting antennas. In order to decode the STBC streams, the interference of SM user needs to be cancelled from the received signals according to (17). Then the error propagation P_{en} (where the subscript n=1,2,3,4) after SIC process is presented ($if\ \hat{s}_n - s_n \neq 0$) according to Table 1. where $P_r\{\hat{s}_1 \neq s_1 \bigcap \hat{s}_2 \neq s_2\}$ represents the probability of the event that both s_1 and s_2 are incorrectly detected by MLD. The error propagation from possible 4 cases are used to calculate the average symbol error rate of STBC user of HMRS system. Because the system has two SM detecting symbols ($\hat{s}_1\ and\ \hat{s}_2$), each case has a different value of error propagation depending on the detectable results of s_1 and s_2. By considering 4 cases of the error propagations in Table 1, the different effective SNRs of STBC user are generated by the error propagations that the effective SNRs are presented in Table 2. From Table 2, the 4 different effective SNRs are used to determine symbol error probability (P_{stcn}) of each

Table 1: *Error propagation of 1SM+1STBC HMRS.*

Case	Error propagation
1) $P_r\{\hat{s}_1 \neq s_1 \bigcap \hat{s}_2 \neq s_2\}$	$P_{e1} = P_{ML}^2$
2) $P_r\{\hat{s}_1 \neq s_1 \bigcap \hat{s}_2 = s_2\}$	$P_{e2} = P_{ML}(1 - P_{ML})$
3) $P_r\{\hat{s}_1 = s_1 \bigcap \hat{s}_2 \neq s_2\}$	$P_{e3} = P_{e2}$
4) $P_r\{\hat{s}_1 = s_1 \bigcap \hat{s}_2 = s_2\}$	$P_{e4} = (1 - P_{ML})^2$

Table 2: *Effective SNRs of 1SM+1STBC HMRS.*

Case	Effective SNR	SER of each case
1	$\gamma_{c1} = \frac{\lVert \mathbf{H}' \rVert_F^2}{\left(\frac{2N}{\bar{\gamma}} + 4^2\right)}$	$P_{stc1} = P_r\{\hat{s}_{STBC} \neq s_{STBC} \mid P_{el}\}$
2	$\gamma_{c2} = \frac{\lVert \mathbf{H}' \rVert_F^2}{\left(\frac{2N}{\bar{\gamma}} + 2^2\right)}$	$P_{stc2} = P_r\{\hat{s}_{STBC} \neq s_{STBC} \mid P_{e2}\}$
3	$\gamma_{c3} = \gamma_{c2}$	$P_{stc3} = P_r\{\hat{s}_{STBC} \neq s_{STBC} \mid P_{e3}\}$
4	$\gamma_{c4} = \gamma \frac{-\lVert \mathbf{H}' \rVert_F^2}{2N}$	$P_{stc4} = P_r\{\hat{s}_{STBC} \neq s_{STBC} \mid P_{e4}\}$

case by applying γ_{cn} in (10), where N=1 (because 2 streams of SM user are cancelled from the received signals), s_{STBC} denotes the transmitted symbol of STBC user (s_3 or s_4) and H' represents the $M \times 2$ MIMO channel of STBC user. Finally, the error propagation and probability of symbol error (P_{stcn}) of all cases from Table 1 and Table 2 are jointly combined to calculate the average SER of STBC user-HMRS, is given by

$$
\begin{aligned}
&P_{e,STBC_{(1SM+1STBC\ HMRS)}} \leq P_{e1}P_{stc1} + P_{e2}P_{stc2} \\
&+ P_{e3}P_{stc3} + P_{e4}P_{stc4} \\
&P_{e,STBC_{(1SM+1STBC\ HMRS)}} \leq \\
&\left\{ C^{-4} \sum_j \sum_{i,i\neq j} \frac{1}{(1+r_{c,ij})^7} \sum_a^3 \binom{7}{a} r_{c,ij}^a \right\}^2 \\
&x \left\{ 1 - \sqrt{\frac{2}{2q_{c1}+2}} \sum_{k=0}^{7} \binom{2k}{k} \left(\frac{\left(1 - \left(\sqrt{\frac{2}{2q_{c1}+2}}\right)^2\right)}{4} \right)^k \right\} \\
&+ 2C^{-4} \sum_j \sum_{i,i\neq j} \frac{1}{(1+r_{c,ij})^7} \sum_a^3 \binom{7}{a} r_{c,ij}^a \\
&x \left\{ 1 - \left\{ C^{-4} \sum_j \sum_{i,i\neq j} \frac{1}{(1+r_{c,ij})^7} \sum_a^3 \binom{7}{a} r_{c,ij}^a \right\} \right\} \\
&x \left\{ 1 - \sqrt{\frac{2}{2q_{c2}+2}} \sum_{k=0}^{7} \binom{2k}{k} \left(\frac{\left(1 - \left(\sqrt{\frac{2}{2q_{c2}+2}}\right)^2\right)}{4} \right)^k \right\} \\
&+ \left\{ 1 - \left\{ C^{-4} \sum_j \sum_{i,i\neq j} \frac{1}{(1+r_{c,ij})^7} \sum_a^3 \binom{7}{a} r_{c,ij}^a \right\} \right\}^2 \\
&x \left\{ 1 - \sqrt{\frac{2}{2q_{c4}+2}} \sum_{k=0}^{7} \binom{2k}{k} \left(\frac{\left(1 - \left(\sqrt{\frac{2}{2q_{c4}+2}}\right)^2\right)}{4} \right)^k \right\}
\end{aligned} \tag{21}
$$

where $q_{cn} = \lVert \mathbf{H}' \rVert_F^2 / \gamma_{cn}$, n=1,2,3,4. The average SER of SM user-HMRS is already determined by (3), then it can be written as $P_{e,SM_{(1SM+1STBC\ HMRS)}} = P_{ML_{N=2,M=4}}$.

Table 3: *error propagation of 2SM+1STBC HMRS*

Error propagation	The number of occurrences of the event
$P_{c1} = P_{ML}^4$	1
$P_{c2} = P_{ML}^3(1 - P_{ML})$	4
$P_{c3} = P_{ML}^2(1 - P_{ML})^2$	6
$P_{c4} = P_{ML}(1 - P_{ML})^3$	4
$P_{c5} = (1 - P_{ML})^4$	1

Table 4: *Effective SNRs of 2SM+1STBC HMRS*

Case	Effective SNR	SER of each case
1	$\gamma_{d1} = \frac{\lVert \mathbf{H}' \rVert_F^2}{\left(\frac{2N}{\bar{\gamma}} + 8^2\right)}$	$P_{s1} = P_r\{\hat{s}_{STBC} \neq s_{STBC} \mid P_{cl}\}$
2	$\gamma_{d2} = \frac{\lVert \mathbf{H}' \rVert_F^2}{\left(\frac{2N}{\bar{\gamma}} + 6^2\right)}$	$P_{s2} = P_r\{\hat{s}_{STBC} \neq s_{STBC} \mid P_{c2}\}$
3	$\gamma_{d3} = \frac{\lVert \mathbf{H}' \rVert_F^2}{\left(\frac{2N}{\bar{\gamma}} + 4^2\right)}$	$P_{s3} = P_r\{\hat{s}_{STBC} \neq s_{STBC} \mid P_{c3}\}$
4	$\gamma_{d4} = \frac{\lVert \mathbf{H}' \rVert_F^2}{\left(\frac{2N}{\bar{\gamma}} + 2^2\right)}$	$P_{s4} = P_r\{\hat{s}_{STBC} \neq s_{STBC} \mid P_{c4}\}$
5	$\gamma_{d5} = \gamma \frac{-\lVert \mathbf{H}' \rVert_F^2}{2N}$	$P_{s5} = P_r\{\hat{s}_{STBC} \neq s_{STBC} \mid P_{c5}\}$

4.3 Performance Analysis of 2SM+1STBC HMRS System

In this mode, MLD and STBC are applied by 2 SM users and 1 STBC user, respectively. All transmitted streams can be detected according HMRS algorithm. By considering from (14), the SER expression after MLD detecting as well as SER of SM user ($P_{e,SM}$) can be determined by (3) where the authors define N=3 and M=4. P_{ML} represents the probability of symbol error equally for each transmitted stream from all transmitting antennas. In order to decoding the STBC streams (s_5 and s_6), the interference of 2 SM users (s_1, s_2, s_3 and s_4) needs to be cancelled from the received signals by (17). Then the error propagation after SIC process is presented according to Table 3.

where P_{c1} represents event that 4 symbols are wrong detected, P_{c2} represents event that 3 symbols are wrong detected, P_{c3} represents event that 2 symbols are wrong detected, P_{c4} represents event that 1 symbols are wrong detected and P_{c5} represents event that 4 symbols are correctly detected. From Table 4,

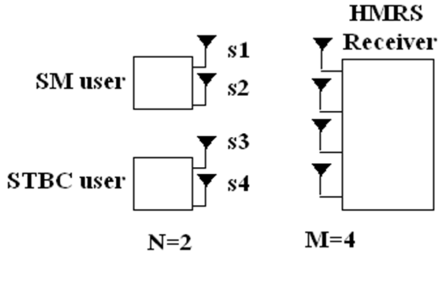

Fig.3: *1SM+1STBC HMRS system.*

the 5 different effective SNRs are used to determine SER (P_{sn}) of each case by applying γ_{dn} in (10), where N =1 (because 4 streams of SM users are cancelled from the received signals). Finally, the error propagation and probability of symbol error (P_{sn}) of all cases from Table 3 and Table 4 are jointly combined to calculate the average SER of STBC user which is given by

$$
\begin{aligned}
&P_{e,STBC_{(2SM+1STBC\ HMRS)}} \leq P_{c1}P_{s1} + 4P_{c2}P_{s2} \\
&+ 6P_{c3}P_{s3} + P_{c4}P_{s4} + P_{c5}P_{s5} \\
&P_{e,STBC_{(2SM+1STBC\ HMRS)}} \leq \\
&\left\{ C^{-6} \sum_{j} \sum_{i,i\neq j} \frac{1}{(1+r_{c,ij})^7} \sum_{a}^{3} \begin{pmatrix} 7 \\ a \end{pmatrix} r_{c,ij}^{a} \right\}^4 \\
&x \left\{ 1 - \sqrt{\frac{2}{2q_{d1}+2}} \sum_{k=0}^{7} \begin{pmatrix} 2k \\ k \end{pmatrix} \left(\frac{\left(1 - \left(\sqrt{\frac{2}{2q_{d1}+2}}\right)^2\right)}{4} \right)^k \right\} \\
&+ 4 \left\{ C^{-6} \sum_{j} \sum_{i,i\neq j} \frac{1}{(1+r_{c,ij})^7} \sum_{a}^{3} \begin{pmatrix} 7 \\ a \end{pmatrix} r_{c,ij}^{a} \right\}^3 \\
&x \left\{ 1 - \left\{ C^{-6} \sum_{j} \sum_{i,i\neq j} \frac{1}{(1+r_{c,ij})^7} \sum_{a}^{3} \begin{pmatrix} 7 \\ a \end{pmatrix} r_{c,ij}^{a} \right\} \right\} \\
&x \left\{ 1 - \sqrt{\frac{2}{2q_{d2}+2}} \sum_{k=0}^{7} \begin{pmatrix} 2k \\ k \end{pmatrix} \left(\frac{\left(1 - \left(\sqrt{\frac{2}{2q_{d2}+2}}\right)^2\right)}{4} \right)^k \right\} \\
&+ 6 \left\{ C^{-6} \sum_{j} \sum_{i,i\neq j} \frac{1}{(1+r_{c,ij})^7} \sum_{a}^{3} \begin{pmatrix} 7 \\ a \end{pmatrix} r_{c,ij}^{a} \right\}^2 \\
&x \left\{ 1 - \left\{ C^{-6} \sum_{j} \sum_{i,i\neq j} \frac{1}{(1+r_{c,ij})^7} \sum_{a}^{3} \begin{pmatrix} 7 \\ a \end{pmatrix} r_{c,ij}^{a} \right\} \right\}^2 \\
&x \left\{ 1 - \sqrt{\frac{2}{2q_{d3}+2}} \sum_{k=0}^{7} \begin{pmatrix} 2k \\ k \end{pmatrix} \left(\frac{\left(1 - \left(\sqrt{\frac{2}{2q_{d3}+2}}\right)^2\right)}{4} \right)^k \right\} \\
&+ 4 \left\{ C^{-6} \sum_{j} \sum_{i,i\neq j} \frac{1}{(1+r_{c,ij})^7} \sum_{a}^{3} \begin{pmatrix} 7 \\ a \end{pmatrix} r_{c,ij}^{a} \right\} \\
&x \left\{ 1 - \left\{ C^{-6} \sum_{j} \sum_{i,i\neq j} \frac{1}{(1+r_{c,ij})^7} \sum_{a}^{3} \begin{pmatrix} 7 \\ a \end{pmatrix} r_{c,ij}^{a} \right\} \right\}^3 \\
&x \left\{ 1 - \sqrt{\frac{2}{2q_{d4}+2}} \sum_{k=0}^{7} \begin{pmatrix} 2k \\ k \end{pmatrix} \left(\frac{\left(1 - \left(\sqrt{\frac{2}{2q_{d4}+2}}\right)^2\right)}{4} \right)^k \right\} \\
&+ \left\{ 1 - \left\{ C^{-6} \sum_{j} \sum_{i,i\neq j} \frac{1}{(1+r_{c,ij})^7} \sum_{a}^{3} \begin{pmatrix} 7 \\ a \end{pmatrix} r_{c,ij}^{a} \right\} \right\}^4 \\
&x \left\{ 1 - \sqrt{\frac{2}{2q_{d5}+2}} \sum_{k=0}^{7} \begin{pmatrix} 2k \\ k \end{pmatrix} \left(\frac{\left(1 - \left(\sqrt{\frac{2}{2q_{d5}+2}}\right)^2\right)}{4} \right)^k \right\}
\end{aligned} \tag{22}
$$

where $q_{dn} = \|\mathbf{H}^{'}\|_F^2/\gamma_{dn}$, n={1,2,3,4,5}. The average SER of SM users is already determined by (3), then it can be written as $P_{e,SM_{(2SM+1STBC\ HMRS)}} = P_{ML_{N=3,M=4}}$.

In case of N is defined more than 3, the average SER can also be analyzed by calculating the number of occurrences of the event for each error propagation case, computing the effective SNR and writing all patterns of error propagation for $2^{2(N-1)}$ events. Then the analytical SER equation can be written by the method in (22).

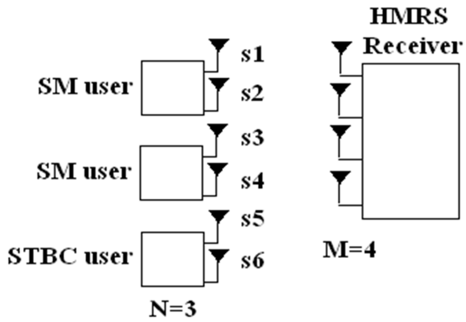

Fig.4: *2SM+1STBC HMRS system.*

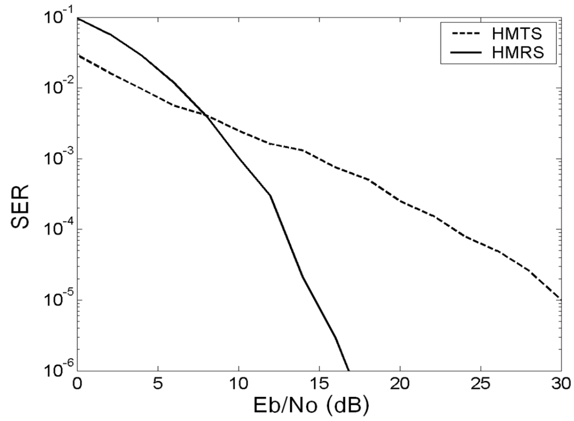

Fig.5: *SER performance of HMRS and HMTS techniques.*

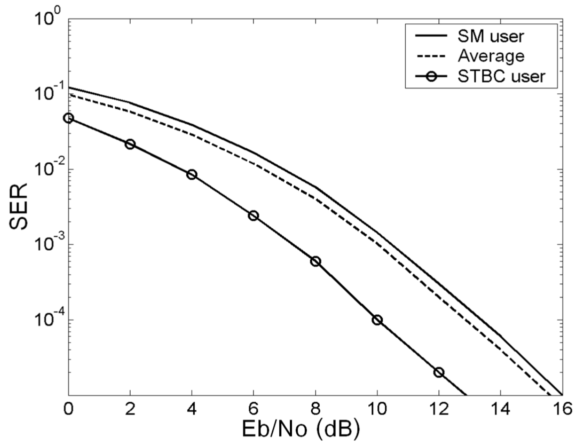

Fig.6: *SER performance of 1SM+1STBC HMRS system.*

5. NUMERICAL RESULTS AND DISCUSSION

In this section, the authors present numerical results for symbol error probability of HMRS systems

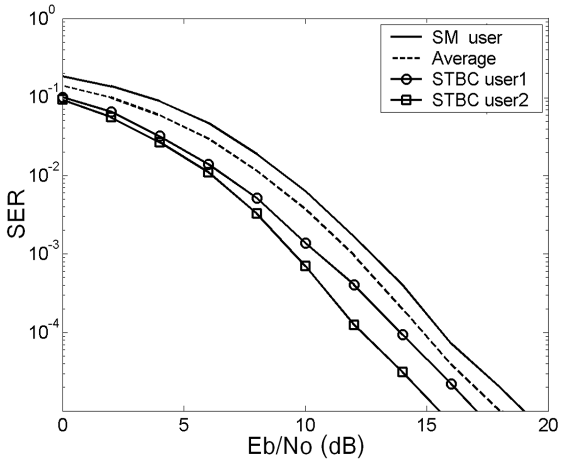

Fig.7: *SER performance of 1SM+2STBC HMRS system.*

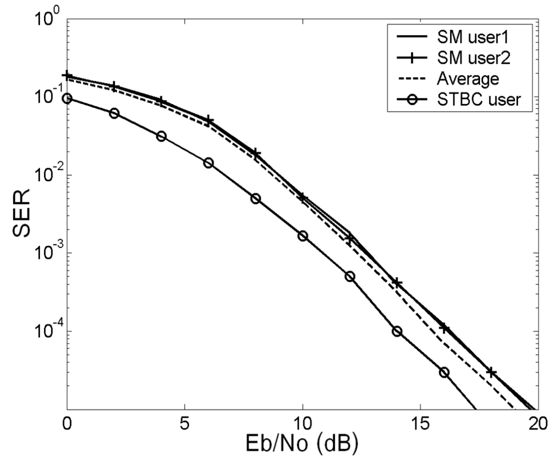

Fig.8: *SER performance of 2SM+1STBC HMRS system.*

including SM and STBC users. The parameter N and M are varied to reveal the HMRS performances that support various types of user transmission modes. The performance comparisons between Monte-Carlo simulation and analysis are also presented to demonstrate the performance accuracy of this work.

5.1 Simulation Results of HMRS

The simulations of multi-user HMRS system in case of N users (N=2, 3 and 4) and M=4 are presented to reveal the effect of increasing the number of users at the transmitting side. The several transmitting configurations are assigned in the Monte-Carlo simulation model to investigate the trend of HMRS performance under the multi-user situation. Fig. 5 illustrates the SER performance comparison of HMRS and HMTS in case of two users. Both SM and STBC users apply BPSK modulation to form the transmitted signals. As the result, HMRS can offer higher performance than HMTS about 8 dB at 10^{-4} SER.

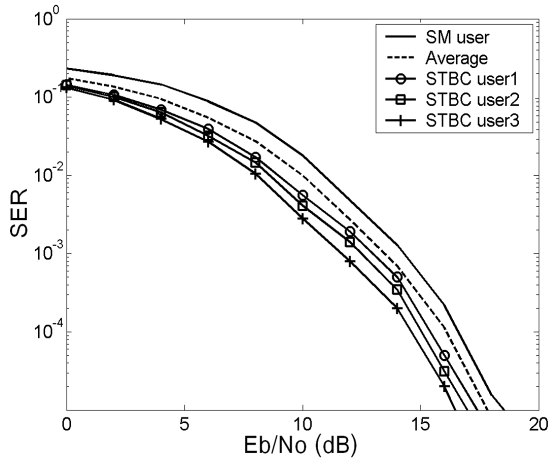

Fig.9: *SER performance of 1SM+3STBC HMRS system.*

Fig.6 presents the SER performance of the 1SM+1STBC HMRS system. The STBC user can achieve a higher performance than SM user because of applying the interference cancellation and the orthogonal structure of STBC code. The dash line shows the average performance of a whole system.

Fig.7 illustrates the SER performance of the 1SM+2STBC HMRS system, the STBC code is used by two users and the SM code is used by one user. The highest performance can be obtained by STBC user2 because the two steps of the interference cancellation are continuously used at the receiver to detect the symbols of STBC user2. The last layer has the minimum level of interference.

Fig.8 presents the SER performance of the 2SM+1STBC HMRS system, the STBC code is used by one user and the SM code is used by two users. The highest performance can be taken by STBC user because the SIC technique is employed before STBC layer is detected. The performances of two SM users are equivalence because the symbols of all SM users can be taken by MLD in one time. The HMRS in Fig. 7 has a higher performance than the HMRS in Fig. 8 because the two steps of the interference cancellation are continuously applied.

Fig.9 shows the SER performance of the 1SM+3STBC HMRS system. 1 SM and 3 STBC users are placed at the transmitting side. The highest performance can be achieved by STBC user3 because the three steps of the interference cancellation are continuously operated at the receiver to detect the symbols of STBC user3. The average performance of this mode has a higher performance than the HMRS in Fig. 10 and Fig. 11 because the interference level of this system can intensively be decreased by using many SIC modules and the orthogonal property of STBC users.

Fig.10 shows the SER performance of the 2SM+2STBC HMRS system. The STBC code is used

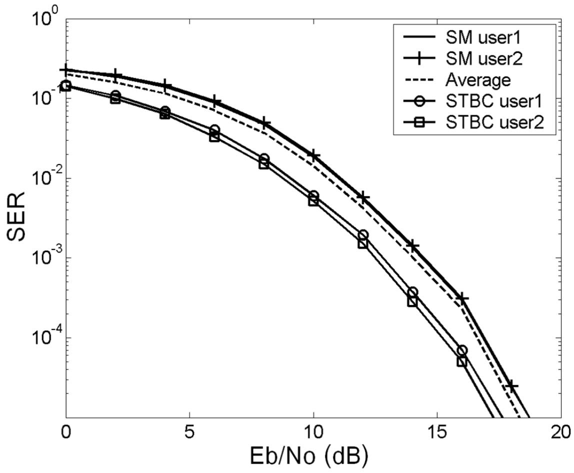

Fig.10: *SER performance of 2SM+2STBC system.*

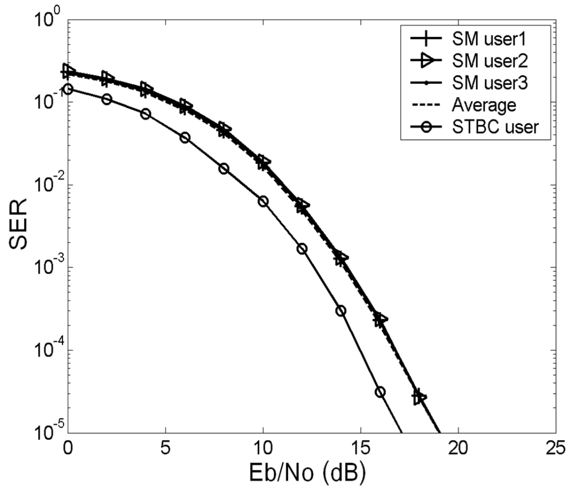

Fig.11: *SER performance of 3SM+1STBC HMRS system.*

by two users and the SM code is used by two users. The highest performance can be achieved by STBC user2 because the two steps of the interference cancellation are continuously applied at the receiver to detect the symbols of STBC user2. The performance of STBC users can also be enhanced by the orthogonal property of STBC code.

Fig.11 shows the SER performance of the 3SM+1STBC HMRS system. The STBC code is used by one user and the SM code is used by three users. The highest performance can be taken by STBC user. The performances of all SM users are equal because the symbols of all SM users can be taken by ML detection in one time.

5.2 Analytical Results of Error Rate Performance

Fig.12 presents the comparing results between analytical SER given by (21) and simulation results. The simulations employ BPSK modulation with one SM

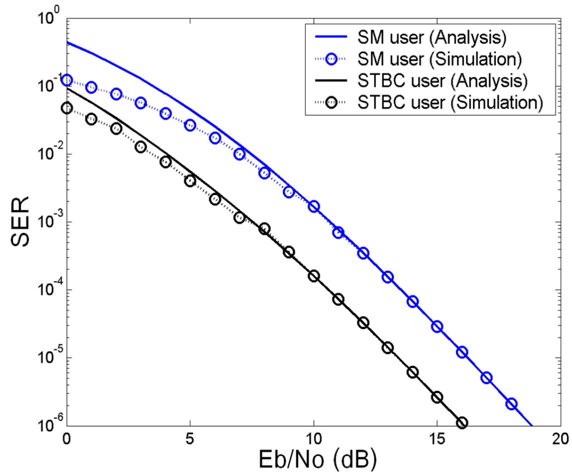

Fig.12: *Comparison between simulation results and analytical results for 1SM+1STBC HMRS system.*

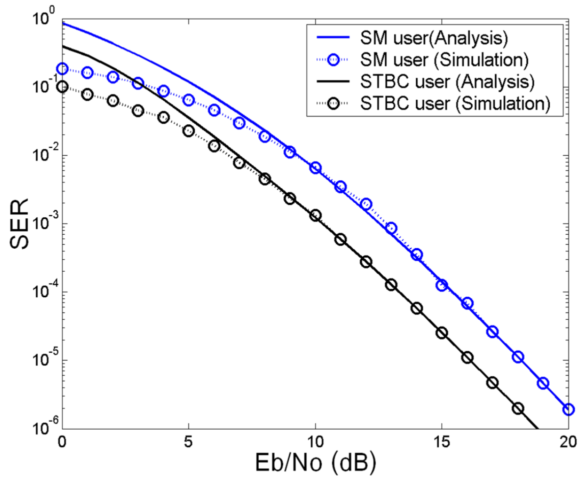

Fig.13: *Comparison between simulation results and analytical results for 2SM+1STBC HMRS system.*

user, one STBC user (1SM+1STBC HMRS system) and perfect CSI. The authors observe that, when the SER of simulation is below 0.01, the maximum relative error between simulation and analytical results is less than 5.3%.

Fig.13 shows the performance comparison between the analytical results given by (22) and simulation results for BPSK modulation with two SM users, one STBC user (2SM+1STBC HMRS system) and perfect CSI. The authors observe that, when the SER of simulation is below 0.01, the maximum relative error between simulation and analytical results is less than 3.6%.

From the results in Fig. 12 and Fig. 13, the relative error between simulation and analytical results is presented due to the approximation of the union bound on the SER in (3), (21) and (22). However, the performances in Fig. 12 and Fig. 13 can properly be used for general wireless systems because the relative error is not presented in the high SNR region, especially at 10^{-4}SER or better level.

6. CONCLUSIONS

In this paper, the investigation of MU-MIMO system with HMRS technique for N users (N=2, 3 and 4) are presented. The study of SER performance when increasing the number of users is reported. From the results, the average SER performance of the HMRS is degraded when the numbers of users is increased. The SER performances of HMRS can be improved when the number of STBC users is more than the number of SM users but the spectral efficiency is also degraded. The SER performances of SM users in any configurations of HMRS are corresponded when they have the same total number of users. When comparing between HMRS and HMTS performances, it can indicate that the HMRS offers a better advantage than HMTS, both in SER performance and the complexity of the detecting procedure under the same spectral efficiency scenario. In case of the 1SM+2STBC HMRS system, the simulation result can reveal that the performance of STBC user1 is close to SM user. The STBC user2 give a higher performance than STBC user1. Therefore, the system needs to detect the symbols of STBC user2 in the second step if the location of STBC user2 is placed far away from the base station more than STBC user1. In case of 2SM+2STBC HMRS system, the equivalent performances of both STBC users are presented. This transmission mode can appropriately be applied in MU-MIMO systems in the situation of the same distance between the location of both STBC users and the base station. In case of the multi-STBC user scenario, the symbol of STBC users can be detected in the end of process if the locations of the STBC users are positioned at the largest distance from the base station. Consequently, the signals of other cells are mildly interfered because this STBC layer can be transmitted by applying the lowest power. From above reasons, each transmission mode can offer the different benefits. Especially, the base station needs to detect the symbols in each STBC layer sequentially by considering the level of interference in each layer and the position of each user. Moreover, the limit of the Multi-user HMRS depends on the number of users. When many users are placed in the system, the complexity of ML detection is intensely increased. Therefore the system needs to choose the number of users carefully. The corresponding performances of analytical result and simulation result can indicate the accuracy of this work.

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