# Aggregating Method of Induction Motor Group Using Energy Conservation Law

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**ABSTRACT**

This paper proposes a simple and efficient method for aggregating a group of induction motors, which are connected at the same bus. The parameters of aggregate motor are derived using a technique based on energy conservation law. An accuracy of the aggregate model parameters is verified by comparing dynamic responses obtained from the aggregate motor with the sum of individual motors. The presented technique gives accurate parameters in which the dynamic responses from the aggregate motor closely reflect the behaviour of a whole group of the motors under study.

**Keywords**: Aggregate model, induction motor.

## 1. INTRODUCTION

In highly stressed area of power system, the type of static voltage-independent load may be inadequate due to its neglect of the dynamic nature [1]. For example, in large industrial plants a significant portion of system load is comprised of an appreciable percentage of large induction motors. Their dynamic responses play a key role in the transient behaviour of entire system [2, 3]. In order to obtain realistic dynamic responses of power system, they must be precisely included into the power system simulation. Since the large industrial plants have composed of large numbers of induction motors, it is not realistic to model every induction motor that is in the system. Hence, aggregate models or single-unit models with minimum order of induction motor are needed to represent a group of motors. Along the chronological order, various methods have been proposed for handing with aggregation of induction motor models [4-16]. Among them, a method in [4] is to replace a group of induction motors by a single equivalent unit whose parameters are identified from the steady-state consideration. In [9], the steady state and the transient starting-up approaches have been used for finding the equivalent of circuit and inertia of aggregate induction motors, respectively. Moreover, in [10] the technique based on Thevenin's theorem and transient properties of induction machine has been employed

to calculate the handy index for grouping the large- and low-slip induction motors due to the difference in the electromagnetic inertia among them. In [11], a new aggregate model based on the transformer-type equivalent circuit has been proposed with a grouping criterion to classify homogeneous motor. In [13], the technique to aggregate the single-cage induction motor using the double-cage model of induction machine is presented. In [12, 14-16], the no-load and locked-rotor methods are commonly used for finding the single-unit equivalent circuit of induction motors.

Although several techniques for grouping induction motor loads have been already presented, more efficient method is still required in order to reduce the complexity of grouping procedures. Hence, this paper proposes a simple and efficient technique based on energy conservation law to find equivalent circuit parameters of a group of parallel induction motor loads. The obtained parameters of the aggregate motor are shown and compared with those appearing in the open literatures. Moreover, the dynamic simulation responses given from the sum of individual motor model and from the aggregate model are compared to verify the computing accuracy of the obtained parameters.

## 2. AGGREGATION TECHNIQUE OF MULTIPLE INDUCTION MOTORS

Generally, the steady-state model of induction motor is represented by the equivalent circuit as shown in Fig. 1. $R_s$ and $R_r$ are stator and rotor resistance. $X_{ls}$ and $X_{lr}$ are stator and rotor leakage reactance. $X_m$ is mutual reactance. Let considering the group of induction motors which are connected at the same buses as shown in Fig. 2. In the grouping procedure, it is initially assumed that all parameters of each motor are known. These parameters are required to be adjusted to the same common MVA base. If the operating slip of each individual motor is not available, it can be alternatively computed using (1) with terminal voltage of 1.0pu ($V_s$) as,

$$T_m - T_e = 0 \qquad (1)$$

where,

$$T_m = T_0(A(1 - s)^2 + B(1 - s) + C) \qquad (2)$$

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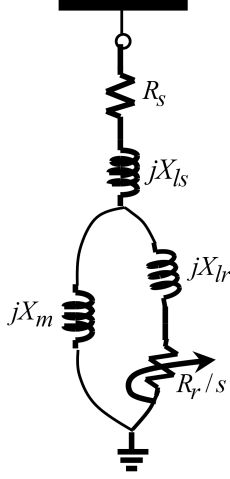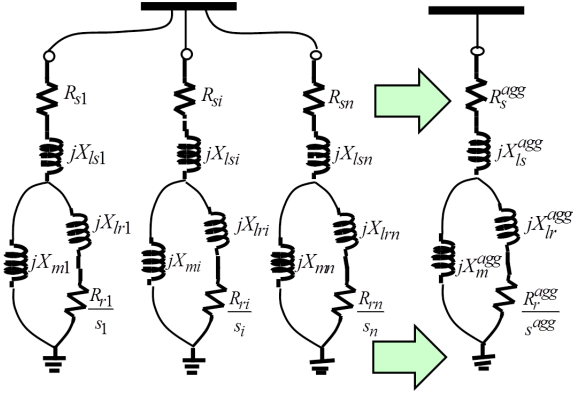**Fig.1:** *Equivalent circuit model of induction motor.*



**Fig.2:** *Aggregate equivalent circuit model.*

$$T_e = \left((X_m V_s)^2 \Big/ \left((R_{th} + \frac{R_r}{s})^2 + (X_{th} + X_{lr})^2\right)(R_s^2 + (X_m + X_{ls})^2)\right)\frac{R_r}{s} \tag{3}$$

$$R_{th} = \frac{R_s X_m^2}{R_s^2 + (X_m + X_{ls})^2} \tag{4}$$

$$X_{th} = \frac{R_s^2 X_m + X_m X_{ls}(X_m + X_{ls})}{R_s^2 + (X_m + X_{ls})^2} \tag{5}$$

The input current, active, and reactive powers of all motors are then computed using the equivalent circuit in Fig. 1. In order to find the aggregate model or single unit model of them, the law of energy conservation is applied in this paper. The apparent power absorbed by the aggregate motor is equal to total power absorbed by all motors. Hence, the total stator and rotor currents of aggregate motor can be expressed by,

$$\widetilde{I}_s^{agg} = \sum_{i=1}^{n} \widetilde{I}_{si} \tag{6}$$

$$\widetilde{I}_r^{agg} = \sum_{i=1}^{n} \widetilde{I}_{ri} \tag{7}$$

The tilde sign applied on the top of variables indicates phasor quantities. Based on the energy conservation law, the circuit parameters of the aggregate motor can be derived as follows,

$$R_s^{agg} = \frac{\sum_{i=1}^{n} |\widetilde{I}_{si}|^2 R_{si}}{|\widetilde{I}_s^{agg}|^2} \tag{8}$$

$$R_r^{agg} = \frac{\sum_{i=1}^{n} |\widetilde{I}_{ri}|^2 R_{ri}}{|\widetilde{I}_r^{agg}|^2} \tag{9}$$

$$X_{ls}^{agg} = \frac{\sum_{i=1}^{n} |\widetilde{I}_{si}|^2 X_{lsi}}{|\widetilde{I}_s^{agg}|^2} \tag{10}$$

$$X_{lr}^{agg} = \frac{\sum_{i=1}^{n} |\widetilde{I}_{ri}|^2 X_{lri}}{|\widetilde{I}_r^{agg}|^2} \tag{11}$$

$$X_m^{agg} = \frac{\sum_{i=1}^{n} |\widetilde{I}_{si} - \widetilde{I}_{ri}|^2 X_{mi}}{|\widetilde{I}_s^{agg} - \widetilde{I}_r^{agg}|^2} \tag{12}$$

with the same technique, the air-gap power of the aggregate motor can be expressed by,

$$P_{ag}^{agg} = \sum_{i=1}^{n} \{Re(\widetilde{V}_{si}\widetilde{I}_{si}^*) - |\widetilde{I}_{si}|^2 R_{si}\} \tag{13}$$

the slip of the aggregate motor can be then computed by,

$$s^{agg} = \frac{|\widetilde{I}_r^{agg}|^2 R_r^{agg}}{P_{ag}^{agg}} \tag{14}$$

the inertia constant of a group of the motor can be found by the kinetic energy conservation law as follows,

$$H^{agg} = \frac{\sum_{i=1}^{n} H_i S_i}{S^{agg}} \tag{15}$$

where,

$$H_i S_i = \frac{1}{2} J_i \omega_{si}^2 \tag{16}$$

$$S^{agg} = \sum_{i=1}^{n} S_i \tag{17}$$

It is noted that $S_i$ is the rated kVA of each motor. For the case where the moment of inertia $J^{agg}$ of the aggregate motor is needed, it can be found by,

$$J^{agg} = \frac{2H^{agg} S^{agg}}{(\omega_m^{agg})^2} \tag{18}$$

where,

$$\omega_m^{agg} = \omega_s(1 - s^{agg})(\frac{2}{P^{agg}}) \tag{19}$$

It should be noted that the pseudo number of pole $P^{agg}$ can be determined from [15]. Next, the mechanical load torque coefficients can be derived from

the assumption that the total amount of mechanical power equals to the mechanical power delivered by all motors in the group. Hence,

$$T_m^{agg}(1 - s^{agg}) = \sum_{i=1}^{n} T_{mi}(1 - s_i) \qquad (20)$$

where,

$$\begin{aligned} T_m^{agg} = & T_0^{agg}(A^{agg}(1 - s^{agg})^2 \\ & + B^{agg}(1 - s^{agg}) + C^{agg}) \end{aligned} \qquad (21)$$

$$T_{mi} = T_{0i}(A_i(1 - s^{agg})^2 + B_i(1 - s^{agg}) + C_i) \qquad (22)$$

$$A^{agg} + B^{agg} + C^{agg} = 1 \qquad (23)$$

By setting the slip in (20) equal to zero, the parameter $T_0^{agg}$ can be given by,

$$T_0^{agg} = \sum_{i=1}^{n} T_{0i}(A_i + B_i + C_i) \qquad (24)$$

After $T_0^{agg}$ is already known, the torque coefficients in (21) can be computed as,

$$A^{agg} = \frac{\sum_{i=1}^{n} T_{0i}A_i(1 - s_i)^3}{T_0^{agg}(1 - s^{agg})^3} \qquad (25)$$

$$B^{agg} = \frac{\sum_{i=1}^{n} T_{0i}B_i(1 - s_i)^2}{T_0^{agg}(1 - s^{agg})^2} \qquad (26)$$

$$C^{agg} = \frac{\sum_{i=1}^{n} T_{0i}C_i(1 - s_i)}{T_0^{agg}(1 - s^{agg})} \qquad (27)$$

## 3. SIMULATION RESULTS

In this section, the aggregate parameter obtained from the proposed technique have been presented and compared with those published in the open literatures. The industrial power system shown in Fig. 3 is chosen as a test system. The system consists of a group of induction motor loads. A group of motors $M_1$-$M_5$, connected at the same bus (bus No. 5), is the main of interest. Their parameters are taken from [11] and listed in Table 1. The mechanical torque characteristics of each motor are uniquely set through coefficients $A$, $B$, and $C$, respectively.

After applying the technique presented through the previous section, a set of parameter of the single-unit equivalent circuit representing of five individual induction motors ($M_1$-$M_5$), is obtained as summarized in Table 1 (row 7). The horse power output of the aggregate motor is 198 in total. Moreover, Table 1 indicates another two sets of the aggregate motor parameters that are derived from two different methods based on the transformer-type equivalent circuit [11] and no-load and lock-rotor technique [16]. It is evident that the parameters of aggregate motor obtained using the proposed technique very much agree with those of the other two published techniques. These
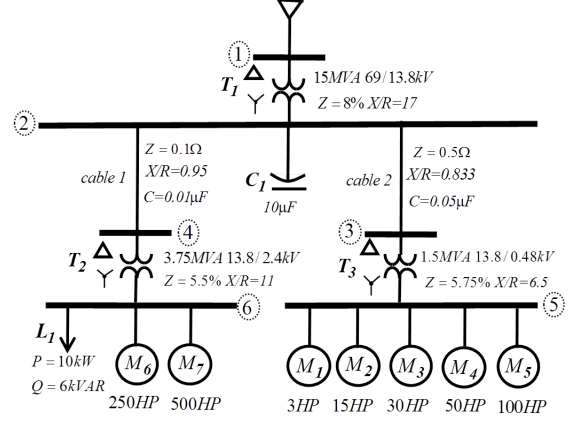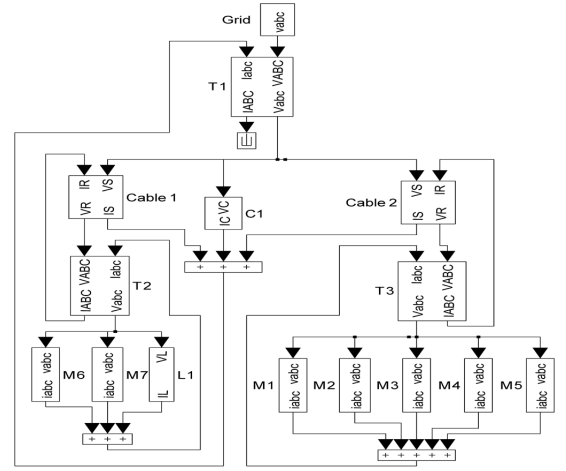


**Fig.3:** *Industrial power system.*



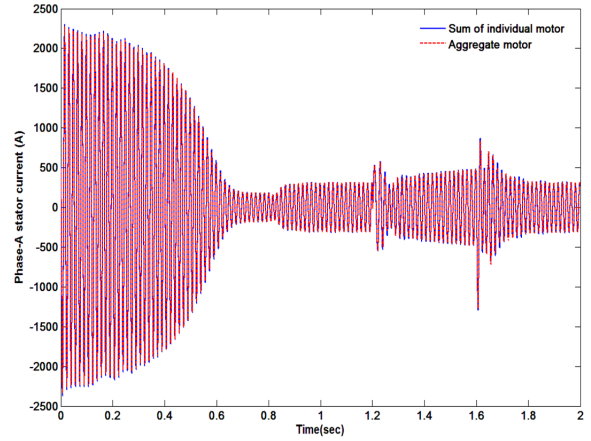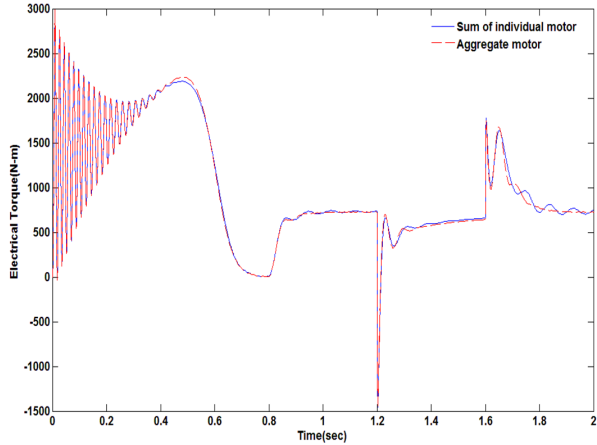**Fig.4:** *Industrial power system with Matlab/Simulink model.*

satisfactory results confirm that the energy conservation law can be one of the simple and effective techniques in which the aggregate model parameters of induction motors is accurately found.

To verify the motor's dynamic responses during transient period, the dynamic model of the industrial power system in Fig. 3 is implemented through Matlab/Simulink environment as illustrated in Fig. 4. The graphical connection of the model in Fig. 4 has been discussed in detail in [17].

It consists of objected-oriented block diagrams of transformers ($T_1$-$T_3$), line cables (Cable 1 and 2), static and induction motor loads ($L_1$ and $M_1$-$M_7$). The fifth-order model of induction motor is fully considered in this dynamic simulation. The system dynamic simulation is started with all state variables initially holding from zero. The sums of actual electrical torque and stator current obtained from the five individual motors are compared with those of the aggregate model, whose parameters are taken from Table 1 (row 7). The current magnitudes in peak value, transient electrical torques, and instantaneous wave-

**Table 1:** *Parameters of the five individual motor $M_1$-$M_5$ and aggregate motor (R and X are in ohm, $N_r$ is in rpm, and J is in kg·m$^2$)*

| HP | $R_s$ | $R_r$ | $X_{1s}$ | $X_{1r}$ | $X_m$ | $J$ | $N_r$ | $A$ | $B$ | $C$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 4.86 | 1.84 | 2.67 | 2.67 | 84.68 | 0.09 | 1760 | 0 | 1 | 0 |
| 15 | 1.48 | 0.31 | 0.18 | 0.18 | 24.89 | 0.50 | 1765 | 0 | 1 | 0 |
| 30 | 0.73 | 0.16 | 0.16 | 0.16 | 14.96 | 1.0 | 1765 | 0 | 1 | 0 |
| 50 | 0.42 | 0.14 | 0.15 | 0.15 | 9.47 | 1.66 | 1750 | 1 | 0 | 0 |
| 100 | 0.25 | 0.08 | 0.10 | 0.10 | 3.97 | 2.7 | 1740 | 0 | 0 | 1 |
| 198 | 0.1174 | 0.0352 | 0.0407 | 0.0404 | 2.0999 | 5.955 | 1748.7 | .253 | .246 | .503 |
| 198[11] | 0.12 | 0.035 | 0.043 | 0.041 | 2.10 | 5.96 | 1749 | - | - | - |
| 198[16] | 0.1183 | 0.0345 | 0.0384 | 0.0384 | 2.1025 | 5.953 | 1749 | - | - | - |
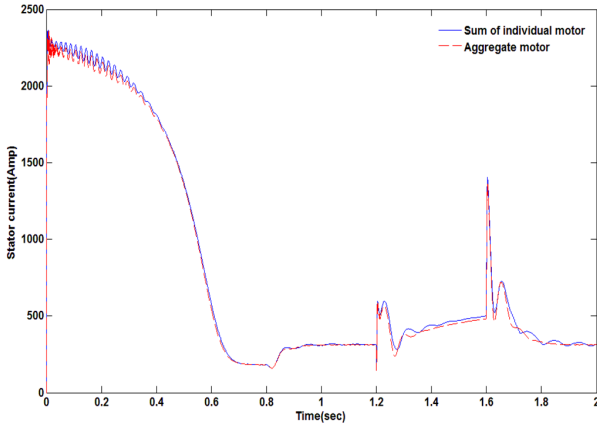


**Fig.5:** *Stator currents of Induction motors.*



**Fig.6:** *Electrical torques of induction motors.*



**Fig.7:** *Phase-A currents of Induction Motor.*

form of phase-A currents are indicated as displayed in Fig. 5-7, respectively.

It is apparent that transient phenomena in the electrical torque and current are initially occurred during free acceleration interval (0.0-0.7sec). All motors draw large amount of input currents in order to develop their electrical torque during this time. The torque and current reach steady-state about 0.75sec along with vanishing of the transient occurrence. It can be clearly observed that the current responses

(Fig. 5) obtained from the sum of individual motor and the aggregate motor are not matched with each other throughout the starting time interval between 0 and 0.4sec, where all motors initially begin to accelerate from standstill. During this period, instantaneous current waveforms in phase-A can be examined through Fig. 7. In order to clearly identify an error between these two current waveforms, the zoom of these currents for a short interval of time 0.23-0.3sec is given as shown in Fig. 8. It is evident that there is only a small difference in the peak amplitudes of these waveforms. Thus, the obtained parameters of aggregate model could be used to describe dynamic behaviours of all motors during free acceleration period.

After the steady-state condition is reached, a step change in the mechanical torque (equal to the rated torque) is applied to all motors $M_1$-$M_5$, beginning at $t$=0.8sec. It can be seen that the electrical current and torque (Fig 5 and 6) is suddenly increased and moved toward a new operating point. The dynamic responses of the current and electrical torque from the aggregate model still agree with those from the entire sum of each motor throughout 0.8-1.2sec interval.

Next, a 40% of voltage sag through the grid source is applied, starting at $t$=1.2sec, for 0.4sec duration. The motor currents (Fig. 5) are abruptly jumped in
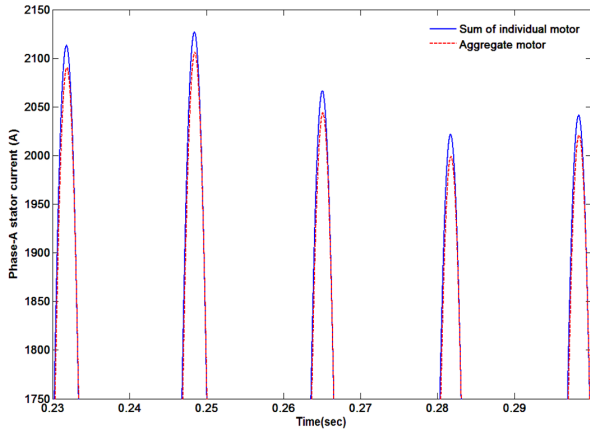
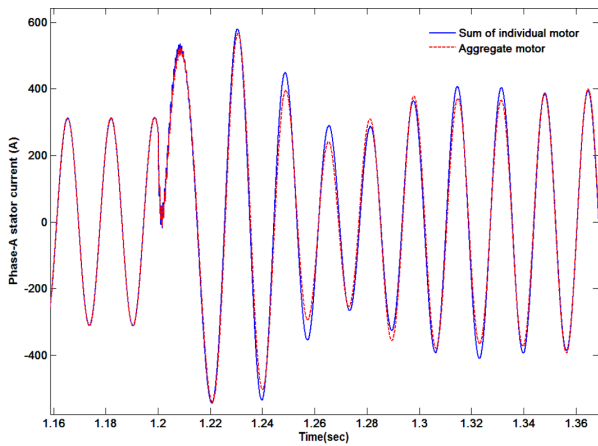***Fig.8:*** *Zoom of phase-A currents during free acceleration period.*



***Fig.9:*** *Zoom of phase-A currents during applying sag period.*

correlation with the negative induced torque (Fig. 6). These results indicate that all motors turn to act as generator. The plot of current waveforms in phase-A at this instant of time is zoomed as illustrated in Fig. 9. It is apparent that the instantaneous current is immediately raised and maintained for a short period of time (between 1.21-1.23sec). The current waveforms taken from the sum of each motor and the aggregate motor are relatively matched throughout this period. After the negative torque is died out, the stator current sharply falls and then gradually increases as the motors attempt to restore their electrical torque. During this transition period, there is a minor difference in the maximum magnitudes as indicated in Fig. 9. Similarly, it can be also found that the torque and current responses (Fig. 5-6) during the decaying interval ($t$>1.65sec) are not fully identical after the voltage sag is already removed.

## 4. CONCLUSION

This paper presents a simple and efficient method based on energy conservation law for aggregating the three-phase induction motor loads. The results show that the parameters of aggregate motor obtained in this paper are very close to the former published results using the transformer equivalent circuit and no-load and lock-rotor techniques. According to the dynamic simulation results, the responses obtained from the aggregate motor closely reflect the behaviour of a whole group of motors in satisfactory manner.

In power system dynamic simulation, it is impossible to incorporate each individual induction motor load into the system model since a large number of the motor's state variables leads to great increases in the model complexity and computational resource requirement. The proposed technique could be applied for finding an aggregate model of industrial loads, consisting of a large number of parallel induction motors. Then, the obtained parameters of aggregate equivalent circuit can be directly put into the conventional dynamic model of induction motor through the commercial software simulation package. It is expected that the aggregate circuit could reduce the model complexity and contribute a great reduction in the computational time, while maintaining acceptable dynamic responses.

## 5. ACKNOWLEDGEMENT

## References

[1] IEEE Committee Report, "Load Representation for Dynamic Performance Analysis," *IEEE Trans. Power Syst.*, vol. 2, pp. 472-481, May 1993.

[2] D. S. Brereton, D. G. Lewis, and C. C. Young, "Representation of Induction-Motor Loads During Power-System Stability Studies," *AIEE Trans. Power. Appar. Syst.*, vol. 76, pp. 451-461, Apr. 1957.

[3] G. J. Rogers, J. D. Manno, and R. T. H. Alden, "An Aggregate Induction Motor Model for Industrial Plants," *IEEE Trans. Power. Appar. Syst.*, vol. 103, pp. 683-690, Apr. 1984.

[4] M. M. Abdel Hakim, and G. J. Berg, "Dynamic single-unit representation of induction motor groups," *IEEE Trans. Power. Appar. Syst.*, vol. 95, pp. 155-165, Jan. 1976.

[5] F. Nozari, M. D. Kankam, and W. W. Price, "Aggregation of induction motors for transient stability load modeling," *IEEE Trans. Power Syst.*, vol. 2, pp. 1096-1103, Nov. 1987.

[6] G. J. Rogers, J. Di Manno, and R. T. H. Alden, "An aggregate induction motor model for industrial plants," *IEEE Trans. Power Appar. Syst.*, vol. 103, pp. 683-690, Apr. 1984.

[7] S. Sriharan, L. H. Tan, and H. M. Ting, "Reduced transient model of a group of induc-

tion motors", *IEEE Trans. Ener. Conv.*, vol. 8, pp.769-777, Dec. 1993.

[8] T. Y. J. Lem and R. T. H. Alden, "Comparison of experimental and aggregate induction motor responses," *IEEE Trans. Power Syst.*, vol. 9, pp. 1895-1900, Nov. 1994.

[9] D. C. Franklin, and A. Morelato, "Improving dynamic aggregation of induction motor models," *IEEE Trans. Power Syst.*, vol. 9, pp. 1934-1941, Nov. 1994.

[10] M. Taleb, M. Akbaba, and E .A. Abdullah, "Aggregation of induction machines for power system dynamic studies," *IEEE Trans. Power Syst.*, vol.9, pp. 2042-2048, Nov. 1994.

[11] E. Pillay, S.M.A. Sabur and M.M. Haq, "A model for induction motor aggregation for power system studies," *Electric Power Syst. Research*, vol.42, pp. 225-228, Sept. 1997.

[12] T. Kataoka, H. Uchida, S. Nishhta, T. Kai and T. Fuaabashi, "A method for aggregation of a group of induction motor loads," *Proc. Int. Conf. Power Syst. Technology*, 2000, pp. 1683-1688.

[13] J. Pedra, L. Sainz, and F. Corcoles, "Study of Aggregate Models for squirrel-cage induction motors", *IEEE Trans. Power Syst.*, vol. 20, pp. 1519-1527, Aug. 2005.

[14] K. W. Louie, J. R. marti, and H. W. Dommel, "Aggregation of induction motors in a power system based on some special operating condition", *Proc. Canadian Conf. Elect. Comput. Eng.*, 2007, pp. 1429-1432.

[15] A. Kunakorn, T. Banyatnopparat, "Representation of a group of three-phase induction motor using per unit aggregation model," *Proc. Int. Conf. Power Syst. Transient*, 2009.

[16] A. Karakas, F. Li, S. Adhikari, "Aggregation of multiple induction motors using matlab-based software package," *IEEE PES Power Syst. Conf. Exposition*, 2009, pp. 1-6.

[17] P. Aree, and W. Nakawiro, "Eectrical Modeling and Simulation of Industrial Power System With Matab/Simulink Program," *Proc. Int. Association Sci. Technology Develop. Conf.*, 2013, pp. 347-352

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