Leakage Inductance Calculations in Different Geometries of Traction Transformers

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ABSTRACT

Calculating leakage inductances between windings of traction transformer is one of the most important schemes in design procedure. In this paper, analytic, semi-analytic and finite element methods are introduced for electromagnetic modeling of different geometries of the traction transformer. Using these methods, the leakage inductances between windings are calculated and the methods have been validated with the help of a traditional analytic method and experimental data gathered from a typical 4000kVA dry-type transformer. The analytic method which usually used for electromagnetic modeling of transformers with concentric windings, assumes the windings to be constructed of straight wires. This method is fast and suitable for calculating the axial magnetic field and leakage inductance between concentric windings. However, in order to increase the accuracy of leakage inductance calculations between pancake windings, a semi-analytic method is proposed. It is shown that this semi-analytic method has a better accuracy in comparison with the analytic method and despite of the longer computing time, it is still a faster method compared to finite element.

Keywords: Analytic Method, Semi-analytic Method, Finite Element, Leakage Inductance, Traction Transformer

1. INTRODUCTION

Along with the rapid development of subways and traction networks, the converter systems have come true and widely used in power systems. The main part of these converter systems is its traction transformer. In order to protect against flame and explosion, nowadays traction transformers are manufactured as dry-type transformers. This type of transformer is usually manufactured from four split windings in which the high voltage windings are in parallel. As it is known, calculating leakage inductances of windings is an important factor in design procedure of multi-winding traction transformers [1-4]. Leakage inductances of a concentric three-winding transformer are discussed in [5]. Many different methods have been presented for inductance calculations in transformers with concentric and pancake windings [6-10]. An analytic method for calculations of leakage inductances of the two-winding transformer have been introduced in [10]; a semi-analytic method has been presented in [11] for determination of inductance between thick coils. Authors introduce a simple analytic method for leakage inductance calculations of concentric and pancake windings in [12]. However, there is a need to have more accurate and fast methods for electromagnetic modeling and thus leakage inductance calculations in different geometries of the multi-winding traction transformer. Therefore in this paper, analytic, semi-analytic and finite element methods are introduced for such modeling and the accuracy of the presented methods is verified by experimental results and introduced method in [12].

Finite element (FE) model [13] is the most precise method for electromagnetic modeling and computing the leakage inductances of the windings with long computation time. When a shorter computation time is essential (such as optimization) the FE is not preferred and analytical methods are recommended. The analytical method (that assumes the windings are substituted by straight wires of rectangular cross section) is a fast method and it is suitable when the axial magnetic field or leakage inductances of concentric windings are required. However, it will be shown that the analytic method is not accurate enough when radial component of the magnetic field or leakage inductances of pancake windings is required. So, a semi-analytic method is introduced to improve the accuracy of the electromagnetic modeling and leakage inductance calculations. In the following it is shown that the introduced semi-analytic method is a powerful and accurate method that this is in addition of its small computation time (comparing to the finite element method).

2. ELECTROMAGNETIC MODELING OF A MULTI-WINDING TRACTION TRANSFORMER

As shown in Fig. 1, three-winding traction transformers are usually manufactured with two general types of winding geometries. In the geometry 1 (Fig.1.a), high voltage winding is located between two concentric low voltage windings (concentric type).

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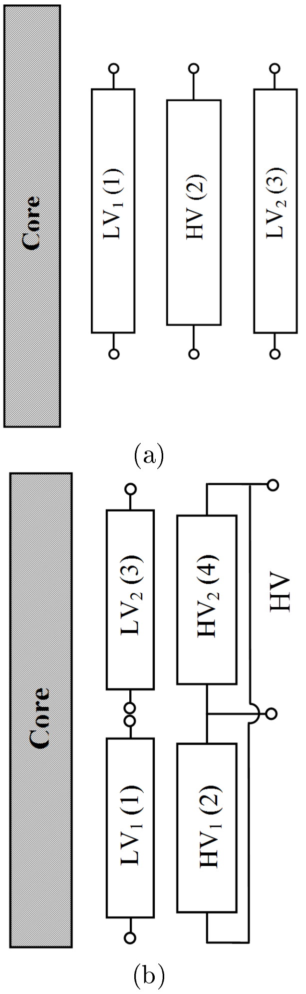

Fig.1: *Schematic view of the traction transformer, a) geometry 1 and b) geometry 2.*

Consequently, in the geometry 2 (Fig.1.a), high voltage winding is constructed of two separated sections that are in parallel and low voltage windings are located axially (pancake type).

In this section, two-dimensional (2D) and three-dimensional (3D) methods are introduced for electromagnetic field modeling of these transformers. The computed magnetic field is used for the inductance calculations in the next section. To obtain an analytic/semi-analytic method, assume that the windings cross sections include a few rectangular areas with homogenous current densities. In this case, magnetic field in an arbitrary point can be computed as sum of the partial fields generated by each current-carrying section and its magnetic images. In the following part, the analytic and the semi-analytic methods are introduced to calculate these partial fields and then image technique is presented for core modeling. Finally, FE method is presented for validation of the introduced methods.

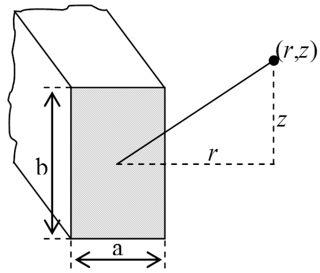

Fig.2: *Straight wire with rectangular cross section.*

2.1 Analytic Method

If the thickness of a winding is much smaller than its radius, the flux density is very close to the one which can be generated by an unwound wire [9]. This simple description allows analytical calculation to be carried out. The elementary field source is an infinite straight wire with a rectangular cross section (Fig. 2) carrying homogenous current density (J).

As shown in (1), vector potential can be obtained by integrating the straight wire (Fig. 2) on its surface.

$$A = \frac{\mu_0 J}{4\pi} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \ln\left[\left(r'-r\right)^2 + \left(z'-z\right)^2\right] dr'dz' \quad (1)$$

where, μ_0 is the permeability of free space and J is the current density of the wire. Note that r, z, a and b are shown in Fig. 3.

Radial and axial components of the flux density can be deduced from A as follows [9] and [10]:

$$\begin{aligned} B_r(r,z) &= \frac{\partial A(r,z)}{\partial z} \\ &= -\frac{\mu_0 J}{4\pi}\left[\left[r' \ln\left(r'^2 + z'^2\right)\right.\right. \\ &\quad \left.\left. + 2z' \tan^{-1}\left(\frac{r'}{z'}\right)\right]_{r'=r-\frac{a}{2}}^{r'=r+\frac{a}{2}}\right]_{z'=z-\frac{b}{2}}^{z'=z+\frac{b}{2}} \end{aligned} \quad (2)$$

$$\begin{aligned} B_z(r,z) &= -\frac{\partial A(r,z)}{\partial r} \\ &= \frac{\mu_0 J}{4\pi}\left[\left[z' \ln\left(r'^2 + z'^2\right)\right.\right. \\ &\quad \left.\left. + 2r' \tan^{-1}\left(\frac{z'}{r'}\right)\right]_{r'=r-\frac{a}{2}}^{r'=r+\frac{a}{2}}\right]_{z'=z-\frac{b}{2}}^{z'=z+\frac{b}{2}} \end{aligned} \quad (3)$$

2.2 Semi-Analytic Method

Analytical calculation is a fast and suitable method if only axial component of the magnetic field exists (as in normal transformers with two concentric windings). However, as the windings are actually wound, assuming them to be constructed of straight wires causes a considerable error in radial field calculations. In order to improve the accuracy of magnetic field calculations, a 3D semi-analytical formulation

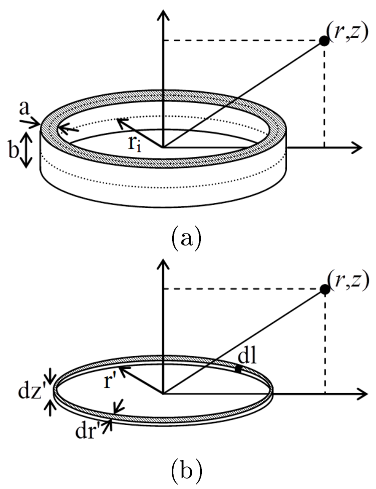

Fig.3: *a) Cylindrical coil and a b) current carrying ring.*

based on actual geometry of windings is introduced. Let us to assume the elementary field source as a cylindrical current carrying coil that is wound with a thin wire (Fig. 3a).

In order to calculate the flux density generated by the coil, it is convenient to apply the Biot-Savart law to a thin current carrying ring (Fig. 3b) as

$$dB = \frac{\mu_0 J}{4\pi} \int_0^{2\pi} r' \frac{(z-z')\sin\phi\, \hat{a}_r + (r' - r.\sin\phi)\ \hat{a}_z}{\left(r'^2 + r^2 - 2r'r\sin\phi + (z-z')^2\right)^{3/2}} d\phi \quad (4)$$

where, r, z, are shown in Fig. 3 and φ is the angular direction across the ring.

By integrating (4) across the radial and axial axes, magnetic field generated by the coil of Fig. 3a is expressed as follows:

$$B = \frac{\mu_0 J}{4\pi} \int_{r_i - a/2}^{r_i + a/2} \int_{-b/2}^{b/2} \int_0^{2\pi} r' \frac{(z-z')\sin\phi\, \hat{a}_r + (r' - r\sin\phi)\ \hat{a}_z}{\left(r'^2 + r^2 - 2r'r\sin\phi + (z-z')^2\right)^{3/2}} d\phi\, dz'\, dr' \quad (5)$$

where, a and b are the conductor's dimensions and $\hat{a}_r$ and $\hat{a}_z$ represent the radial and axial directions.

Relation (5) can be simplified as the following:

$$B = \frac{\mu_0 J}{4\pi} \int_{r_i - a/2}^{r_i + a/2} \int_0^{2\pi} \left[F(\phi, r', b/2) - F(\phi, r', -b/2)\right]\ d\phi dr' \quad (6)$$

where,

$$F(\phi, r', z') = \frac{r'\sin\phi\, \hat{a}_r + \frac{r'(r' - r\sin\phi)(z-z')}{r'^2 + r^2 - 2r'r\sin\phi}\, \hat{a}_z}{\left(r'^2 + r^2 - 2r'r\sin\phi + (z-z')^2\right)^{3/2}} \quad (7)$$

Since (6) cannot be easily evaluated analytically, a numerical technique is employed. By numerical integration, a semi-analytic formulation can be obtained as follows:

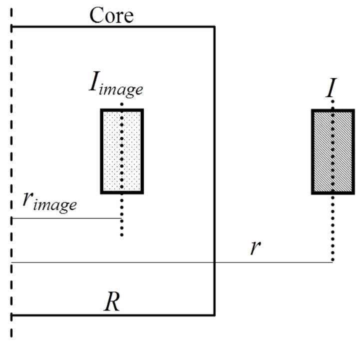

Fig.4: *Image of a conductor due to the magnetic core.*

$$B = \frac{\mu_0 J}{4\pi} \sum_{j=1}^{n_r} \sum_{k=1}^{n_\phi} \left[F(\phi_k, r_j, b/2) - F(\phi_k, r_j, -b/2)\right] \Delta\phi \Delta r \quad (8)$$

where, r_j, is the radius of the j^{th} conducting part in radial direction and φ_k is the k^{th} conducting part in angular direction angular direction.

2.3 Magnetic Images

As the magnetic permeability of core is very high, an infinite value can be assumed for it. Therefore, magnetic field is perpendicular to the core and surface of the core acts as a mirror for magnetic field lines. Consider a wire carrying current I in parallel to the core surface. The impact of the core on the magnetic field distribution can be simulated by an image wire as shown in Fig. 4; ignoring the iron core, its magnetic field is added to the field generated by the main conductor to simulate the field distribution outside the core.

If a 2D (analytic) method is used, core surface can be assumed as a flat mirror that causes the image wire to be located symmetrically on the other side of the core surface [10]. For a 3D (semi-analytic) method, surface of the core may be assumed as a curved mirror that causes the image wire to be located in a radius calculated as follows [7]:

$$r_{image} = \frac{R \times r}{2r - R} \quad (9)$$

where, R and r are shown in Fig. 4.

Current in the image wire flows in the same direction as the main conductor and its current must be adjusted for better accuracy. The ratio of I_{image}/I in the conductors near the core is higher than this value for the conductors far from the core surface [7]. In this paper, the image currents have been set at $1.5I$ for the main leg and $0.5I$ for the yokes (in 2D formulation).

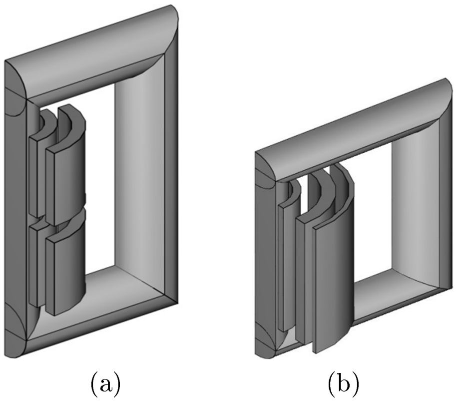

Fig.5: *FE models of the traction transformer, a) geometry 1 [12] and b) geometry 2.*

It is important to note that in analytic and semi-analytic methods the impact of the core is not modeled properly; but in many cases, these methods are fast and accurate enough.

2.4 Finite Element (FE) Method

Most accurate method for electromagnetic modeling of the traction transformer is finite element method [13]. In order to simplify the model and to shorten the computation time, it is convenient to model the three-phase transformer through its single-phase equivalent model [7]. Here, it is assumed that the windings can be modeled as units carrying uniform current densities. For a quasi-static electromagnetic field and by neglecting the eddy currents term, electromagnetic problem can be modeled using the Poisson's equation (10).

$$\nabla^2 A - \mu\varepsilon \frac{\partial^2 A}{\partial t^2} = -\mu \cdot J \tag{10}$$

where, A is the magnetic vector potential, μ is permeability of the medium, ε is permittivity of the medium and t is the time.

Here, FE method is applied to solve the Poisson's equation and model the electromagnetic field of the traction transformer. Fig.5 shows the 3D FE models for electromagnetic modeling of a single-phase four-winding traction transformer.

3. LEAKAGE INDUCTANCE CALCULATIONS

3.1 Based on the Presented Electromagnetic Methods

In order to measuring leakage inductance in a pair of windings in multi-winding transformers (Fig.1), it is convenient to use test circuit given in the Fig.6. If the nominal currents applied to the windings i and j (Ii for winding i, -Ij for winding j and zero for the other two windings), and the magnetic flux density

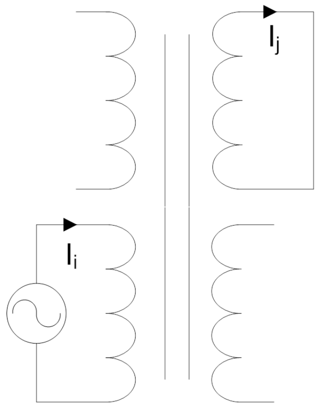

Fig.6: *Circuit for measuring leakage inductance in a pair of windings.*

Table 1: *Typical Dry-Type Traction Transformer's Nominal Data.*

S = 4MVA/2×2MVA	Dd0/y11
V_{HV} = 20 kV	I_{HV} = 115.5 A
V_{LV1} = 750 V	I_{LV1} = 1540 A
V_{LV2} = 750 V	I_{LV2} = 1540 A

Table 2: *Geometry 1 (Concentric Winding): Designed for the Typical Transformer Given in Table 1.*

Winding	HV	LV_1	LV_2
Rated Power	4 MVA	2 MVA	2 MVA
Rated Voltage	20 kV	750V	750 V
Rated Current	115.5 A	1.54 kA	1.54 kA
Connection	D	Y	d
Layer Number	1	3	3
Axial Channel	-	3×10 mm	1×12 mm
Disk Number	11	1	1
Radial Channel Between Disks	9×12 mm 1×26 mm	-	-
Height	1230 mm	1240 mm	1240 mm
Internal Diameter	654 mm	332 mm	929 mm
External Diameter	799 mm	454 mm	982 mm
Conductor	Aluminum foil		
Length of core window	1015 mm		
Height of core window	1410 mm		
Diameter of core	320 mm		

(B) is calculated using the presented methods, leakage inductance (seen from side j) is calculated as follows [12]:

$$L_{lij} = \frac{1}{\mu_0 I_j^2} \int_{vol} B^2 \, dv \tag{11}$$

3.2 Simple Analytic Method [12]

Using design parameters, leakage inductance of a pair of windings can be evaluated as follows:

$$L_{lij} = \sqrt{\left(L_{lij}^r\right)^2 + \left(L_{lij}^z\right)^2} \tag{12}$$

Table 3: *Geometry 2 (Pancake Windings): Designed for the Typical Transformer Given in Table 1.*

Winding	HV_1	HV_2	LV_1	LV_2
Rated Power	2 MVA	2 MVA	2 MVA	2 MVA
Rated Voltage	20 kV	20 kV	750 V	750 V
Rated Current	57.7 A	57.7 A	1.54 kA	1.54 kA
Connection	D	D	Y	d
Layer Number	1	1	4	4
Axial Channel	-	-	3×14 mm	2×14 mm 1×12 mm
Disk Number	11	11	1	1
Radial Channel Between Disks	5×12 mm 4×12 mm 1×26 mm	5×12 mm 4×14 mm 1×26 mm	-	-
Height	934 m	934 mm	950 mm	650 mm
Internal Diameter	632 mm	632 mm	375 mm	374 mm
External Diameter	764 mm	764 mm	498 mm	498 mm
Conductor	Aluminum foil			
Length of core window	820 mm			
Height of core window	2160 mm			
Diameter of core	320 mm			
Axial Distance between LV_s	110 mm			

Table 4: *Leakage Reactances of Traction Transformer with Concentric Windings, P.U. (%).*

	Method in [12]	FE	Analytic	Semi-Analytic
LV_1-HV	5.89	5.89	5.85	5.88
LV_1-LV_2	17.02	17.1	16.8	16.85
LV_2-HV	6.02	6.01	5.97	5.97

where L^r_{lij} and L^z_{lij} are the inductances due to the radial and axial leakage fluxes which are computed as follows:

$$L^r_{lij} = \frac{2\mu_0 N_0^2 \pi D_{ij}}{(b_i + b_j)/K_{rog-r}} \left(\frac{h_i}{3} + \frac{h_j}{3} + h_{ij}\right), \quad \begin{matrix} i = 1,2 \\ j = 3,4 \end{matrix} \tag{13}$$

$$L^z_{lij} = \frac{2\mu_0 N_0^2 \pi D_{ij}}{(h_i + h_j)/K_{rog-a}} \left(\frac{b_i}{3} + \frac{b_j}{3} + b_{ij}\right), \quad \begin{matrix} i = 1,3 \\ j = 2,4 \end{matrix} \tag{14}$$

where D_{ij} is the mean diameter of windings i and j, b_i is the width of winding i, b_{ij} is the axial gap between windings i and j, h_i is the height of winding i, h_{ij} is the radial gap between windings i and j, and $K_{rog-a/r}$ are the Rogofsky factors.

4. COMPARISION OF DIFFERENT GEIOMETRIES IN THREE-WINDING TRACTION TRANSFORMER

In this section a typical dry-type traction transformer is selected according to Table 1 and two different geometries are designed for these ratings as shown Tables 2 and 3.

Using these geometries, flux density is calculated using the analytic and semi-analytic methods. Leakage inductances are computed using the calculated flux density and the results are verified against the FE, the experimental results and the method presented in [12].

Tables 4 and 5 show the calculated leakage inductances of the mentioned transformers and verify the results of the presented methods against FE and experimental results. Consequently, tables 6 and 7 show the computational errors related to the different methods. Note that the method presented in [12] is a simple and very fast method (consumes less than a second). Consequently, introduced analytical and semi-analytical methods consume around 1 and 2.5 minutes respectively. On the other hand FE method is a very time-consuming method and consumes more than 10 minutes.

The analytic methods are fast and precise method for axial magnetic field calculation (when there is no significant radial field). However, as the winding diameter decreases, this method is not accurate required to compute the radial magnetic field (or inductance between pancake windings), and therefore semi-analytic method must be used. However, if the computation time must be short; FE is not a preferred method and in an automatic transformer de-

Table 5: *Leakage Reactances of Traction Transformer with Pancake Windings, P.U. (%).*

Windings	Measured	Method in [12]	FE	Analytic	Semi-Analytic
LV_1-HV_1	5.8	5.90	5.80	5.79	5.80
LV_1-LV_2	73	56	77	62	80
LV_2-HV_1	72	72	76	60	78
LV_1-HV_2	74	73	77	60	79
HV_1-HV_2	72	88	77	61	80
LV_2-HV_2	5.94	6.0	5.97	6.02	5.9

Table 6: *Computational errors (in comparison with FE) in Transformer with Concentric Windings.*

	Method in [12]	Analytic	Semi-Analytic
LV_1-HV	0%	0.68%	0.17%
LV_1-LV_2	0.47%	1.75%	1.5%
LV_2-HV	0.17%	0.67%	0.67%

Table 7: *Computational errors (in comparison with measured) in Transformer with Pancake Windings.*

Windings	Measured	Method in [12]	FE	Analytic	Semi-Analytic
LV_1-HV_1	5.8	1.7%	0%	0.17%	0%
LV_1-LV_2	73	23%	5%	15%	9.6%
LV_2-HV_1	72	0%	5.5%	16.6%	8.3%
LV_1-HV_2	74	1.35%	4%	18.9%	6.8%
HV_1-HV_2	72	22%	6.9%	15.3%	11%
LV_2-HV_2	5.94	1%	0.5%	1.35%	0.67%

sign process, the introduced semi-analytic method can be used to shorten the total computation time. Note that the analytical calculations (presented in this paper and [12]) are 2D methods and the semi-analytical calculation is a 3D method. This justifies the better accuracy of the introduced semi-analytical method in leakage inductance calculations of the pancake windings.

Note that when the winding's height is increased against its width, the accuracy of all methods and especially 2D methods may be affected. In the highest height/weight ratio the radial magnetic field will be increased and the 2D methods are not preferred. But the accuracy of 3D FE and semi-analytical methods will not decrease considerably.

As it can be seen from Table 5, in the split-winding transformer, inductances between common axis windings (i.e. LV1-LV2) are much higher than the leakage inductances of concentric windings (i.e. LV1-HV1) of the same heights. Comparing Tables 4 and 5 shows that leakage inductance between low voltage windings are much higher in transformer with split high voltage windings. In the traction transformer it is preferred to be designed with high leakage inductance between the low voltage windings and so the transformer with split high voltage windings is a better choice.

5. CONCLUSION

In this paper analytic and semi-analytic methods were introduced for electromagnetic modeling of traction transformers with different types of geometries. Using the presented methods, the leakage inductances were estimated and the results had been compared to FE method and experimental results.

Analytical calculation is a very fast and accurate method if axial magnetic field is required. However, assuming the windings as straight wires with infinite lengths causes a considerable error in estimating the radial field and its related inductances. In order to improve the accuracy, a 3D semi-analytic method was introduced for inductance calculations. The semi-analytic method has a better accuracy, when a much longer computation time is acceptable, in comparison with the analytic method. However, it is still a faster method if compared to FE method under optimization process.

As seen, it is better to choose a traction transformer with two split high voltage windings (split-winding transformer) to have a higher leakage inductance between low voltage windings. In the split-winding traction transformer, leakage inductances of axial windings are about ten times of leakage inductances in radial windings and adding a middle yoke can improve and increase this ratio. The effect of this middle yoke on the leakage inductances of radial windings is not considerable in spite of increase in leakage inductances of axial windings.

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