Comparative Experimental Exploration of Robust Norm Functions for Iterative Super Resolution Reconstructions under Noise Surrounding

Vorapoj Patanavijit[*1], Non-member

ABSTRACT

In DIP (Digital Image Processing) research society, the multi-frame SRR (Super Resolution Reconstruction) algorithm has grown to be the momentous theme in the last ten years because of its cost effectiveness and its superior spectacle. Consequently, for a multi-frame SRR algorithm which is commonly comprised of a Bayesian ML (Maximum Likelihood) approach and a regularization technique into the unify SRR framework, numerous robust norm functions (which have both redescending and non-redescending influence functions) have been commonly comprised in the unify SRR framework for increasingly against noise or outlier. First, this paper presents the mathematical model of several iterative SRR based on a Bayesian ML (Maximum Likelihood) approach and a regularization technique. Three groups of robust norm functions (a zero-redescending influence function (Tukey's Biweight, Andrew's Sine and Hampel), a nonzero-redescending influence function (Lorentzian, Leclerc, Geman&McClure, Myriad and Meridian) and a non-redescending influence function (Huber)) are mathematically incorporated into the SRR framework. The close form solutions of the SRR framework based on these robust norm functions have been concluded. Later, the experimental section utilizes two standard images of Lena and Susie (40th) for pilot studies and fraudulent noise patterns of noiseless, AWGN, Poisson, Salt&Pepper, and Speckle of several magnitudes used to contaminate these two standard images. In order to acquire the maximum PSNR, the comparative experimental exploration has been done by comprehensively tailoring all experimental parameters such as step-size, regularization parameter, norm constant parameter.

Keywords: DIP (Digital Image Processing), SRR (Super Resolution Reconstruction), Robust Norm Function, Bayesian ML, Regularization Techniques

Manuscript received on July 1, 2015 ; revised on July 28, 2015.

* The authors are with Department of Electrical Engineering at the Assumption University, Bangkok, Thailand E-mail: Patanavijit@yahoo.com[1]

1. RELATED WORKS

In digital image research society during the last ten years, many researchers pay a great plenty of work force on the multi-frame SRR (Super Resolution Reconstruction) algorithm [1-4] thereby the multi-frame SRR algorithm has grown to be the momentous theme because of its cost effectiveness and its superior spectacle. For enriching the image quality and reconstructing a HR (High Resolution) image (so called a SR image) , which is commonly better resolution and less noise corruption, by SRR principle idea, supplementary data of each LR (Low Resolution) images can be mathematically merged.

In fact, the real system noise (which contaminates the captured LR images) or the error of SRR modelling (from the simplification of a mathematical model of SRR), is commonly unfamiliar thereby these outlier (such as noise or error) is usually modelled by a random statistical pattern. Due to the mathematical tractable and computational complexity, noise is first commonly mathematically expressed as Gaussian distribution pattern then the classical norm function (such as L1 [5] and L2 [6]) successfully estimates the original HR image from a group of contaminated LR (Low Resolution) images however if the real noise are an impulsive or non Gaussian distribution pattern then the estimated HR image, which is created by the SRR algorithm based on these classical norm functions, has generally poor performance.

From the concept of a robust signal processing, the robust norm estimation [7-8] has been applied to the digital image processing since 1996. The Lorentzian norm function [9], Huber norm function [10] and Tukey's Biweigth norm function [11] have been first applied to the multi-frame SRR framework based on stochastic Bayesian approach and regularization technique since 2006. Next, the Hampel norm function [12], Andrew's Sine norm function [13], Geman & Mcclure norm function [14] and Leclerc norm function [15] have been applied to this multi-frame SRR framework since 2008-2009. Finally, Myriad norm function [16] and Meridian norm function [17], which are completely mathematical proved for non-Gaussian outlier [18-19], have been applied to this multi-frame SRR framework since 2010-2011. Although numerous robust norm functions have been applied to the multi-frame SRR framework, there is no performance and no comparative experimental exploration of each robust norm functions when it is applied in SRR framework thus this paper thoroughly presents comparative experimental exploration of an iterative SRR algorithm based on several robust norm functions such as zero-redescending influence functions (Tukey's Biweight, Andrew's Sine and Hampel), nonzero-redescending influence functions (Lorentzian, Leclerc, Geman&McClure, Myriad and Meridian) and non-redescending influence functions (Huber).

In the remaining part of this paper, section 2 first depicts the mathematical problem model of SRR (Super Resolution Reconstruction) and the classical solution of this mathematical problem. Later, the mathematical combination of robust norm functions (zero-redescending influence functions (Tukey's Biweight, Andrew's Sine and Hampel), nonzero-redescending influence functions (Lorentzian, Leclerc, Geman&McClure, Myriad and Meridian) and non-redescending influence functions (Huber)) and the SRR algorithm based on Bayesian ML (Maximum Likelihood) approach and a regularization technique are presented. Subsequently, section 3 thoroughly depicts the comparative experimental exploration of the multi-frame SRR frameworks based on numerous robust norm functions in order to efficiently evaluate the experimental performance impact of the robust norm function. Finally, Section 4 depicts the experimental discussion of simulated results and its conclusion.

2. MATHMATICAL MODEL OF SRR USING ROBUST NORM FUNCTIONS

To turn the calculated memory down and reduce processing time, the estimated HR image and all LR images are split into a lot of overlapping small sized squares and arranged in the lexicography format as $\underline{X}(q^2M^2 \times 1)$ for HR image and $\underline{Y}_k(t)$ $(M^2 \times 1)$ for each LR frames respectively. (where is the index of observed LR frames, which are the input of the SRR algorithm and is the spatial up-sampling index). By SRR principle idea, the mathematical association between the estimated HR $\underline{X}$ and each measured LR $\underline{Y}_k(t)$ is principle depicted as the forthcoming equation.

$$\underline{Y}_k = D_k H_k F_k \underline{X} + \underline{V}_k \quad ; k = 1, 2, \ldots, N \qquad (1)$$

The alignment of the estimated HR $\underline{\mathrm{X}}$ to the position of each measured LR $\underline{Y}_k(t)$ is depicted as the distort matrix $\underline{F}_k$ $(q^2M^2 \times q^2M^2)$. This paper depicts the Gaussian blur matrix $\underline{H}_k$ $(q^2M^2 \times q^2M^2)$ to be the space time-invariant property. This paper depicted the spatial down-sampling matrix and a noise vector as $\underline{D}_k$ $(M^2 \times q^2M^2)$ and $\underline{V}_k$ $(M^2 \times 1)$respectively.

2.1 The Classical Norm Function for SRR Algorithm

This section presents the classical norm functions (L1 and L2) that have been regularly used for SRR.

2.1.1 L2 Norm Function for SRR Algorithm

Because of its statistic simplicity and low complexity, the L2 norm function [5] as shown in Fig. 1(a) is the first norm function that has been undertaken for SRR algorithm since 1997. By the above reason, the SRR algorithm will become the minimized equation as forthcoming.

$$\underline{X} = \underset{\underline{X}}{ArgMin} \left\{ \sum_{k=1}^{N} \| D_k H_k F_k \underline{X} - \underline{Y}_k \|_2^2 + \lambda \cdot (\Gamma \underline{X})^2 \right\} \qquad (2)$$

Evaluating the mathematical close-form solution by the nonlinear optimization expertise, the mathematical close-form solution of the former equation can be depicted as the forthcoming equation.

$$\underline{\hat{X}}_{n+1} = \underline{\hat{X}}_n + \beta \cdot \left\{ \sum_{k=1}^{N} F_k^T H_k^T D_k^T \left(\underline{Y}_k - D_k H_k F_k \underline{\hat{X}}_n \right) - \left(\lambda \cdot \left(\Gamma^T \Gamma \right) \underline{\hat{X}}_n \right) \right\} \qquad (3)$$

where λ is the regularization constrain parameter $(0 \le \lambda \le 1)$, β is the gradient descent step-size constant parameter of an iterative computational method $(0 \le \beta \le 1)$, and Γ is the Laplacian regularized function which is defined as $\Gamma_{\mathrm{KERNEL}} = \frac{1}{8} \left[\begin{array}{ccccccccccc} 1 & 1 & 1 & ; & 1 & -8 & 1 & ; & 1 & 1 & 1 \end{array} \right]$.

2.1.2 Norm Function for SRR Algorithm

Later, because of its noise tolerance and statistic simplicity, the L1 norm function [6] as shown in Fig. 1(b) is the second norm function that has been undertaken for SRR algorithm (for non-Gaussian outlier) since 2004. By the above reason, the SRR algorithm will become the minimized equation as forthcoming.

$$\underline{X} = \underset{\underline{X}}{ArgMin} \left\{ \sum_{k=1}^{N} \| D_k H_k F_k \underline{X} - \underline{Y}_k \| + \lambda \cdot (\Gamma \underline{X})^2 \right\} \qquad (4)$$

Evaluating the mathematical close-form solution by the nonlinear optimization expertise, the mathematical close-form solution of the former equation can be depicted as the forthcoming equation.

$$\underline{\hat{X}}_{n+1} = \underline{\hat{X}}_n + \beta \cdot \left\{ \begin{array}{l} \left(\sum_{k=1}^{N} F_k^T H_k^T D_k^T \, sign \left(D_k H_k F_k \underline{\hat{X}}_n - \underline{Y}_k \right) \right) \\ - \left(\lambda \cdot \left(\Gamma^T \Gamma \right) \underline{\hat{X}}_n \right) \end{array} \right\} \qquad (5)$$

2.2 Robust Norm Function for SRR Algorithm: Zero-Redescending Influence Function

This section presents the robust norm functions that have the influence function to be zero when the error rises beyond a fixed point. (so called zero-redescending influence function)

2.2.1 TukeyâĂŹs Biweigth Norm Function for SRR Algorithm

The Tukey's Biweigth norm function [11] as shown in Fig. 1(c) is one of the robust norm functions that has been undertaken for SRR algorithm (for non-Gaussian outlier) since 2006. The SRR algorithm will become the minimized equation as forthcoming.

$$\underline{X} = \underset{\underline{X}}{ArgMin}\left\{\sum_{k=1}^{N}\rho_{TUKEY}\left(D_k H_k F_k \underline{X} - \underline{Y}_k\right) + \lambda\cdot(\Gamma\underline{X})^2\right\} \quad (6)$$

$$\rho_{TUKEY}(x) = \begin{cases} \frac{x^2}{T^2_{TUKEY}} - \frac{x^4}{T^4_{TUKEY}} + \frac{x^6}{3T^6_{TUKEY}} & ; |x| \le T_{TUKEY} \\ \frac{1}{3} & ; otherwise \end{cases} \quad (7)$$

Where T is the soft threshold of the robust norm function.

Evaluating the mathematical close-form solution by the nonlinear optimization expertise, the mathematical close-form solution of Eq. (6) can be depicted as the forthcoming equation.

$$\hat{\underline{X}}_{n+1} = \hat{\underline{X}}_n + \beta\cdot\left\{\begin{array}{l}\sum_{k=1}^{N} F_k^T H_k^T D_k^T \cdot \psi_{TUKEY}\left(\underline{Y}_k - D_k H_k F_k \hat{\underline{X}}_n\right) \\ -\left(\lambda\cdot\left(\Gamma^T\Gamma\right)\hat{\underline{X}}_n\right)\end{array}\right\} \quad (8)$$

$$\psi_{TUKEY}(x) = f'_{TUKEY}(x) = \begin{cases} x\left[1-(x/T_{TUKEY})^2\right]^2 & ; |x| \le T_{TUKEY} \\ 0 & ; otherwise \end{cases} \quad (9)$$

2.2.2 AndrewâĂŹs Sine Norm Function for SRR Algorithm

The Andrew's Sine norm function [13] as shown in Fig. 1(d) is one of the robust norm functions that has been undertaken for SRR algorithm (for non-Gaussian outlier) since 2008. The SRR algorithm will become the minimized equation as forthcoming.

$$\underline{X} = \underset{\underline{X}}{ArgMin}\left\{\sum_{k=1}^{N}\rho_{\substack{ANDREW\\SINE}}\left(D_k H_k F_k \underline{X} - \underline{Y}_k\right) + \lambda\cdot(\Gamma\underline{X})^2\right\} \quad (10)$$

$$\rho_{\substack{ANDREW\\SINE}}(x) = \begin{cases} \left(T^2_{\substack{ANDREW\\SINE}}\right)\sin^2\left(x \Big/ 2T_{\substack{ANDREW\\SINE}}\right) & ; |x| \le \pi T_{\substack{ANDREW\\SINE}} \\ T^2_{\substack{ANDREW\\SINE}} & ; |x| > \pi T_{\substack{ANDREW\\SINE}} \end{cases} \quad (11)$$

Evaluating the mathematical close-form solution by the nonlinear optimization expertise, the mathematical close-form solution of Eq. (10) can be depicted as the forthcoming equation.

$$\hat{\underline{X}}_{n+1} = \hat{\underline{X}}_n + \beta\cdot\left\{\begin{array}{l}\sum_{k=1}^{N} F_k^T H_k^T D_k^T \cdot \psi_{\substack{ANDREW\\SINE}}\left(\underline{Y}_k - D_k H_k F_k \hat{\underline{X}}_n\right) \\ -\left(\lambda\cdot\left(\Gamma^T\Gamma\right)\hat{\underline{X}}_n\right)\end{array}\right\} \quad (12)$$

$$\begin{aligned}\psi_{\substack{ANDREW\\SINE}}(x) &= \rho'_{\substack{ANDREW\\SINE}}(x) \\ &= \begin{cases} T_{\substack{ANDREW\\SINE}}\sin\left(x \Big/ T_{\substack{ANDREW\\SINE}}\right) & ; |x| \le \pi T_{\substack{ANDREW\\SINE}} \\ 0 & ; |x| > \pi T_{\substack{ANDREW\\SINE}} \end{cases}\end{aligned} \quad (13)$$

2.2.3 Hampel Norm Function for SRR Algorithm

The Hampel norm function [12] as shown in Fig. 1(e) is one of the robust norm functions that has been undertaken for SRR algorithm (for non-Gaussian outlier) since 2008. The SRR algorithm will become the minimized equation as forthcoming.

$$\underline{X} = \underset{\underline{X}}{ArgMin}\left\{\sum_{k=1}^{N}\rho_{HAMPEL}\left(D_k H_k F_k \underline{X} - \underline{Y}_k\right) + \lambda\cdot(\Gamma\underline{X})^2\right\} \quad (14)$$

$$\rho_{HAMPEL}(x) = \begin{cases} x^2 & ; |x| \le T_{HAMPEL} \\ 2T_{HAMPEL}|x| - T^2_{HAMPEL} & ; T_{HAMPEL} < |x| \le 2T_{HAMPEL} \\ 4T^2_{HAMPEL} - \frac{(3T_{HAMPEL} - |x|)^2}{4T^2_{HAMPEL}} & ; 2T_{HAMPEL} < |x| \le 3T_{HAMPEL} \\ & ; |x| > 3T_{HAMPEL} \end{cases} \quad (15)$$

Evaluating the mathematical close-form solution by the nonlinear optimization expertise, the mathematical close-form solution of Eq.(14) can be depicted as the forthcoming equation.

$$\hat{\underline{X}}_{n+1} = \hat{\underline{X}}_n + \beta\cdot\left\{\begin{array}{l}\sum_{k=1}^{N} F_k^T H_k^T D_k^T \cdot \psi_{HAMPEL}\left(\underline{Y}_k - D_k H_k F_k \hat{\underline{X}}_n\right) \\ -\left(\lambda\cdot\left(\Gamma^T\Gamma\right)\hat{\underline{X}}_n\right)\end{array}\right\} \quad (16)$$

$$\begin{aligned}\psi_{HAMPEL}(x) &= \rho'_{HAMPEL}(x) \\ &= \begin{cases} 2x & ; |x| \le T_{HAMPEL} \\ 2T_{HAMPEL}\,\mathrm{sign}(x) & ; T_{HAMPEL} < |x| \le 2T_{HAMPEL} \\ 2\left(3T_{HAMPEL} - |x|\right)\mathrm{sign}(x) & ; 2T_{HAMPEL} < |x| \le 3T_{HAMPEL} \\ 0 & ; |x| > 3T_{HAMPEL} \end{cases}\end{aligned} \quad (17)$$

2.3 Robust Norm Function for SRR Algorithm: Nonzero-Redescending Influence Function

This section presents the robust norm functions that have the influence function to be decreasing but nonzero when the error rises beyond a fixed point. (so called nonzero-redescending influence function)

2.3.1 Lorentzian Norm Function for SRR Algorithm

The Lorentzian norm function [9] as shown in Fig. 1(f) is the first robust norm function of this group that has been undertaken for SRR algorithm (for non-Gaussian outlier) since 2006 because of its statistical performance. The SRR algorithm will become the minimized equation as forthcoming.

$$\underline{X} = \underset{\underline{X}}{ArgMin}\left\{\sum_{k=1}^{N}\rho_{LOR}\left(D_k H_k F_k \underline{X} - \underline{Y}_k\right) + \lambda\cdot(\Gamma\underline{X})^2\right\} \quad (18)$$

$$\rho_{LOR}(x) = \log\left[1 + \frac{1}{2}\left(\frac{x}{T_{LOR}}\right)^2\right] \tag{19}$$

Evaluating the mathematical close-form solution by the nonlinear optimization expertise, the mathematical close-form solution of Eq.(18) can be depicted as the forthcoming equation.

$$\underline{\hat{X}}_{n+1} = \underline{\hat{X}}_n + \beta \cdot \left\{ \begin{array}{l} \sum_{k=1}^{N} F_k^T H_k^T D_k^T \cdot \psi_{LOR}\left(\underline{Y}_k - D_k H_k F_k \underline{\hat{X}}_n\right) \\ -\left(\lambda \cdot \left(\Gamma^T \Gamma\right) \underline{\hat{X}}_n\right) \end{array} \right\} \tag{20}$$

$$\psi_{LOR}(x) = \rho'_{LOR}(x) = \frac{2x}{2T_{LOR}^2 + x^2} \tag{21}$$

2.3.2 Geman & Mcclure Norm Function for SRR Algorithm

The Geman & Mcclure norm function [14] as shown in Fig. 1(g) is one of the robust norm functions that been undertaken for SRR algorithm (for non-Gaussian outlier) since 2008. The SRR algorithm will become the minimized equation as forthcoming.

$$\underline{X} = \underset{\underline{X}}{ArgMin}\left\{ \sum_{k=1}^{N} \rho_{GM}\left(D_k H_k F_k \underline{X} - \underline{Y}_k\right) + \lambda \cdot \left(\Gamma \underline{X}\right)^2 \right\} \tag{22}$$

$$\rho_{GM}(x) = T_{GM}^2\left(\frac{x^2}{T_{GM}^2 + x^2}\right) \tag{23}$$

Evaluating the mathematical close-form solution by the nonlinear optimization expertise, the mathematical close-form solution of Eq.(22) can be depicted as the forthcoming equation.

$$\underline{\hat{X}}_{n+1} = \underline{\hat{X}}_n + \beta \cdot \left\{ \begin{array}{l} \sum_{k=1}^{N} F_k^T H_k^T D_k^T \cdot \psi_{GM}\left(\underline{Y}_k - D_k H_k F_k \underline{\hat{X}}_n\right) \\ -\left(\lambda \cdot \left(\Gamma^T \Gamma\right) \underline{\hat{X}}_n\right) \end{array} \right\} \tag{24}$$

$$\psi_{GM}(x) = \rho'_{GM}(x) = T_{GM}^4 \frac{2x}{\left(T_{GM}^4 + x^2\right)^2} \tag{25}$$

2.3.3 Leclerc Norm Function for SRR Algorithm

The Leclerc norm function [15] as shown in Fig. 1(h) is one of the robust norm functions that has been undertaken for SRR algorithm (for non-Gaussian outlier) since 2009. The SRR algorithm will become the minimized equation as forthcoming.

$$\underline{X} = \underset{\underline{X}}{ArgMin}\left\{ \sum_{k=1}^{N} \rho_{LEC}\left(D_k H_k F_k \underline{X} - \underline{Y}_k\right) + \lambda \cdot \left(\Gamma \underline{X}\right)^2 \right\} \tag{26}$$

$$\rho_{LEC}(x) = 1 - \exp\left(-\frac{x^2}{T_{LEC}^2}\right) \tag{27}$$

Evaluating the mathematical close-form solution by the nonlinear optimization expertise, the mathematical close-form solution of Eq.(26) can be depicted as the forthcoming equation.

$$\underline{\hat{X}}_{n+1} = \underline{\hat{X}}_n + \beta \cdot \left\{ \begin{array}{l} \sum_{k=1}^{N} F_k^T H_k^T D_k^T \cdot \psi_{LEC}\left(\underline{Y}_k - D_k H_k F_k \underline{\hat{X}}_n\right) \\ -\left(\lambda \cdot \left(\Gamma^T \Gamma\right) \underline{\hat{X}}_n\right) \end{array} \right\} \tag{28}$$

$$\psi_{LEC}(x) = \rho'_{LEC}(x) = \left(\frac{2x}{T_{LEC}^2}\right)\exp\left(-\frac{x^2}{T_{LEC}^2}\right) \tag{29}$$

2.3.4 Myriad Norm Function for SRR Algorithm

The Myriad norm function [16,18] as shown in Fig. 1(i) is one of the robust norm functions that has been undertaken for SRR algorithm (for non-Gaussian outlier) since 2010. The SRR algorithm will become the minimized equation as forthcoming.

$$\underline{X} = \underset{\underline{X}}{ArgMin}\left\{ \sum_{k=1}^{N} \rho_{MYRIAD}\left(D_k H_k F_k \underline{X} - \underline{Y}_k\right) + \lambda \cdot \left(\Gamma \underline{X}\right)^2 \right\} \tag{30}$$

$$\rho_{MYRIAD}(x) = T_{MYRIAD}^2 \log\left(T_{MYRIAD}^2 + x^2\right) \tag{31}$$

Evaluating the mathematical close-form solution by the nonlinear optimization expertise, the mathematical close-form solution of Eq.(30) can be depicted as the forthcoming equation.

$$\underline{\hat{X}}_{n+1} = \underline{\hat{X}}_n + \beta \cdot \left\{ \begin{array}{l} \sum_{k=1}^{N} F_k^T H_k^T D_k^T \cdot \psi_{MYRIAD}\left(\underline{Y}_k - D_k H_k F_k \underline{\hat{X}}_n\right) \\ -\left(\lambda \cdot \left(\Gamma^T \Gamma\right) \underline{\hat{X}}_n\right) \end{array} \right\} \tag{32}$$

$$\psi_{MYRIAD}(x) = \rho'_{MYRIAD}(x) = \frac{2T_{MYRIAD}^2 x}{\left(T_{MYRIAD}^2 + x^2\right)} \tag{33}$$

2.3.5 Meridian Norm Function for SRR Algorithm

The Meridian norm function [17,19] as shown in Fig. 1(j) is one of the robust norm functions that use to be undertaken for SRR algorithm (for non-Gaussian outlier) in 2011. The SRR algorithm will become the minimized equation as forthcoming.

$$\underline{X} = \underset{\underline{X}}{ArgMin}\left\{ \sum_{k=1}^{N} \rho_{MER}\left(D_k H_k F_k \underline{X} - \underline{Y}_k\right) + \lambda \cdot \left(\Gamma \underline{X}\right)^2 \right\} \tag{34}$$

$$\rho_{MER}(x) = T_{MER}^2 \log\left(T_{MER}^2 + |x|\right) \tag{35}$$

Evaluating the mathematical close-form solution by the nonlinear optimization expertise, the mathematical close-form solution of Eq.(34) can be depicted as the forthcoming equation.

$$\underline{\hat{X}}_{n+1} = \underline{\hat{X}}_n + \beta \cdot \left\{ \begin{array}{l} \sum_{k=1}^{N} F_k^T H_k^T D_k^T \cdot \psi_{MER}\left(\underline{Y}_k - D_k H_k F_k \underline{\hat{X}}_n\right) \\ -\left(\lambda \cdot \left(\Gamma^T \Gamma\right) \underline{\hat{X}}_n\right) \end{array} \right\} \tag{36}$$

$$\psi_{MER}(x) = \rho'_{MER}(x) = \frac{T_{MER}^2 \operatorname{sgn}(x)}{\left(T_{MER}^2 + |x|\right)} \tag{37}$$

2.4 Robust Norm Function for SRR Algorithm: Non-Redescending Influence Function

This section presents the robust norm functions that have the influence function to be nondecreasing when the error rises beyond a fixed point. (so called non-redescending influence function)

2.4.1 Huber Norm Function for SRR Algorithm

The Huber norm function [10] as shown in Fig. 1(k) is one of the robust norm functions that has been undertaken for SRR algorithm (for non-Gaussian outlier) since 2006. The SRR algorithm will become the minimized equation as forthcoming.

$$\underline{X} = \underset{\underline{X}}{ArgMin}\left\{\sum_{k=1}^{N} \rho_{HUBER}\left(D_k H_k F_k \underline{X} - \underline{Y}_k\right) + \lambda \cdot \left(\Gamma \underline{X}\right)^2\right\} \quad (38)$$

$$\rho_{HUBER}(x) = \begin{cases} x^2 & ; |x| \le T_{HUBER} \\ T_{HUBER}^2 + 2T_{HUBER}\left(|x| + T_{HUBER}\right) & ; |x| > T_{HUBER} \end{cases} \quad (39)$$

Evaluating the mathematical close-form solution by the nonlinear optimization expertise, the mathematical close-form solution of Eq.(38) can be depicted as the forthcoming equation.

$$\hat{\underline{X}}_{n+1} = \hat{\underline{X}}_n + \beta \cdot \left\{ \begin{array}{l} \sum_{k=1}^{N} F_k^T H_k^T D_k^T \cdot \psi_{HUBER}\left(\underline{Y}_k - D_k H_k F_k \hat{\underline{X}}_n\right) \\ -\left(\lambda \cdot \left(\Gamma^T \Gamma\right) \hat{\underline{X}}_n\right) \end{array} \right\} \quad (40)$$

$$\psi_{HUBER}(x) = \rho'_{HUBER}(x) = \begin{cases} 2x & ; |x| \le T_{HUBER} \\ 2T_{HUBER} \cdot \text{sign}(x) & ; |x| > T_{HUBER} \end{cases} \quad (41)$$

3. COMPARATIVE EXPERIMENTAL EXPLORATION

All experiments were figured out by MATLAB software. In this simulation, an HR image is divided into the overlapped square of 8x8 ($M = 8$) and each square is arranged in the lexicography format of 64x1. The dimension of small size square area of LR image with overlapping are adjusted to be 16x16 ($q = 8$) and arranged in the lexicography format of 256x1.

The two groups of four tested LR images are synthesized from Susie (40^th) (176x144) and Lena (512x512) as following.

The original HR image ($\underline{X}$) is translated by a single pixel in the vertical direction to be translated HR image ($F_k\underline{X}$).

The translated HR image ($F_k\underline{X}$) is blurred by matrix H_k as 3x3 Gaussian blurred with SD=1 to be blurred translated HR image($H_k F_k \underline{X}$).

The blurred translated HR image ($H_k F_k \underline{X}$) is downsampling by 2 with matrix D_k for both vertical and horizontal directions to be blurred translated LR image ($D_k H_k F_k \underline{X}$).

The blurred translated LR image ($D_k H_k F_k \underline{X}$) is corrupted by 5 different noise models: one noiseless, five AWGN, one Poisson noise, three Speckle noise and three Salt and Pepper noise to be noisy blurred translated LR image ($D_k H_k F_k \underline{X} + \underline{V}_k$).

The above synthesized process with different translation in vertical and horizontal directions was implemented for synthesizing a group of four LR images (88x72 for Susie and 256x256 for Lena) from the original HR image.

This simulation experiment applies the iterative SRR algorithm, based on Bayesian ML (Maximum Likelihood) approach and a regularization technique, using several robust norm functions to the two groups (Y_k) of four tested LR images: Lena and Susie (40th Frame) in order to reconstruct the estimated original HR (High Resolution) image ($\hat{\underline{X}}_n$) (so called a SR image).

(The parameters in this simulations (such as step-size β, regularization parameter λ, norm constant parameter T and number of iteration n) in this simulation were set such as the reconstructed SR image has the highest PSNR and the maximum visual quality. Thereby, for guaranteeing justice, each simulation was restated several times with different set of simulated parameters and the reconstructed SR image the highest PSNR and the most sight sensing attraction from each simulating was nominated [9-18]).

The comparative experimental exploration results of the two groups of four tested LR images: Lena and Susie (40th Frame) are illustrated as Table 1 and Table 2 respectively (for objective measurement). Due to limitation of publication pages, only the experimental results of Lena image are illustrated in Fig. 2. From these result, we can summarize the general remark as following.

3.1 Results of Noiseless Case

For the average PSNR point of view in Table 1 (Lena) and Table 2 (Susie), the robust norm estimator, which is a nonzero-redescending influence function (Lorentzian, Leclerc, Geman&McClure, Myriad and Meridian) (Lena : 31.9282 dB and Susie : 35.2779 dB) and non-redescending influence function (Huber) (Lena : 32.0186 dB and Susie : 35.1436 dB) give the better average PSNR than zero-redescending influence functions (Tukey's Biweight, Andrew's Sine and Hampel) (Lena : 31.5448 dB and Susie : 34.7454 dB) and the classical norm estimator (L1 and L2) (Lena : 29.8594 dB and Susie : 33.1844 dB). Moreover, all robust norm functions dramatically give the better average PSNR than the classical norm estimator (L1 and L2) up to 2-3 dB.

For the highest PSNR point of view, the Meridian norm estimator (Lena : 32.1825 dB and Susie : 35.7149 dB), gives the highest PSNR. Later, Huber norm estimator (Lena : 32.0186 dB and Susie : 35.1436 dB), Myriad norm estimator (Lena : 32.0007 dB and Susie : 35.3357 dB) and Lorentzian norm estimator (Lena : 31.9565 dB and Susie : 35.2853 dB) give the better PSNR than other robust norms.

Next, the visual experimental results of Lena image is demonstrated in Fig. 2 (a-1) to Fig. 2(a-13) and the SR images from each groups of influence robust functions are closely similar but the quality of the SR images from the robust estimation is obviously better than the SR images from the classical estimation (L1 and L2).

3.2 Results of AWGN Case

For the average PSNR of point of view in Table 1 (Lena) and Table 2 (Susie), the robust norm estimator, which is a non-redescending influence function (Huber) (Lena : 28.5227 dB and Susie : 30.8137 dB) gives the better average PSNR than a nonzero-redescending influence function (Lorentzian, Leclerc, Geman&McClure, Myriad and Meridian) (Lena : 28.0976 dB and Susie : 29.9930 dB) and zero-redescending influence functions (Tukey's Biweight, Andrew's Sine and Hampel) (Lena : 27.8038 dB and Susie : 29.6249 dB) and the classical norm estimator (L1 and L2) (Lena : 27.5287 dB and Susie : 29.4263 dB).

For the highest PSNR point of view, the robust Huber norm estimator (Lena : 28.5227 dB and Susie : 30.8137 dB) gives the highest PSNR. Later, L2 norm estimator (Lena : 28.3755 dB and Susie : 30.3403 dB) give the better PSNR than other robust norms.

Next, the visual experimental results of Lena image for this case is demonstrated in Fig. 2 (b-1) to Fig. 2(b-13), Fig. 2 (c-1) to Fig. 2(c-13), Fig. 2 (d-1) to Fig. 2(d-13), Fig. 2 (e-1) to Fig. 2(e-13) and Fig. 2 (f-1) to Fig. 2(f-13) for SNR=25dB, SNR=22.5dB, SNR=20dB, SNR=17.5dB, and SNR=15dB, respectively.

3.3 Results of Poisson Noise Case

For the average PSNR point of view in Table 1 (Lena) and Table 2 (Susie), the robust norm estimator, which is a non-redescending influence function (Huber) (Lena : 28.7282 dB and Susie : 30.8496 dB) gives the better average PSNR than nonzero-redescending influence function (Lorentzian, Leclerc, Geman&McClure, Myriad and Meridian) (Lena : 28.4238 dB and Susie : 30.3185 dB), zero-redescending influence functions (Tukey's Biweight, Andrew's Sine and Hampel) (Lena : 28.4103 dB and Susie : 30.3997 dB) and the classical norm estimator (L1 and L2) (Lena : 27.8397 dB and Susie : 29.8416 dB). Moreover, all robust norm functions dramatically give the better average PSNR than the classical norm estimator (L1 and L2) up to 0.5 dB.

For the highest PSNR point of view, the robust Huber norm estimator (Lena : 28.7282 dB and Susie : 30.8496 dB) and the Andrew's Sine norm estimator (Zero-Redescending Influence Function) (Lena : 28.7302 dB and Susie : 30.8360 dB), give the highest PSNR. Later, Hampel norm estimator (Lena : 28.7130 dB and Susie : 30.7853 dB) and L2 norm estimator (Lena : 28.7190 dB and Susie : 30.7634 dB) gives the better PSNR than other norms.

Next, the visual experimental results of Lena image for this case is demonstrated in Fig. 2 (g-1) to Fig. 2(g-13) and the SR images from each group of influence robust functions are closely similar but the quality of the SR images from the robust estimation is obviously better than the SR images from the classical estimation (L1 and L2).

3.4 Results of S&P Noise Case

For the average PSNR point of view in Table 1 (Lena) and Table 2 (Susie), the robust norm estimator, which is a nonzero-redescending influence function (Lorentzian, Leclerc, Geman&McClure, Myriad and Meridian) (Lena : 31.3045 dB and Susie : 35.0194 dB) gives the better average PSNR than zero-redescending influence functions (Tukey's Biweight, Andrew's Sine and Hampel) (Lena : 31.0740 dB and Susie : 34.4791 dB), non-redescending influence function (Huber) (Lena : 30.9305 dB and Susie : 34.4073 dB) and the classical norm estimator (L1 and L2) (Lena : 27.1214 dB and Susie : 29.0485 dB). Moreover, all robust norm functions dramatically give the better average PSNR than the classical norm estimator (L1 and L2) up to 3.5-5.0 dB.

For the highest PSNR point of view, the robust Meridian norm estimator (Lena : 31.4342 dB and Susie : 35.1881 dB) and the Geman & Mcclure norm estimator (Lena : 31.5845 dB and Susie : 35.1932 dB) give the highest PSNR in all three noise cases (SNR=0.005, D=0.010, D=0.015). Later, Tukey's Biweigth norm, Lorentzian, Leclerc and Myriad gives the better PSNR than other robust norms.

Next, the visual experimental results of Lena image for this case is demonstrated in Fig. 2 (h-1) to Fig. 2(h-13), Fig. 2 (i-1) to Fig. 2(i-13) and Fig. 2 (j-1) to Fig. 2(j-13) for D=0.005, D=0.010, D=0.015, respectively. The SR images from each group of influence robust functions are closely similar but the quality of the SR images from the robust estimation is obviously better than the SR images from the classical estimation (L1 and L2).

3.5 Results of Speckle Noise Case

For the average PSNR point of view in Table 1 (Lena) and Table 2 (Susie), the robust norm estimator, which is a non-redescending influence function (Huber) (Lena : 26.3659 dB and Susie : 29.5089 dB) gives the better average PSNR than a nonzero-redescending influence function (Lorentzian, Leclerc, Geman&McClure, Myriad and Meridian) (Lena : 25.6570 dB and Susie : 28.6674 dB) and zero-redescending influence functions (Tukey's Biweight,

Table 1: *The performance and comparative exploration of reconstructed SR image: Lena (in PSNR).*

Noise Model		Reconstructed SR Image (PSNR in dB)										
	LR Frame	L1	L2	Tukey	Andrew	Hampel	Lorentz	G&M	Leclerc	Myriad	Mer.	Huber
Noiseless	28.8634	28.8634	30.8553	31.5889	31.5579	31.4877	31.9565	31.8216	31.6798	32.0007	32.1825	32.0186
AWGN												
SNR=25	27.8884	27.9490	29.6579	29.1331	29.7392	29.7453	29.7359	29.6364	29.6833	29.7312	29.1246	29.7224
SNR=22.5	27.2417	27.4918	29.1611	28.4460	29.1980	29.1916	29.1927	28.9743	29.0421	29.1613	28.5588	29.1935
SNR=20	26.2188	26.7854	28.6024	27.5795	28.6288	28.6089	28.5610	28.1350	28.2238	28.4625	27.8683	28.6305
SNR=17.5	24.9598	26.0348	27.8153	26.5722	27.8765	27.8186	27.7621	27.1096	27.1801	27.6065	27.0903	27.8725
SNR=15	23.3549	25.1488	26.6406	25.8894	26.7178	26.6117	26.7566	25.9241	25.9488	26.6049	26.3668	27.1945
Poisson	26.5116	26.9604	28.7190	27.7876	28.7302	28.7130	28.6735	28.4313	28.4046	28.5916	28.0182	28.7282
S&P:												
D=0.005	26.8577	27.1149	28.8495	31.3146	30.9544	30.9745	31.1843	31.5831	31.3655	31.3578	31.4675	30.9462
D=0.010	25.2677	26.0569	28.0346	31.3423	30.9482	30.9721	31.0524	31.5814	31.3597	31.3528	31.4608	30.9329
D=0.015	24.219	25.3534	27.3188	31.2792	30.9435	30.9652	30.0229	31.5890	31.3573	31.4592	31.3744	30.9124
Speckle:												
V=0.03	23.5294	25.3133	26.6956	25.8825	26.1051	25.9440	26.0696	26.0561	25.8783	25.9141	25.9389	26.6723
V=0.05	21.7994	24.4215	25.3165	25.2894	25.2729	25.2726	25.3136	25.5008	25.2822	25.3078	25.3088	26.0595

Table 2: *The performance and comparative exploration of reconstructed SR image: Susie (in PSNR).*

Noise Model		Reconstructed SR Image (PSNR in dB)										
	LR Frame	L1	L2	Tukey	Andrew	Hampel	Lorentz	G&M	Leclerc	Myriad	Mer.	Huber
Noiseless	32.1687	32.1687	34.2000	34.7056	34.7837	34.7470	35.2853	35.1087	34.9449	35.3357	35.7149	35.1436
AWGN												
SNR=25	30.1214	30.3719	32.3688	31.3532	32.3923	31.6115	32.2341	32.0580	32.2053	32.2933	31.4804	32.3936
SNR=22.5	29.0233	29.6481	31.6384	30.4980	31.7038	31.6813	31.4751	31.0733	31.2254	31.4217	30.7234	31.6806
SNR=20	27.5316	28.7003	30.6898	29.3590	30.7257	30.6642	30.5472	29.8789	30.0717	30.3597	29.9152	30.7518
SNR=17.5	25.7322	27.5771	29.3375	28.6044	29.4251	29.3112	29.4712	28.6165	28.6075	29.3263	29.1792	30.0448
SNR=15	23.7086	26.2641	27.6671	27.6932	27.7981	27.6565	28.1516	27.6848	27.6858	28.1426	28.0866	29.1977
Poisson	27.9071	28.9197	30.7634	29.5778	30.8360	30.7853	30.6934	30.0703	30.2880	30.5118	30.029	30.8496
S&P:												
D=0.005	29.0649	29.5041	31.5021	34.5282	34.4748	34.4785	34.7155	35.2115	34.9323	34.9379	35.2243	34.4428
D=0.010	26.4446	27.7593	29.8395	34.5169	34.4742	34.4803	34.7194	35.1929	35.1510	34.9357	35.1774	34.4171
D=0.015	25.276	26.9247	28.7614	34.5018	34.4497	34.4483	34.6991	35.1752	35.1510	34.9046	35.1626	34.3620
Speckle:												
V=0.01	27.6166	28.8289	30.6139	29.3607	30.4604	30.4150	29.8499	29.3449	29.3499	29.7793	29.4441	30.4619
V=0.02	25.3563	27.5527	28.9409	28.4824	28.6021	28.4404	28.5018	28.4641	28.4703	28.4945	28.5026	29.3749
V=0.03	24.0403	26.8165	27.7654	27.9699	27.9403	27.9409	27.9468	27.9544	27.9614	27.9627	27.9846	28.6900

Fig.1: *The characteristic of the norm functions and the influence functions.*

Fig.2: *The characteristic of the norm functions and the influence functions (Cont.).*

Fig.3: *The characteristic of the norm functions and the influence functions (Cont.).*

Fig.4: *The comparative experimental exploration results of SRR algorithms using several robust norm functions (The lowest in position portrait on our simulated outcome of each sub- portrait is the absolute difference between it's correspond the highest in position (SR) portrait and the original HR; The absolute difference is multiplied by five in order to obviously sight sensing.).*

Fig.5: *The comparative experimental exploration results of SRR algorithms using several robust norm functions (The lowest in position portrait on our simulated outcome of each sub- portrait is the absolute difference between it's correspond the highest in position (SR) portrait and the original HR; The absolute difference is multiplied by five in order to obviously sight sensing.).*

Fig.6: *The comparative experimental exploration results of SRR algorithms using several robust norm functions (The lowest in position portrait on our simulated outcome of each sub- portrait is the absolute difference between it's correspond the highest in position (SR) portrait and the original HR; The absolute difference is multiplied by five in order to obviously sight sensing.).*

Fig.7: *The comparative experimental exploration results of SRR algorithms using several robust norm functions (The lowest in position portrait on our simulated outcome of each sub- portrait is the absolute difference between it's correspond the highest in position (SR) portrait and the original HR; The absolute difference is multiplied by five in order to obviously sight sensing.).*

Fig.8: *The comparative experimental exploration results of SRR algorithms using several robust norm functions (The lowest in position portrait on our simulated outcome of each sub- portrait is the absolute difference between it's correspond the highest in position (SR) portrait and the original HR; The absolute difference is multiplied by five in order to obviously sight sensing.).*

Fig.9: *The comparative experimental exploration results of SRR algorithms using several robust norm functions (The lowest in position portrait on our simulated outcome of each sub- portrait is the absolute difference between it's correspond the highest in position (SR) portrait and the original HR; The absolute difference is multiplied by five in order to obviously sight sensing.).*

Andrew's Sine and Hampel) (Lena : 25.4530 dB and Susie : 28.6403 dB) and the classical norm estimator (L1 and L2) (Lena : 25.4367 dB and Susie : 28.4197 dB).

For the highest PSNR point of view, the robust Huber norm estimator (Lena : 26.3659 dB and Susie : 29.5089 dB) gives the highest PSNR. Later, L2 norm estimator (Lena : 26.0061 dB and Susie : 29.1067 dB) give the better PSNR than other norms. Next, the visual experimental results of Lena image for this case is demonstrated in Fig. 2 (k-1) to Fig. 2(k-13) for V=0.05.

4. THE EXPERIMENTAL CONCLUSION AND DISCUSSION

This paper thoroughly presents an experimental exploration of an iterative SRR framework based on several robust norm functions such as a zero-redescending influence function (Tukey's Biweight, Andrew's Sine and Hampel), a nonzero-redescending influence function (Lorentzian, Leclerc, Geman&McClure, Myriad and Meridian) and a non-redescending influence functions (Huber). This paper utilizes two standard images Lena and Susie (40th) for pilot studies and fraudulent noise patterns of noiseless, AWGN, Poisson, Salt&Pepper, and Speckle of several magnitudes are used to contaminate these two standard images. Both of the subjective outcomes (visual results) and objective outcomes (PSNR in dB) are thoroughly presented and the experimental discussion also explains in the experimental section.

References

[1] D. Rajan, S. Chaudhuri, and M. V. Joshi, "Multi-objective super resolution concepts and examples," *IEEE Signal Processing Mag.*, vol. 20, no. 3, pp. 49-61, May. 2003.

[2] Moon Gi Kang, and Subhasis Chaudhuri, "Super-Resolution Image Reconstruction," *IEEE Signal Processing Mag.*, vol. 20, no. 3, pp. 21-36, May. 2003.

[3] M. K. Ng, and Nirmal K. Bose, "Mathematical analysis of super-resolution methodology," *IEEE Signal Processing Mag.*, vol. 20, no. 3, pp. 62-74, May. 2003.

[4] S. C. Park, M. K. Park, and M. G. Kang, "Super-Resolution Image Reconstruction : A Technical Overview," *IEEE Signal Processing Mag.*, vol. 20, no. 3, pp. 21-36, May. 2003.

[5] R. R. Schultz, and R. L. Stevenson, "Extraction of High-Resolution Frames from Video Sequences," *IEEE Trans. Image Process.* , vol. 5, no. 6,pp. 996-1011, Jun. 1996.

[6] S. Farsiu, M. D. Robinson, M. Elad, and P. Milanfar, "Fast and Robust Multiframe Super Resolution," *IEEE Trans. Image Process.* , vol. 13, no. 10,pp. 1327-1344, Oct. 2004.

[7] M. J. Black, and A. Rangarajan, "On The Unification Of Line Processes, Outlier Rejection and Robust Statistics with Applications in Early Vision," *Int. J. of Comput. Vision*, vol. 19, no. 1, pp. 57-91, Jul. 1996.

[8] M. J. Black, G. Sapiro, D. H. Marimont, and D. Herrger, "Robust Anisotropic Diffusion," *IEEE*

Trans. Image Process., vol. 7, no. 3, pp. 421-432, May. 1998.

[9] V. Patanavijit, and S. Jitapunkul, "A Lorentzian Stochastic Estimation for an Robust and Iterative Multiframe Super-Resolution Reconstruction," *TENCON 2006. 2006 IEEE Region 10 Conf.*, Wan Chai, Hong Kong, 2006, pp. 1-4.

[10] V. Patanavijit, and S. Jitapunkul, "A Robust Iterative Multiframe Super-Resolution Reconstruction using a Huber Statistical Estimation Technique," *Communications and Networking in China, 2006. ChinaCom '06. 1st Int. Conf. on*, Beijing, China, 2006, pp. 1-3.

[11] V. Patanavijit, and S. Jitapunkul, "A Robust Iterative Multiframe Super-Resolution Reconstruction using a Bayesian Approach with Tukey's Biweigth," *Proceeding of IEEE Int. Conf. on Signal Processing 2006*, Guilin, China, 2006.

[12] Vorapoj Patanavijit, "A robust iterative multiframe SRR using stochastic regularization technique based on hampel estimation," *Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology, 2008. ECTI-CON 2008. 5th Int. Conf. on*, Krabi, Thailand, 2008, pp. 473-476.

[13] Vorapoj Patanavijit, "Andrew's Sine Estimation for a Robust Iterative Multiframe Super-Resolution Reconstruction using Stochastic Regularization Technique," *Circuits and Systems and TAISA Conf., 2008. NEWCAS-TAISA 2008. 2008 Joint 6th Int. IEEE Northeast Workshop on*, Montreal, Canada, 2008, pp. 145-148.

[14] Vorapoj Patanavijit, "A Robust Iterative Multiframe SRR using Stochastic Regularization Technique Based on Geman & Mcclure Estimation,"*Proc. of The Nat. Conf. on Information Technology 2008*, Bangkok, Thailand, 2008, pp. 241-247.

[15] Vorapoj Patanavijit, "Multiframe Resolution-Enhancement using A Robust Iterative SRR based on Leclerc Stochastic Technique," *Proc. of The 32nd Electrical Engineering Conf.*, Prachinburi, Thailand, 2009.

[16] Vorapoj Patanavijit, "A Robust Resolution-Enhancement using Recursive Multiframe Super Resolution Reconstruction based on Myriad Norm Estimation Technique with Myriad-Tikhonov Regularization," *Proc. of The 33nd Electrical Engineering Conf.*, Ching Mai, Thailand, 2010.

[17] Vorapoj Patanavijit, "A Recursive Resolution-Enhancement using Multiframe SRR based on Meridian Filter with Meridian-Tikhonov Regularization," *Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology, 2011 8th Int. Conf. on*, Khon Kaen, Thailand, 2011, pp. 1047-1050.

[18] Juan G, "Gonzalez and Gonzalo R. Arce, Statistically-Efficient Filtering in Impulsive Environments: Weighted Myriad Filters," *EURASIP Journal on Applied Signal Processing 2002*, 2002.

[19] Tuncer Can Aysal, and Kenneth E. Barner, "Meridian Filtering for Robust Signal Processing," *IEEE Trans. Signal Process.*, vol. 55, no. 8,pp. 3949-3962, Aug. 2007.

[20] Vorapoj Patanavijit, "Performance and Comparative Exploration of Reconstructed Quality for An Iterative SRR Algorithm Based on Robust Norm Functions Under Several Noise Surrounding," *Asia-Pacific Signal and Information Processing Association, 2014 Annual Summit and Conf.*, Khon Kaen, Cambodia, 2014, pp. 1-6.

Vorapoj Patanavijit He has served as a full-time lecturer at Department of Computer and Network Engineering, Faculty of Engineering, Assumption University since 1998 where he is currently an Assistance Professor (Electrical Engineering) in 2009. He has authored and co-authored over 100 national/international peer-reviewed publications in Digital Signal Processing (DSP) and Digital Image Processing (DIP). He has participated in more than 6 projects and research programs funded by public and private organizations. He has served as a Technical Program Committees (TPC) on Signal Processing of ECTI (Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology) Association, Thailand since 2012. Since 2006, he has served as a technical reviewer of IEEE Transactions on Image Processing, IEEE Journal of Selected Topics in Signal Processing (J-STSP), IET in Image Processing (IEEE), EURASIP Journal on Applied Signal Processing (JASP), Digital Signal Processing (Elsevier Ltd.), Neural Networks (Elsevier Ltd.), The Visual Computer (Springer), Journal of Electronic Imaging (SPIE), IEICE Journal Electronics Express (ELEX) and ECTI Transactions on CIT (ECTI Thailand). He works in the field of signal processing and multidimensional signal processing, specializing, in particular, on Image/Video Reconstruction, SRR (Super-Resolution Reconstruction), Compressive Sensing, Enhancement, Fusion, Denoising, Inverse Problems, Motion Estimation, Optical Flow Estimation and Registration.