Policies for Channel Allocation in Cognitive Radio Networks using Game Theory

Amulya Bhattarai *1, **Chalie Charoenlarpnopparut** *2, **Prapun Suksompong** *3, and **Patrachart Komolkiti** **4, Non-members

ABSTRACT

Cognitive radio networks evolve according to the actions of selfish users who act independently. The eventual state reached in most of the cases by such a system is a Nash Equilibrium (NE). Multiple NE with different qualities may exist. In this work we consider the problem of allocating channels to multiple transceivers. Based on policies derived from various network metrics we develop a way to initialize the allocation and choose the appropriate channel for each user to push the system towards a higher normalized cumulative total throughput of the CRN. A novel equation to allocate channels is also derived. The improvement is confirmed by our simulation results.

Keywords: Cognitive radio network (CRN), Policy for channel allocation, Game theory, Nash Equilibirum, Refine utility terms.

1. INTRODUCTION

With the drastic increment of data transmission over wireless mediums, researchers are looking for new techniques to enhance the capacity of wireless communications systems [1], [17]. Fixed allocation of resources for spectrum, power or transmission is inefficient, as much of the resources are unused by licensed users most of the time [2].

Dynamic resource allocation enhances the efficiency of a wireless system [4], as the the system adopts to changes in the network dynamically; this is the core idea of Cognitive Radio Network (CRN). CRN utilizes intelligent radios which are capable of sensing the environment, reasoning based on the observed information and previous data, and adopting the transmission parameters that result in an efficient system [13]. Transmission parameters such as bandwidth [5], transmission power [3], modulation schemes can be dynamically and strategically varied in relation to the CRN environment changes, which enhances the efficiency and performances of the CRN [18].

CRN consists of primary users (PUs) who are the licensed users and secondary users (SUs). SUs are intelligent users who make use of the unused resources allocated to PUs without interfering with or in the absence of the PUs [7]. Some CRNs only have SUs [6]. Others, including members of our research group, are working on the sensing of the PU [9] and resource allocation to PU [3]; this work focuses on the resource allocation to the SUs, after the sensing of PU and resource allocation to the PU is complete.

CRN can be categorized into various groups. In our work, CRN is categorized into three groups, as shown in Fig. 1. The first group computation refers to where the CRN runs its algorithms. In some cases a central entity collects data from all users and does all the computation, whereas in other cases all the computation is done in a distributed way by the individual users [15]. The second group considers the nature of the user; users can be selfish, altruistic or partially selfish, depending upon how their payoff is defined. The third group considers the information exchange between the users. If there is no information exchange, the user will be able to scan limited information, whereas in a complete information exchange scheme all users have access to the network metrics of all users. We focus on a distributed CRN where the users are selfish and there is partial information exchange. However, for comparative study and analysis, different groups, such as centralized and co-operative CRN, can be emulated by varying individual user's payoff and other parameters, which are delineated in detail in our work.

A global optimal allocation obtained from a centralized and co-operative CRN is impractical in a distributed CRN, as the selfish and independent SUs will deviate from such a solution if they find a way to increase their own payoff [14]. Thus the global optimum is attained for a short time. This deviation does not occur if the system is operated at one of the equilibrium states, Nash Equilibrium [16]. The concept of Nash Equilibrium (NE) was initially proposed by John Nash and further refined by others in the years to follow [21]. By definition,

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* The authors are with School of Information, Computer and Communication Technology, Sirindhorn International Institute of Technology, Thammasat University, Pathum Thani 12000, Thailand , E-mail : abhattarai@student.siit.tu.ac.th 1, chalie@siit.tu.ac.th 2, prapun@siit.tu.ac.th 3
** The author is with the Department of Computer Engineering, Vincent Mary School of Engineering, Assumption University, Samuthprakarn 10540, Thailand, E-mail : patrachart@eng.au.edu 4

at NE, none of the users will benefit by unilaterally deviating from their specific strategy [18]. NE is a natural and eventual solution reached by selfish and independent users. Each user tries to maximize its own payoff irrespective of others, and after a considerable amount of time the system can reach NE. In some cases, due to system evolutionary paths, it may fall into a loop, and thus never reach NE, and the users end up oscillating between strategy sets.

There can be several NEs, especially in a big network and reaching a good NE in a distributed way is a challenge [22], [23]. Defining the payoff of an individual user in a partially selfish or altruistic way can lead to better NE. In our previous work [11], we added interference terms in the utility of the individual user and were able to achieve a higher performance index of the CRN [8]. Several techniques have been explored to reach NE: best response (BR) technique [24], exhaustive search, etc. An exhaustive search technique for large networks is computationally overwhelming and impractical. In our previous work [11] we developed simple policies for channel allocation which saves time and computational resources. Developing policies for channel allocation can provide a good starting point for the best response algorithm.

The objective of our work is to develop practical channel allocation policies and refine the individual users payoff in order to achieve higher CRN performance in a distributed system. The performances of the CRN considered in this paper is normalized cumulative total throughput.

In Section II, the system model of our CRN is explained. The distribution of links and channels in the CRN, the mathematical formulation of different performance metrics in the CRN, and the assumptions and scope of the work, are illustrated. In Section III, Game Theory is analysed in detail. The definition of NE, ways of reaching NE, different ways of expressing payoff, which can emulate selfish or altruistic users, etc., are covered. In Section IV, the method used to derive policy and refine utility is discussed, and in Section V, policies are developed, based on the network metrics, and utility terms are refined. Section VI illustrates different kinds of algorithms for channel allocation in a distributed system mainly based on the best-response technique. Simulation results are presented in Section VII, which verify the performance achieved by using algorithms that use policy and refined utility. These are better than other algorithms, in a distributed CRN with partial information exchange. Section VIII concludes the work.

The objective of the work is to develop policies and refine the utility terms of the users so that we can achieve higher normalized cumulative total throughput for a distributed CRN system.

2. SYSTEM MODEL

This section discusses the essential components of the model. Initially, the links (receivers and transmitters) and channels in our model, are described. Then mathematical equations that describe the received power, interference and throughput at the links and network, are presented. Finally the assumptions made in our model are delineated.

2.1 Links and Channels

In our CRN, there are several Transmitters (Tx) and Receivers (Rx). One Tx establishes communication with an Rx which is referred to as a link. There can be many number of links in a CRN. Fig. 12 shows an example of ten links in a CRN. The co-ordinates of the Tx and Rx are randomly generated.

Limited resources that are available in the CRN have to be shared by the links. The resources that we try to allocate in this work are channel, spectra. N is the total number of links and C is the total number of channels available in the CRN. We assume there are less channels than the number of links, $C < N$; and that each link can occupy only one channel at a time.

2.2 Mathematical Formulations

The power received at the receiver from a transmitter, using the free space ideal propagation model, can be expressed as

$$P_r = \frac{G_r G_t P_t}{\left(\frac{4\pi d}{\lambda}\right)^2}, \tag{1}$$

where G_r and G_t are the antenna gain at the receiving and transmitting sides, respectively, and P_r and P_t are the power at the receiver and transmitter, respectively. The distance between the transmitter and the receiver is d, and λ is the wavelength of the electromagnetic wave used in the communication system [12]. The received power at receiver link j from the transmitter link i can be expressed as

$$p_{ij} = A/(d_{ij}^2), \tag{2}$$

where d_{ij} is the distance between the transmitter link i and receiver link j (the 1st index for the p, and d denote the transmitter link, and the 2nd index represents the receiver link). And $A = \frac{G_r G_t P_t}{\left(\frac{4\pi}{\lambda}\right)^2}$. In our work, the transmission power of all antennas and the gain of the antennas is a constant, unity. Similarly, as we are operating in a narrow-band spectrum, we consider the wavelength of the electromagnetic wave used in the CRN to be constant. Hence A is a constant. In order to make things simpler we let $A = 1$, or it is normalized to 1. The main idea is to explore the received power, with the variation of the link distance.

Various path-loss models can be used, a prominent and simple one being $L_p = kd^{\alpha}$, where k and α are propagation constants, and d is the distance between the transmitter and the receiver. According to the environment, different values of α, ranging from 2 to 5 can be used [12]. In our work $\alpha = 2$ is used, which describes a simple scenario, free space propagation as illustrated in (2).

In order to avoid very high or infinite values of p_{ij} in (2) when d_{ij} is very small, we consider $p_{ij} = 10$ when $d_{ij} < 0.316$ units, avoiding the near-field effects.

Then the Signal to Interference plus Noise Ratio (SINR) at the receiver link i is

$$\mathrm{SINR_i} = \frac{\mathrm{p_{ii}}}{\sum_{\mathrm{j=1\neq\, i}}^{\mathrm{N}} \mathrm{p_{ji}} + \sigma_{\mathrm{n}}^2}, \tag{3}$$

where p_{ii} is the power received at link i from transmitter link i, and σ_n^2 is the noise power. In this work noise power equals 0.1, which is 10dB smaller than the transmit power. In our other works noise power is 0.001, which is 30dB smaller than the transmit power. The results obtained have similar conclusions.

When links use different channels, there is no interference. Interference occurs only when links use the same channel.

The throughput of a link i with AWGN channel with $\mathrm{SINR_i}$, based on Shannon Capacity, can be expressed as [25]

$$T_i = \log_2(1 + \mathrm{SINR_i}) \tag{4}$$

Here, the throughput is in bit/s/Hz, that is the throughput is normalized over band-width.

We want to start with a simple model as the coverage area of our work is small and focus on the methodology and see if tangible performance is enhanced by the derived policy.

The total throughput of a network with N links is then the sum of all the N individual link throughputs in the CRN

$$T_{tot} = \sum_{i=1}^{N} T_i. \tag{5}$$

T_{tot} varies as different links choose different strategies. In an exhaustive search analysis, all the different possible pure strategies are listed, and T_{tot}^{max} is the maximum T_{tot}, which is illustrated in Fig. 2, 4.

Next we define the normalized total cumulative throughput. In a CRN where the users try to reach an equilibrium point in a distributed manner, it is essential to compute the cumulative total throughput especially when we need to make a comparison between different algorithms and their performance from the 1st iteration until a certain iteration. Normalized cumulative total throughput at the mth iteration is expressed as

$$T_{nor.cum.tot}^{m} = \frac{1}{m} \sum_{k=1}^{m} T_{tot}^{k}, \tag{6}$$

where T_{tot}^{k} is the total throughput at the kth iteration.

2.3 Assumptions

There are four prominent assumptions that have been made in our work, which are briefly described below.

As discussed in the Section 1. CRN has been categorized into three groups in our work, as shown in Fig. 1. The three assumptions made in our work are related to the categories mentioned in the figure.

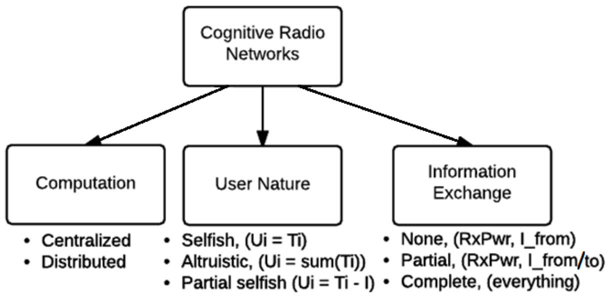

Fig.1: *CRN can be categorized into various groups. In our work, CRN is categorized into three groups: where the computation is performed, the nature of the user, and the information exchange between the users.*

The first assumption is that the users compute the resource allocation schemes themselves. Resource allocation in networks can be done in various ways. CRN central resource allocation is not practical and not popular, as the individual users are independent, and hence can deviate from the global central solution. In our work we assume the users choose the strategies independently.

The second assumption is that the SUs are rational and selfish. Users will choose the strategies in order to increase their own payoff. Payoff can be defined in different ways, which are illustrated briefly in Fig. 1. By defining the user's utility as an individual throughput, a selfish behavior can be emulated; similarly, if a user's utility is defined as the total throughput of the network, a co-operative scheme can be emulated. We consider the nature of the users to be partially selfish, but incorporate other scenarios for the purpose of analysis and comparison.

The third assumption is regarding the information sharing between users. Users might not share any information at all or share all possible information, as illustrated in Fig. 1. Our work focuses on partial information exchange. Each user can measure the received power and interference from other links, and the interference it gives to other links is exchanged with other users. This is further discussed in Section 3.3

The fourth assumption is that the best response method leads to NE (when there exists a NE in the system). If we let the users to independently choose the strategy that maximizes their individual payoff, after a considerable amount of time the system will naturally and eventually reach NE. NE is not an artificial imposition on the users or the CRN system, rather it is an equilibrium solution, which the system will converge to eventually; NE is further discussed in Section 3.1. In bigger networks there can be multiple NE's. Some NE's result in high network performances, whereas others do not.

To be able to allocate the resources to the CRN, users with the above mentioned objectives need to make use of certain mathematical tools. One prominent mathematical tool, which has been extensively used in resource allocation problems in CRN, is Game Theory, which is discussed in the next section.

3. GAME FORMULATIONS

Game theory is a mathematical tool that is used to analyse the strategic interactions of users, especially with conflicting interests. Over the years game theory has been used in CRN by users for resource allocation problems.

Generally in a game, there are N players, S_i is the set of strategies of player i, and U_i is the utility or outcome set of user i, which is a function of the strategy played by all users, $< N, S_i, U_i(S_i, S_{-i}) >$ where $i \in N$ and S_{-i} represents the strategies played by all users apart from i [21], [24]. In our work, the players are the links. Each link chooses to transmit in a particular channel, and as there are less channels than users, and different strategy set results in different individual payoff. Each user can choose a particular strategy, and different strategies result in different payoff or utility. Selfish, rational, and independent users will want to maximize their individual payoff. Over iterations, each user will try to maximize their individual payoff, and eventually they might converge to an equilibrium point.

3.1 Nash Equilibrium (NE)

Nash Equilibrium (NE) is a natural and eventual equilibrium state reached by selfish and independent users who choose strategies that will maximize their individual payoff. At NE, each user will not be able to increase its payoff by unilaterally changing its strategy [21], provided the other users are playing the best response strategies as well, which is expressed mathematically as

$$U(S_i^*, S_{-i}^*) \geq U(S_i, S_{-i}^*) \quad \forall i \in N. \tag{7}$$

where S_i^* and S_{-i}^* represents the best response strategy of user i, and all others users except i, respectively.

Nash Equilibrium is an important concept and equilibrium solution, especially in a system where selfish users can make independent decisions. A centralized global solution may result in a higher overall performance, but once the selfish users are allowed to make their independent choices, they will drift away from the centralized global solution. However, once a system converges to a NE, none of the selfish users will change their strategies, as by doing so they will get a lower individual payoff, which is illustrated in (7).

In a system, there can be multiple NEs. Some NE are better, whereas others may not be as good. Depending upon how we define the utility, we may converge to get different results and NE points.

3.2 Ways to reach NE

There are various ways we can reach NE. Two prominent ones that are used in this work are explained below.

First let us delve into the exhaustive search technique. This technique explores all the possible configurations, and hence it gives us an exhaustive analysis of the various performance metrics of the CRN [24]. For example, for the scenario, "3links 2channels" there are $2^3 = 8$ different channel allocation possibilities, which is shown in Fig. 2, along with the individual and total SINR. To check if a configuration is a NE or not, we have to make sure that each user cannot get a higher payoff by deviating from the NE strategy set, as shown in Fig. 3, and explained in detail in our previous work [11].

No	Ch Con	SINR User 1	SINR User2	SINR User3	Sum_SINR	NE?
1	111	2.1687	0.2064	0.0823	2.4574	NO
2	112	2.8571	0.2273	1.1111	4.1955	NO
3	121	4.7368	2.5	0.101	7.3379	NO
4	122	10	1.1842	0.3175	11.5017	YES
5	211	10	1.1842	0.3175	11.5017	YES
6	212	4.7368	2.5	0.101	7.3379	NO
7	221	2.8571	0.2273	1.1111	4.1955	NO
8	222	2.1687	0.2064	0.0823	2.4574	NO

Fig.2: *Exhaustive search analysis of a "random: 3links 2channels" scenario, which shows all the possible channel allocation configurations* $2^3 = 8$.

As we move to bigger networks, exhaustive search becomes cumbersome, for "5links 3channels" there are $3^5 = 253$ different channel configurations, whereas for "10links 4channels", there are $4^{10} = 1,048,576$ different channel configurations, and for bigger networks the exhaustive search analysis becomes impossible.

Secondly, let us delve into the best-response

	Ch Config u1,u2,u3	SINR,u1	SINR,u2	SINR,u3	
4	122	10	1.1842	0.3175	IsN.E
	222	2.1687	0.2064	0.0823	
	112	2.8571	0.2273	1.1111	
	121	4.7368	2.5	0.101	

Fig.3: *Verifying the channel configuration "122" from Fig. 2 is a NE.*

technique. This technique is practiced in every-day life to reach an equilibrium point where many independent and selfish users exist. A user chooses the strategy that gives the highest payoff. Then, the next user chooses a strategy that gives that particular user the highest payoff; all users keep doing this until they reach an equilibrium point. Different versions of the best-response technique are possible, which are further explained in Section 6.

MSA (Method of Successive Average); Algebriac techniques are also other ways by which we can reach NE, which are discussed in detail in our previous work [11].

3.3 Utility varied to portray different CRN scenarios

By defining the utility in different ways, it is possible to portray different CRN scenarios, as illustrated in Fig. 1, which is further explained below.

First, we shall discuss the distributed CRN, where all the individual users are independent. There is no central controller enforcing rules and regulations on the individual users. Each user is selfish and tries to maximize its own throughput. The individual utility is defined as

$$U_i = T_i \tag{8}$$

There is no information exchange between the users. A user can measure the received power and interference it receives, and hence compute individual SINR and throughput.

The second scenario is a centralized and co-operative CRN which generally has a central unit, which controls/oversees the activities of all users. Users report the information they sense locally to the central unit, and the central unit decides the individual user's strategy that is beneficial for the entire network [19]. In [20], individual utility is defined as the total throughput of the network

$$U_i = \sum_{n=1} (T_i). \tag{9}$$

Each user in the CRN tries to maximize the total throughput of the system, and hence the users are considered altruist. The users share all information with each other. The solutions reached in such a network is also referred as global optimal solutions.

The third scenario is a distributed CRN with partial information exchange. The users exchange some vital information with each other while they are still independent and selfish. In our previous work [10], we modified the utility presented in [6] to include terms I_{from} and I_{to}, which are the sum of interference user i receives from other users, and gives to other users, respectively. Each user tries to maximize its individual utility U_i (10), which results in maximizing the received power and minimizing the interference terms. In this way, a higher overall performance index is achieved.

$$U_i = \alpha \cdot P_r - \beta \cdot I_{from} - \gamma \cdot I_{to} \tag{10}$$

where α, β and γ are constants which are further discussed in Section 5.3 In the next section, we study the characteristics of network metrics, based on which we will develop policies and refine utility terms constants, to achieve higher performance.

4. METHODOLOGY

In this section, the methodology used to develop policies and refine utility for enhancing the performance metrics of the CRN, are explained. Initially, the characteristics of network metrics, especially SINR, I_{from}, and I_{to} are analysed. Are there any patterns and specific probability distribution of the network metrics at NE? In the second part, policies are developed based on the distribution and characteristics of the network metrics. In the third part, we refine the terms in the utility of a user based on the distribution and characteristics of the network metrics obtained in the earlier parts. The next section, describes the three steps mentioned in detail.

5. FORMULATING POLICY AND REFINING UTILITY TERMS

In this section we formulate policy and refine the utility terms. However, before doing so, we analyse the characteristics of SINR, I_{from}, and I_{to} at NE, as mentioned in Section 4. which is helpful for formulating the policies and refining the utility terms.

5.1 Distribution of network metrics

Received power, interference from other users, and interference to other users, are important network metrics in CRN. In this subsection, we will analyze the histogram distribution of network metrics at NE. Based on the distribution, it is possible to develop policies and refine the utility terms. To obtain the pdf as mentioned in the flow chart, it is essential to perform the exhaustive search analysis. Fig. 4 shows the exhaustive search analysis of a "random: 5links 3channels" scenario.

There are all together $3^5 = 243$ different channel configurations. Out of the 243 different possibilities,

Ch. No.	Ch. Config.	Throughput (T)						NE?	max tot T
		User 1	User 2	User 3	User 4	User 5	Sum/Tot		
1	11111	0.147	0.0815	0.0774	0.3492	0.0286	0.6836	NO	NO
2	11112	0.1573	0.0879	0.1188	0.4151	0.2489	1.028	NO	NO
3	11113	0.1573	0.0879	0.1188	0.4151	0.2489	1.028	NO	NO
4	11121	0.1749	0.2437	0.086	1.0504	0.0292	1.5841	NO	NO
5	11122	0.1898	0.3118	0.1404	0.7056	0.2113	1.5589	NO	NO
.							.		
.							.		
41	12222	0.5822	0.0843	0.0911	0.4321	0.0572	1.2469	NO	NO
42	12223	0.5822	0.0912	0.1544	0.5381	0.2489	1.6148	NO	NO
43	12231	0.4606	0.359	0.1929	1.0504	0.0464	2.1092	NO	YES
44	12232	0.5822	0.2715	0.1032	1.0504	0.0597	2.0669	NO	NO
45	12233	0.5822	0.359	0.1929	0.7056	0.2113	2.0509	YES	NO
46	12311	0.3064	0.4214	0.229	0.5073	0.0449	1.5091	NO	NO
47	12312	0.3555	0.3056	0.229	0.6607	0.1165	1.6673	NO	NO
.							.		
.							.		
240	33323	0.1749	0.2437	0.086	1.0504	0.0292	1.5841	NO	NO
241	33331	0.1573	0.0879	0.1188	0.4151	0.2489	1.028	NO	NO
242	33332	0.1573	0.0879	0.1188	0.4151	0.2489	1.028	NO	NO
243	33333	0.147	0.0815	0.0774	0.3492	0.0286	0.6836	NO	NO

Fig.4: *Exhaustive search analysis of a "random: 5links 3channels" scenario, which shows all the possible channel allocation configurations,* $3^5 = 243$.

which are listed, 6 or 12, or 24, or at times none results in NE, depending upon the Tx-Rx co-ordinates and the location of the links. The right-hand most column of Fig. 4 shows if the particular channel configurations is at NE or not. A total of 10,000 "random: 5links 3channels" scenarios were simulated, which is used to generate the different pdf of SINR, I_{from}, and I_{to} for "Before Channel Allocation" and "After Channel Allocation", as illustrated in the flow chart in Fig. 5.

5.1.1 Before Channel Allocation

A "random: 5links 3channels" scenario is simulated, and all users are given the same channel (for e.g "u1,u2,u3,u4,u5" = "1,1,1,1,1"). The network metrics of each user, when the users use the same channel, is measured. Then, based on the exhaustive search analysis, the channel allocation at NE is determined. Is a particular user getting a single channel or sharing a channel at NE? Based on this, the network metrics are segregated into the respective bins. Hence, we consider this case "NE, before channel allocation". A histogram of the network metrics is shown in Fig. 6.

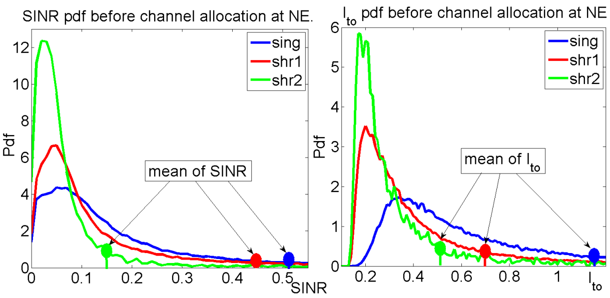

Fig.6: *Pdf of SINR, and* I_{to} *at initial channel allocation of "1,1,1,1,1", which later relates to the case with a user getting a single channel or a shared channel at NE.*

5.1.2 After Channel Allocation

The values of the network metrics at NE are taken and split into single and shared categories, depending upon how each of the users are allocated channels at NE. Hence, we consider this case "NE, after channel allocation". A histogram of the network metrics is shown in Fig. 7.

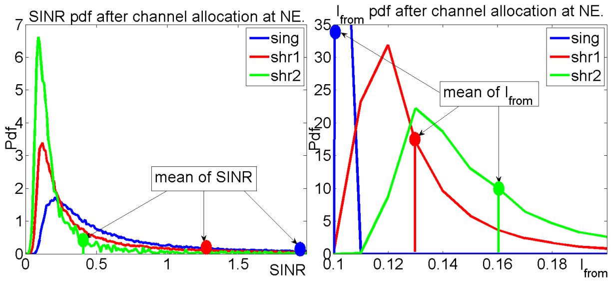

Fig.7: *Pdf of SINR and* I_{from} *at NE; a user getting a single channel or shared channel at NE.*

Similarly, "random: 3links 2channels" and "random: 10links 4channels" were also analysed, which produce similar results. Hence, we included only the results from "random: 5links 3channels" cases. The histogram plots presented in this section shows how the links that get a single channel at NE has higher SINR, I_{from}, and I_{to} initially (when all links use the same channel) than the links that share a channel. Additionally at NE, links getting a single channel have much higher SINR and lower I_{from} and I_{to} than links that share a channel. This idea is further discussed in the next section, and channel allocation policies are formulated.

5.2 Formulating policy

In our previous work [11], a simple policy was developed, based on the distance of the links, as shown in Fig. 8. After pursuing the research further and from the distribution of network metrics in Section 5. several important ideas on channel allocation can be inferred:

	channel	channel	channel	channel
Fixed: 3 link 2 ch	one link	two links		
Fixed: 5 link 3 ch	one link	one link	three links	
Random: 5 link 3 ch	one link	one/ two links	three/ two links	
Random: 10 link 4 ch	two link	two links	three links	three links
	short link	→		long link

Fig.8: *A sample of a policy for channel allocation from our previous work [11], based on the distance of a link.*

- Before channel allocation (when all links share the same channel), links with high SINR, high to/from

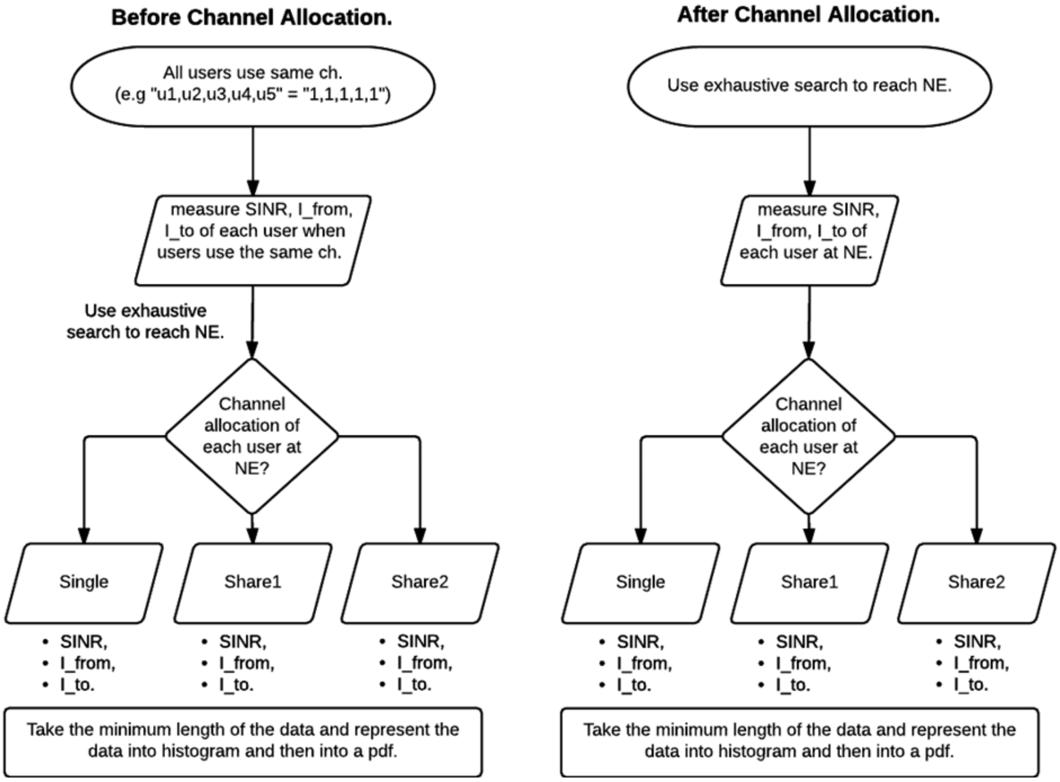

Fig.5: *Flow chart showing how the pdf of the network metrics like SINR, I_{from}, and I_{to} are obtained for "before channel allocation" and "after channel allocation" cases.*

interference tend to get good channels (less shared channels); hence, the policy is to allocate good channels (less shared channels) to links that have high SINR and high to/from interference, and to allocate bad channels (more shared channels) to links that have low SINR and low to/from interference.

- After channel allocation at NE, links with high SINR and low to/from interference tend to be more at NE than links which have low SINR and high to/from interference.

Based on the policy derived, we propose an equation that distributes the links into different channels, provided we know the link's SINR, I_{from}, and I_{to} values before channel allocation. The following steps are taken to derive the equation:

- If all channels share an equal number of links, then the number of links in channel i can be expressed as, $c_i = N/C$, where N is the total number of links and C is the total number of channels available in the CRN.
- We want to give preference to links that have high SINR and high interference terms. So, we sort the links based on these network metrics and allocate channels.
- c_1, c_2, have less links, which have high SINR and interference terms; whereas c_{C-1}, c_C, have more links with low SINR and Interference terms. Hence, as the prefix of c, i increases, the value of c_i increases as well so we have, $c_i = N/C+\theta i$. How fast c_i increases with an increment of i depends on θ, a constant which can be adjusted; in our current work, $\theta = 1$.
- In order to normalize c_i such that the $\sum_{i=1}^{C} c_i = N$, we need to divide the equation for c_i by the sum of c_i, and multiply by N, which results in, $c_i = \frac{N/C+\theta i}{\sum_{j=1}^{C}(N/C+\theta j}N$.
- Since the number of links in each channel can be an integer value only, the symbol $\lceil..\rfloor$ is used in the equation, which denotes nearest integer function.

Hence the final equation, that represents the number of links in the ith channel, can be expressed as

$$c_i = \lceil \left(\frac{\frac{N}{C} + \theta \cdot i}{\sum_{j=1}^{C} \left(\frac{N}{C} + \theta \cdot j \right)} N \right) \rfloor, \tag{11}$$

$$\sum_{i=1}^{C} c_i = N \tag{12}$$

The channel allocation for different scenarios based on (11), are as follows:

- 3links 2channels: $c_1 = 1$; $c_2 = 2$.
- 5links 3channels: $c_1 = 1$; $c_2 = 2$; $c_3 = 2$.
- 10links 4channels: $c_1 = 2$; $c_2 = 2$; $c_3 = 3$; $c_4 = 3$.

The channel allocation result obtained from (11) and from simulation of many random links for "3links 2channels", "5links 3channels" and "10links 4channels" is the same.

Fig. 9 shows how the links are distributed among the different channels for "random: 100links 20channels", based on (11). Links with higher priority get less shared channels, and links with lower priority get channel that are shared with many other links.

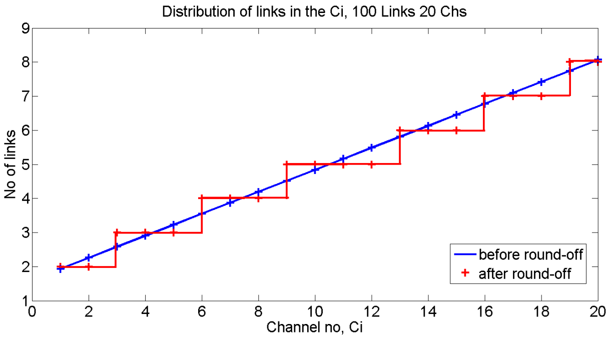

Fig.9: *Shows how the channel allocation is done for a "random: 100links 20channels" scenario, based on the proposed policy equation (11).*

5.3 Refining Utility

As mentioned in Section 1. and 3.3, there are many ways payoff can be defined, which can emulate different CRN scenarios and lead to better NE solutions. In our previous work [11] payoff was simply defined as the individual SINR. However, in [10], individual payoff was defined as received power minus the interference to/from other links, as in (10). In our previous work [11], the constants α, β, and γ in (10) were all one. But in this work the constants have been fine-tuned to be, $\alpha = 1$, $\beta = 0.4$, and $\gamma = 0.4$, which give better performance for the distributed CRN we are working with. It is expected that these constants are a function of N, C, and size of the CRN, and further analysis will be done in our future work.

6. ALGORITHMS

As discussed in Section 3.2, the best-response technique is a practical and preferable scheme used to reach a distributed solution among selfish users. The best response technique can be classified into various categories. Listed below are several versions of best response techniques with different utility terms that portray different CRN scenarios, as illustrated in Fig. 1. In the next section we compare the performance of these algorithms.

As shown in Fig. 10, different algorithms are shown in a tree diagram.

6.1 Random: Initial Channel Allocation

Depending on how we allocate the initial channel at the beginning of the algorithm, we split our algorithms into two categories, random and jump start which are discussed in detail in the following subsections.

6.1.1 Utility, T_i (R_uT)

This category can be further divided into three sub-categories, depending upon how the users choose the best-response strategy.

Best Response, Simultaneous (R_uT_S):

Step 1: Users select a random channel allocation.
Step 2: User1, user2, ..., userN all scan the environment and then select the best-strategy simultaneously. When all the users complete their best-strategy selection and implementation, one iteration is complete.
Step 3: All the users repeat Step 2 until the $iter_{max}$, final iteration.

Best Response, Round Robin (R_uT_Rr):

Step 2: User1 scans the environment and then selects the best-strategy. After user1 makes the best-strategy decision and implements it, user2 scans the environment and makes the best-strategy choice. This goes on until the Nth user. When the Nth user completes its best-strategy selection and implementation, one iteration is complete.
Note: Step 1 and *Step 3* are the same as in algorithm R_uT_S.

Best Response, Random (R_uT_Rn):

Step 2: Out of the N links, one link randomly scans the environment, and then selects the best-strategy. One iteration is complete after one random user chooses the best-response strategy and implements it.
Note: Step 1 and *Step 3* are the same as in algorithm R_uT_S.

In the three aforementioned algorithms each individual user tries to maximize its own throughput, that is $U_i = T_i$.
Out of the three best response techniques illustrated, the round-robin algorithm converges to the distributed solution fastest, as in every iteration, user1 to userN choose their best response strategy, and the other users know the strategy chosen by the previous users. In our work, the round-robin technique is used as the default technique, especially when it is not mentioned.

6.1.2 Utility, Pr_i, I_{from} and I_{to} (R_uPI)

This is the same as in algorithm R_uT_Rr. However, in this algorithm, each individual user tries to maximize its received power and minimize the interference to and from other users, $U_i = \alpha \cdot \hat{p}_i - \beta \cdot I_{from} - \gamma \cdot I_{to}$.

6.1.3 Utility, T_{tot} (R_uTt)

This is the same as in algorithm R_uT_Rr. However, in this algorithm each individual user tries to maximize the total throughput of the CRN, $U_i = T_{tot} = \sum_{i=1}^{N} T_i$.

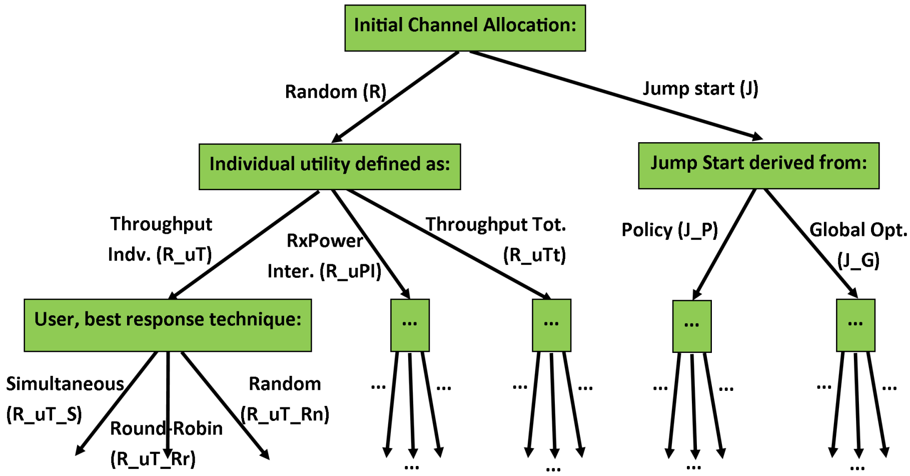

Fig.10: *Tree diagram of the different algorithms used in the work. The initial channel allocation can be Random, Global Optimal, or a good Jump Start, based on the derived policy. The best response technique is used to reach a distributed solution.*

6.2 Jump Start: Initial Channel Allocation

As shown in Fig. 10, this section considers the algorithms that does not use random initial channel allocation. Based on a policy or network metrics distribution or exhaustive search, an initial channel allocation is assigned to the users. So, the users have a good starting point. But the users use best-response technique in later iterations.

Fig. 11 shows how the distribution and characteristics of network metrics can be used to allocate channels to the users to reach a better solution faster. Algorithm J_P is uses this technique to some extent; in our future work we intend to fully develop the channel allocation, based on the process mentioned in the flow chart, which includes machine learning techniques.

6.2.1 Policy (J_P)

Step 1: Users allocate a channel based on the policy derived in Section 5.2
Step 2 on-wards: Use algorithm (R_uPI).
This algorithm shows the benefits of using the policy. Although the policy might not be an equilibrium point, it is a good starting point to reach a better NE point.

6.2.2 Global Optimal (J_G)

Step 1: A central entity allocates channels to individual users, such that the maximum total throughput of the network, is obtained. Channel allocation at maximum total throughput is determined via the exhaustive search technique.
Step 2 On-wards: Use algorithm (R_uT_Rr).
This algorithm starts from the best possible global solution and lets the selfish independent users make their own decisions, until they reach an equilibrium point.

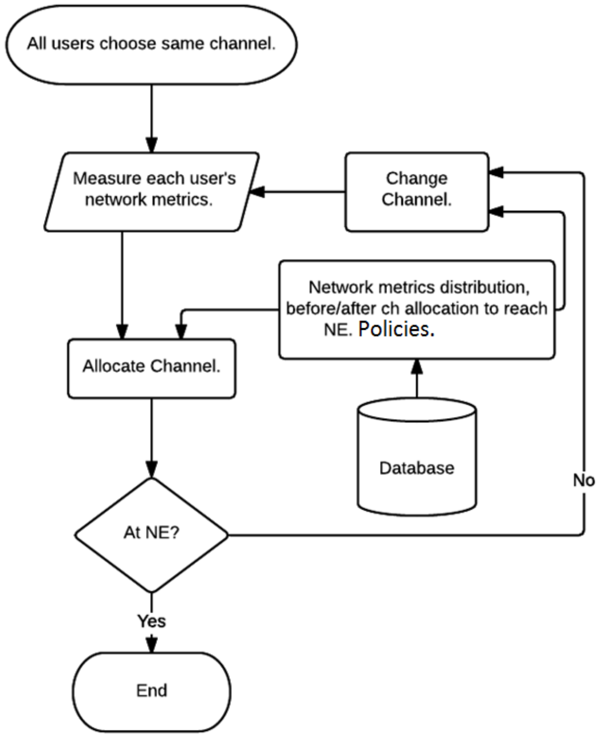

Fig.11: *Flow chart showing how the channels are allocated, based on the distribution of network metrics (policy).*

7. SIMULATION RESULTS

In this section, the best response algorithms mentioned in the Section 6. with different utility terms and policies, are implemented in MATLAB and their performances, are compared. Fig. 14, 15, 16, and 17 show the normalized cumulative throughput (6) on the y-axis and the iteration number on the x-axis. This shows how the different algorithms converge to different values over the iterations. Initially, we shall have a detailed look at a single scenario of "random: 10links 4channels". Then, we move on to the cases where we analyze the average of thousand of "random: 5links 3channels", "random: 10links 4channels" and "random: 100links 20channels" scenarios.

7.1 One random scenario "10links 4channels".

Fig. 12 is an example of a "random: 10links 4channels" scenario. An exhaustive search analysis of this scenario gives a total of $4^{10} = 1048576$ channel configurations, which are shown in Fig. 13-(top). An exhaustive search analysis gives a thorough analysis of the case. There are a total of 24 channel configurations that result in maximum total throughput, Fig. 13-(bottom). Similarly there are 144 channel configurations that results in NE, Fig. 13-(middle). There are multiple NE. Some NE are better than others in that they produce larger $T^m_{nor.cum.tot}$.

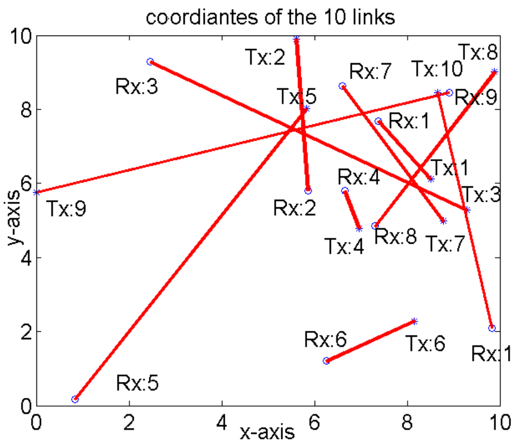

Fig.12: *CRN scenario "random: 10links 4channels".*

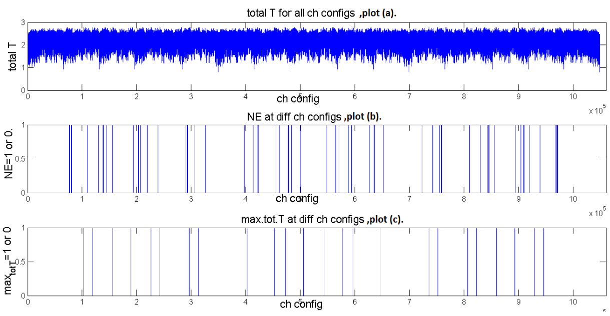

Fig.13: *Exhaustive search of a "random: 10links 4channels" scenario. There are a total of* $4^{10} = 1048576$ *channel allocations possible, which are shown on the x-axis. The y-axis at the top plot shows the* T_{tot}*; the middle plot shows if the configuration is NE or not; the bottom plot shows if the configuration is maximum* T_{tot}*, or not?*

Fig. 14 compares the normalized cumulative throughput obtained from different algorithms. The highest normalized cumulative throughput at the last iteration is obtained, when all users are playing a co-operative game. All users try to maximize the total throughput of the system, algorithm (R_uTt). This is as expected, as all the users are sharing all information, and their objective is to maximize the total overall throughput. Algorithm (R_uPI) gives the second highest performance, which represents the refined utility case with the weighted to/from interference and received power terms, as in (10). This is a distributed scheme, and there is partial sharing of the interference information. If each user tries to minimize interference to other links and enhance the received power, it is going to result in higher performance. Using the policy gives a good jump start, and the performance achieved by algorithm (J_P) is evident in the figure. Although the global optimal value obtained from the exhaustive search technique is the best, which is evident in the first iteration of algorithm (J_G), over the iterations the selfish users with independence deviate from the global optimal solution by choosing strategies that maximize their personal utility. Hence, the overall performance deteriorates.

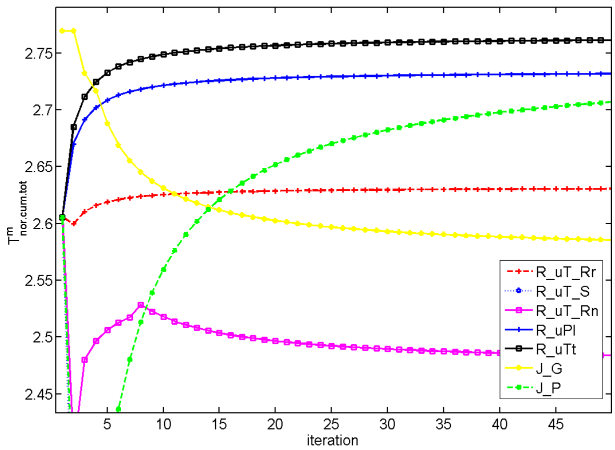

Fig.14: *Comparing the performance of different algorithms for a "random: 10links 4channels" scenario, discussed in Section 7.1*

7.2 Average of 1000 scenarios

To verify that the performance of the algorithms is consistent over different random scenarios, one thousand random scenarios with different link sizes were simulated, and the average of the 1000 random scenarios for different link sizes are compared below.

1000 "random: 5links 3channels" scenarios are simulated and the average performance of the different algorithms are computed, which are then plotted in Fig. 15, 16, and 17. It is evident from the figures that the best performance achieved is of algorithm (R_uTt), which represents complete co-operation and information exchange among users, and each user strives to enhance the total throughput $\left(U_i = \sum_{i=1}^{N}(T_i)\right)$ of the system. Algorithm (R_uPI), which incorporates the refined utility terms with weighted to/from interference and received power terms as in (10), has the next best performance, followed by algorithm (J_P), which incorporates the developed policy. It is evident from the figures that the same trend is visible for 1000 "random: 10links 4channels" scenarios and 100 "random: 100links 20channels" scenarios as well.

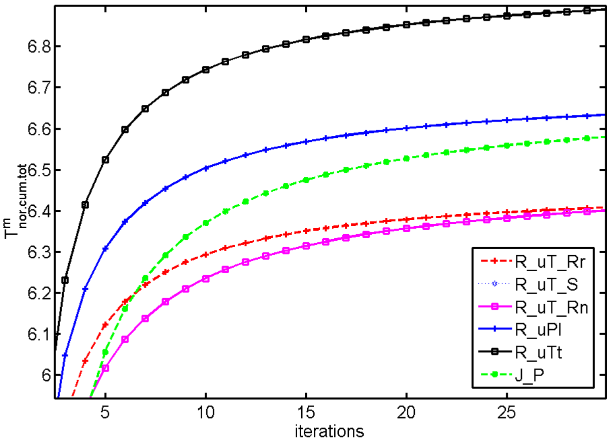

Fig.15: *Comparing the performance of different algorithms, based on the 1000 "random: 5links 3channels" scenarios.*

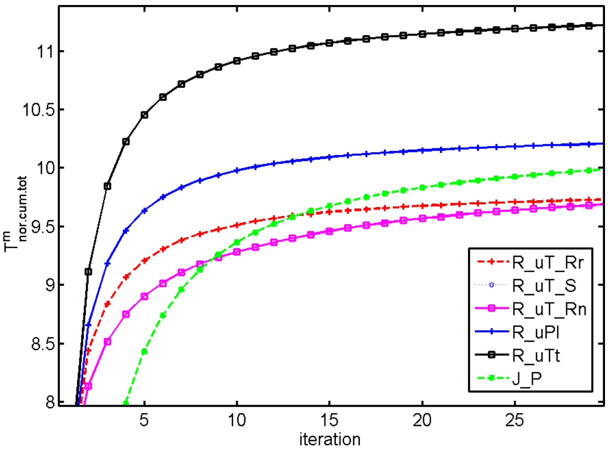

Fig.16: *Comparing the performance of different algorithms, based on the 1000 "random: 10links 4channels" scenarios.*

Based on the simulation results, the ideal solution, based on centralized and fully co-operative network (R_uTt), is considered as the bench-mark, which however is not practical in reality. Our work focuses on distributed CRN with selfish users and algorithms based on proposed policy (J_P) and refined utility terms (R_uPI). We have obtained a 10% to 20% better performance index, depending on the the network size than the general best response techniques (R_uT_Rr,R_uT_S,R_uT_Rn).

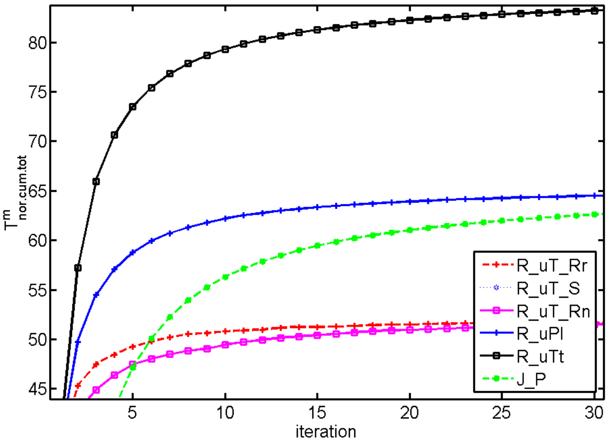

Fig.17: *Comparing the performance of different algorithms, based on the 1000 "random: 100links 20channels" scenarios.*

8. CONCLUSION

Cognitive Radio Networks that incorporate dynamic resource allocation are more efficient than traditional fixed resource allocation schemes. Game theory is used to allocate the channels to different links in a distributed CRN. By defining the individual payoff of a user differently, various CRN scenarios can be emulated, ranging from centralized and co-operative, to distributed and selfish. The ideal solution, based on centralized and fully co-operative CRN, is shown as a bench-mark, which is not practical in reality, as the selfish and independent SUs will deviate from such a solution if they find a way to increase their own payoff. We focus on distributed CRN with partially selfish users with partial information exchange. The distribution (pdf) and characteristics of networks metrics, SINR, interference from/to other links before channel allocation, and after channel allocation to reach NE, were analysed and policies were developed for channel allocation. A novel equation for allocating channels is also presented. Various algorithms based on different best response techniques, which incorporate the policies and refined utility terms, were developed to reach a distributed solution. By simulation results, the performance index, and normalized cumulative total throughput, based on the policies and refined utility, is 10% to 20% better, depending on the the network size, than the general best response techniques.

In our future work, we plan to further develop the policy for channel allocation and refine the utility terms, to obtain higher performance in the distributed CRN scenarios with selfish users. We are also trying to use machine learning techniques to help us extract patterns and develop policies for channel/resource allocation in such CRNs.

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Amulya Bhattarai received his B.Eng. in Electrical and Electronics Engineering from Assumption University, Thailand in 2004. M.S. in Communication, Systems and Electronics from Jacobs University, Germany in 2008. Currently he is doing his Ph.D in Computer and Communication Technology from SIIT, Thammasat University. He has been a lecturer at Assumption University, Thailand since 2008. His re-search interests are in smart antennas, cognitive radios, game theory, and machine learning.

Chalie Charoenlarpnopparut received his B.Eng in Electrical Engineering, Chulalongkorn University, Bangkok, Thailand and his M.S and Ph.D in Electrical Engineering from The Pennsylvania State University, PA, USA. He has been an Associate Professor at the School of Information, Computer, and Communication Technology (ICT) at SIIT, Thammasat University since 2002. His research interests are in multidimensional systems and signal processing, robust control, image processing, wavelet and filter bank, signal processing for communication, convolutional code design.

Prapun Suksompong received his B.S. in Electrical and Computer Engineering, M.S. in Electrical and Computer Engineering and Ph.D. in Electrical and Computer Engineering, Cornell University, USA. He has been an Assistant Professor at the School of Information, Computer, and Communication Technology (ICT) at SIIT, Thammasat University since 2008. His research interest are in wireless communications, indoor positioning principles and localization techniques, computational neuroscience, energy-efficient coding and Poisson process and Poisson convergence.

Patrachart Komolkiti received his B.Eng degree from Chulalongkorn University, Bangkok, Thailand, in 1997, his M.S. degree from Northwestern University, USA, in 1998, and his Ph.D. from Chulalongkorn University, Bangkok, Thailand, in 2011, all in Electrical Engineering. He joined the department of Computer Engineering, Assumption University in November 2003, where he is currently serving as the department chairperson. His research interests include Wireless Networks, QoS, and Resource Allocation.