

Sum-Rate Upper Bounds for the Two-User Gaussian X Channel with Limited Receiver Cooperation

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ABSTRACT

In this paper, we propose sum-rate upper bounds in terms of the sum of 3 rates and the sum of 4 rates for the two-user Gaussian X channel with limited receiver cooperation. These upper bounds are derived by using the Fano's inequality, the data processing inequality and Genie-aided techniques. Then, we explore the generalized degrees of freedom (GDoF) of the proposed upper bounds with a symmetric channel setting. By setting a certain set of parameters, the received results show that our results are identical to the existing results for both non-cooperation and limited receiver cooperation cases. Moreover, we obtain that the GDoF computed from our proposed upper bounds are greater than the GDoF of those in the non-cooperation case.

Keywords: Genie-aided techniques, Receiver cooperation, Sum-rate upper bound, X channel.

1. INTRODUCTION

In modern communication systems, communications between transmitter-receiver pairs through a common communication medium where each transmitter sends an independent message to each receiver are called a X channel.

Vishwanath *et al.* [1] first introduce the X channel. The best known achievable region for the X channel is proposed by Koyluoglu *et al.* [2]. However, characterizing of this achievable region is extremely complicated [4]. In [3], Huang *et al.* provide the sum capacity of the deterministic X channel and Gaussian X channel under a symmetric channel setting. They also explore the GDoF of the symmetric Gaussian X channel from their sum capacity of the Gaussian X channel. Next, [4] gives outer bounds on the capacity region of X channel for the strong and mixed X channel. Recently, Niesen and Maddah-Ali [5] find an

approximately optimal communication scheme and show that it achieves the capacity of the Gaussian X channel to within a constant gap.

In [1-5] above, there are no communications between transmitters or receivers. Nowadays, cooperation between transmitters or receivers which is allowed by exchanging a certain amount of information at the limited rate due to physical constraints has become the necessary composition of modern communication systems. For example, the base stations in a cellular network can be connected via wireline backhaul links [6]. It is widely known that cooperation can alleviate interference by forming distributed multiple antenna arrays or called as distributed multiple-input multiple-output (MIMO) systems [7-8].

Cooperation research has been studied in [9-13]. Willems [9] introduces initially conferencing among encoders and then characterizes the capacity region of multiple access channels (MAC). Next, one-sided Gaussian interference channels with unidirectional and bidirectional rate-limited conferencing between decoders are considered in [10-11], respectively. In [7], Wang and Tse consider the two-user Gaussian interference channel with rate-limited receiver cooperation and characterize its entire capacity region to within a constant gap. After that, Ashraphijuo *et al.* [12] extend the work [7] to the MIMO case. Very recently, Tan-a-ram and Benjapolakul [13] first study the two-user Gaussian X channel with limited receiver cooperation. They propose to use a strategy consisting the transmission scheme based on the message splitting [14] and the cooperative protocol based on the two-round strategy [7] for the general case of this channel and then derive an achievable rate region based on their proposed strategy.

In the perspective of information theory, it is widely known that characterizing upper bounds which are the subset of an outer bound or an outer bound which is also called an approximate capacity region gives the better understanding of communication limits in the channel which is considered.

From the literature review of the X channel and the cooperation above, however, knowledge of the Gaussian X channel with cooperation is presently little known despite the two-user case. In this paper, therefore, we consider the two-user Gaussian X channel with limited receiver cooperation where the rates of exchanging information between receivers are lim-

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ited. We give sum-rate upper bounds and then show the benefits of receiver cooperation for this channel.

The contributions of this paper are given below.

1. We derive upper bounds in terms of the sum of 3 rates and the sum of 4 rates for the two-user Gaussian X channel with limited receiver cooperation, i.e., (1)–(13) in Theorem 1, using the Fano's inequality, the data processing inequality and Genie-aided techniques [3], [7], [15]. We show through the comparisons that our proposed upper bounds (5)–(8) and (9)–(10) are the same as the previous known results in both the non-cooperation [3], [16] and the receiver cooperation [7], [11], respectively, by setting a certain set of parameters. This indicates that our results are more generalized than those in the existing works. Furthermore, bounds (1)–(4) and (11)–(12) are the novel results for this channel scenario and are proposed for the first time in this paper.
2. We give approximate sum capacity bounds under a symmetric channel setting, i.e., $\text{SNR}_1 = \text{SNR}_2 = \text{SNR}$, $\text{INR}_1 = \text{INR}_2 = \text{INR}$, and $\text{C}_{12}^B = \text{C}_{21}^B = \text{C}^B$. We reveal that approximate symmetric sum capacity bounds (18)–(20) are the same as the existing results in both the non-cooperation [3], [15] and the receiver cooperation [7] by setting a certain set of parameters. This implies that our results are more generalized than those in several communication scenarios. In addition, other symmetric sum capacity bounds, i.e., (17) and (21), are the novel results and are also proposed for the first time in this paper.
3. We explore the GDoF from the proposed approximate symmetric sum capacity bounds. The obtained results reveal that gains from receiver cooperation are obtained efficiently when the GDoF is approximately proportional to the normalized capacity of the receiver-cooperative link (κ). However, it is found that the GDoF does not increase, i.e., it reaches full receiver cooperation, when $\kappa = \frac{1}{2}, 1, \frac{3}{2}, 2$ for $\alpha = \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, 2, \frac{5}{2}, 3$, respectively.

The rest of this paper is organized as follows: We introduce the channel model and provide sum-rate upper bounds for the two-user Gaussian X channel with limited receiver cooperation in Section II and III, respectively. The effectiveness of the proposed upper bounds is verified in Section IV. In Section V, we explore the GDoF under a symmetric channel setting. Conclusion of this paper is in Section VI.

Notations: $h(\cdot)$ and $I(\cdot)$ denote the differential entropy of a continuous random variable or vector, and mutual information, respectively. For a real number a , $(a)^+ := \max(0, a)$ denotes its positive part. \mathbb{C} denotes the set of all complex numbers. $\mathcal{CN}(0, 1)$ denotes complex Gaussian random variable with zero mean and unit variance. Let x^N denote the sequence $\{x[1], \dots, x[N]\}$ where $[\cdot]$ denote time indices. Unless indicated, all logarithms $\log(\cdot)$ are of the base 2.

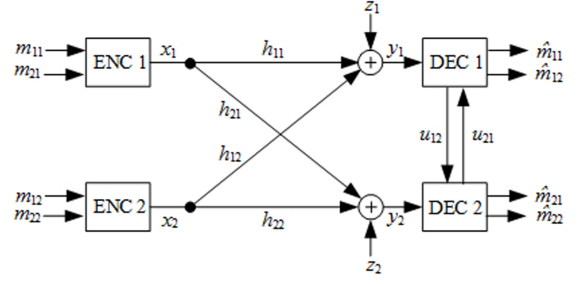


Fig. 1: Two-user Gaussian X channel with limited receiver cooperation

2. CHANNEL MODEL

Since the two-user Gaussian X channel is physically the same as the two-user Gaussian interference channel [3]; therefore, we can describe the two-user Gaussian X channel with limited receiver cooperation using the channel model in [7] as shown in Fig. 1. This channel model consists of two transmitter-receiver pairs where each transmitter has an independent message for each receiver. These two pairs communicate each other via two main non-interference links¹ as follows:

Transmitter-Receiver Links: These links are modeled as the *normalized* Gaussian X channel

$$\begin{aligned} y_1 &= h_{11}x_1 + h_{12}x_2 + z_1 \\ y_2 &= h_{21}x_1 + h_{22}x_2 + z_2 \end{aligned}$$

where the additive noise processes $\{z_i[n]\}$, ($i = 1, 2$) are independent and identically distributed (i.i.d.) $\mathcal{CN}(0, 1)$ over time.

For $i, j = 1, 2$ and $i \neq j$, suppose that there are four independent messages where m_{ji} denotes the message sent from transmitter i to receiver j . Hence, transmitter i encodes message m_{ii} and m_{ji} into a block codeword $\{x_i[n]\}_{n=1}^N$ with the average transmit power constraints

$$\frac{1}{N} \sum_{n=1}^N |x_i[n]|^2 \leq 1, \quad i = 1, 2,$$

for arbitrary block length N . Note that the outcome of each encoder depends only on its own messages. SNR and INR can be defined as follows:

$$\text{SNR}_i = |h_{ii}|^2, \text{ and } \text{INR}_i = |h_{ij}|^2, \quad i, j = 1, 2 \text{ and } i \neq j$$

Receiver-Cooperative Links: These links are noiseless with finite capacities C_{ij}^B from the receiver i to j , for $(i, j) = (1, 2), (2, 1)$. For any time index $n = 1, 2, \dots, N$, encoding at each receiver is causal in the sense that the cooperation signal from receiver 1 to 2, $u_{12}[n]$, is only a function

¹This paper focuses only on the out-of-band limited receiver cooperation; therefore, signals in transmitter-receiver links do not interfere with those in receiver-cooperative links

of $\{y_1[1], \dots, y_1[n-1], u_{21}[1], \dots, u_{21}[n-1]\}$. Similarly, $u_{21}[n]$ is only a function of $\{y_2[1], \dots, y_2[n-1], u_{12}[1], \dots, u_{12}[n-1]\}$.

3. UPPER BOUNDS FOR THE TWO-USER GAUSSIAN X CHANNEL WITH LIMITED RECEIVER COOPERATION

This section proposes sum-rate upper bounds in terms of the sum of 3 rates and the sum of 4 rates for the two-user Gaussian X channel with limited receiver cooperation. In deriving these upper bounds, we first use the mutual information to upper bound the rates through the Fano's inequality and the data processing inequality. Second, we divide them into two parts: 1) Terms which are related to those in the Gaussian X channel without receiver cooperation and 2) Terms which present gains from receiver cooperation. Finally, the genie-aided techniques [3], [7], [15] are utilized to upper bound terms in the first part, where genies give useful side information signals which are carefully selected to the receivers. The obtained results are given in the following theorem.

Theorem 1: The nonnegative rate quadruple $(R_{11}, R_{12}, R_{21}, R_{22})$ of the two-user Gaussian X channel with limited receiver cooperation defined in Section 2, satisfies the inequalities (1)–(13) as shown at the bottom of the next page.

Proof : Details of this proof are given in Appendix. For $i, j = 1, 2$ and $i \neq j$, the concise ideas for deriving these bounds have the following details:

First of all, bounds (1)–(4) correspond to the Z-channel bounds. A genie gives interfering information s_{ji}^N and m_{ji} to receiver i and x_j^N and m_{ii} to receiver j for bounds (1) and (4) and s_{jj}^N and m_{jj} to receiver i and x_i^N and m_{ij} to receiver j for bounds (2) and (3). The gain of receiver cooperation is the sum of C_{ij}^B and C_{ji}^B .

Bounds (5)–(8) correspond to the Huang-Cadambe-Jafar (HCJ) upper bounds for the Gaussian X channel without cooperation [5]. For these bounds, a genie gives y_i^N , m_{ii} and m_{ij} to receiver j . Therefore, the gain from receiver j to i is absorbed into the power gain and the other gain is upper bounded by C_{ij}^B .

Bounds (9)–(10) correspond to the Etkin-Tse-Wang (ETW) upper bounds for the interference channel without cooperation [15] which is extended to the Gaussian X channel without cooperation [5]. In the genie-aided channel, a genie gives side information \tilde{s}_{jj}^N and m_{jj} to receiver i for (9) and \tilde{s}_{ji}^N and m_{ji} to receiver i for (10). The gain of receiver cooperation is upper bounded by $C_{12}^B + C_{21}^B$.

Bounds (11)–(12) on $R_{11} + R_{12} + R_{21} + R_{22}$ are derived by giving side information y_j^N and \tilde{s}_{ji}^N to receiver i and \tilde{s}_{ij}^N to receiver j . Since a genie gives y_j^N to receiver i , therefore, the gain from receiver j to i is absorbed into the power gain and the other gain is

upper bounded by C_{ij}^B .

Bound (13) is straightforward cut-set upper bound of the sum of 4 rates.

Note that the derivation of all bounds works for all SNR's and INR's. The power gain which is mentioned in (5)–(8) and (11)–(12) is the gain in the saturation region where receiver cooperation is inefficient [7]. \square

4. EFFECTIVENESS OF THE PROPOSED SUM-RATE UPPER BOUNDS

This section verifies the proposed bounds in Theorem 1 by comparing some of these results in the special cases with the existing results [3], [7], [11], [16] in the case of non-cooperation and receiver cooperation.

4.1 Comparisons of our results with the existing results

We compare the upper bounds in Theorem 1 with the previous known results in the following two cases.

1. Non-cooperation case:

- Setting $C_{12}^B = C_{21}^B = 0$ in (5)–(8) and (9)–(10), we obtain that these bounds are identical to the results of Lemma 5.2 and Theorem 5.3, respectively, in [3].
- Setting $m_{12} = m_{21} = \phi$ and $C_{12}^B = C_{21}^B = 0$ in (6)–(7), (10), we obtain that these bounds and the results in [16] are the same.

2. Receiver cooperation case:

- Setting $m_{12} = m_{21} = \phi$, we obtain that (6)–(7), (10) and (13) are identical to the inequalities (8), (7), (6) and (9), respectively, of an outer bound in [7, Lemma 5.1].
- Setting $m_{12} = m_{21} = \phi$ and assigning that receiver 1 suffers from interference and noise but receiver 2 suffers only from noise, (6) and (13) with some modifications and disappearance of h_{12} are identical to the results in [11, Theorem 2].
- Comparing (9) in [7] with (13) which is not specified with any parameters, we see obviously that both inequalities are the same.

4.2 Discussions

The received results in Section 4.1 show that some of our proposed bounds are the same as the upper bounds in [3], [7], [11], [16] by setting a certain set of parameters properly. This implies that our results are more generalized than those in several communication scenarios. This obtained results also correspond to the fact that the two-user X channel is a generalization of the two-user interference channel [3].

In addition, other upper bounds, i.e., (1)–(4) and (11)–(12) are the novel results for the research X channel and are proposed for the first time in this paper.

In the next section, we provide the better understanding with the effect of receiver cooperation by exploring the GDoF.

5. GENERALIZED DEGREES OF FREEDOM UPPER BOUND

In this section, we first give the definition of the GDoF of the sum capacity from [7] and then make use Theorem 1 to calculate an approximate symmetric sum capacity for the two-user Gaussian X with limited receiver cooperation. Finally, the GDoF is explored by using the calculated approximate symmetric sum capacity.

5.1 Generalized Degrees of Freedom

The GDoF of the sum capacity [7] is defined as

$$d(\alpha, \kappa) := \lim_{\substack{\text{fix } \alpha, \kappa \\ \text{SNR} \rightarrow \infty}} \frac{C_\Sigma(\text{SNR}, \text{INR}, C^B)}{\log \text{SNR}} \quad (14)$$

where

$$\lim_{\text{SNR} \rightarrow \infty} \frac{\log \text{INR}}{\log \text{SNR}} = \alpha, \quad \lim_{\text{SNR} \rightarrow \infty} \frac{C^B}{\log \text{SNR}} = \kappa$$

and $C_\Sigma(\text{SNR}, \text{INR}, C^B) = R_{11} + R_{12} + R_{21} + R_{22}$.

Note that α and κ are called the normalized interference level and the normalized capacity of the receiver-cooperative link, respectively.

Furthermore, we use approximations [15]

$$\log(1 + \text{SNR} + \text{INR}) \approx \max(\log \text{SNR}, \log \text{INR}) \quad (15)$$

$$\log\left(1 + \frac{\text{SNR}}{1 + \text{INR}}\right) \approx \left(\log\left(\frac{\text{SNR}}{\text{INR}}\right)\right)^+ \quad (16)$$

to give an expansion of the sum capacity which is accurate to the first order approximation.

5.2 Approximate Symmetric Sum Capacity

Using the results in Theorem 1, the approximate symmetric sum capacity C_Σ can be computed and is presented in the following Corollary:

Corollary 1 (*Approximate Symmetric Sum Capacity*) The results are shown in (17)–(22).

Note that each of the inequalities in Corollary 1 is calculated from Theorem 1 using the relationship $C_\Sigma = R_{11} + R_{12} + R_{21} + R_{22}$ with the constraints $\text{SNR}_1 = \text{SNR}_2 = \text{SNR}$, $\text{INR}_1 = \text{INR}_2 = \text{INR}$ and $C_{12}^B = C_{21}^B = C^B$.

Remark 1 (Effectiveness of the approximate symmetric sum capacity). Similar to the verification in Section 4, we compare the approximate symmetric sum capacity in Corollary 1 with the existing results under the symmetric channel setting in the following two cases.

1. Non-cooperation case:
 - Setting $C^B = 0$ in (18)–(20), we easily see that these sum capacity bounds are the same as the results in [3].

- Setting $m_{12} = m_{21} = \phi$ and $C^B = 0$ in (19), we obviously see that this sum capacity bound is the same as Theorem 1 in [15].

2. Receiver cooperation case:

- Setting $m_{12} = m_{21} = \phi$ in (19), we obtain that this sum capacity bound is the same as (6) in [Lemma 5.1, 7].
- Setting $m_{12} = m_{21} = \phi$ in (22), we obtain that this sum capacity bound is the same as (9) in [Lemma 5.1, 7].

Remark 2 (Discussion). The obtained results as shown in Remark 1 reveal that some of our proposed symmetric sum capacity bounds are also identical to those in [3], [15], [7] by setting a certain set of parameters properly. This indicates that our results are more generalized than those in several communication scenarios. Furthermore, other symmetric sum capacity bounds, i.e., (17) and (21), are the novel results for the two-user Gaussian X channel with limited receiver cooperation and are also proposed for the first time in this paper.

Next, we provide the GDoF using the results (17)–(22).

5.3 GDoF of the symmetric sum capacity

The GDoF which is translated from the approximate symmetric sum capacity in Corollary 1 is given in the following theorem.

Theorem 2: The GDoF of the symmetric sum capacity is given as follows:

For $0 \leq \alpha < 1$,

$$\begin{aligned} d(\alpha, \kappa) &= \min\left\{2 + 2\kappa, 2 - \frac{2\alpha}{3} + \frac{4\kappa}{3}, 2, \right. \\ &\quad 2\max(\alpha, (1 - \alpha)^+) + 2\kappa, \\ &\quad (2 - \alpha)^+ + \max(\alpha, (1 - \alpha)^+) - (1 - \alpha)^+ + \kappa, \\ &\quad \left. \frac{1}{3}\left[4 + 2\alpha + 2\max(\alpha, (1 - \alpha)^+) + 8\kappa\right]\right\} \quad (23) \end{aligned}$$

For $\alpha \geq 1$,

$$\begin{aligned} d(\alpha, \kappa) &= \min\left\{2\alpha + 2\kappa, 2\max(1, (\alpha - 1)^+) + 2\kappa, \right. \\ &\quad 2\alpha - \frac{2}{3} + \frac{4\kappa}{3}, 2\alpha - (1 - \alpha)^+ + \kappa, 2\alpha \\ &\quad \left. \frac{1}{3}\left[4\alpha + \max(4, 2 + 2(\alpha - 1)^+) + 8\kappa\right]\right\} \quad (24) \end{aligned}$$

Note that both (23)–(24) are calculated directly from Corollary 5.2 using the approximations in (15)–(16) and utilizing Lemma 7.2 in [7] for considering the phases of channel coefficients, i.e., terms involving with $|h_{11}h_{22} - h_{12}h_{21}|^2$.

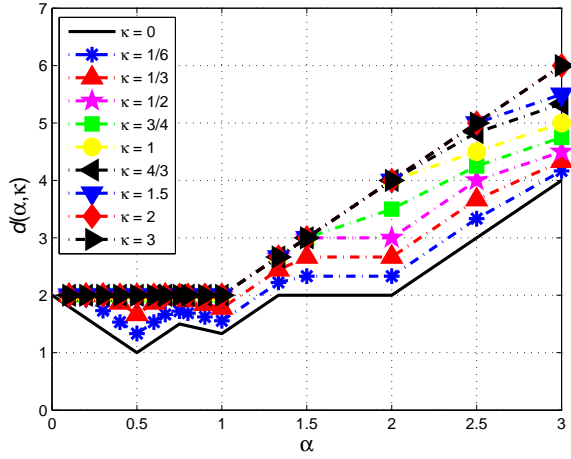


Fig.2: Plotting the GDoF versus the normalized interference level α .

Remark 3 (Interpretation). Based on the definition of the GDoF in (14), the GDoF of the symmetric sum capacity as shown in (23)–(24) provides a sense of how interference, which is in terms of α , and receiver cooperation, which is in terms of κ , affects this communication channel. This implies that the GDoF in (23)–(24) can indicate the performance of the symmetric two-user Gaussian X channel with limited receiver cooperation from characteristics of the obtained gains from receiver cooperation as follows:

1. By comparing with the non-cooperation case ($\kappa = 0$), we can observe the obtained gains from receiver cooperation in the considered range of α at different values of κ .
2. At each α , we can obtain the range of κ which receiver cooperation is efficient, i.e., the GDoF is approximately proportional to κ , and the range of κ which receiver cooperation is inefficient, i.e., the GDoF is a constant value and independent to κ .

Details of two main points above are provided obviously in Section 5.4

5.4 Gains from limited receiver cooperation

In this section, we show the behavior of gains from receiver cooperation for the two-user Gaussian X channel with limited receiver cooperation via the viewpoint of α and κ , respectively.

5.4.1 Gains from limited receiver cooperation in the viewpoint of α

Using (23)–(24), the GDoF $d(\alpha, \kappa)$ versus the normalized interference level α is plotted in Fig. 2.

From Fig. 2, we see that

- For all α 's $\in [0, 3]$, it is obviously seen that the gains from limited receiver cooperation are received when the normalized capacity of the receiver-cooperative link $\kappa > 0$. However, we observe that these obtained gains do not equal at the different values of α .

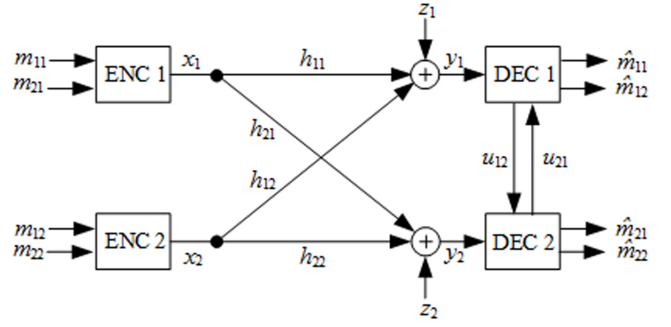


Fig.3: Gain from receiver cooperation when considering at $\alpha = \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, 2, \frac{5}{2}$ and 3.

- Full receiver cooperation can be obtained, i.e., the system reaches the saturation. To consider this point, we divide α into 2 ranges as follows:

1. For $0 \leq \alpha < 1$: The GDoF curve changes from a sawtooth curve to a linear line with a slope of 0 when κ increases from 0 to $\frac{1}{2}$ and the GDoF $d(\alpha, \kappa) = 2$ at $\kappa = \frac{1}{2}$ for all α 's $\in [0, 1]$. However, it reaches full receiver cooperation performance at the GDoF $d(\alpha, \kappa) = 2$ even though $\kappa > \frac{1}{2}$. In addition, we observe that the GDoF curve can reach full receiver cooperation in some ranges of α when $\kappa < \frac{1}{2}$, that is,

- $d(\alpha, \kappa = \frac{1}{3}) = 2$ when $0 \leq \alpha \leq 0.3$ and $\alpha = \frac{2}{3}$;
- $d(\alpha, \kappa = \frac{1}{6}) = 2$ when $0 \leq \alpha \leq 0.1$

2. For $1 \leq \alpha < 3$: The GDoF curve changes from a step to a linear line with a slope of 2 when κ increases from 0 to 2. Furthermore, we obtain that full receiver cooperation performance can be achieved when

- $\kappa \geq \frac{1}{2}$ for $1 < \alpha \leq \frac{3}{2}$;
- $\kappa \geq 1$ for $\frac{3}{2} < \alpha \leq 2$;
- $\kappa \geq \frac{3}{2}$ for $2 < \alpha \leq \frac{5}{2}$;
- $\kappa \geq 2$ for $\frac{5}{2} < \alpha \leq 3$.

Therefore, from the obtained results above, we can be seen obviously that the GDoF computed from our proposed upper bounds ($\kappa > 0$) are greater than the GDoF of those in the non-cooperation case, i.e., $\kappa = 0$.

5.4.2 Gains from limited receiver cooperation in the viewpoint of κ

We present the obtained gains from receiver cooperation by plotting the GDoF $d(\alpha, \kappa)$ versus κ . Especially, considering at $\alpha = \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, 2, \frac{5}{2}$ and 3. This result is shown in Fig. 3.

From Fig. 3, at $\alpha = \frac{1}{2}$, we observe that the GDoF curve increases linearly and its slope = 2 when κ increases from 0 to $\frac{1}{2}$ and we obtain the GDoF $d(\alpha = \frac{1}{2}, \kappa = \frac{1}{2}) = 2$. However, the GDoF curve achieves the saturation when $\kappa \geq \frac{1}{2}$, i.e., $d(\frac{1}{2}, \kappa) = 2$ for $\frac{1}{2} \leq \kappa \leq 4$. Similarly, for $\alpha = \frac{2}{3}, \frac{3}{2}, 2, \frac{5}{2}, 3$, the

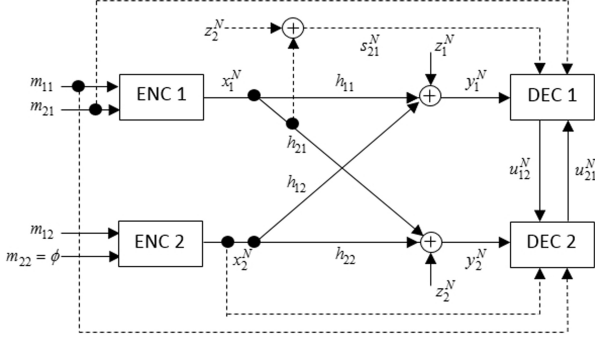


Fig. 4: Side information structure for bound (1).

GDoF curves increase linearly until they achieve the saturation at $\kappa = \frac{1}{3}, \frac{1}{2}, 1, \frac{3}{2}, 2$, respectively.

These received results above correspond to the results in [7], i.e., the receiver cooperation is efficient in the linear region where the GDoF is proportional κ with a positive slope.

6. CONCLUSION

This paper provides sum-rate upper bounds to better understand the communication limits of the two-user Gaussian X channel with limited receiver cooperation. The received results show that the existing results in the non-cooperation and receiver cooperation cases can be obtained by setting properly a certain set of parameters in our proposed upper bounds. Furthermore, characterizing the GDoF of the symmetric sum capacity shows clearly the benefit of receiver cooperation compared with the GDoF of non-cooperation case. However, each GDoF curve reaches its saturated point $\kappa^* = \frac{1}{2}, \frac{1}{3}, \frac{1}{2}, 1, \frac{3}{2}, 2$ for $\alpha = \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, 2, \frac{5}{2}, 3$, respectively.

7. APPENDIX

Proof of Theorem 1

This appendix gives the details for proving Theorem 1 which are based on the genie-aided techniques [3], [7], [15]. For this proof, we define auxiliary information s_{ij} and side information \tilde{s}_{ij} as follows:

$$s_{ij} := h_{ij}x_j + z_i, \quad \tilde{s}_{ij} := h_{ij}x_j + \tilde{z}_i$$

where, for $i, j = 1, 2$, z_i and \tilde{z}_i are i.i.d $\mathcal{CN}(0, 1)$ and independent of everything else. Both s_{ij} and \tilde{s}_{ij} have the same marginal distribution.

Bounds (1) on $R_{11} + R_{12} + R_{21}$, (2) on $R_{11} + R_{12} + R_{22}$, (3) on $R_{11} + R_{21} + R_{22}$ and (4) on $R_{12} + R_{21} + R_{22}$:

Proof : In this proof, we show only (1) and other bounds can be shown similarly. To upper bound $R_{11} + R_{12} + R_{21}$, we set message $m_{22} = \phi$. A genie gives side information s_{21}^N and m_{21} to receiver 1 and x_2^N and m_{11} to receiver 2 (refer to Fig. 4). If

(R_{11}, R_{12}, R_{21}) is achievable, we can write

$$\begin{aligned} & N(R_{11} + R_{12} + R_{21} - \epsilon_N) \\ & \stackrel{(a)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N) + I(m_{21}; y_2^N, u_{12}^N) \\ & \stackrel{(b)}{\leq} I(m_{11}, m_{12}; y_1^N) + I(m_{11}, m_{12}; u_{21}^N | y_1^N) \\ & \quad + I(m_{21}; y_2^N) + I(m_{21}; u_{12}^N | y_2^N) \\ & \stackrel{(c)}{\leq} I(m_{11}, m_{12}; y_1^N) + H(u_{21}^N) + I(m_{21}; y_2^N) + H(u_{12}^N) \\ & \stackrel{(d)}{\leq} I(m_{11}, m_{12}; y_1^N, s_{21}^N, m_{21}) + I(m_{21}; y_2^N, x_2^N, m_{11}) \\ & \quad + NC_{21}^B + NC_{12}^B \\ & = I(m_{11}, m_{12}; y_1^N, s_{21}^N | m_{21}) + I(m_{21}; y_2^N | x_2^N, m_{11}) \\ & \quad + NC_{21}^B + NC_{12}^B \\ & = I(m_{11}, m_{12}; s_{21}^N | m_{21}) + I(m_{11}, m_{12}; y_1^N | s_{21}^N, m_{21}) \\ & \quad + I(m_{21}; y_2^N | x_2^N, m_{11}) + NC_{21}^B + NC_{12}^B \\ & = h(s_{21}^N | m_{21}) - h(s_{21}^N | m_{21}, m_{11}, m_{12}) + h(y_1^N | s_{21}^N, m_{21}) \\ & \quad - h(y_1^N | s_{21}^N, m_{21}, m_{11}, m_{12}) + h(y_2^N | x_2^N, m_{11}) \\ & \quad - h(y_2^N | x_2^N, m_{11}, m_{21}) + NC_{21}^B + NC_{12}^B \\ & \leq h(s_{21}^N | m_{21}) - h(z_2^N) + h(y_1^N | s_{21}^N) - h(z_1^N) \\ & \quad + h(s_{21}^N | m_{11}) - h(z_2^N) + NC_{21}^B + NC_{12}^B \\ & \stackrel{(e)}{\leq} N\{\text{RHS of (1)}\} \end{aligned}$$

where $\epsilon_N \rightarrow 0$ as $N \rightarrow \infty$. (a) is due to Fano's inequality. (b) is due to the chain rule. (c) is due to the fact that $I(m_{11}, m_{12}; u_{21}^N | y_1^N) \leq H(u_{21}^N)$ and $I(m_{21}; u_{12}^N | y_2^N) \leq H(u_{12}^N)$. (d) is due to the genie providing side information s_{21}^N and m_{21} to receiver 1 and x_2^N and m_{11} to receiver 2 and $H(u_{ij}^N) \leq NC_{ij}^B$. (e) is due to the fact that i.i.d. Gaussian distribution maximises differential entropy under covariance constraints.

Hence, similarly if a genie provides side information s_{22}^N and m_{22} to receiver 1, x_1^N and m_{12} to receiver 2 and setting $m_{21} = \phi$ for $R_{11} + R_{12} + R_{22}$, x_2^N and m_{21} to receiver 1, s_{11}^N and m_{11} to receiver 2 and setting $m_{12} = \phi$ for $R_{11} + R_{21} + R_{22}$, and x_1^N and m_{22} to receiver 1, s_{12}^N and m_{12} to receiver 2 and setting $m_{11} = \phi$ for $R_{12} + R_{21} + R_{22}$, we have shown bounds (1)–(4). \square

Bounds (5) on $R_{11} + R_{12} + R_{21}$, (6) on $R_{11} + R_{12} + R_{22}$, (7) on $R_{11} + R_{21} + R_{22}$ and (8) on $R_{12} + R_{21} + R_{22}$:

Proof : In this proof, we show only (5) and other bounds can be shown similarly. To upper bound $R_{11} + R_{12} + R_{21}$, we set message $m_{22} = \phi$. Let a genie give side information y_1^N , m_{11} and m_{12} to receiver 2 (refer to Fig. 5). If (R_{11}, R_{12}, R_{21}) is achievable, we

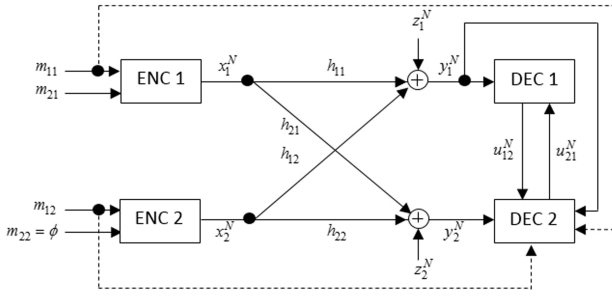


Fig. 5: Side information structure for bound (5).

obtain

$$\begin{aligned}
 & N(R_{11} + R_{12} + R_{21} - \epsilon_N) \\
 & \stackrel{(a)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N) + I(m_{21}; y_2^N, u_{12}^N) \\
 & \stackrel{(b)}{\leq} I(m_{11}, m_{12}; y_1^N) + I(m_{11}, m_{12}; u_{21}^N | y_1^N) \\
 & \quad + I(m_{21}; y_2^N, u_{12}^N, y_1^N, m_{11}, m_{12}) \\
 & \stackrel{(c)}{\leq} I(m_{11}, m_{12}; y_1^N) + H(u_{21}^N) \\
 & \quad + I(m_{21}; y_2^N, u_{12}^N, y_1^N | m_{11}, m_{12}) \\
 & \stackrel{(d)}{\leq} I(m_{11}, m_{12}; y_1^N) + NC_{21}^B + I(m_{21}; y_1^N, y_2^N | m_{11}, m_{12}) \\
 & = h(y_1^N) - h(y_1^N | m_{11}, m_{12}) + h(y_1^N, y_2^N | m_{11}, m_{12}) \\
 & \quad - h(y_1^N, y_2^N | m_{11}, m_{12}, m_{21}) + NC_{21}^B \\
 & \leq h(y_1^N) + h(y_2^N | y_1^N, m_{11}, m_{12}, x_2^N) \\
 & \quad - h(y_1^N, y_2^N | m_{11}, m_{12}, m_{21}, x_1^N, x_2^N) + NC_{21}^B \\
 & \leq h(y_1^N) + h(s_{21}^N | s_{11}^N) - h(z_1^N, z_2^N) + NC_{21}^B \\
 & \stackrel{(e)}{\leq} N\{\text{RHS of (5)}\}
 \end{aligned}$$

where $\epsilon_N \rightarrow 0$ as $N \rightarrow \infty$. (a) is due to Fano's inequality. (b) is due to the genie providing side information x_2^N to receiver 2, i.e., conditioning reduces entropy. (c) is due to the fact that $I(m_{11}, m_{12}; u_{21}^N | y_1^N) \leq H(u_{21}^N)$ and all messages m_{11}, m_{12}, m_{21} are independent. (d) is due to the fact that u_{ij}^N is a function of (y_1^N, y_2^N) and $H(u_{ij}^N) \leq NC_{ij}^B$. (e) is due to the fact that i.i.d. Gaussian distribution maximises differential entropy under covariance constraints.

Hence, similarly if a genie provides side information y_1^N , m_{11} and m_{12} to receiver 2 and setting $m_{21} = \phi$ for $R_{11} + R_{12} + R_{22}$, y_2^N , m_{21} and m_{22} to receiver 1 and setting $m_{12} = \phi$ for $R_{11} + R_{21} + R_{22}$, and y_2^N , m_{21} and m_{22} to receiver 1 and setting $m_{11} = \phi$ for $R_{12} + R_{21} + R_{22}$, we have shown bounds (5)–(8). \square

Bounds (9)–(10) on $R_{11} + R_{12} + R_{21} + R_{22}$:

Proof : To upper bound (9), a genie gives side information \tilde{s}_{22}^N and m_{22} to receiver 1 and \tilde{s}_{11}^N and m_{11} to receiver 2 (refer to Fig. 6). If $(R_{11}, R_{12}, R_{22}, R_{22})$

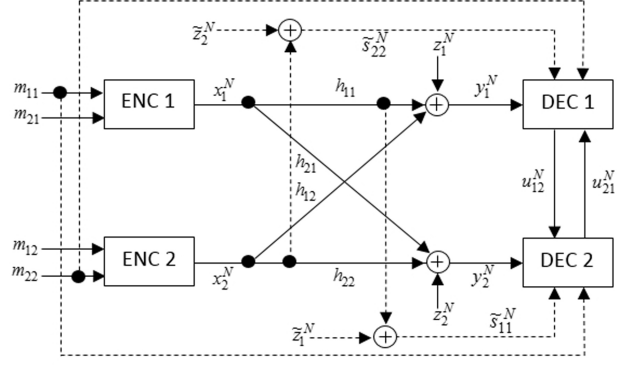


Fig. 6: Side information structure for bound (9).

is achievable, we have

$$\begin{aligned}
 & N(R_{11} + R_{12} + R_{21} + R_{22} - \epsilon_N) \\
 & \stackrel{(a)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N) + I(m_{21}, m_{22}; y_2^N, u_{12}^N) \\
 & \stackrel{(b)}{\leq} I(m_{11}, m_{12}; y_1^N) + I(m_{11}, m_{12}; u_{21}^N | y_1^N) \\
 & \quad + I(m_{21}, m_{22}; y_2^N) + I(m_{21}, m_{22}; u_{12}^N | y_2^N) \\
 & \stackrel{(c)}{\leq} I(m_{11}, m_{12}; y_1^N) + H(u_{21}^N) \\
 & \quad + I(m_{21}, m_{22}; y_2^N) + H(u_{12}^N) \\
 & \stackrel{(d)}{\leq} I(m_{11}, m_{12}; y_1^N, \tilde{s}_{22}^N, m_{22}) + NC_{21}^B \\
 & \quad + I(m_{21}, m_{22}; y_2^N, \tilde{s}_{11}^N, m_{11}) + NC_{12}^B \\
 & \stackrel{(e)}{=} I(m_{11}, m_{12}; y_1^N, \tilde{s}_{22}^N | m_{22}) + NC_{21}^B \\
 & \quad + I(m_{21}, m_{22}; y_2^N, \tilde{s}_{11}^N | m_{11}) + NC_{12}^B \\
 & \stackrel{(f)}{=} I(m_{11}, m_{12}; \tilde{s}_{22}^N | m_{22}) + I(m_{11}, m_{12}; y_1^N | m_{22}, \tilde{s}_{22}^N) \\
 & \quad + I(m_{21}, m_{22}; \tilde{s}_{11}^N | m_{11}) + I(m_{21}, m_{22}; y_2^N | m_{11}, \tilde{s}_{11}^N) \\
 & \quad + NC_{21}^B + NC_{12}^B \\
 & \stackrel{(g)}{\leq} h(\tilde{s}_{22}^N | m_{22}) - h(\tilde{s}_{22}^N | x_2^N, m_{11}) + h(y_1^N | \tilde{s}_{22}^N, m_{22}) \\
 & \quad - h(y_1^N | \tilde{s}_{22}^N, x_2^N, m_{11}) + h(\tilde{s}_{11}^N | m_{11}) - h(\tilde{s}_{11}^N | x_1^N, m_{22}) \\
 & \quad + h(y_2^N | \tilde{s}_{11}^N, m_{11}) - h(y_2^N | \tilde{s}_{11}^N, x_1^N, m_{22}) \\
 & \quad + NC_{21}^B + NC_{12}^B \\
 & \leq h(\tilde{s}_{22}^N | m_{22}) - h(\tilde{z}_2^N) + h(y_1^N | \tilde{s}_{22}^N) - h(s_{11}^N | m_{11}) \\
 & \quad + h(\tilde{s}_{11}^N | m_{11}) - h(\tilde{z}_1^N) + h(y_2^N | \tilde{s}_{11}^N) - h(s_{22}^N | m_{22}) \\
 & \quad + NC_{21}^B + NC_{12}^B \\
 & = h(y_1^N | \tilde{s}_{22}^N) - h(\tilde{z}_2^N) + h(y_2^N | \tilde{s}_{11}^N) - h(\tilde{z}_1^N) + NC_{21}^B \\
 & \quad + NC_{12}^B \\
 & \stackrel{(h)}{\leq} N\{\text{RHS of (9)}\}
 \end{aligned}$$

where $\epsilon_N \rightarrow 0$ as $N \rightarrow \infty$. (a) is due to Fano's inequality. (b) is due to chain rule. (c) is due to the fact that $I(m_{ii}, m_{ij}; u_{ji}^N | y_i^N) \leq H(u_{ji}^N)$. (d) is due to the genie providing side information \tilde{s}_{ii}^N and m_{ii} to receiver j , for $i, j = 1, 2$ and the fact that

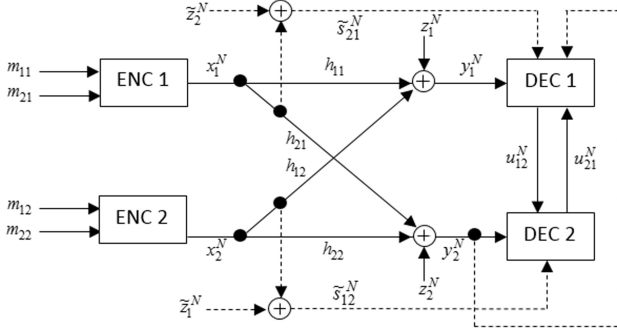


Fig. 7: Side information structure for bound (11).

$H(u_{ji}^N) \leq NC_{ji}^B$. (e) is due to the fact that all messages are independent. (f) is due to chain rule. (g) is due to the fact that x_i^N is a function of messages (m_{ii}, m_{ji}) . (h) is due to the fact that i.i.d. Gaussian distribution maximises differential entropy under covariance constraints. Hence, and similarly if a genie gives side information \tilde{s}_{ji}^N and m_{ji} to receiver i , we have shown bounds (9)–(10). \square

Bounds (11)–(12) on $R_{11} + R_{12} + R_{21} + R_{22}$:

Proof : To upper bound (11), a genie gives side information y_2^N and \tilde{s}_{21}^N to receiver 1 and \tilde{s}_{12}^N to receiver 2 (refer to Fig. 7). If $(R_{11}, R_{12}, R_{22}, R_{22})$ is achievable, we obtain

$$\begin{aligned}
 & N(R_{11} + R_{12} + R_{21} + R_{22} - \epsilon_N) \\
 & \stackrel{(a)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N) + I(m_{21}, m_{22}; y_2^N, u_{12}^N) \\
 & \stackrel{(b)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N, y_2^N, \tilde{s}_{21}^N) \\
 & \quad + I(m_{21}, m_{22}; y_2^N, \tilde{s}_{12}^N) + I(m_{21}, m_{22}; u_{12}^N | y_2^N) \\
 & \stackrel{(c)}{\leq} I(m_{11}, m_{12}; y_1^N, u_{21}^N, y_2^N | \tilde{s}_{21}^N) + I(m_{11}, m_{12}; \tilde{s}_{21}^N) \\
 & \quad + I(m_{21}, m_{22}; y_2^N, \tilde{s}_{12}^N) + H(u_{12}^N) \\
 & \stackrel{(d)}{\leq} I(m_{11}, m_{12}; y_1^N, y_2^N | \tilde{s}_{21}^N) + h(\tilde{s}_{21}^N) - h(\tilde{s}_{21}^N | m_{11}, m_{12}) \\
 & \quad + h(y_2^N, \tilde{s}_{12}^N) - h(y_2^N, \tilde{s}_{12}^N | m_{21}, m_{22}) + NC_{12}^B \\
 & \leq h(y_1^N, y_2^N | \tilde{s}_{21}^N) - h(y_1^N, y_2^N | \tilde{s}_{21}^N, m_{11}, m_{12}) + h(\tilde{s}_{21}^N) \\
 & \quad - h(\tilde{z}_2^N) + h(\tilde{s}_{12}^N) + h(y_2^N | \tilde{s}_{12}^N) - h(s_{21}^N, \tilde{z}_1^N) + NC_{12}^B \\
 & \leq h(y_1^N, y_2^N | \tilde{s}_{21}^N) - h(s_{12}^N, s_{22}^N) + h(\tilde{s}_{21}^N) - h(\tilde{z}_2^N) \\
 & \quad + h(\tilde{s}_{12}^N) + h(y_2^N | \tilde{s}_{12}^N) - h(s_{21}^N) - h(\tilde{z}_1^N) + NC_{12}^B \\
 & = h(y_1^N, y_2^N | \tilde{s}_{21}^N) + h(y_2^N | \tilde{s}_{12}^N) - h(s_{22}^N | s_{12}^N) - h(\tilde{z}_2^N) \\
 & \quad - h(\tilde{z}_1^N) + NC_{12}^B \\
 & \stackrel{(e)}{\leq} N\{\text{RHS of (11)}\}
 \end{aligned}$$

where $\epsilon_N \rightarrow 0$ as $N \rightarrow \infty$. (a) is due to Fano's inequality. (b) is due to chain rule and the fact that a genie gives side information y_2^N and \tilde{s}_{21}^N to receiver 1 and \tilde{s}_{12}^N to receiver 2. (c) is due to chain rule and the fact that $I(m_{21}, m_{22}; u_{12}^N | y_2^N) \leq H(u_{12}^N)$. (d) is

due to the fact that u_{21}^N is a function of (y_1, y_2) and $H(u_{12}^N) \leq NC_{12}^B$. (e) is due to the fact that i.i.d. Gaussian distribution maximises differential entropy under covariance constraints.

Hence, and similarly if a genie gives side information y_1^N and \tilde{s}_{12}^N to receiver 2 and \tilde{s}_{21}^N to receiver 1, we have shown bounds (11)–(12). \square

Bound (13) on $R_{11} + R_{12} + R_{21} + R_{22}$:

Proof : This is the straightforward cut-set upper bound: if $(R_{11}, R_{12}, R_{22}, R_{22})$ is achievable, we obtain

$$\begin{aligned}
 & N(R_{11} + R_{12} + R_{21} + R_{22} - \epsilon_N) \\
 & \leq I(m_{11}, m_{12}, m_{21}, m_{22}; y_1^N, y_2^N) \\
 & = h(y_1^N, y_2^N) - h(z_1^N, z_2^N) \\
 & \leq N\{\text{RHS of (13)}\}
 \end{aligned}$$

where $\epsilon_N \rightarrow 0$ as $N \rightarrow \infty$.

Hence, we have shown bounds (13). \square

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tions of communication networks in smart grid.

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$$R_{11} + R_{12} + R_{21} \leq \log(1 + \text{INR}_2) + \log\left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{1 + \text{INR}_2}\right) + C_{12}^B + C_{21}^B \quad (1)$$

$$R_{11} + R_{12} + R_{22} \leq \log(1 + \text{SNR}_2) + \log\left(1 + \text{SNR}_1 + \frac{\text{INR}_1}{1 + \text{SNR}_2}\right) + C_{12}^B + C_{21}^B \quad (2)$$

$$R_{11} + R_{21} + R_{22} \leq \log(1 + \text{SNR}_1) + \log\left(1 + \text{SNR}_2 + \frac{\text{INR}_2}{1 + \text{SNR}_1}\right) + C_{12}^B + C_{21}^B \quad (3)$$

$$R_{12} + R_{21} + R_{22} \leq \log(1 + \text{INR}_1) + \log\left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{1 + \text{INR}_1}\right) + C_{12}^B + C_{21}^B \quad (4)$$

$$R_{11} + R_{12} + R_{21} \leq \log(1 + \text{SNR}_1 + \text{INR}_1) + \log\left(1 + \frac{\text{INR}_2}{1 + \text{SNR}_1}\right) + C_{21}^B \quad (5)$$

$$R_{11} + R_{12} + R_{22} \leq \log(1 + \text{SNR}_1 + \text{INR}_1) + \log\left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_1}\right) + C_{21}^B \quad (6)$$

$$R_{11} + R_{21} + R_{22} \leq \log(1 + \text{SNR}_2 + \text{INR}_2) + \log\left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2}\right) + C_{12}^B \quad (7)$$

$$R_{12} + R_{21} + R_{22} \leq \log(1 + \text{SNR}_2 + \text{INR}_2) + \log\left(1 + \frac{\text{INR}_1}{1 + \text{SNR}_2}\right) + C_{12}^B \quad (8)$$

$$R_{11} + R_{12} + R_{21} + R_{22} \leq \log\left(1 + \text{SNR}_1 + \frac{\text{INR}_1}{1 + \text{SNR}_2}\right) + \log\left(1 + \text{SNR}_2 + \frac{\text{INR}_2}{1 + \text{SNR}_1}\right) + C_{12}^B + C_{21}^B \quad (9)$$

$$R_{11} + R_{12} + R_{21} + R_{22} \leq \log\left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{1 + \text{INR}_2}\right) + \log\left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{1 + \text{INR}_1}\right) + C_{12}^B + C_{21}^B \quad (10)$$

$$\begin{aligned} R_{11} + R_{12} + R_{21} + R_{22} &\leq \log\left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} + \text{INR}_1 + \text{SNR}_2 + \frac{\text{INR}_1}{1 + \text{INR}_2} + \frac{|h_{11}h_{22} - h_{12}h_{21}|^2}{1 + \text{INR}_2}\right) \\ &\quad + \log\left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{1 + \text{INR}_1}\right) - \log\left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_1}\right) + C_{12}^B \end{aligned} \quad (11)$$

$$\begin{aligned} R_{11} + R_{12} + R_{21} + R_{22} &\leq \log\left(1 + \text{SNR}_1 + \frac{\text{INR}_1}{1 + \text{INR}_1} + \frac{\text{SNR}_2}{1 + \text{INR}_1} + \text{INR}_2 + \frac{|h_{11}h_{22} - h_{12}h_{21}|^2}{1 + \text{INR}_1}\right) \\ &\quad + \log\left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{1 + \text{INR}_2}\right) - \log\left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_2}\right) + C_{21}^B \end{aligned} \quad (12)$$

$$R_{11} + R_{12} + R_{21} + R_{22} \leq \log(1 + \text{SNR}_1 + \text{INR}_1 + \text{SNR}_2 + \text{INR}_2 + |h_{11}h_{22} - h_{12}h_{21}|^2) \quad (13)$$

$$C_\Sigma \leq \frac{1}{3} \left[2\log(1 + \text{SNR}) + 2\log\left(1 + \text{SNR} + \frac{\text{INR}}{1 + \text{SNR}}\right) + 2\log(1 + \text{INR}) + 2\log\left(1 + \text{INR} + \frac{\text{SNR}}{1 + \text{INR}}\right) + 8C^B \right] \quad (17)$$

$$C_\Sigma \leq \frac{1}{3} \left[4\log(1 + \text{SNR} + \text{INR}) + 2\log\left(1 + \frac{\text{INR}}{1 + \text{SNR}}\right) + 2\log\left(1 + \frac{\text{SNR}}{1 + \text{INR}}\right) + 4C^B \right] \quad (18)$$

$$C_\Sigma \leq 2\log\left(1 + \text{INR} + \frac{\text{SNR}}{1 + \text{INR}}\right) + 2C^B \quad (19)$$

$$C_\Sigma \leq 2\log\left(1 + \text{SNR} + \frac{\text{INR}}{1 + \text{SNR}}\right) + 2C^B \quad (20)$$

$$\begin{aligned} C_\Sigma &\leq \log\left(1 + \frac{\text{SNR}}{1 + \text{INR}} + \text{INR} + \text{SNR} + \frac{\text{INR}}{1 + \text{INR}} + \frac{|h_{11}h_{22} - h_{12}h_{21}|^2}{1 + \text{INR}}\right) + \log\left(1 + \text{INR} + \frac{\text{SNR}}{1 + \text{INR}}\right) \\ &\quad - \log\left(1 + \frac{\text{SNR}}{1 + \text{INR}}\right) + C^B \end{aligned} \quad (21)$$

$$C_\Sigma \leq \log(1 + 2\text{SNR} + 2\text{INR} + |h_{11}h_{22} - h_{12}h_{21}|^2) \quad (22)$$