Moving Reference Planes of Unit Cells of Reciprocal Lossy Periodic Transmission-Line Structures

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ABSTRACT

An analysis of moving reference planes of unit cells of reciprocal lossy periodic transmission-line (TL) structures (RLSPTLSs) by using the equivalent bi-characteristic-impedance transmission line (BCITL) model is presented. Applying the BCITL theory, only the equivalent BCITL parameters (characteristic impedances for wave propagating in forward and reverse directions and associated complex propagation constant) are of interest. In the analysis, an arbitrary infinite RLSPTLS is firstly considered by shifting a reference position of unit cells along TLs. Then, a semi-infinite terminated RLSPTLS is subsequently investigated in term of associated load reflection coefficients. It is found that the equivalent BCITL characteristic impedances of the original and shifted unit cells, as well as the associated load reflection coefficients of both unit cells, are mathematically related by the bilinear transformation. However, the equivalent BCITL complex propagation constant remains unchanged. Numerical results are provided to show the validity of the proposed technique.

Keywords: Reference Planes of Unit Cell, Reciprocal Lossy Periodic Transmission-Line Structures, Bi-Characteristic-Impedance Transmission line, Bilinear Transformation.

1. INTRODUCTION

Periodic structures of transmission lines (TLs) have been the subject of study for a long time, and have several practical applications in microwave technology; e.g., microwave filters, slow wave components, traveling-wave amplifiers, phase shifters, antennas, electromagnetic-bandgap (EBG) structures and metamaterials [1]-[7]. In general, problems associated with periodic structures have been analyzed based on the Floquet's theorem [8]. Recently, lossless periodic TL structures have been discussed about the analysis and their useful applications using the conjugately characteristic-impedance TL (CCITL) theory [6]-[7], [9]-[11]. However, CCITLs cannot be used to model lossy periodic TL structures to obtain higher accuracy in some practical applications. Therefore, it is required to generalize the CCITL model for these lossy structures. Lately, the generalized model based on bi-characteristic-impedance TLs (BCITLs) was presented to analyze terminated finite reciprocal lossy periodic TL structures (RLSPTLSs) [12]-[13]. In applying the BCITL model, only the equivalent quantities associated with each unit cell of RLSPTLSs are employed instead. Therefore, it is an alternative way for the analysis and design of RLSPTLSs. For the equivalent BCITL model, the characteristic impedances for waves propagating in the forward and reverse directions, defined as Z_0^+ and Z_0^- respectively, are generally different, and the corresponding complex propagation constants for waves propagating in the forward (γ^+) and reverse (γ^-) directions, respectively. It should be pointed out that CCITLs are the special case of BCITLs when Z_0^+ and Z_0^- are complex conjugate of each other.

For a unit cell of periodic structures related to a moving reference plane, published papers [14]-[15] and the book [16] discovered that the propagation wave number is not a function of the position of the reference plane and the associated characteristic impedances depend on the choice of the reference position of the unit cell, respectively. However, published papers in [14]-[15] are derived based on Maxwell's equations via the scattering-matrix approach, which is unnecessarily complicated. In addition, the finding in [16] is valid for a specific case only. Furthermore, it is not obvious for RLSPTLS how to define a unit cell properly for convenience in analysis. Recently, a moving reference plane of unit cell of lossy periodic transmission-line (TL) structures is studied numerically by using standard formulas of BCITL model [17]. It is found that the original and shifted unit cells are related. Nevertheless, in [17] could not point out for RLSPTLS, how they are related. For more information of the relationship between the original and shifted unit cells, this paper aims to provide alternative and simple derivations associated with the BCITL parameters for an arbitrary unit cell of RLSPTLSs when a reference position of unit cells is shifted along TLs of interest, including the relevant derivation of the associated load reflection coefficient. In addition, numerical results are

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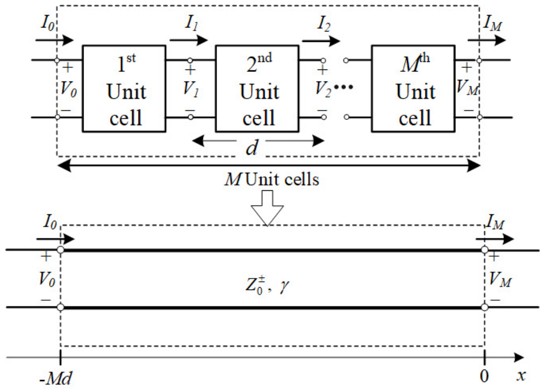


Fig.1: *A finite RLSPTLS of M unit cells and its equivalent BCITL model.*

provided to verify the proposed technique.

This paper is organized as follows. Section 2 presents the analysis of moving reference planes associated with unit cells of infinite RLSPTLSs using the equivalent BCITL model, where the equivalent complex propagation constant and the equivalent characteristic impedances are considered. The analysis for semi-infinite terminated RLSPTLSs is provided in Section 3, where the associated load reflection coefficient is discussed. An example of RLSPTLSs is shown in Section4 to show the validity of the proposed solutions. Finally, conclusions are given in Section 5.

2. ANALYSIS OF MOVING REFERENCE PLANES FOR INFINITE RLSPTLSS

In [12], a finite RLSPTLS of M unit cells at each unit-cell terminal can be effectively modeled as a BCITL of length $l = Md$, as shown in Fig. 1, where d is the length of each unit cell. Note that V_m and I_m are the phasor voltage and the phasor current at the terminal of the m^{th} unit cell (where m = 1, 2, ..., M), respectively. Generally, reciprocal BCITLs possess the complex propagation constant γ with characteristic impedances Z_0^+ and Z_0^- . Note that both forward and reverse propagation constants are identical ($\gamma^+ = \gamma^- = \gamma$) for *reciprocal* lossy periodic TL structures. In this section, an infinite RLSPTLS is considered which can be obtained from Fig.1 by letting both ends approach infinity.

Using the $ABCD$ matrix technique, the equivalent characteristic impedances $Z_0^\pm$ can be expressed in terms of the total $ABCD$ parameters of the unit cell of interest as [13]

$$Z_0^\pm = \frac{\mp 2B}{A - D \mp j\sqrt{4 - (A+D)^2}} \tag{1}$$

In addition, the equivalent complex propagation constant γ can be determined from the following dispersion relation [1]:

$$\cosh \gamma d = \frac{A+D}{2} \tag{2}$$

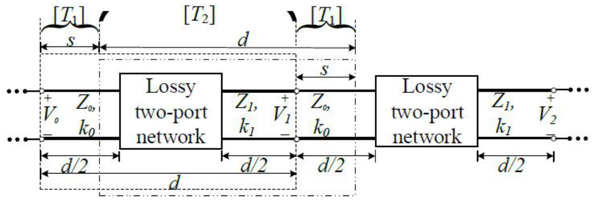


Fig.2: *A single unit cell of an infinite RLSPTLS obtained by shifting a reference position s along TLs.*

To analyze RLSPTLSs of the moving reference planes, an original unit cell of length d is initially considered for convenience as shown in Fig. 2 (on the left end). In Fig. 2, the RLSPTLS consists of two TLs, with the unloaded propagation constants k_0 and k_1, and the characteristic impedances Z_0 and Z_1, respectively, loaded with an arbitrary lossy two-port network at the center. It should be pointed out that the lossy two-port network is dimensionless. The reference position (s) of the unit cell is shifted along TLs of interest, where $0 \le s \le d$. Since it is on the same RLSPTLS, γ is expected to remain unchanged. To clarify this, let us consider Fig. 2, where the original unit cell can be considered as being composed of a cascade of two two-port networks possessing the transmission matrices, $[T_1]$ and $[T_2]$. In addition, a shifted unit cell in Fig. 2 is composed of a cascade of two two-port networks possessing the transmission matrices, $[T_2]$ and $[T_1]$, where

$$[T_i] = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \tag{3}$$

for $i = 1$ and 2, and A_i, B_i, C_i and D_i are $ABCD$ parameters. Note that the total transmission matrix of the original (when $s = 0$ and $s = d$) and shifted unit cells can be written explicitly as (4) and (5), respectively:

$$[T]_0 = [T_1]\,[T_2] = \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & C_1B_2 + D_1D_2 \end{bmatrix} \tag{4}$$

$$[T]_s = [T_2]\,[T_1] = \begin{bmatrix} A_2A_1 + B_2C_1 & A_2B_1 + B_2D_1 \\ C_2A_1 + D_2C_1 & C_2B_1 + D_2D_1 \end{bmatrix} \tag{5}$$

where the subscripts o and s in this paper are associated with the original and shifted unit cells, respectively. From (4) and (5), it is obvious that both cases provide the same γ computed using (2) as follows:

$$\gamma = \frac{1}{d}\cosh^{-1}\left(\frac{A_1A_2 + B_1C_2 + C_1B_2 + D_1D_2}{2}\right) \tag{6}$$

However, it can be shown using (1), (4) and (5) that the equivalent characteristic impedances $Z_0^{\pm}$ of the original and shifted unit cells are distinct in general ($Z_{0,o}^{\pm} \neq Z_{0,s}^{\pm}$) depending on the reference planes, where $Z_{0,o}^{\pm}$ and $Z_{0,s}^{\pm}$ are those of the original and shifted unit cells, respectively. Specifically, it is found that $Z_{0,o}^{\pm}$ and $Z_{0,s}^{\pm}$ are mathematically related by the bilinear transformation as follows [18]:

$$Z_{0,s}^{\pm} = \frac{aZ_{0,o}^{\pm} + b}{cZ_{0,o}^{\pm} + d_0} \tag{7}$$

with $ad_0 - bc \neq 0$ in general. Note that $b = 0$ for this case, and

$$a = \mp (A_2B_1 + B_2D_1), \tag{8}$$

$$c = B_2C_1 - B_1C_2, \tag{9}$$

$$d_0 = \mp (A_1B_2 + B_1D_2) \tag{10}$$

The bilinear transformation possesses several interesting properties, which can be found in [18] for more details.

For a reciprocal lossless CCITL, which is a special case of BCITLs, it is found that $Z_{0,o}^{\pm}$ and $Z_{0,s}^{\pm}$ are different and mathematically related by a bilinear transformation as shown in (7) as well. In addition, both original and shifted unit cells, operated in passbands, provide the identical real propagation constant, denoted as β, as expected as shown below:

$$\beta = \frac{1}{d}\cos^{-1}\left(\frac{A_1A_2 + B_1C_2 + C_1B_2 + D_1D_2}{2}\right) \tag{11}$$

3. ANALYSIS OF MOVING REFERENCE PLANES FOR SEMI-INFINITE TERMINATED RLSPTLSS

Consider a semi-infinite RLSPTLS, terminated in a load impedance Z_L, as shown in Fig. 3, where M approaches infinity. At the terminal of an arbitrary unit cell in Fig. 3, the load reflection coefficient of the original and shifted unit cells are mathematically defined in terms of associated load impedances as (12) and (13), respectively [13]

$$\Gamma_{L,o} = \frac{Z_{L,o}Z_{0,o}^{-} - Z_{0,o}^{+}Z_{0,o}^{-}}{Z_{L,o}Z_{0,o}^{+} + Z_{0,o}^{+}Z_{0,o}^{-}} \tag{12}$$

$$\Gamma_{L,s} = \frac{Z_{L,s}Z_{0,s}^{-} - Z_{0,s}^{+}Z_{0,s}^{-}}{Z_{L,s}Z_{0,s}^{+} + Z_{0,s}^{+}Z_{0,s}^{-}} \tag{13}$$

where $Z_{L,o}$ and $Z_{L,s}$ are the load impedances seen at the end terminal of the original and shifted unit cells respectively, and $Z_{L,o}$ is equal to Z_L in Fig. 3. It should be pointed out that $Z_{L,o}$ and $Z_{L,s}$ are also

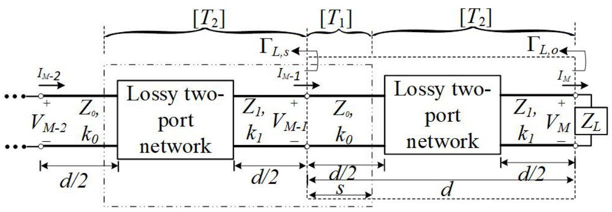


Fig.3: *A single unit cell of a semi-infinite RLSPTLS, terminated in a load impedance Z_L, obtained by shifting a reference position s along TLs (M approaches infinity).*

mathematically related by the bilinear transformation as follows:

$$Z_{L,s} = \frac{A_2V_L + B_2I_L}{C_2V_L + D_2I_L} = \frac{A_2Z_{L,o} + B_2}{C_2Z_{L,o} + D_2} \tag{14}$$

with $A_2D_2 - B_2C_2 \neq 0$ in general. Note that in V_L and I_L in (14) are equal to V_M and I_M in Fig. 3, respectively. It is pointed out that $\Gamma_{L,o}$ and $\Gamma_{L,s}$ are related by the bilinear transformation due to the bilinear transformation relationships between $\Gamma_{L,o}$ and $Z_{L,o}$, and $\Gamma_{L,s}$ and $Z_{L,s}$ as follows:

$$\Gamma_{L,s} = \frac{a_1\Gamma_{L,o} + b_1}{c_1\Gamma_{L,o} + d_1}, \tag{15}$$

$$\Gamma_{L,o} = \frac{a_oZ_{L,o} + b_o}{c_oZ_{L,o} + d_{0,o}}, \tag{16}$$

$$\Gamma_{L,s} = \frac{a_sZ_{L,s} + b_s}{c_sZ_{L,s} + d_{0,s}}, \tag{17}$$

with $a_1d_1 - b_1c_1 \neq 0$,$a_0d_0 - b_0c_0 \neq 0$ and $a_sd_{0,s} - b_sc_s \neq 0$ in general. These constant coefficients are defined in Table 1.

In (7), it should be pointed out that the equivalent characteristic impedances $Z_{0,o}^{\pm}$ of the original unit cell remain unchanged when moving reference planes due to the cascading property of the transmission matrix. Thus, once $Z_{0,o}^{\pm}$ are known, only simple parameters in (7)-(17) are computed as moving reference planes. This is an advantages of using (7) and (15)-(17) in computing $Z_{0,s}^{\pm}$ and $\Gamma_{L,s}$ instead of (1) and (12) when studying effects of moving reference planes. In addition, these proposed formulas provide more inside between associated BCITL parameters of original and shifted unit cells.

4. AN EXAMPLE OF RLSPTLSS

Consider an example of RLSPTLSs, the reciprocal lossy lumped two-port network implemented by two identical lossy TLs with the attenuation constant of 0.73 neper/m and the propagation constant of 115.43 radian/m. It is periodically loaded with equal series resistors of 200 Ωand a shunt resistor of 75 Ω at the center of the TLs as shown in Fig. 3 when

Table 1: *Constant coefficients for the bilinear transformation pairs*

Bilinear transformation pairs	Constant coefficients
$\Gamma_{L,s}$ and $\Gamma_{L,o}$	$a_1 = Z_{0,s}^{-}\left(B_L Z_{0,o}^{+} - A_L Z_{0,o}^{+} Z_{0,o}^{-}\right) + Z_{0,s}^{+} Z_{0,s}^{-}\left(C_L Z_{0,o}^{+} Z_{0,o}^{-} - D_L Z_{0,o}^{+}\right)$ $b_1 = -Z_{0,s}^{-}\left(B_L Z_{0,o}^{-} + A_L Z_{0,o}^{+} Z_{0,o}^{-}\right) + Z_{0,s}^{+} Z_{0,s}^{-}\left(C_L Z_{0,o}^{+} Z_{0,s}^{-} + D_L Z_{0,o}^{-}\right)$ $c_1 = Z_{0,s}^{+}\left(B_L Z_{0,o}^{+} - A_L Z_{0,o}^{+} Z_{0,o}^{-}\right) + Z_{0,s}^{+} Z_{0,s}^{-}\left(D_L Z_{0,o}^{+} - C_L Z_{0,o}^{+} Z_{0,o}^{-}\right)$ $d_1 = -Z_{0,s}^{+}\left(B_L Z_{0,o}^{+} + A_L Z_{0,o}^{+} Z_{0,o}^{-}\right) - Z_{0,s}^{+} Z_{0,s}^{-}\left(C_L Z_{0,o}^{+} Z_{0,o}^{-} + D_L Z_{0,o}^{-}\right)$
$\Gamma_{L,o}$ and $Z_{L,o}$	$a_o = Z_{0,o}^{-}$, $b_o = -Z_{0,o}^{+} Z_{0,o}^{-}$ $c_o = Z_{0,o}^{+}$ and $d_{0,o} = Z_{0,o}^{+} Z_{0,o}^{-}$
$\Gamma_{L,s}$ and $Z_{L,s}$	$a_s = Z_{0,s}^{-}$, $b_s = -Z_{0,s}^{+} Z_{0,s}^{-}$ $c_s = Z_{0,s}^{+}$ and $d_{0,s} = Z_{0,s}^{+} Z_{0,s}^{-}$

the lossy two-port network is replaced by T-network lumped element. The characteristic impedance of TLs is 61.24+j0.26 Ω and the length d of the unit cell is 6 cm. It should be pointed out that only the operating frequency of 1.5 GHz is considered in this paper. This RLSPTLS is terminated in a 50 Ω-load impedance. In this example, it is assumed that each reciprocal unit cell is linear for the range of input voltages and currents of interest.

Using the standard formula (1) and the proposed bilinear-transformation formulas of (7)-(10), the equivalent BCITL characteristic impedances can be readily computed. Figure 4 shows the plot of the magnitude and the argument of $Z_0^{\pm}$ versus s/d. Note that ϕ^+ and ϕ^- are the arguments of Z_0^+ and Z_0^-, respectively. For the original unit cell when $s/d = 0$, $\left|Z_0^{\pm}\right|$ and $\phi^{\pm}$ are identical and equal to 144.59 Ω and -47.41 degrees, respectively due to the symmetrical unit cell. It is found that $Z_0^{\pm}$ are different when varying s/d as expected. The magnitudes of $Z_0^{\pm}$ can be varied significantly when moving reference planes as shown in Fig. 4 (a). In addition, the arguments of $Z_0^{\pm}$ are different as expected since the considered unit cell is unsymmetrical as shown in Fig. 4 (b). When s/d = 0 and s/d =1, all considered parameters are identical as expected because the shifted unit cell becomes the original unit cell again. Using either standard or proposed formulas, they provide the identical equivalent BCITL characteristic impedances. However, the proposed formulas provide more useful information of the relationship between equivalent BCITL characteristic impedances of the original and shifted unit cells.

Similarly, using the standard formula (13) and the proposed bilinear-transformation formulas of (15)-(17), the load reflection coefficient of the shifted unit cell can be readily computed. Note that $\Gamma_{L,bilinear}$ in Fig.5 is computed by using (15). It is found that both

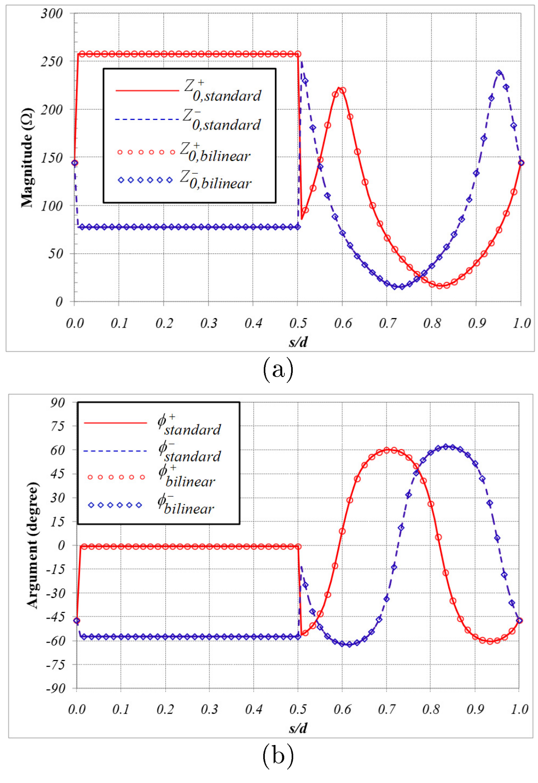
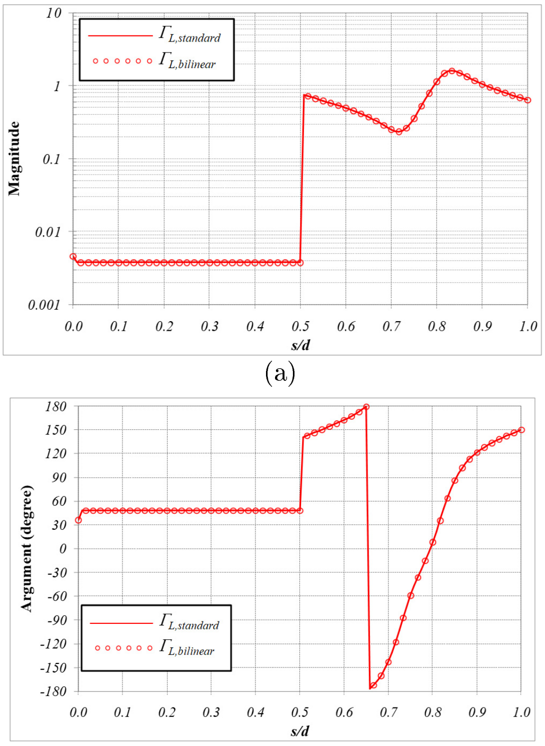


Fig.4: *Equivalent BCITL characteristic impedances: (a) $\left|Z_0^{\pm}\right|$ and (b) $\phi^{\pm}$.*

approaches provide identical results as shown in Fig. 5, but the proposed formulas provide the relationship between the load reflection coefficients of the original and shifted unit cells. It should be pointed out that Γ_L of the shifted unit cell can be computed by using the relationship between $\Gamma_{L,s}$ and $Z_{L,s}$ in (17) as well depending on what parameters are known. Note that the equivalent BCITL complex propagation constants of unit cells of the example remain unchanged



(b)

Fig.5: *Reflection coefficient at the load: (a) magnitude and (b) argument.*

as moving reference planes as expected, specifically $\gamma = 4.14 + j16.57m^{-1}$.

5. CONCLUSIONS

In this paper, analytical results of a moving reference plane of an arbitrary unit cell of RLSPTLSs by using the equivalent BCITL model are presented. Interestingly, the equivalent BCITL characteristic impedances of original and shifted unit cells are mathematically related by the bilinear transformation, and they are generally different depending on the reference position as expected. In addition, it is found that the associated load impedances of the original and shifted unit cells, as well as the associated load reflection coefficients and the associated load impedances of each unit cell, are mathematically related by the bilinear transformation. Therefore, the associated load reflection coefficients of the original and shifted unit cells are mathematically related by the bilinear transformation as well. In the analysis, both standard and proposed formulas are used in a computation. It is found that both approaches provide identical results. However, the proposed formulas provide more insight about the relationship between associated BCITL parameters of the original and shifted unit cells. Furthermore, the equivalent BCITL complex propagation constant remains unchanged for both original and shifted unit cells as expected.

6. ACKNOWLEDGEMENT

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