A Method for Designing Modified PID Controllers for Time-delay Plants and Their Application

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ABSTRACT

In this paper, we examine a method for designing modified proportional-integral-derivative (PID) controllers for use in stable and/or minimum-phase time-delay plants. The PID controller structure is the one used most widely in industrial applications. Recently, the parameterization of all stabilizing PID controllers has been considered. However, no method which guarantees the stability of a PID control system for any stable and/or minimum-phase time-delay plants, and in which the admissible sets of P-, I- and D-parameters are independent from each other, has been published. In this paper, we propose a method for designing modified PID controllers such that the modified PID controller makes the feedback control system for any stable and/or minimum-phase time-delay plant stable and the admissible sets of P-, I- and D-parameters are independent from each other. Numerical examples and application in a heat flow experiment are shown to illustrate the effectiveness of the proposed method.

Keywords: PID control, time-delay plant, stability, admissible set, heat flow experiment

1. INTRODUCTION

The proportional-integral-derivative (PID) controller is the most widely used controller structure in industrial applications [1–3]. Its structural simplicity and ability to solve many practical control problems have contributed to this wide acceptance.

Several papers on tuning methods for PID parameters have been published [4–14] but these methods do not guarantee the stability of a closed-loop system. Several methods of designing PID controllers to guarantee the stability of closed-loop systems have been proposed [15–18]. However, it is difficult to tune PID parameters to meet control specifications using these methods. If the admissible sets of PID parameters that would guarantee the stability of a closed-loop system can be determined, we can easily design stabilizing PID controllers to meet control specifications.

Obtaining admissible sets of PID parameters that guarantee the stability of a closed-loop system is a parameterization problem [3, 19, 20]. The parameterization of all stabilizing PID controllers is considered in [3, 19, 20]. However, the difficulty remaining with these methods is that the admissible sets of P-, I- and D-parameters in [3, 19, 20] are related to each other. In other words, if the P-parameter is changed, then the admissible sets of I- and D-parameters also change. From a practical point of view, it is desirable for the admissible sets of P-, I- and D-parameters to be independent from each other. Yamada and Moki first approached this problem and proposed a method for designing modified PI controllers for any minimum-phase system such that the admissible sets of P- and I-parameters were independent from each other [21]. Yamada expanded the results in [21] and proposed a method for designing modified PID controllers for minimum-phase plants such that the admissible sets of P-, I- and D-parameters were independent from each other [22]. For stable plants, a method for designing modified PID controllers was considered in [23, 24]. However, no paper gives a method for designing modified PID controllers for any stable and/or minimum-phase time-delay plant such that the stability of the control system is guaranteed and the admissible sets of P-, I- and D-parameters are independent.

In this paper, we expand the results in [21–24] and propose a method for designing modified PID controllers such that the controller makes the feedback control system stable for any stable and/or minimum-phase time-delay plant and the admissible sets of P-, I- and D-parameters are independent. This paper is organized as follows: In Section 2., the problem considered in this paper is clarified. In Section 3., we describe the basic concepts of designing modified PID controllers such that the controller makes the feedback control system stable for any stable and/or minimum-phase time-delay plant and the admissible sets of P-, I- and D-parameters are independent. In Section 4. and Section 5., we propose methods of designing modified PID controllers for use in stable plants and minimum-phase plants, respectively. In Section 6., a simple numerical example is shown. In Section 7., an application in a heat flow experiment is

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shown to illustrate the effectiveness of the proposed method.

Notation

R	The set of real numbers.
$R(s)$	The set of real rational functions with s.
RH_∞	The set of stable proper real rational functions.
$\mathcal{U}$	The set of unimodular functions on RH_∞. That is, $U(s) \in \mathcal{U}$ implies both $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$.

2. PROBLEM FORMULATION

Consider the feedback control system in Fig. 1. Here, $G(s)e^{-sT}$ is the single-input/single-output time-delay plant, $G(s) \in R(s)$ is assumed to be strictly proper and have no zero on the origin, $T > 0$ is the time delay, $C(s)$ is the controller, $r \in R$ is the reference input, $u \in R$ is the control input, $y \in R$ is the output, $d \in R$ is disturbance and $e \in R$ is the error.

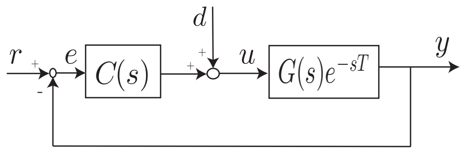

Fig.1: *Feedback control system.*

When the controller $C(s)$ can be written as

$$C(s) = a_P + \frac{a_I}{s} + a_D s, \tag{1}$$

the controller $C(s)$ is a PID controller [1–3, 19, 20], where $a_P \in R$ is the P-parameter, $a_I \in R$ is the I-parameter and $a_D \in R$ is the D-parameter. a_P, a_I and a_D are set so that the feedback control system in Fig. 1 has desirable characteristics such as steady state control and transient control. For simplicity, we call $C(s)$ in (1) the conventional PID controller. The transfer function from r to y in Fig. 1 is written as

$$y = \frac{G(s)\left(a_P + \frac{a_I}{s} + a_D s\right)e^{-sT}}{1 + G(s)\left(a_P + \frac{a_I}{s} + a_D s\right)e^{-sT}} r. \tag{2}$$

It is obvious that when a_P, a_I and a_D are set at random, the stability of the feedback control system in Fig. 1 is not guaranteed. If a stabilizing PID controller $C(s)$ in (1) for $G(s)$ exists, we can use the results in [3, 19, 20] to design a PID controller $C(s)$ in (1) to stabilize the feedback control system in Fig. 1. In other words, the admissible sets of a_P, a_I and a_D able to stabilize the feedback control system in Fig. 1 can be obtained using results from [3, 19, 20]. According to these authors, the admissible sets of a_P, a_I and a_D are related to each other i.e. if a_P is changed then the admissible sets of a_I and a_D also change. However, from a practical point of view, it is desirable for the admissible sets of a_P, a_I and a_D to be independent. Furthermore, the transfer function in (2) generally has an infinite number of poles. When the transfer functions from r to y and from d to y have an infinite number of poles, it is difficult to obtain desirable input–output, disturbance attenuation and other such control characteristics.

The purpose of this paper is to propose a method for designing modified PID controllers $C(s)$ in Fig. 1, with the following characteristics.

1. The stability of the feedback control system in Fig. 1 is guaranteed.
2. The transfer functions from r to y and from d to y in Fig. 1 have a finite number of poles.
3. The admissible sets of P-parameters a_P, I-parameters a_I and D-parameters a_D that guarantee stability of the feedback control system in Fig. 1 are independent.
4. The roles of P-parameters a_P, I-parameters a_I and D-parameters a_D in the modified PID controller are equivalent to those of the conventional PID controller in (1).

3. BASIC CONCEPTS

In this section, we describe the basic concept of designing modified PID controllers $C(s)$ with the characteristics described in Section 2.

In order to design modified PID controllers $C(s)$ that can be applied to any stable and/or minimum-phase time-delay plants and ensure the transfer functions from r to y and d to y have a finite number of poles, we adopted the parameterization of all stabilizing modified Smith predictors for any stable and/or minimum-phase time-delay plant in [25]. The reason for not adopting the Smith predictor, which is an effective time-delay compensator for a time-delay plant, in [26] but adopting the parameterization of all stabilizing modified Smith predictors in [25] is that the Smith predictor in [26] cannot be applied to unstable time-delay plants but the parameterization of all stabilizing modified Smith predictors in [25] can be applied to any stable and/or minimum-phase time-delay plant. The basic concept of designing modified PID controllers $C(s)$ with the characteristics described in Section 2., using the results in [25], is summarized as follows.

1. When $G(s)$ is stable.

According to [25], the parameterization of all stabilizing modified Smith predictors for stable time-delay plants is written as

$$C(s) = \frac{Q(s)}{1 - Q(s)G(s)e^{-sT}}, \tag{3}$$

where $Q(s) \in RH_\infty$ is any function. Using the controller in (3), the transfer functions from r to y and d to y in Fig. 1 are written as

$$y = Q(s)G(s)e^{-sT}r \tag{4}$$

and

$$y = (1 - Q(s)G(s)e^{-sT})G(s)e^{-sT}d, \tag{5}$$

respectively. Assuming that $Q(s) \in RH_\infty$, the transfer functions in (4) and (5) have a finite number of poles. In addition, it is obvious that the stability of the feedback control system in Fig. 1 is guaranteed. In order to design modified PID controllers with the characteristics described inSection 2., the free parameter $Q(s)$ in (3) is set so that $C(s)$ in (3) has the same characteristics as the conventional PID controller $C(s)$ in (1). Thus, we now describe the role of the conventional PID controller $C(s)$ in (1), in order to clarify the conditions that the modified PID controller $C(s)$must satisfy. The P-parameters a_P, I-parametersa_I and D-parameters a_D are determined according to

$$a_P = \lim_{s\to\infty}\left\{-s^2\frac{d}{ds}\left(\frac{1}{s}C(s)\right)\right\}, \tag{6}$$

$$a_I = \lim_{s\to 0}\{sC(s)\} \tag{7}$$

and

$$a_D = \lim_{s\to\infty}\frac{d}{ds}\{C(s)\}, \tag{8}$$

respectively, from (1), using $C(s)$. Therefore, if the controller $C(s)$ in (3) holds (6), (7) and (8), the role of controller $C(s)$ in (3) is equivalent to that of the conventional PID controller $C(s)$ in (1). In other words, we can design a stabilizing modified PID controller such that the role of controller $C(s)$ in (3) is equivalent to that of the conventional PID controller $C(s)$ in (1).

2. When $G(s)$ is of minimum-phase.

According to [25], the parameterization of all stabilizing modified Smith predictors for minimum-phase time-delay plants is written as

$$C(s) = \frac{C_f(s)}{1 - C_f(s)G(s)e^{-sT}}, \tag{9}$$

$$C_f(s) = \frac{\bar{G}_u(s)}{G_u(s)}\left(1 + \frac{Q(s)}{G_u(s)}\right), \tag{10}$$

where $Q(s) \in RH_\infty$ is any function, $\bar{G}_u(s) \in \mathcal{U}$ satisfies

$$\bar{G}_u(s_i) = \frac{1}{G_s(s_i)e^{-s_iT}} \quad (i = 1, \ldots, n), \tag{11}$$

n is the number of unstable poles of $G(s)$, $s_i (i = 1, \ldots, n)$ are the unstable poles of $G(s)$, $G_s(s)$ is a stable minimum-phase function of $G(s)$, i.e. when$G(s)$ is factorized as

$$G(s) = G_u(s)G_s(s), \tag{12}$$

$G_u(s)$ is the unstable biproper minimum-phase function and $G_s(s)$ is the stable minimum-phase function. Using the controller in (9), the transfer functions from r to y and d to y are written as

$$y = \bar{G}_u(s)\left(1 + \frac{Q(s)}{G_u(s)}\right)G_s(s)e^{-sT}r \tag{13}$$

and

$$\begin{aligned} y \quad = \quad & \left\{1 - \bar{G}_u(s)\left(1 + \frac{Q(s)}{G_u(s)}\right)\right. \\ & \left.\cdot G_s(s)e^{-sT}\right\}G(s)e^{-sT}d, \end{aligned} \tag{14}$$

respectively. From $Q(s) \in RH_\infty$, $\bar{G}_u(s) \in \mathcal{U}$, $G_s(s) \in RH_\infty$ and $1/G_u(s) \in RH_\infty$, the transfer functions in (13) and (14) have finite numbers of poles. In addition, it is obvious that the stability of the feedback control system in Fig. 1 is guaranteed. If the controller $C(s)$ in (9) holds (6), (7) and (8), the role of controller $C(s)$ in (3) is equivalent to that of the conventional PID controller $C(s)$ in (1). In other words, we can design a stabilizing modified PID controller such that the role of controller $C(s)$ in (9) is equivalent to that of the conventional PID controller $C(s)$ in (1).

In Section 4. and Section 5., we use the ideas discussed above to describe a method for designing the modified PID controller $C(s)$ in (3) or (9) that works as a modified PID controller. In the following, we call $C(s)$

1. the modified P controller if $C(s)$ in (3) or (9) satisfies (6),
2. the modified I controller if $C(s)$ in (3) or (9) satisfies (7),
3. the modified D controller if $C(s)$ in (3) or (9) satisfies (8),
4. the modified PI controller if $C(s)$ in (3) or (9) satisfies (6) and (7),
5. the modified PD controller if $C(s)$ in (3) or (9) satisfies (6) and (8),
6. the modified PID controller if $C(s)$ in (3) or (9) satisfies (6), (7) and (8).

4. MODIFIED PID CONTROLLER, WHEN $G(S)$ IS STABLE

In this section, we describe a method for designing the modified PID controller $C(s)$ in (3) that works as a modified PID controller.

4.1 Modified P controller

In this subsection, we discuss a method for designing a modified P controller $C(s)$ in (3) that holds (6), makes the feedback control system in Fig. 1 stable and is applicable to any stable time-delay plant $G(s)e^{-sT}$.

The modified P controller $C(s)$ satisfying (6) is written as (3), where

$$Q(s) = a_P. \tag{15}$$

Since $Q(s)$ in (15) is included in RH_∞, the controller $C(s)$ in (3) with (15) makes the feedback control system in Fig. 1 stable for any stable time-delay plant $G(s)e^{-sT}$ independent from a_P.

4.2 Modified I controller

In this subsection, we discuss a method for designing a modified I controller $C(s)$ in (3) that holds (7), makes the feedback control system in Fig. 1 stable and is applicable to any stable time-delay plant $G(s)e^{-sT}$.

The modified I controller $C(s)$ satisfying (7) is written as (3), where

$$Q(s) = \frac{q_0 + q_1 s}{\tau_0 + \tau_1 s}, \tag{16}$$

$$q_0 = \frac{\tau_0}{G(0)}, \tag{17}$$

$$\begin{aligned} q_1 = & \frac{\tau_1}{G(0)} \\ & - \frac{\tau_0}{G^2(0)}\left(\frac{1}{a_I} - TG(0) + \frac{d}{ds}\left\{G(s)\right\}\bigg|_{s=0}\right) \end{aligned} \tag{18}$$

and $\tau_i \in R > 0 (i = 0, 1)$. For $\tau_i > 0 (i = 0, 1)$, $Q(s)$ in (15) is included in RH_∞. This implies that the controller $C(s)$ in (3) with (16) makes the feedback control system in Fig. 1 stable for any stable time-delay plant $G(s)e^{-sT}$ independent from a_I.

4.3 Modified D controller

In this subsection, we discuss a method for designing a modified D controller $C(s)$ in (3) that holds (8), makes the feedback control system in Fig. 1 stable and is applicable to any stable time-delay plant $G(s)e^{-sT}$.

The modified D controller $C(s)$ satisfying (8) is written as (3), where

$$Q(s) = a_D s. \tag{19}$$

Since $Q(s)$ in (19) is improper, $Q(s)$ in (19) is not included in RH_∞. In order for $Q(s)$ to be included in RH_∞, (19) must be modified according to

$$Q(s) = \frac{a_D s}{1 + \tau_D s}, \tag{20}$$

where $\tau_D \in R > 0$. For $\tau_D > 0$ in (20), $Q(s)$ in (20) is included in RH_∞. This implies that the controller $C(s)$ in (3) with (20) makes the feedback control system in Fig. 1 stable for any stable time-delay plant $G(s)e^{-sT}$ independent from a_D.

4.4 Modified PI controller

In this subsection, we discuss a method for designing a modified PI controller $C(s)$ in (3) that holds (6) and (7), makes the feedback control system in Fig. 1 stable and is applicable to any stable time-delay plant $G(s)e^{-sT}$.

The modified PI controller $C(s)$ satisfying (6) and (7) is written as (3), where

$$Q(s) = \frac{q_0 + q_1 s + q_2 s^2}{\tau_0 + \tau_1 s + \tau_2 s^2}, \tag{21}$$

$$q_0 = \frac{\tau_0}{G(0)}, \tag{22}$$

$$\begin{aligned} q_1 = & \frac{\tau_1}{G(0)} \\ & - \frac{\tau_0}{G^2(0)}\left\{\frac{1}{a_I} - TG(0) + \frac{d}{ds}\left(G(s)\right)\bigg|_{s=0}\right\}, \end{aligned} \tag{23}$$

$$q_2 = \tau_2 a_P \tag{24}$$

and $\tau_i \in R > 0 (i = 0, 1, 2)$. For $\tau_i > 0 (i = 0, 1, 2)$, $Q(s)$ in (21) is included in RH_∞. This implies that the controller $C(s)$ in (3) with (21) makes the feedback control system in Fig. 1 stable for any stable time-delay plant $G(s)e^{-sT}$ independent from a_P and a_I.

4.5 Modified PD controller

In this subsection, we discuss a method for designing a modified PD controller $C(s)$ in (3) that holds (6) and (8), makes the feedback control system in Fig. 1 stable and is applicable to any stable time-delay plant $G(s)e^{-sT}$.

The modified PD controller $C(s)$ satisfying (6) and (8) is written as (3), where

$$Q(s) = a_P + a_D s. \tag{25}$$

Since $Q(s)$ in (25) is improper, $Q(s)$ in (25) is not included in RH_∞. In order for $Q(s)$ to be included in RH_∞, (25) must be modified according to

$$Q(s) = a_P + \frac{a_D s}{1 + \tau_D s}, \tag{26}$$

where $\tau_D \in R > 0$. For $\tau_D > 0$ in (26), $Q(s)$ in (26) is included in RH_∞. This implies that the controller $C(s)$ in (3) with (26) makes the feedback control system in Fig. 1 stable for any stable time-delay plant $G(s)e^{-sT}$ independent from a_P and a_D.

4.6 Modified PID controller

In this subsection, we discuss a method for designing a modified PID controller $C(s)$ in (3) that holds (6), (7) and (8), makes the feedback control system in Fig. 1 stable and is applicable to any stable time-delay plant $G(s)e^{-sT}$.

The modified PID controller $C(s)$ satisfying (6), (7) and (8) is written as (3), where

$$Q(s) = \frac{q_0 + q_1 s + q_2 s^2}{\tau_0 + \tau_1 s + \tau_2 s^2} + q_3 s, \tag{27}$$

$$q_0 = \frac{\tau_0}{G(0)}, \tag{28}$$

$$\begin{aligned} q_1 &= \frac{\tau_1}{G(0)} - q_3\tau_0 \\ &\quad -\frac{\tau_0}{G^2(0)}\left\{\frac{1}{a_I} - TG(0) + \frac{d}{ds}\left(G(s)\right)\bigg|_{s=0}\right\}, \end{aligned} \tag{29}$$

$$q_2 = \tau_2 a_P, \tag{30}$$

$$q_3 = a_D \tag{31}$$

and $\tau_i \in R > 0 (i = 0, 1, 2)$. Since $Q(s)$ in (27) is improper, $Q(s)$ in (27) is not included in RH_∞. In order for $Q(s)$ to be included in RH_∞, (27) must be modified according to

$$Q(s) = \frac{q_0 + q_1 s + q_2 s^2}{\tau_0 + \tau_1 s + \tau_2 s^2} + \frac{q_3 s}{1 + \tau_D s}, \tag{32}$$

where $\tau_D \in R > 0$. For $\tau_i > 0 (i = 0, 1, 2)$ and $\tau_D > 0$ in (32), $Q(s)$ in (32) is included in RH_∞. This implies that the controller $C(s)$ in (3) with (32) makes the feedback control system in Fig. 1 stable for any stable time-delay plant $G(s)e^{-sT}$ independent from a_P, a_I and a_D.

5. MODIFIED PID CONTROLLER, WHEN $G(S)$ IS OF MINIMUM-PHASE

In this section, we describe a method for designing the modified PID controller $C(s)$ in (9) that works as a modified PID controller.

5.1 Modified P controller

In this subsection, we discuss a method for designing a modified P controller $C(s)$ in (9) that holds (6), makes the feedback control system in Fig. 1 stable and is applicable to any minimum-phase time-delay plant $G(s)e^{-sT}$.

The modified P controller $C(s)$ satisfying (6) is written as (9), where

$$Q(s) = \lim_{s\to\infty}\left(\frac{G_u^2(s)}{\bar{G}_u(s)}a_P - G_u(s)\right) = \text{const.} \tag{33}$$

Since $Q(s)$ in (33) is included in RH_∞, the controller $C(s)$ in (9) with (33) makes the feedback control system in Fig. 1 stable for any minimum-phase time-delay plant $G(s)e^{-sT}$ independent from a_P.

5.2 Modified I controller

In this subsection, we discuss a method for designing a modified I controller $C(s)$ in (9) that holds (7), makes the feedback control system in Fig. 1 stable and is applicable to any minimum-phase time-delay plant $G(s)e^{-sT}$.

The modified I controller $C(s)$ satisfying (7) is written as (9), where

$$Q(s) = \frac{q_0 + q_1 s}{\tau_0 + \tau_1 s}, \tag{34}$$

$$q_0 = \tau_0\left(\frac{G_u(0)}{\bar{G}_u(0)G_s(0)} - G_u(0)\right), \tag{35}$$

$$\begin{aligned} q_1 &= \left(\frac{2\tau_0}{\bar{G}_u(0)G_s(0)} - \tau_0\right)\frac{d}{ds}\left(G_u(s)\right)\bigg|_{s=0} \\ &\quad - \frac{\tau_0 G_u(0)\,\frac{d}{ds}\left(\bar{G}_u(s)\right)\Big|_{s=0}}{\bar{G}_u^2(0)G_s(0)} - \frac{\tau_0\,\frac{d}{ds}\left(G(s)\right)\Big|_{s=0}}{\bar{G}_u(0)G_s^2(0)} \\ &\quad - \left\{\tau_1 + \frac{\tau_0\left(\tau_1 - T\right)}{\bar{G}_u(0)G_s(0)}\right\}G_u(0) - \frac{\tau_0}{a_I\bar{G}_u(0)G_s^2(0)} \end{aligned} \tag{36}$$

and $\tau_i \in R > 0 (i = 0, 1)$. For $\tau_i > 0 (i = 0, 1)$, $Q(s)$ in (34) is included in RH_∞. This implies that the controller $C(s)$ in (9) with (34) makes the feedback control system in Fig. 1 stable for any minimum-phase time-delay plant $G(s)e^{-sT}$ independent from a_I.

5.3 Modified D controller

In this subsection, we discuss a method for designing a modified D controller $C(s)$ in (9) that holds (8), makes the feedback control system in Fig. 1 stable and is applicable to any minimum-phase time-delay plant $G(s)e^{-sT}$.

The modified D controller $C(s)$ satisfying (8) is written as (9), where

$$Q(s) = q_0 s \tag{37}$$

and

$$q_0 = a_D \lim_{s\to\infty}\left(\frac{G_u^2(s)}{\bar{G}_u(s)}\right). \tag{38}$$

Since $Q(s)$ in (37) is improper, $Q(s)$ in (37) is not included in RH_∞. In order for $Q(s)$ to be included in RH_∞, (37) must be modified according to

$$Q(s) = \frac{a_D s}{1 + \tau_D s}, \tag{39}$$

where $\tau_D \in R > 0$. For $\tau_D > 0$ in (39), $Q(s)$ in (39) is included in RH_∞. This implies that the controller $C(s)$ in (9) with (39) makes the feedback control system in Fig. 1 stable for any minimum-phase time-delay plant $G(s)e^{-sT}$ independent from a_D.

5.4 Modified PI controller

In this subsection, we discuss a method for designing a modified PI controller $C(s)$ in (9) that holds (6) and (7), makes the feedback control system in Fig. 1 stable and is applicable to any minimum-phase time-delay plant $G(s)e^{-sT}$.

The modified PI controller $C(s)$ satisfying (6) and (7) is written as (9), where

$$Q(s) = \frac{q_0 + q_1 s + q_2 s^2}{\tau_0 + \tau_1 s + \tau_2 s^2}, \tag{40}$$

$$q_0 = \tau_0 \left(\frac{G_u(0)}{\bar{G}_u(0) G_s(0)} - G_u(0) \right), \tag{41}$$

$$\begin{aligned} q_1 = & \left(\frac{2\tau_0}{\bar{G}_u(0)G_s(0)} - \tau_0 \right) \frac{d}{ds} \left(G_u(s) \right) \Big|_{s=0} \\ & - \frac{\tau_0 G_u(0) \left. \frac{d}{ds} \left(\bar{G}_u(s) \right) \right|_{s=0}}{\bar{G}_u^2(0) G_s(0)} - \frac{\tau_0 \left. \frac{d}{ds} \left(G(s) \right) \right|_{s=0}}{\bar{G}_u(0) G_s^2(0)} \\ & - \left\{ \tau_1 + \frac{\tau_0 (\tau_1 - T)}{\bar{G}_u(0) G_s(0)} \right\} G_u(0) - \frac{\tau_0}{a_I \bar{G}_u(0) G_s^2(0)}, \end{aligned} \tag{42}$$

$$q_2 = \tau_2 \lim_{s \to \infty} \left(\frac{G_u^2(s)}{\bar{G}_u(s)} a_P - G_u(s) \right) \tag{43}$$

and $\tau_i \in R > 0 (i = 0, 1, 2)$. For $\tau_i > 0 (i = 0, 1, 2)$, $Q(s)$ in (40) is included in RH_∞. This implies that the controller $C(s)$ in (9) with (40) makes the feedback control system in Fig. 1 stable for any minimum-phase time-delay plant $G(s)e^{-sT}$ independent from a_P and a_I.

5.5 Modified PD controller

In this subsection, we discuss a method for designing a modified PD controller $C(s)$ in (9) that holds (6) and (8), makes the feedback control system in Fig. 1 stable and is applicable to any minimum-phase time-delay plant $G(s)e^{-sT}$.

The modified PD controller $C(s)$ satisfying (6) and (8) is written as (9), where

$$Q(s) = q_0 + q_1 s, \tag{44}$$

$$\begin{aligned} q_0 = & \lim_{s \to \infty} \left[\frac{G_u^2(s)}{\bar{G}_u(s)} a_P - G_u(s) \right. \\ & + \frac{a_D s^2 G_u(s)}{\bar{G}_u(s)} \left\{ \frac{G_u(s) \frac{d}{ds} \left(\bar{G}_u(s) \right)}{\bar{G}_u(s)} \right. \\ & \left. \left. - 2 \frac{d}{ds} \left(G_u(s) \right) \right\} \right] \end{aligned} \tag{45}$$

and

$$q_1 = a_D \lim_{s \to \infty} \left(\frac{G_u^2(s)}{\bar{G}_u(s)} \right). \tag{46}$$

Since $Q(s)$ in (44) is improper, $Q(s)$ in (44) is not included in RH_∞. In order for $Q(s)$ to be included in RH_∞, (44) must be modified according to

$$Q(s) = q_0 + \frac{q_1 s}{1 + \tau_D s}, \tag{47}$$

where $\tau_D \in R > 0$. For $\tau_D > 0$ in (47), $Q(s)$ in (47) is included in RH_∞. This implies that the controller $C(s)$ in (9) with (47) makes the feedback control system in Fig. 1 stable for any minimum-phase time-delay plant $G(s)e^{-sT}$ independent from a_P and a_D.

5.6 Modified PID controller

In this subsection, we discuss a method for designing a modified PID controller $C(s)$ in (9) that holds (6), (7) and (8), makes the feedback control system in Fig. 1 stable and is applicable to any minimum-phase time-delay plant $G(s)e^{-sT}$.

The modified PID controller $C(s)$ satisfying (6), (7) and (8) is written as (9), where

$$Q(s) = \frac{q_0 + q_1 s + q_2 s^2}{\tau_0 + \tau_1 s + \tau_2 s^2} + q_3 s, \tag{48}$$

$$q_0 = \tau_0 \left(\frac{G_u(0)}{\bar{G}_u(0) G_s(0)} - G_u(0) \right), \tag{49}$$

$$\begin{aligned} q_1 = & \left(\frac{2\tau_0}{\bar{G}_u(0)G_s(0)} - \tau_0 \right) \frac{d}{ds} \left(G_u(s) \right) \Big|_{s=0} \\ & - \frac{\tau_0 G_u(0) \left. \frac{d}{ds} \left(\bar{G}_u(s) \right) \right|_{s=0}}{\bar{G}_u^2(0) G_s(0)} - \frac{\tau_0 \left. \frac{d}{ds} \left(G(s) \right) \right|_{s=0}}{\bar{G}_u(0) G_s^2(0)} \\ & - \left\{ (\tau_1 + \frac{\tau_0 (\tau_1 - T)}{\bar{G}_u(0) G_s(0)} \right\} G_u(0) - \frac{\tau_0}{a_I \bar{G}_u(0) G_s^2(0)} \\ & + a_D \lim_{s \to \infty} \left(\frac{G_u^2(s)}{\bar{G}_u(s)} \right), \end{aligned} \tag{50}$$

$$q_2 = \lim_{s\to\infty}\left[\frac{G_u^2(s)}{\bar{G}_u(s)}a_P - G_u(s) + \frac{a_D s^2 G_u(s)}{\bar{G}_u(s)}\left\{\frac{G_u(s)\frac{d}{ds}\left(\bar{G}_u(s)\right)}{\bar{G}_u(s)} - 2\frac{d}{ds}\left(G_u(s)\right)\right\}\right], \quad (51)$$

$$q_3 = a_D \lim_{s\to\infty}\left(\frac{G_u^2(s)}{\bar{G}_u(s)}\right) \quad (52)$$

and $\tau_i \in R > 0 (i = 0, 1, 2)$. Since $Q(s)$ in (48) is improper, $Q(s)$ in (48) is not included in RH_∞. In order for $Q(s)$ to be included in RH_∞, (48) must be modified according to

$$Q(s) = \frac{q_0 + q_1 s + q_2 s^2}{\tau_0 + \tau_1 s + \tau_2 s^2} + \frac{q_3 s}{1 + \tau_D s}, \quad (53)$$

where $\tau_D \in R > 0$. For $\tau_i > 0 (i = 0, 1, 2)$ and $\tau_D > 0$ in (53), $Q(s)$ in (53) is included in RH_∞. This implies that the controller $C(s)$ in (9) with (53) makes the feedback control system in Fig. 1 stable for any minimum-phase time-delay plant $G(s)e^{-sT}$ independent from a_P, a_I and a_D.

6. NUMERICAL EXAMPLE

In this section, we use a numerical example to show the effectiveness of the proposed method.

Consider the problem of designing a modified PID controller $C(s)$ for an unstable minimum-phase time-delay plant $G(s)e^{-sT}$ written as

$$G(s)e^{-sT} = \frac{100}{s^3 - 0.1s^2 - 0.7s - 0.2}e^{-0.15s}, \quad (54)$$

where $T = 0.15$[sec] and

$$G(s) = \frac{100}{s^3 - 0.1s^2 - 0.7s - 0.2}. \quad (55)$$

Note that $G(s)e^{-sT}$ in (54) has no stabilizing conventional PID controller.

Since $G(s)$ in (55) is unstable and of minimum-phase, we design a modified PID controller using the method described in Section 5.6 a_P, a_I and a_D are settled by

$$\begin{cases} a_P = 55 \\ a_I = 0.0002 \\ a_D = 0.15 \end{cases}. \quad (56)$$

q_0, q_1, q_2 and q_3 are determined by (49), (50), (51) and (52), respectively, where

$$\begin{cases} \tau_0 = 1 \\ \tau_1 = 2 \\ \tau_2 = 1 \end{cases} \quad (57)$$

and $\tau_D = 1$.

Using the parameters mentioned above, the modified PID controller $C(s)$ is designed according to (9) and (53).

The step response of the control system using the modified PID controller $C(s)$ is shown in Fig. 2. Figure 2 shows that the modified PID controller $C(s)$ makes the feedback control system stable.

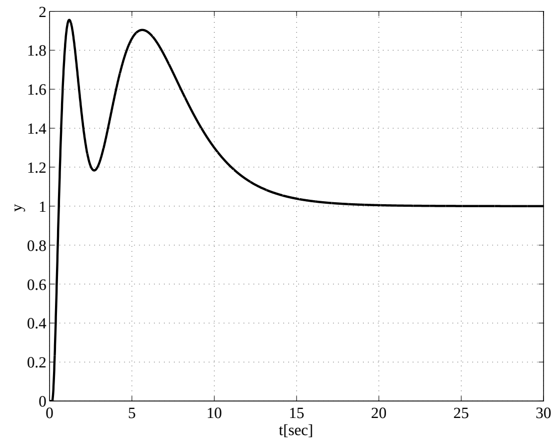

Fig.2: *Step response of the control system using a modified PID controller.*

On the other hand, using a conventional PID controller with (56), the step response of the control system is shown in Fig. 3. Figure 3 shows that the conventional PID control system is unstable. This is obvious, since no stabilizing conventional PID controller for the plant $G(s)e^{-sT}$ in (54) exists. Contrary to this, the stability of the modified PID control system is guaranteed to be independent from a_P, a_I and a_D. Therefore, we find that the modified PID controller can be applied to plants that have no stabilizing conventional PID controller.

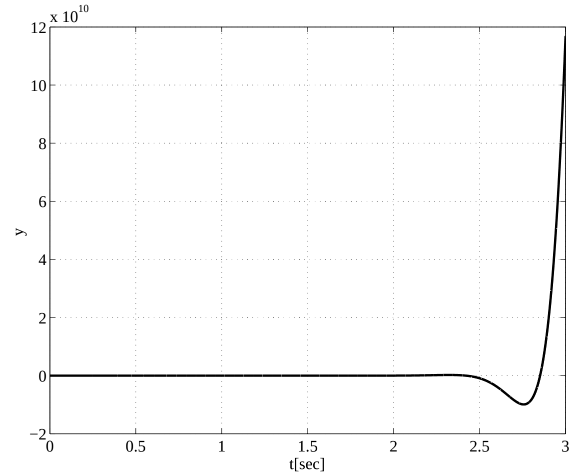

Fig.3: *Step response of the control system using a conventional PID controller.*

When a_P, a_I and a_D in the modified PID controller are varied, the step responses can be compared. First, the step responses for various a_P, with $a_P = 30$, $a_P = 40$ and $a_P = 50$, are shown in Fig. 4. Here, the solid line, dotted line and broken line

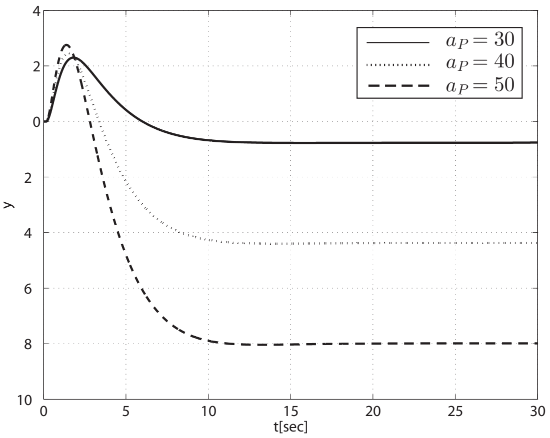

Fig.4: *Step response using modified P controller with* $a_P = 30, 40, 50$.

show the step responses of the modified PID control system using $a_P = 30$, $a_P = 40$ and $a_P = 50$, respectively. Figure 4 shows that as the value of a_P increased, the overshoot became larger and the rise time became shorter. Since these characteristics are equivalent to those of the conventional PID controller, the role of the P-parameter a_P in the modified PID controller is equivalent to that in a conventional PID controller. Secondly, the step responses for various a_I, with $a_I = 0.00001$, $a_I = 0.00005$ and $a_I = 0.0001$, are shown in Fig. 5. Here, the solid line,

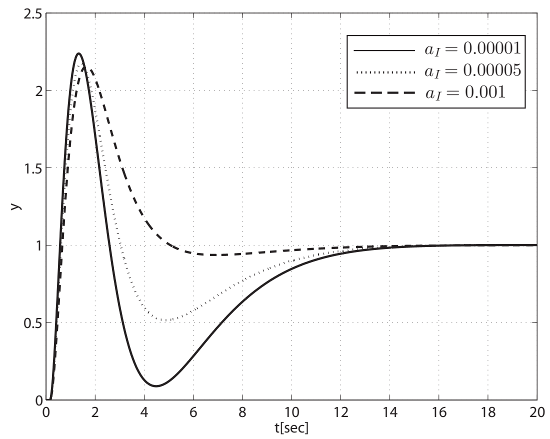

Fig.5: *Step response using modified I controller with* $a_I = 0.00001, 0.00005, 0.0001$.

dotted line and broken line show the step responses of the modified PID control system using $a_I = 0.00001$, $a_I = 0.00005$ and $a_I = 0.0001$, respectively. Figure 5 shows that as the value of a_I increased, the overshoot became smaller and convergence became more rapid. Since these characteristics are equivalent to those of the conventional PID controller, the role of the I-parameter a_I in the modified PID controller is equivalent to that in a conventional PID controller. Thirdly, the step responses for various a_D, with $a_D = 1$, $a_D = 50$ and $a_D = 100$, are shown in Fig. 6. Here, the solid line, dotted line and broken

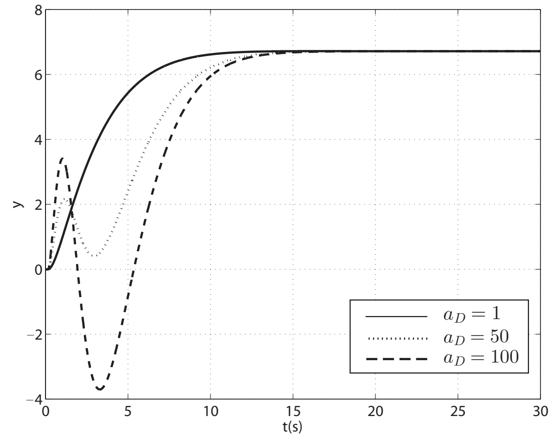

Fig.6: *Step response using modified D controller with* $a_D = 1, 50, 100$.

line show the step responses of the modified PID control system using $a_D = 1$, $a_D = 50$ and $a_D = 100$, respectively. Figure 6 shows that as the value of a_D increased, the response was smoothed. Since this characteristic is equivalent to that of the conventional PID controller, the role of the D-parameter a_D in the modified PID controller is equivalent to that in a conventional PID controller.

Thus, we have shown that we can easily design a stabilizing modified PID controller for an unstable minimum-phase time-delay plant, with the same characteristics as a conventional PID controller, and guarantee the stability of the feedback control system.

7. APPLICATION IN A HEAT FLOW EXPERIMENT

In this section, we show an application of the proposed modified PID controller in a heat flow experiment, to illustrate its effectiveness. The heat flow apparatus is shown in Fig. 7. The heat flow apparatus comprises a duct equipped with a heater and blower at one end and three temperature sensors located along the duct, as shown in Fig. 7. V_h and V_b denote the voltage to the heater and blower, respectively. S_1, S_2 and S_3 are terminals for measurement of temperatures at Sensor 1, Sensor 2 and Sensor 3. We denote temperature measurements at Sensor i as T_i.

The problem considered in this section is to design

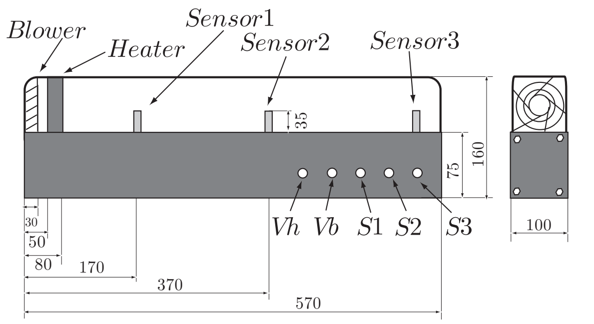

Fig. 7: *Apparatus used in the heat flow experiment.*

a modified PID controller to make T_3, the temperature at Sensor 3, remain steady at 30[deg]. When we set $V_b = 5[V]$, we found that the transfer function from V_h to T_3 could be written as

$$T_3 = \frac{5.30}{1 + 18.15s} e^{-1.25s} V_h. \tag{58}$$

T_3 and V_h are considered the output y and control input u, respectively, in the control system in Fig. 1. From (58), $G(s)e^{-sT}$ in Fig. 1 can be written as

$$G(s)e^{-sT} = \frac{5.30}{1 + 18.15s} e^{-1.25s}, \tag{59}$$

where $T = 1.25$[sec] and

$$G(s) = \frac{5.30}{1 + 18.15s}. \tag{60}$$

The reference input r in Fig. 1 is set so that $r(t) = 30$[deg].

Since $G(s)$ in (60) is stable, we designed a modified PID controller using the method described in Section 4. a_P, a_I and a_D are set according to

$$\left\{ \begin{array}{rcl} a_P & = & 4.000 \\ a_I & = & 0.140 \\ a_D & = & 0.005 \end{array} \right. . \tag{61}$$

q_0, q_1, q_2 and q_3 are determined by (28), (29), (30) and (31), respectively, where

$$\left\{ \begin{array}{rcl} \tau_0 & = & 1 \\ \tau_1 & = & 2 \\ \tau_2 & = & 1 \end{array} \right. \tag{62}$$

and $\tau_D = 1$. Using the parameters given above, the modified PID controller $C(s)$ was designed according to (3) and (32).

The experimental step response of the control system using this modified PID controller $C(s)$ is shown in Fig. 8. Figure 8 shows that the output $y = T_3$ followed the step reference input $r = 30$ with negligible steady-state error and high convergence speed, compared to open-loop response.

When a_P, a_I and a_D in the modified PID controller are varied, the step responses can be examined. First, the step responses for various a_P, with

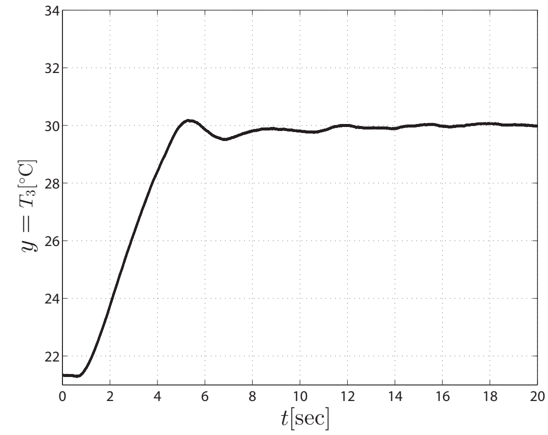

Fig. 8: *Experimental step response using the modified PID controller.*

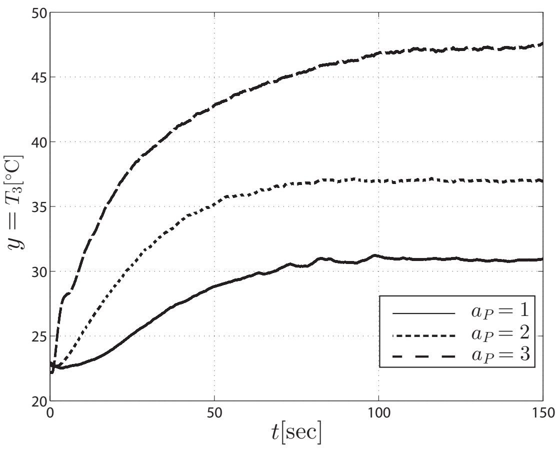

Fig. 9: *Step response using modified P controller with $a_P = 1, 2, 3$.*

$a_P = 1$, $a_P = 2$ and $a_P = 3$, are shown in Fig. 9. Here, the solid line, dotted line and broken line show the step response of the modified PID control system using $a_P = 1$, $a_P = 2$ and $a_P = 3$, respectively. Figure 9 shows that as the value of a_P increased, the overshoot became larger and the rise time became shorter. Since those characteristics are equivalent to those of the conventional PID controller, the role of the P-parameter a_P in the modified PID controller is equivalent to that in the conventional PID controller. Secondly, the step responses for various a_I, with $a_I = 0.1$, $a_I = 0.2$ and $a_I = 0.3$, are shown in Fig. 10. Here, the solid line, dotted line and broken line show the step response of the modified PID control system using $a_I = 0.1$, $a_I = 0.2$ and $a_I = 0.3$, respectively. Figure 10 shows that as the value of a_I increased, convergence became more rapid. Since this characteristic is equivalent to that of the conventional PID controller, the role of the I-parameter a_I in the modified PID controller is equivalent to that in the conventional PID controller. Thirdly, the step

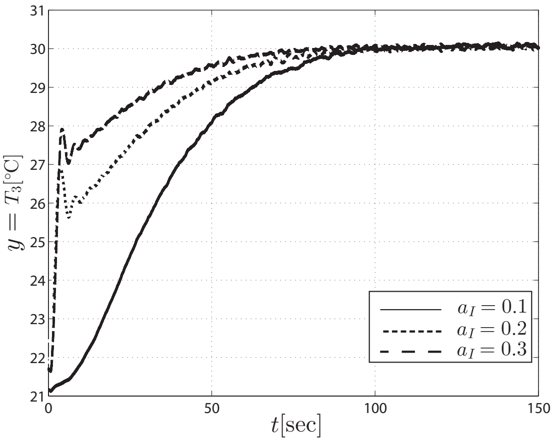

Fig.10: *Step response using modified I controller with* $a_I = 0.1, 0.2, 0.3$.

responses for various a_D, with $a_D = 1$, $a_D = 2$ and $a_D = 3$, are shown in Fig. 11. Here, the solid line,

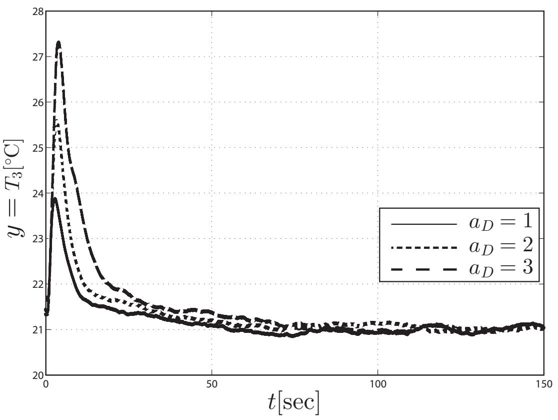

Fig.11: *Step response using modified D controller with* $a_D = 1, 2, 3$

dotted line and broken line show the step response of the modified PID control system using $a_D = 1$, $a_D = 2$ and $a_D = 3$, respectively. Figure 11 shows that as the value of a_D increased, the response was smoothed. Since this characteristic is equivalent to that of the conventional PID controller, the role of the D-parameter a_D in the modified PID controller is equivalent to that in the conventional PID controller.

On the basis of these results, we found that the proposed modified PID controller provided effective control in this heat flow experiment.

8. CONCLUSIONS

In this paper, we proposed a method for designing modified PID controllers such that the modified PID controller makes the feedback control system for any stable and/or minimum-phase time-delay plants asymptotically stable and the admissible sets of P-, I- and D-parameters are independent from each other. A numerical example and application in a heat flow experiment were also shown to illustrate the effectiveness of the proposed method.

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