Proposal for Simple Repetitive Controllers

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ABSTRACT

The modified repetitive control system is a type of servomechanism for periodic reference input. Even if the plant does not include a time delay, the transfer functions from both the reference input and the disturbance of the output of the modified repetitive control system generally have an infinite number of poles, and for this reason it is difficult to settle the input–output and the disturbance attenuation characteristics of a modified repetitive control system. From a practical point of view, the input–output and the disturbance attenuation characteristics of a control system need to be settled easily, and thus, it is desirable that the transfer functions from both the reference input and the disturbance to the output have a finite number of poles. In order to easily specify the input–output and the disturbance attenuation characteristics, we propose the concept of a simple repetitive control system in which the controller works as a stabilizing modified repetitive controller and the transfer functions to the output from both the reference input and the disturbance have a finite number of poles. In addition, the parametrization of all stabilizing simple repetitive controllers is clarified. A simple design procedure for a stabilizing simple repetitive controller is presented.

Keywords: repetitive control, stability, parametrization, finite number of poles

1. INTRODUCTION

The repetitive control system is a type of servomechanism for periodic reference input. That is, the repetitive control system follows the periodic reference input without steady-state error, even if a periodic disturbance or uncertainty exists in the plant [1-15]. It is difficult to design stabilizing controllers for the plant because a repetitive control system that follows any periodic reference input without steady-state error is a neutral type of time-delay control system [11]. In order to design a repetitive control system that follows any periodic reference input without steady-state error, the plant needs to be biproper [3-11]. In practice, the plant is strictly proper. Many design methods of repetitive control systems for strictly proper plants are given in [3-11]. These studies are divided into two types. One uses a low-pass filter [3-10] and the other uses an attenuator [11]. The latter is difficult to design because it uses a state variable time delay in the repetitive controller [11]. The former, which is called modified repetitive control system [3-10], has a simple structure and is easily designed.

On the other hand, there exists an important control problem, namely the parametrization problem, to find all stabilizing controllers [19-22]. First, the parametrization of all stabilizing modified repetitive controllers that follow the periodic reference input with a small steady-state error even if there exists a periodic disturbance or the uncertainty of the plant was studied by [4]. In [4], since the stability-sufficient condition of a repetitive control system is decided as H_∞ a norm problem, the parametrization for a repetitive control system is given by resolving the interpolation problem of Nevanlinna-Pick. Katoh and Funahashi give the parametrization of all stabilizing repetitive controllers for minimum phase systems by solving the Bezout equation exactly [13]. However, the latter authors [13] assume the plant to be asymptotically stable. This implies that they give the parametrization of all causal repetitive controllers for an asymptotically stable and minimum phase plant, i.e. they do not give the explicit parametrization for minimum phase systems [13]. In addition, in [13] it is assumed that the relative degree of the low-pass filter in the repetitive compensator is equal to that of the plant. Extending the results in [13], Yamada and Okuyama give the parametrization of all stabilizing repetitive controllers for minimum phase systems [16]. Yamada et al. give the parametrization of all such controllers for a certain class of nonminimum phase systems [17]. They obtained the parametrization of all repetitive controllers using fusion of the parallel compensation technique and the solution of the Bezout equation. However, they give the parametrization of all repetitive controllers only for a limited class of nonminimum phase systems. Yamada et al. give the complete parametrization of all stabilizing modified repetitive controllers for nonminimum phase single-input/single-output systems [18]. They obtained the parametrization of all repetitive controllers using fusion of the parallel compensation technique and the solution of Bezout equation. In this way, the parametrization of all stabilizing modified repetitive controllers has been considered.

However, using the modified repetitive controllers in [3-18], even if the plants do not include time delay,

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the transfer functions from both the periodic reference input and the disturbance to the output have an infinite number of poles, and in such a case it is difficult to specify the input–output and the disturbance attenuation characteristics. From the practical point of view, it is desirable that both these characteristics are easily specified. For this purpose it is desirable that the transfer functions from both the periodic reference input and the disturbance to the output have a finite number of poles. However no paper has examined a design method for modified repetitive control systems in this way.

In this paper, we propose the concept of simple repetitive control systems, such that the controller works as a modified repetitive controller and the transfer functions from both the periodic reference input and the disturbance to the output have a finite number of poles. In addition, the parametrization of all stabilizing simple repetitive controllers is clarified. This paper is organized as follows: In Section 2., the concept of the simple repetitive controllers is proposed and the problem considered in this paper is described. In Section 3., the parametrization of all stabilizing simple repetitive controllers is clarified. In Section 4., the control characteristics are written using simple repetitive controllers. In Section 5., we present a design procedure for stabilizing simple repetitive controllers. In Section 6., we show a numerical example to illustrate the effectiveness of the proposed method.

Notation

R	The set of real numbers.
$R(s)$	The set of real rational functions with s.
RH_∞	The set of stable proper real rational functions.

2. SIMPLE REPETITIVE CONTROL SYSTEMS AND PROBLEM FORMULATION

Consider the unity feedback control system given by

$$\begin{cases} y &= G(s)u + d \\ u &= C(s)(r-y) \end{cases}, \tag{1}$$

where $G(s) \in R(s)$ is the plant, $C(s)$ is the controller, $u \in R$ is the control input, $y \in R$ is the output, $d \in R$ is the disturbance, and $r \in R$ is the periodic reference input with period $T > 0$ satisfying

$$r(t+T) = r(t) \ \ (\forall t \geq 0). \tag{2}$$

According to [3-10], when the plant $G(s)$ has a periodic disturbance d with period T and an uncertainty, in order for the output y to follow the periodic reference input r with period T with a small steady-state error, the controller $C(s)$ must be written by

$$C(s) = C_1(s) + C_2(s)C_r(s), \tag{3}$$

where $C_1(s) \in R(s)$, $C_2(s) \neq 0 \in R(s)$. $C_r(s)$ is an internal model for the periodic reference input r with period T and is written by

$$C_r(s) = \frac{e^{-sT}}{1-q(s)e^{-sT}}, \tag{4}$$

where $q(s)$ is a proper low-pass filter satisfying $q(0) = 1$. According to [3-10], if the low-pass filter $q(s)$ satisfies

$$1 - q(j\omega_i) \simeq 0 \quad (i = 0, \ldots, N_{max}), \tag{5}$$

where ω_i is the frequency component of the periodic reference input r written by

$$\omega_i = \frac{2\pi}{T} i \quad (i = 0, \ldots, N_{max}) \tag{6}$$

and N_{max} is the maximum frequency component of the periodic reference input r, then the output y in (1) follows the periodic reference input r with a small steady-state error. The controller written by (3) is called by the modified repetitive controller [3-10].

Using the modified repetitive controller $C(s)$ in (3), the transfer functions from the periodic reference input r and from the disturbance d to the output y in (1) are written as

$$\begin{aligned} \frac{y}{r} &= \frac{C(s)G(s)}{1+C(s)G(s)} \\ &= \frac{\left\{C_1(s) - (C_1(s)q(s) - C_2(s))e^{-sT}\right\} G(s)}{\left[1 + C_1(s)G(s) - \{(1+C_1(s)G(s))q(s) - C_2(s)G(s)\} e^{-sT}\right]} \end{aligned} \tag{7}$$

and

$$\begin{aligned} \frac{y}{d} &= \frac{1}{1+C(s)G(s)} \\ &= \frac{1-q(s)e^{-sT}}{\left[1 + C_1(s)G(s) - \{(1+C_1(s)G(s))q(s) - C_2(s)G(s)\} e^{-sT}\right]}, \end{aligned} \tag{8}$$

respectively. Generally, the transfer functions from the periodic reference input r to the output y in (7) and from the disturbance d to the output y in (8) have an infinite number of poles. When the transfer functions from the periodic reference input r and from the disturbance d to the output y have an infinite number of poles, it is difficult to specify the characteristics of the input–output and the disturbance attenuation. From the practical point of view, it is desirable that the input–output and the disturbance attenuation characteristics are easily specified. In order to easily specify the input–output and the disturbance attenuation characteristics it is desirable that the transfer functions from both the periodic reference input r and the disturbance d to the output y have a finite number of poles.

From the above practical requirement, we propose the concept of a simple repetitive controller, as follows:

Definition 1: (simple repetitive controller)
A controller is a "simple repetitive controller", if the following expressions hold true:

1. The controller $C(s)$ works as a modified repetitive controller. That is, the controller $C(s)$ is written by (3), where $C_1(s) \in R(s)$, $C_2(s) \neq 0 \in R(s)$, $C_r(s)$ is written by (4) and $q(s) \in R(s)$ satisfies $q(0) = 1$.
2. The controller $C(s)$ causes the transfer function from the periodic reference input r to the output y in (1) and that from the disturbance d to the output y in (1) to have a finite number of poles.

The problem considered in this paper is to propose the parametrization of all stabilizing simple repetitive controllers. That is, we find all controllers written in the form of (3) to make the transfer functions from the periodic reference input r to the output y in (1) and from the disturbance d to the output y in (1), to be written as

$$y = \left(\bar{G}_{r1}(s) + \bar{G}_{r2}(s)e^{-sT}\right) r \tag{9}$$

and

$$y = \left(\bar{G}_{d1}(s) + \bar{G}_{d2}(s)e^{-sT}\right) d, \tag{10}$$

respectively, where $\bar{G}_{ri}(s) \in RH_\infty (i = 1, 2)$ and $\bar{G}_{di}(s) \in RH_\infty (i = 1, 2)$.

3. THE PARAMETRIZATION

In this section, we propose the parametrization of all stabilizing simple repetitive controllers, which is summarized in the following theorem.

Theorem 1: A controller $C(s)$ is a stabilizing simple repetitive controller if and only if $C(s)$ is written by

$$C(s) = \frac{X(s) + D(s)(Q(s) + \bar{Q}(s)e^{-sT})}{Y(s) - N(s)(Q(s) + \bar{Q}(s)e^{-sT})}, \tag{11}$$

where $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \frac{N(s)}{D(s)}, \tag{12}$$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are functions satisfying

$$X(s)N(s) + Y(s)D(s) = 1, \tag{13}$$

$Q(s) \in RH_\infty$ and $\bar{Q}(s) \neq 0 \in RH_\infty$ are any functions satisfying

$$\frac{N(0)\bar{Q}(0)}{Y(0) - N(0)Q(0)} = 1. \tag{14}$$

The proof of this theorem requires the following lemma.

Lemma 1: A unity feedback control system in (1) is internally stable if and only if $C(s)$ is written by

$$C(s) = \frac{X(s) + D(s)Q(s)}{Y(s) - N(s)Q(s)}, \tag{15}$$

where $N(s) \in RH_\infty$, $D(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying (12), $X(s)$ and $Y(s)$ are RH_∞ functions satisfying (13) and $Q(s) \in RH_\infty$ is any function [22].

Using Lemma 1, we shall show the proof of Theorem 1.

Proof: First, the necessity clause is shown. That is, we show that if the controller $C(s)$ in (3) causes the control system in (1) to be stable and the transfer function from r to y of the control system in (1) to have a finite number of poles, then $C(s)$ takes the form (11). From the assumption that the controller $C(s)$ in (3) causes the transfer function from r to y of the control system in (1) to have a finite number of poles,

$$\begin{aligned}\frac{C(s)G(s)}{1 + C(s)G(s)} &= \frac{\left\{C_1(s) - (C_1(s)q(s) - C_2(s))e^{-sT}\right\} G(s)}{\left[1 + C_1(s)G(s) - \{(1 + C_1(s)G(s))q(s) - C_2(s)G(s)\right\} e^{-sT}\right]}\end{aligned} \tag{16}$$

has a finite number of poles. This implies that

$$C_2(s) = \frac{(1 + C_1(s)G(s))q(s)}{G(s)} \tag{17}$$

is satisfied, that is, $C(s)$ is necessarily

$$C(s) = \frac{C_1(s)G(s) + q(s)e^{-sT}}{G(s)(1 - q(s)e^{-sT})}. \tag{18}$$

From the assumption that $C(s)$ in (3) causes the control system in (1) to be stable, $C(s)G(s)/(1 + C(s)G(s))$, $C(s)/(1+C(s)G(s))$, $G(s)/(1+C(s)G(s))$, and $1/(1+C(s)G(s))$ are stable. From simple manipulation and (18), we have

$$\frac{C(s)G(s)}{1 + C(s)G(s)} = \frac{C_1(s)G(s) + q(s)e^{-sT}}{1 + C_1(s)G(s)}, \tag{19}$$

$$\frac{C(s)}{1 + C(s)G(s)} = \frac{C_1(s)G(s) + q(s)e^{-sT}}{(1 + C_1(s)G(s))G(s)}, \tag{20}$$

$$\frac{G(s)}{1 + C(s)G(s)} = \frac{(1 - q(s)e^{-sT})G(s)}{1 + C_1(s)G(s)} \tag{21}$$

and

$$\frac{1}{1 + C(s)G(s)} = \frac{1 - q(s)e^{-sT}}{1 + C_1(s)G(s)}. \tag{22}$$

From the assumption that all the transfer functions in (19), (20), (21), and (22) are stable, $C_1(s)G(s)/(1+C_1(s)G(s))$, $C_1(s)/(1+C_1(s)G(s))$, $G(s)/(1+C_1(s)G(s))$, and $1/(1+C_1(s)G(s))$ are stable. This means that $C_1(s)$ is an internally stabilizing controller for $G(s)$. From Lemma 1, $C_1(s)$ must take the form

$$C_1(s)=\frac{X(s)+D(s)Q(s)}{Y(s)-N(s)Q(s)}, \tag{23}$$

where $Q(s)\in RH_\infty$.

From the assumption that the transfer functions in (19) and (22) are stable, $q(s)/(1+C_1(s)G(s))$ is stable. This implies that the unstable poles of $q(s)$ are included in those of $C_1(s)$. That is, $q(s)$ is written by

$$q(s)=\frac{\hat{Q}(s)}{Y(s)-N(s)Q(s)}, \tag{24}$$

where $\hat{Q}(s)\neq 0\in RH_\infty$.

Since the transfer function in (20) is stable, $q(s)/\{(1+C_1(s)G(s))G(s)\}$ is stable. From (23) and (24), we have

$$\frac{q(s)}{(1+C_1(s)G(s))\,G(s)}=\frac{D^2(s)\hat{Q}(s)}{N(s)}. \tag{25}$$

From the assumption that $N(s)$ and $D(s)$ are coprime and the transfer function in (25) is stable, $\hat{Q}(s)$ is written in the form

$$\hat{Q}(s)=N(s)\bar{Q}(s), \tag{26}$$

where $\bar{Q}(s)\neq 0\in RH_\infty$. Substituting (17), (23), (24) and (26) for (3), we have (11). In this way, it is shown that if the controller $C(s)$ in (3) causes the control system in (1) to be stable and the transfer function from r to y of the control system in (1) to have a finite number of poles, then $C(s)$ is written as (11). In addition, since $q(0)=1$ holds true, from (24)and (26), (14) is satisfied. Thus, the necessity clause has been shown.

Next, the sufficiency clause is shown. That is, if $C(s)$ takes the form (11), then the controller $C(s)$ causes the control system in (1) to be stable, and the transfer function from r to y of the control system in (1) to have a finite number of poles and works as a stabilizing modified repetitive controller. After simple manipulation, we have

$$\frac{C(s)G(s)}{1+C(s)G(s)} = \left(X(s)+D(s)Q(s)+D(s)\bar{Q}(s)e^{-sT}\right)N(s), \tag{27}$$

$$\frac{C(s)}{1+C(s)G(s)} = \left(X(s)+D(s)Q(s)+D(s)\bar{Q}(s)e^{-sT}\right)D(s), \tag{28}$$

$$\frac{G(s)}{1+C(s)G(s)} = \left(Y(s)-N(s)Q(s)-N(s)\bar{Q}(s)e^{-sT}\right)N(s) \tag{29}$$

and

$$\frac{1}{1+C(s)G(s)} = \left(Y(s)-N(s)Q(s)-N(s)\bar{Q}(s)e^{-sT}\right)D(s). \tag{30}$$

Since $X(s)\in RH_\infty$, $Y(s)\in RH_\infty$, $N(s)\in RH_\infty$, $D(s)\in RH_\infty$, $Q(s)\in RH_\infty$ and $\bar{Q}(s)\in RH_\infty$, the transfer functions in (27), (28), (29) and (30) are stable.

Next, we show that the transfer function from r to y has a finite number of poles. This transfer function is written by (27). The poles of (27) are those of $X(s)$, $N(s)$, $D(s)$, $Q(s)$, and $\bar{Q}(s)$, since the poles of $D(s)\bar{Q}(s)e^{-sT}$ are equal to those of $D(s)\bar{Q}(s)$. $X(s)$, $N(s)$, $D(s)$, $Q(s)$, and $\bar{Q}(s)$ have a finite number of poles because $X(s)\in RH_\infty$, $N(s)\in RH_\infty$, $D(s)\in RH_\infty$, $Q(s)\in RH_\infty$, and $\bar{Q}(s)\in RH_\infty$. Therefore, the transfer function from r to y of the control system in (1) has a finite number of poles.

Next, we show that the controller in (11) works as a modified repetitive controller. The controller in (11) is rewritten in the form of (3), where

$$C_1(s)=\frac{X(s)+D(s)Q(s)}{Y(s)-N(s)Q(s)}, \tag{31}$$

$$C_2(s)=\frac{\bar{Q}(s)}{\left(Y(s)-N(s)Q(s)\right)^2} \tag{32}$$

and

$$q(s)=\frac{N(s)\bar{Q}(s)}{Y(s)-N(s)Q(s)}. \tag{33}$$

From the assumption of $\bar{Q}(s)\neq 0$ and (32), $C_2(s)\neq 0$ holds true. In addition, from (33) and the assumption in (14), $q(0)=1$ is satisfied. These expressions imply that the controller in (11) works as a modified repetitive controller. Thus, the sufficiency clause has been shown.

We have thus proved Theorem 1.

Note 1: From the proof of Theorem 1, we note that in order to design simple repetitive control systems, the relative degree of $q(s)$ should be greater than or equal to that of $G(s)$.

4. CONTROL CHARACTERISTICS

In this section, we describe the control characteristics of the control system in (1) using the stabilizing simple repetitive controller in (11).

First, we mention the input–output characteristic. The transfer function from the periodic reference input r to the error $e = r - y$ is written by

$$\begin{aligned}\frac{e}{r} &= \left(1 - \frac{N(s)\bar{Q}(s)}{Y(s) - N(s)Q(s)}e^{-sT}\right) \\ &\quad \cdot (Y(s) - N(s)Q(s))\, D(s). \qquad (34)\end{aligned}$$

From (34), for ω_i in (6), which is the frequency component of the periodic reference input r, if

$$\begin{aligned}&1 - \frac{N(j\omega_i)\bar{Q}(j\omega_i)}{Y(j\omega_i) - N(j\omega_i)Q(j\omega_i)}e^{-j\omega_i T} \\ &= 1 - \frac{N(j\omega_i)\bar{Q}(j\omega_i)}{Y(j\omega_i) - N(j\omega_i)Q(j\omega_i)} \\ &\simeq 0, \qquad (35)\end{aligned}$$

then the output y follows the periodic reference input r with a small steady-state error.

Note 2: From the assumption that the plant $G(s)$ is strictly proper, that is $N(s)$ is strictly proper,

$$\lim_{\omega\to\infty} \frac{N(j\omega)\bar{Q}(j\omega)}{Y(j\omega) - N(j\omega)Q(j\omega)} = 0 \qquad (36)$$

or equivalently,

$$\begin{aligned}&\lim_{i\to\infty} \frac{e(j\omega_i)}{r(j\omega_i)} \\ &= \lim_{i\to\infty} \left\{\left(1 - \frac{N(j\omega_i)\bar{Q}(j\omega_i)}{Y(j\omega_i) - N(j\omega_i)Q(j\omega_i)}e^{-j\omega_i T}\right)\right. \\ &\quad \left. \cdot (Y(j\omega_i) - N(j\omega_i)Q(j\omega_i))\, D(j\omega_i)\right\} \\ &= \lim_{i\to\infty} Y(j\omega_i)D(j\omega_i) \\ &= 1 \neq 0 \qquad (37)\end{aligned}$$

holds true. This implies that the steady-state error increases with increasing frequency of the reference input r. The gain plot of $e/r = (1 - N(s)\bar{Q}(s)/(Y(s) - N(s)Q(s))e^{-sT})(Y(s) - N(s)Q(s))D(s)$ brings a steady-state error to the periodic reference input $r(s)$.

Next, we mention the disturbance attenuation characteristic. The transfer function from the disturbance d to the output y is written by

$$\begin{aligned}\frac{y}{d} &= \left(1 - \frac{N(s)\bar{Q}(s)}{Y(s) - N(s)Q(s)}e^{-sT}\right) \\ &\quad \cdot (Y(s) - N(s)Q(s))\, D(s). \qquad (38)\end{aligned}$$

From (38), for the frequency component ω_i in (6) of the disturbance d that is same as that of the periodic reference input r, if

$$\begin{aligned}&1 - \frac{N(j\omega_i)\bar{Q}(j\omega_i)}{Y(j\omega_i) - N(j\omega_i)Q(j\omega_i)}e^{-j\omega_i T} \\ &= 1 - \frac{N(j\omega_i)\bar{Q}(j\omega_i)}{Y(j\omega_i) - N(j\omega_i)Q(j\omega_i)} \\ &\simeq 0, \qquad (39)\end{aligned}$$

then the disturbance d is attenuated effectively. When the frequency component ω of the disturbance d is different from that of the periodic reference input r, that is $\omega \neq \omega_i$, the disturbance d cannot be attenuated even if

$$1 - \frac{N(j\omega)\bar{Q}(j\omega)}{Y(j\omega) - N(j\omega)Q(j\omega)} \simeq 0, \qquad (40)$$

because

$$e^{-j\omega T} \neq 1 \qquad (41)$$

and

$$1 - \frac{N(j\omega)\bar{Q}(j\omega)}{Y(j\omega) - N(j\omega)Q(j\omega)}e^{-j\omega T} \neq 0. \qquad (42)$$

In order to attenuate the frequency component ω of the disturbance d that is different from that of the periodic reference input r, we need to settle $Q(s)$, satisfying

$$(Y(j\omega) - N(j\omega)Q(j\omega))\, D(j\omega) \simeq 0. \qquad (43)$$

Note 3: From the same reason as in Note 2,

$$\begin{aligned}&\lim_{\omega\to\infty} \frac{y(j\omega)}{d(j\omega)} \\ &= \lim_{\omega\to\infty} \left\{\left(1 - \frac{N(j\omega)\bar{Q}(j\omega)}{Y(j\omega) - N(j\omega)Q(j\omega)}e^{-j\omega T}\right)\right. \\ &\quad \left. \cdot (Y(j\omega) - N(j\omega)Q(j\omega))\, D(j\omega)\right\} \\ &= \lim_{\omega\to\infty} Y(j\omega)D(j\omega) \\ &= 1 \neq 0 \qquad (44)\end{aligned}$$

holds true. This implies that the steady-state error increases with increasing frequency of the disturbance $d(s)$. The gain plot of $y/d = (1 - N(s)\bar{Q}(s)/(Y(s) - N(s)Q(s))e^{-sT})(Y(s) - N(s)Q(s))D(s)$ brings a steady-state error to the disturbance $d(s)$.

From the above discussion, we find that the role of $\bar{Q}(s)$ is to specify the input–output characteristic for the periodic reference input r and that of $Q(s)$ is to specify the disturbance attenuation characteristic for the frequency component of the disturbance d that is different from that of the periodic reference input r.

5. DESIGN PROCEDURE

In this section, a design procedure of stabilizing a simple repetitive controller satisfying Theorem 1 is presented.

The procedure is summarized as follows.

Procedure

Step 1) Obtain the coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ of $G(s)$ satisfying (12).

Step 2) $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ is settled satisfying (13).

Step 3) $Q(s) \in RH_\infty$ is settled so that for the frequency component of the disturbance d, ω_d, $|(Y(j\omega_d) - N(j\omega_d)Q(j\omega_d))D(j\omega_d)|$ is effectively small. In order to design $Q(s)$ to make $|(Y(j\omega_d) - N(j\omega_d)Q(j\omega_d))D(j\omega_d)|$ effectively small, $Q(s)$ is settled by

$$Q(s) = \frac{Y(s)}{N_o(s)}\bar{q}_d(s), \quad (45)$$

where $N_o(s) \in RH_\infty$ is the outer function of $N(s)$ satisfying

$$N(s) = N_i(s)N_o(s), \quad (46)$$

$N_i(s) \in RH_\infty$ is the inner function satisfying $N_i(0) = 1$ and $|N_i(j\omega)| = 1(\forall\omega \in R)$, $\bar{q}_d(s)$ is a low-pass filter satisfying $\bar{q}_d(0) = 1$, as

$$\bar{q}_d(s) = \frac{1}{(1 + s\tau_d)^{\alpha_d}} \quad (47)$$

is valid, α_d is an arbitrary positive integer to make$\bar{q}_d(s)/N_o(s)$ proper and $\tau_d \in R$ is any positive real number satisfying

$$1 - N_i(j\omega_d)\frac{1}{(1 + j\omega_d\tau_d)^{\alpha_d}} \simeq 0. \quad (48)$$

Step 4) $\bar{Q}(s) \in RH_\infty$ is settled so that for the frequency component $\omega_i(i = 0, \ldots, N_{max})$of the periodic reference input r, $1 - N(j\omega_i)\bar{Q}(j\omega_i)/(Y(j\omega_i) - N(j\omega_i)Q(j\omega_i)) \simeq 0$ is satisfied. In order to design $\bar{Q}(s)$ to hold $1 - N(j\omega_i)\bar{Q}(j\omega_i)/(Y(j\omega_i) - N(j\omega_i)Q(j\omega_i)) \simeq 0$,$\bar{Q}(s) \in RH_\infty$ is settled by

$$\bar{Q}(s) = \frac{Y(s) - N(s)Q(s)}{N_o(s)}\bar{q}_r(s), \quad (49)$$

where $\bar{q}_r(s)$ is a low-pass filter satisfying $\bar{q}_r(0) = 1$, as

$$\bar{q}_r(s) = \frac{1}{(1 + s\tau_r)^{\alpha_r}} \quad (50)$$

is valid, α_r is an arbitrary positive integer to make$\bar{q}_r(s)/N_o(s)$ proper and $\tau_r \in R$ is any positive rea lnumber satisfying

$$\begin{aligned} 1 - N_i(j\omega_i)&\frac{1}{(1 + j\omega_i\tau_r)^{\alpha_r}} \\ \simeq\ & 0(i = 0, \ldots, N_{max}). \end{aligned} \quad (51)$$

Using the above-mentioned procedure, $q(s)$ in (4) is written by

$$\begin{aligned} q(s) &= N_i(s)\bar{q}_r(s) \\ &= N_i(s)\frac{1}{(1 + s\tau_r)^{\alpha_r}}. \end{aligned} \quad (52)$$

Therefore, for ω_i in (6), which is the frequency component of the periodic reference input r, if τ in (52) is settled satisfying

$$\begin{aligned} 1 - q(j\omega_i) &= 1 - N_i(j\omega_i)\frac{1}{(1 + j\omega_i\tau_r)^{\alpha_r}} \\ &\simeq 0(i = 0, \ldots, N_{max}), \end{aligned} \quad (53)$$

then the output y follows the periodic reference input r with a small steady-state error.

6. NUMERICAL EXAMPLE

In this section, a numerical example is given to illustrate the effectiveness of the proposed method.

Consider the problem of obtaining the parametrization of all stabilizing simple repetitive controllers for the plant $G(s)$ written by

$$G(s) = \frac{s - 100}{(s + 1)(s - 1)} \quad (54)$$

that follows the periodic reference input r with period $T = 2$[sec].

A pair of coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ of $G(s)$ in (54) satisfying (12) is given by

$$N(s) = \frac{s - 100}{(s + 3)(s + 2)} \quad (55)$$

and

$$D(s) = \frac{(s + 1)(s - 1)}{(s + 3)(s + 2)}. \quad (56)$$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ satisfying (13) are derived as

$$X(s) = -\frac{0.7075(s + 1.056)}{(s + 3)(s + 2)} \quad (57)$$

and

$$Y(s) = \frac{s^2 + 10s + 38.71}{(s + 3)(s + 2)}. \quad (58)$$

From Theorem 1, the parametrization of all stabilizing simple repetitive controllers for $G(s)$ in (54) is given by (11), where $Q(s) \in RH_\infty$ and $\bar{Q}(s) \in RH_\infty$ are any functions satisfying (14).

In order for the disturbances $d = \sin(\pi t)$ and $d = \sin(\pi t/2)$ to be attenuated effectively and for the output y to follow the periodic reference input $r = \sin(\pi t)$ with a small steady-state error, $Q(s)$ and $\bar{Q}(s)$ are settled by (45) and (49), respectively, where

$$\bar{q}_d(s) = \frac{1}{0.0001s + 1}, \quad (59)$$

$$\bar{q}_r(s) = \frac{1}{0.0001s + 1}, \quad (60)$$

$$N_i(s) = -\frac{s - 100}{s + 100} \quad (61)$$

and

$$N_o(s) = -\frac{s + 100}{(s + 3)(s + 2)}. \quad (62)$$

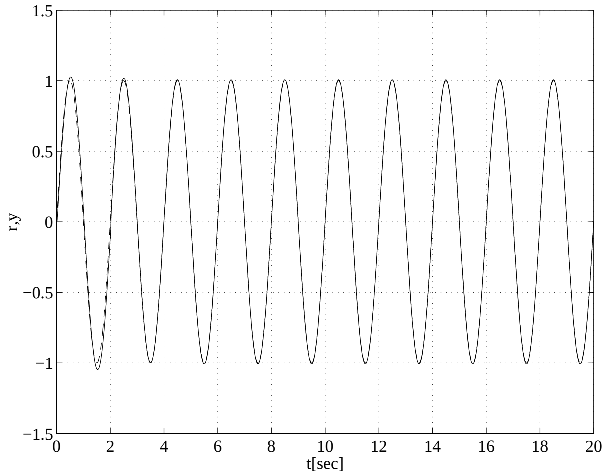

Fig.1: *The response of the output y for the periodic reference input $r = \sin(\pi t)$*

Using the above-mentioned parameters, we have the simple repetitive controller. When the designed simple repetitive controller $C(s)$ is used, the response of the output y in (1) for the periodic reference input $r = \sin(\pi t)$ is shown in Fig. 1. Here, the dotted line shows the response of the periodic reference input r and the solid line shows that of the output y. Figure 1 shows that the output y follows the periodic reference input r with a small steady-state error.

Next, the disturbance attenuation characteristic is shown using the designed simple repetitive controller $C(s)$. The response of the output y for the disturbance $d = \sin(\pi t)$, for which the frequency component is equivalent to that of the periodic reference input r is shown in Fig. 2. Here, the dotted line shows the response of the disturbance d and the solid line shows the output y. Figure 2 shows that the disturbance d is attenuated effectively. Finally, the response of the output y for the disturbance $d = \sin(\pi t/2)$ for which the frequency component is different from that of the periodic reference input r is shown in Fig. 3. Here, the dotted line shows the response of the disturbance d and the solid line shows that of the output y. Figure 3 shows that the disturbance d is attenuated effectively. A stabilizing simple repetitive controller can be easily designed in the way shown here.

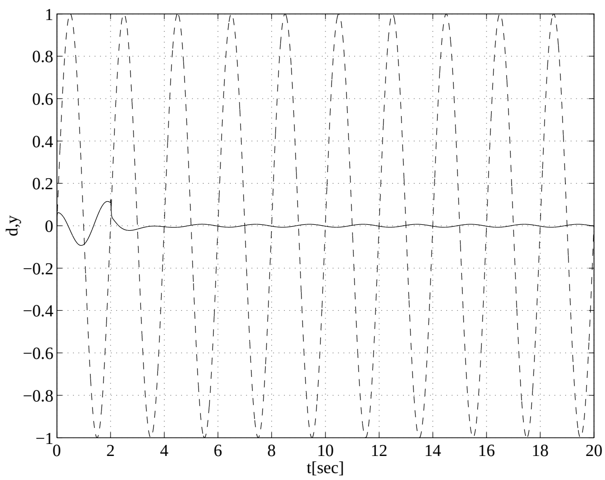

Fig.2: *The response of the output y for the disturbance $d = \sin(\pi t)$*

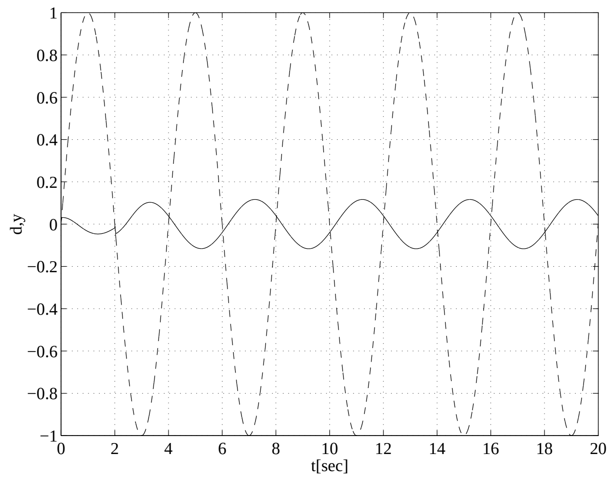

Fig.3: *The response of the output y for the disturbance $d = \sin\left(\frac{\pi}{2}t\right)$*

7. CONCLUSIONS

In this paper, we proposed a simple repetitive control system and the parametrization of all stabilizing simple repetitive controllers. The control characteristics of a simple repetitive control system are presented, as well as a design procedure for a simple repetitive controller. Finally, a numerical example illustrated the effectiveness of the proposed method. Practical applications of the simple repetitive control are expected, since this system has merits, namely the fact that the transfer function from the reference input to the output has a finite number of poles and that it can be easily designed.

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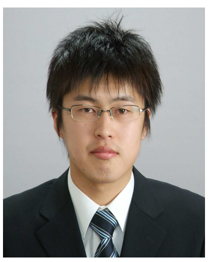

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