Discrete-Time Feedback Error Learning and Nonlinear Adaptive Controller

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ABSTRACT

In this study, the technique of Discrete-Time Feedback Error Learning (DTFEL) is investigated from the viewpoint of adaptive control. First, the relationship between DTFEL and nonlinear discrete-time adaptive control with adaptive feedback linearization is discussed. It shows that DTFEL can be interpreted as a form of nonlinear adaptive control. Second, Lyapunov analysis of the controlled system is presented. It suggests that the condition of strictly positive realness (s.p.r.) associated with the tracking error dynamics is a sufficient condition for asymptotic stability of the closed-loop dynamics. Specifically, for a class of second order SISO systems, it is shown that the control problems is reduced to select the feedback gain that satisfies the s.p.r. Finally, numerical simulations is presented to illustrate the stability properties of DTFEL obtained from mathematical analysis.

Keywords: Discrete-time System, Nonlinear System, Feedback Error Learning, Strictly Positive Realness, Learning Control, Adaptive Control, Feedback and Feedforward Control.

1. INTRODUCTION

This study presents a reformulation and formal stability analysis of the discrete-time feedback error learning (DTFEL) scheme which is developed from the continuous-time feedback error learning (FEL) scheme [1],[4]. The objective is to study and design the controller for a class of nonlinear systems from a viewpoint of the adaptive control theory. Originally, FEL was proposed from a biological perspective to establish a computational model of the cerebellum for learning motor control with internal models in the central nervous system (CNS) [3]. In here, it is inspired by the insight of the close relationship between FEL and adaptive control algorithms which is gained during our recent development of a new adaptive control framework with advanced statistical learning. From a control theoretic viewpoint, FEL can be conceived of as an adaptive control technique [5]. Stability analysis of FEL for a class of linear systems and a two-link planar robot arm in a horizontal plane are presented by Miyamura and Kimura [5] and Ushida and Kimura [8], respectively. However, the plant dynamics considered in [5] are confined to a restricted class of linear systems (stable and stably invertible), and these studies do not address practical issues, e.g. as to how to select feedback gains to ensure the stability in FEL. Nakanishi [6] proposed a more general treatment of the formulation and stability properties of FEL for a class of continuous-time nonlinear systems. However, the controller is now computer-based which is usually not suitable to apply the theoretical knowledge of the continuous FEL directly. Although, there are many researches studied about DTFEL [9],[10], the plant dynamics considered there are still confined to a restricted class of linear systems. This research studies and analyzes the discrete-time version of the original FEL controller specialized for nonlinear systems.

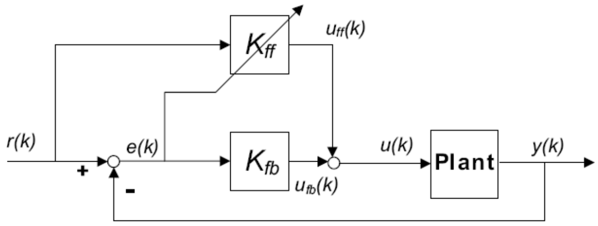

Fig.1: *Original DTFEL Scheme*

Fig. 1 depicts the block diagram of the original DTFEL scheme which was originally proposed for inverse model learning with an adaptive feedforward component. This study focuses on an adaptive state feedback controller. In this formulation, the actual state is used to compute basis functions of a function approximator for parameter updated and cancelation of nonlinearities. The major improvement of this scheme is due to adding the new input (an additional auxiliary signal u_{ad}) to the scheme. With this input, the stability of the DTFEL system can be guaranteed.

This study is organized as follows: First, all mathematical preliminaries, required to analyze the stability of DTFEL system, are summarized. Subsequently, the structure of the control system and function approximation of unknown nonlinearities in the plant dynamics as considered in this study are presented. Then, the adaptive feedback formulation of DTFEL

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is discussed. Next, a Lyapunov stability analysis of DTFEL is also provided. Also, a sufficient condition on the choice of feedback gains to guarantee the stability of learning based on the Lyapunov stability analysis, which is associated with strictly positive realness (s.p.r.) of the tracking error dynamics, is provided. Finally, the numerical simulated examples to illustrate the theoretical stability properties of DTFEL is presented.

Notation:

Throughout this study, a fairly standard notation is used. The overview is as follow.

$\Re$	Set of real numbers.
$\Re^n$	Real n-space.
$\gamma_{\min}[P]$	The smallest eigenvalue of P.
$\|A\|$	$= \sqrt{\mathrm{tr}(\mathrm{A^TA})} = \sqrt{\sum_{i,j} a_{ij}^2}$ Frobenius norm.
(A, B, C, D)	$= D + C(zI - A)^{-1}B$ Minimal realization.
p.r.	positive real.
s.p.r.	strictly positive real.
PE	Persistently Exciting.

2. MATHEMATICAL PRELIMINARIES

In this section, the mathematical requirement to analyze the DTFEL in the next section is discussed. The main and most important area is to study the strictly positive real system.

Definition 1: [7] A square matrix $H(z)$ of real rational functions is a positive real (p.r.) matrix if
(d1) $H(z)$ has elements analytic in $|z| > 1$.
(d2) $H^T(z^*)+H(z)$ is positive, semidefinite and Hermitian for $|z| > 1$.

Condition (d2) can be replaced by
(d3) The poles of the elements of $H(z)$ on $|z| = 1$ are simple and the associated residue matrixes of $H(z)$ at these poles are 0.
(d4) $H(e^{j\theta})+H^T(e^{-j\theta})$ is a positive semidefinite Hermitian matrix for all real θ for which $H(e^{j\theta})$ exists.

Definition 2: [7] A rational transfer matrix $H(z)$ is a strictly positive real (s.p.r.) matrix if $H(\rho z)$ is p.r. for some $0 < \rho < 1$.

Given *Definition 2*, a necessary and sufficient condition in the frequency domain for s.p.r. transfer matrices in the class $\mathfrak{H}$ can be defined as following.

Definition 3: [7] An $n \times n$ rational matrix $H(z)$ is said to belong to class $\mathfrak{H}$ if $H(z) + H^T(z^{-1})$ has rank n almost everywhere in the complex z-plane.

Theorem 1: [7] Consider the $n\times n$ rational matrix $H(z) \in \mathfrak{H}$ given in *Definition 3*. Then, $H(z)$ is a s.p.r. matrix if and only if
(a) All elements of $H(z)$ are analytic in $|z| \geqslant 1$,
(b) $H(e^{j\theta}) + H^T(e^{-j\theta}) > 0, \forall\theta \in [0, 2\pi]$.

Lemma 1: [7](Discrete-time version of Kalman-Yakubovich-Popov)[7] Assume that the rational transfer matrix $H(z)$ has poles that lie in $|z| < \gamma$, where $0 < \gamma < 1$ and (A, B, C, D) is a minimal realization of $H(z)$. Then, $H(\gamma z)$ is s.p.r., if and only if real matrices $P = P^T > 0$, Q and K exist such that

$$\begin{aligned} A^TPA - P &= -QQ^T - (1-\gamma^2)P, \\ A^TPB &= C^T - QK, \\ K^TK &= D + D^T - B^TPB. \end{aligned}$$

For the formal case, i.e. the transfer matrix is stable or $\gamma = 1$, the following lemma is applied.

Lemma 2: (Discrete-time version of Kalman-Yakubovich-Popov) [7] Assume that the rational transfer matrix $H(z)$ has poles that lie in $|z| < 1$, and (A, B, C, D) is a minimal realization of $H(z)$. Then, $H(z)$ is s.p.r., if and only if real matrices $P = P^T > 0$, $L = L^T > 0$, q, ε and ν exist such that

$$\begin{aligned} A^TPA - P &= -qq^T - \varepsilon L, \\ A^TPB &= \frac{C}{2} + \nu q, \\ B^TPB &= D - \nu^2. \end{aligned}$$

Definition 4: [2] An input sequence $x(k)$ is said to be persistently exciting(PE) if $\gamma > 0$ and an integer $k_1 \geqslant 1$ such that

$$\gamma_{\min}\left[\sum_{k=k_0}^{k_1+L-1} \phi(k)\phi^T(k)\right] > \gamma, \forall k_0 \geqslant 0. \quad (1)$$

For the state-space approach, there are two valuable theorems explaining the property of s.p.r and p.r. as follows.

Theorem 2: [7] Let (A, B, C) be minimal and let $D = C(A + I)^{-1}B$, and B be of full rank. If $H(z) = C(zI - A)^{-1}B + D$ is p.r., then
(i) all the eigenvalues of A are in $|z| < 1$ and the eigenvalues of A, on $|z| = 1$ are simple,
(ii) $C(A + I)^{-2}B = (C(A + I)^{-2}B)^T > 0$,
(iii) $C(A + I)^{-1}(A - I)(A + I)^{-2}B$
$+(C(A + I)^{-1}(A - I)(A + I)^{-2}B)^T \leqslant 0$.

For the case of second and first order systems, the following theorem gives the necessary and sufficient conditions of p.r. system.

Theorem 3: [7] Let $n = 1$ and $m = 1$ or $n = 2$ and $m = 1$, and let (A, B, C) be minimal, $D = C(A + I)^{-1}B$, and B be of full rank. Then, $H(z) = C(zI - A)^{-1}B + D$ is p.r. if and only if conditions (i), (ii) and (iii) of *Theorem 2* hold.

3. PLANT DYNAMICS AND FUNCTION APPROXIMATION

The general structure of the control system of interest is a class of nonlinear MIMO systems of the form

$$\mathbf{x(k+1)} = \mathbf{f(x)} + \mathbf{G(x)u}, \quad (2)$$
$$\mathbf{z} = \mathbf{h(x)}, \quad (3)$$

where $\mathbf{x} \in \Re^{\mathbf{n}}$ is a state, $\mathbf{z} \in \Re^{\mathbf{p}}$ is an output, $\mathbf{u} \in \Re^{\mathbf{m}}$ is an input, $\mathbf{f} : \Re^{\mathbf{n}} \to \Re^{\mathbf{n}}$, $\mathbf{G} : \Re^{\mathbf{n}} \to \Re^{\mathbf{n \times m}}$ are nonlinear functions, and $\mathbf{h} : \Re^{\mathbf{n}} \to \Re^{\mathbf{p}}$ denotes a mapping from the state to the output.

In this study, the initial mathematical development is to consider a simplified nth order SISO system of the form:

$$\begin{aligned} x_1(k+1) &= x_2(k) \\ &\vdots \\ x_{n-1}(k+1) &= x_n(k) \\ x_n(k+1) &= f(\mathbf{x},\mathbf{u}) + \mathbf{u} \end{aligned} \quad (4)$$

where $x = x_1$, $\mathbf{x} = [\mathbf{x_1}, \ldots, \mathbf{x_n}]^{\mathbf{T}} \in \Re^{\mathbf{n}}$, $\mathbf{u} = [\mathbf{u(k-1)}, \mathbf{u(k-2)}, \ldots]^{\mathbf{T}} \in \Re^{\mathbf{q}}$, and $u \in \Re$.

Suppose that $f(\mathbf{x},\mathbf{u})$ can be represented in a linearly parameterized form as

$$f\mathbf{x}, \mathbf{u} = \phi^T \mathbf{x}, \mathbf{u}\theta + \mathbf{\Delta x}, \mathbf{u}, \quad (5)$$

where ϕ is the vector of nonlinear basis functions defined by $\phi^T(\mathbf{x},\mathbf{u}) = [\phi_{\mathbf{1}}^{\mathbf{T}}, \ldots, \phi_{\mathbf{N}}^{\mathbf{T}}]^{\mathbf{T}}$, θ is the parameter vector defined by $\theta = [\theta_{\mathbf{1}}^{\mathbf{T}}, \ldots, \theta_{\mathbf{N}}^{\mathbf{T}}]^{\mathbf{T}}$ and $\Delta(\mathbf{x},\mathbf{u})$ is the approximation error.

If the structure of f is known, i.e. all the correct basis functions are known, $\Delta(\mathbf{x},\mathbf{u})$ will be zero. In this study, a perfect approximation where $\Delta = 0$ is assumed.

4. DTFEL FORMULATION

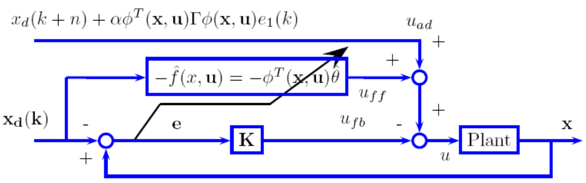

Fig.2: *Block Diagram of a DTFEL for Nonlinear Systems*

This section considers the adaptive formulation of DTFEL as depicted in *Fig. 2*. The relationship between discrete-time nonlinear adaptive control and DTFEL, and the stability properties of DTFEL with respect to the choice of feedback gains are discussed, consequently.

Consider a control law

$$u = u_{ad} + u_{ff} - u_{fb} \quad (6)$$
$$:= \left(x_d(k+n) + \alpha\phi^T(\mathbf{x},\mathbf{u})\mathbf{\Gamma}\phi(\mathbf{x},\mathbf{u})\mathbf{e_1(k)}\right) - \hat{f}(\mathbf{x},\mathbf{u}) - \mathbf{Ke}, \quad (7)$$

where

$$u_{ad} = x_d(k+n) + \alpha\phi^T(\mathbf{x},\mathbf{u})\mathbf{\Gamma}\phi(\mathbf{x},\mathbf{u})\mathbf{e_1(k)}, \quad (8)$$
$$u_{ff} = -\hat{f}(\mathbf{x},\mathbf{u}), \quad (9)$$
$$u_{fb} = \mathbf{Ke}, \quad (10)$$

$\alpha < -\frac{1}{2}$ is a negative real number and Γ is a positive definite adaptation gain matrix. $\mathbf{K} = [\mathbf{K_1}, \mathbf{K_2}, \ldots, \mathbf{K_n}]$ denotes the feedback gain row vector chosen such that the polynomial $z^n = \sum_{i=1}^{n} K_i z^{i-n} = 0$ has all roots in the unit circle of the complex number z-plane, and $\mathbf{e} = [\mathbf{e(k)}, \mathbf{e(k-1)}, \ldots, \mathbf{e(k-n)}]$ is the tracking error vector with $e = x - x_d$ and $x_d(k)$ denotes a desired trajectory. $\hat{f}$ is the estimate of f defined by

$$\hat{f}(\mathbf{x}) = \phi^T(\mathbf{x},\mathbf{u})\hat{\theta}, \quad (11)$$

where $\hat{\theta}$ is an estimate of θ. The tracking error dynamics with the estimate of f can be expressed in the controllable canonical form of the state space representation as:

$$\mathbf{e(k+1)} = \mathbf{Ae(k)} + \mathbf{bv(k)},$$
$$e_1(k) = \mathbf{ce(k)} + \mathbf{dv(k)}, \quad (12)$$

where

$$v(k) = -\phi^{\mathbf{T}}\tilde{\theta}(\mathbf{k}) + \alpha\phi^{\mathbf{T}}(\mathbf{x},\mathbf{u})\mathbf{\Gamma}\phi(\mathbf{x},\mathbf{u})\mathbf{e_1(k)}; \quad \alpha < -\frac{1}{2}, \Gamma = \Gamma^T > 0,$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 1 \\ -K_1 & -K_2 & \cdots & -K_n \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad (13)$$
$$\mathbf{c} = [\mathbf{K_p}, \mathbf{K_d}], \quad \mathbf{d} = \mathbf{c(A+I)^{-1}b},$$

and $\tilde{\theta}$ is the parameter error vector defined as $\tilde{\theta} = \hat{\theta} - \theta$. Note that $\mathbf{A}$ is Hurwitz. Define the sliding surface

$$u_{fb} = \mathbf{Ke}, \quad (14)$$

where $\mathbf{K} = [\mathbf{K_1}, \ldots, \mathbf{K_n}]$ and $K_i > 0$ is chosen such that $(\mathbf{A}, \mathbf{b}, \mathbf{c})$ is minimal (controllable and observable) and $H(z) = \mathbf{c(zI-A)^{-1}b}$ is strictly positive real (s.p.r.). This filtered tracking error will be used in the tracking error-based parameter update and the strictly positive real assumption will be necessary in the Lyapunov stability analysis. If the tracking error-based parameter adaptation law is selected as

$$\Delta\hat{\theta}(k) \triangleq \hat{\theta}(k+1) - \hat{\theta}(k) = \Gamma\phi(\mathbf{x},\mathbf{u})\mathbf{Ke}, \quad (15)$$

where Γ is a positive definite adaptation gain matrix, it is possible to prove the desirable stability properties of the adaptive DTFEL controller as discussed in the later section.

4.1 Lyapunov Stability Analysis

Lyapunov stability analysis is a tool for determining the stability of the DTFEL system. Consider the Lyapunov function

$$V(\mathbf{e},\tilde{\theta}) = \mathbf{2e^T Pe} + \tilde{\theta}^{\mathbf{T}}\mathbf{\Gamma^{-1}}\tilde{\theta}, \quad (16)$$

where P is a positive definite matrix. By applying the positive real property in *Lemma 2* and inserting the error dynamics (12), the adaptive law (15), the function $\Delta V(\mathbf{e(k)},\tilde{\theta}(\mathbf{k})) \triangleq \mathbf{V}(\mathbf{e(k+1)},\tilde{\theta}(\mathbf{k+1})) - \mathbf{V}(\mathbf{e(k)},\tilde{\theta}(\mathbf{k}))$ can be calculated as

$$\begin{aligned} &\Delta V(\mathbf{e(k)},\tilde{\theta}(\mathbf{k})) \\ &= -2\mathbf{e^T}\varepsilon\mathbf{Le} - \mathbf{2}|\mathbf{q^T e} - \nu\mathbf{v}|^{\mathbf{2}} + (\mathbf{2}\alpha+\mathbf{1})\phi^{\mathbf{T}}\mathbf{\Gamma}\phi\mathbf{e_1^2} \\ &< 0 \text{ if } \alpha < -\frac{1}{2}. \end{aligned} \quad (17)$$

(The full computation of this proof is shown in Appendix.)

This Lyapunov analysis implies that the tracking error, $\mathbf{e}$, converges to zero. For asymptotic parameter error convergence to zero, ϕ needs to satisfy the so-called persistent excitation (PE) condition described in *Definition 4*.

4.2 Choice of feedback gains in DTFEL

Given that DTFEL is equivalent to the tracking error-based adaptive controller for the plant dynamics (4), in order to ensure the stability of DTFEL, the feedback gains K_i in DTFEL must be chosen so that the s.p.r. condition holds for the pair $(\mathbf{A},\mathbf{b},\mathbf{c},\mathbf{d})$:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 1 \\ -K_1 & -K_2 & \cdots & -K_n \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (18)$$
$$\mathbf{c} = [\mathbf{K_p},\mathbf{K_d}], \quad \mathbf{d} = \mathbf{c(A+I)^{-1}b}.$$

For the general nth order systems, the s.p.r. condition is somewhat abstract and the criterion as to how to select the feedback gains K_i to satisfy the s.p.r. condition for $(\mathbf{A},\mathbf{b},\mathbf{c},\mathbf{d})$ is not obvious. However, for the case of second order SISO systems with

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -K_p & -K_d \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad (19)$$
$$\mathbf{c} = [\mathbf{K_p},\mathbf{K_d}], \quad \mathbf{d} = \mathbf{c(A+I)^{-1}b},$$

it is possible to derive a condition for the choice of K_i to guarantee the stability of DTFEL in a very simple form using *Theorem 3*.

The conditions in this theorem are:

(i) All the eigenvalues of A are in $|z| < 1$ and the eigenvalues of A, on $|z| = 1$ are simple,

(ii) $C(A+I)^{-2}B = (C(A+I)^{-2}B)^T > 0$,

(iii) $C(A+I)^{-1}(A-I)(A+I)^{-2}B$
$+(C(A+I)^{-1}(A-I)(A+I)^{-2}B)^T \leqslant 0$.

These three conditions give the constraint for selecting K_p and K_d for the second order SISO systems to guarantee the stability of DTFEL. Interestingly, these conditions do not depend on the nonlinearity $f(\mathbf{x})$. Note that the state-space matrix $(\mathbf{A},\mathbf{b},\mathbf{c},\mathbf{d})$ in (19) satisfies the assumption required in *Theorem 3*. In this assumption, the output matrix d must satisfy $d = \mathbf{c(A+I)^{-1}b}$, unless the definiteness of the system cannot be guaranteed. Therefore, K_p and K_d must be chosen so that

$$d = \mathbf{c(A+I)^{-1}b}. \quad (20)$$

To satisfy condition (i), using Jury's stability test,

$$K_p + K_d > -1 \quad (21)$$
$$K_p - K_d > -1. \quad (22)$$

To satisfy condition (ii), K_p and K_d must follow

$$\begin{aligned} C(A+I)^{-2}B &> 0 \\ -(2K_p - K_d)/(-K_d + 1 + K_p)^2 &> 0. \end{aligned} \quad (23)$$

To satisfy condition (iii), K_p and K_d must be restricted to

$$\begin{aligned} &C(A+I)^{-1}(A-I)(A+I)^{-2}B \\ &+(C(A+I)^{-1}(A-I)(A+I)^{-2}B)^T \leqslant 0, \end{aligned}$$

or

$$\frac{-2(-3K_dK_p - 4K_p + 4K_p^2 + K_d^2 + K_d)}{(-K_d + 1 + K_p)^3} \leqslant 0. \quad (24)$$

Equations (20),(21),(22),(23), and (24) are the constraints for selecting the values of K_p and K_d of a DTFEL for Nonlinear System (DTFELN).

5. SIMULATION RESULTS

In this section, the simulation results are illustrated to demonstrate the effectiveness of the theoretical results obtained in this study. Three main simulations have been done in order to illustrate the improvement of the system. The dynamics of the controlling plant is

$$x = Pu + 0.1\sin(u),$$

where the P stands for the linear component of the plant defined as

$$P(z) = \frac{1}{z+0.5},$$

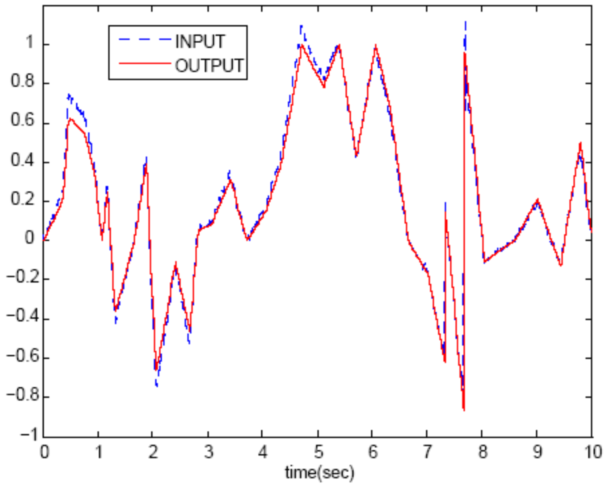

Fig.3: *The Simulation Result of The DTFELN System with* **K** *Satisfied s.p.r. Condition.*

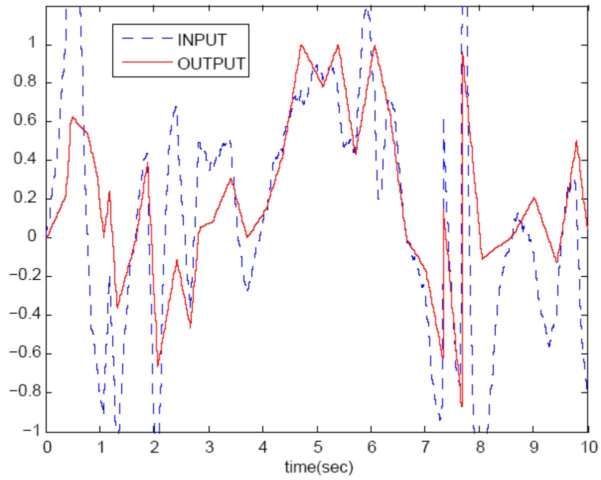

Fig.4: *The Simulation Result of The DTFELN System with* **K** *Not Satisfied s.p.r. Condition.*

and $0.1\sin(u)$ stands for the nonlinear term. In this study, it is assumed that the system nonlinear term $\sin(u)$ is known, however the effective gain of this term, i.e. the coefficient, is unknown.

Note that in this part, plant is strictly-proper. The simulations are done in the present of large disturbance. In *Fig. 3*, the DTFELN, with constant gain $\mathbf{K} = [\mathbf{1} \quad \mathbf{0.8}]^{\mathbf{T}}$ satisfying s.p.r. conditions as described in the previous section, is selected to control the system. The tracking performance between the input signal $r(k)$ and the output signal $y(k)$ is shown. This figure shows the rapid convergence of the signal.

The simulation in *Fig. 4* shows the response of the same system but $\mathbf{K} = [\mathbf{2} \quad \mathbf{3}]^{\mathbf{T}}$ does not satisfy s.p.r. condition. It is clearly that the system is no longer stable.

Finally, the second-order system

$$P_2(z) = \frac{(z-0.1)(z-0.3)(z+0.8)}{(z-0.5)(z-0.7)(z+0.9)},$$

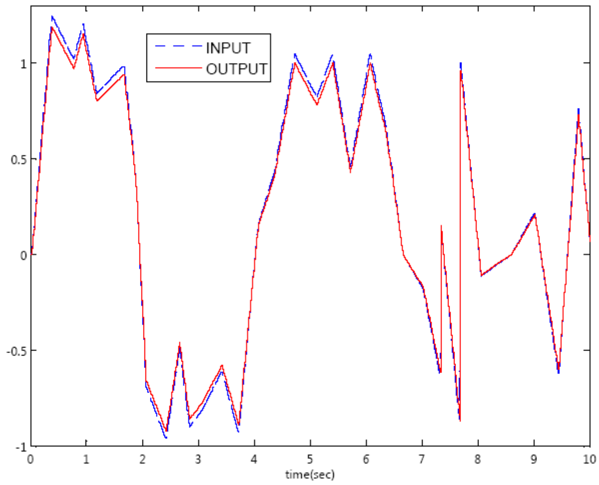

Fig.5: *The Simulation Result of The DTFELN System for the second order System.*

which is a second order system, is controlled. The simulation result of the controlled system is shown in *Fig. 5*. It can be seen that the with the proper selected of **K**, the system are stable guaranteeing the theoretical of DTFELN.

6. CONCLUSION

In this study, the "Discrete-Time Feedback Error Learning" (DTFEL) is demonstrated. This approach is based on a nonlinear adaptive control viewpoint for a class of nth order nonlinear systems.

First, the adaptive control and DTFEL algorithms in an adaptive feedback formulation are considered. It is shown that DTFEL can be viewed as a form of tracking error-based adaptive control. A Lyapunov analysis suggests that s.p.r. is a sufficient condition to guarantee asymptotic stability of DTFEL. Specifically, for a class of second order SISO systems, it can be derived that this condition simplifies to a set of constraints for selecting the values of feedback gain to guarantee asymptotic stability of DTFEL.

The numerical simulation results demonstrate the good tracking performance of the controlled system. They also show the significant difference in the stability property between the system with the proper selection of feedback gain and the one that the value of feedback gain does not satisfy the s.p.r. condition.

The integrated controller for DTFEL systems, the stability of DTFEL for the plant with time-delay could be the future works. The new algorithms for solving the control problems and improving the system performance are also the open-problems in this studied field.

7. ACKNOWLEDGEMENT

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APPENDIX

Full Calculation for finding ΔV
Consider the Lyapunov function

$$V(\mathbf{e},\tilde{\theta}) \;=\; 2\mathbf{e}^T\mathbf{P}\mathbf{e}+\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}$$

By applying the positive real property in *Lemma 2* and inserting the error dynamics (12), the adaptive law (15), the function

$$\Delta V(\mathbf{e}(k),\tilde{\theta}(k)) \triangleq V(\mathbf{e}(k+1),\tilde{\theta}(k+1)) - V(\mathbf{e}(k),\tilde{\theta}(k))$$

can be calculated as following: Let calculate each term in $V(\mathbf{e},\tilde{\theta})$ separately.

For the first term:

$$\begin{aligned}
&\mathbf{e}^T(k+1)P\mathbf{e}(k+1)-\mathbf{e}^T(k)P\mathbf{e}(k)\\
&\quad= (A\mathbf{e}+bv)^TP(A\mathbf{e}+bv)-\mathbf{e}^TP\mathbf{e}\\
&\quad= (\mathbf{e}^TA^TP+v^Tb^TP)(A\mathbf{e}+bv)-\mathbf{e}^TP\mathbf{e}\\
&\quad= \mathbf{e}^TA^TPA\mathbf{e}+v^Tb^TPA\mathbf{e}+\mathbf{e}^TA^TPbv+v^Tb^TPbv-\mathbf{e}^TP\mathbf{e}\\
&\quad= \mathbf{e}^T(A^TPA-P)\mathbf{e}+v^Tb^TPA\mathbf{e}+\mathbf{e}^TA^TPbv+v^Tb^TPbv\\
&\quad= \mathbf{e}^T(-qq^T-\varepsilon L)\mathbf{e}+v^Tb^TPA\mathbf{e}+\mathbf{e}^TA^TPbv+v^Tb^TPbv\\
&\quad= -\mathbf{e}^Tqq^T\mathbf{e}-\mathbf{e}^T\varepsilon L\mathbf{e}+v^T(A^TPb)^T\mathbf{e}+\mathbf{e}^TA^TPbv+v^T(b^TPb)v\\
&\quad= -\mathbf{e}^Tqq^T\mathbf{e}-\mathbf{e}^T\varepsilon L\mathbf{e}+v^T(\frac{c}{2}+\nu q)^T\mathbf{e}+\mathbf{e}^TA^TPbv+v^T(d-\nu^2)v\\
&\quad= -\mathbf{e}^Tqq^T\mathbf{e}-\mathbf{e}^T\varepsilon L\mathbf{e}-\frac{1}{2}v^Tc^T\mathbf{e}+v^Tq^T\nu\mathbf{e}+\mathbf{e}^TA^TPbv+v^Tdv-v^T\nu^2v\\
&\quad= -\mathbf{e}^T\varepsilon L\mathbf{e}-\big[(q^T\mathbf{e})^Tq^T\mathbf{e}+(\nu v)^T(q^T\mathbf{e})+(q^T\mathbf{e})^T(\nu v)+(\nu v)^T(\nu v)\big]\\
&\qquad\;-\frac{1}{2}v^Tc^T\mathbf{e}-\mathbf{e}^TA^TPbv+v^Tdv+\mathbf{e}^Tq\nu v\\
&\quad= -\mathbf{e}^T\varepsilon L\mathbf{e}-|q^T\mathbf{e}-\nu v|^2-\frac{1}{2}v^Tc^T\mathbf{e}-\mathbf{e}^TA^TPbv-\big[v^T(dv+c^T\mathbf{e})\big]\\
&\qquad\;+\big[v^Tc^T\mathbf{e}\big]+\mathbf{e}^Tq\nu v\\
&\quad= -\mathbf{e}^T\varepsilon L\mathbf{e}-|q^T\mathbf{e}-\nu v|^2+\frac{1}{2}v^Tc^T\mathbf{e}-\mathbf{e}^TA^TPbv-\big[v^T(e_1)\big]+\mathbf{e}^Tq\nu v\\
&\quad= -\mathbf{e}^T\varepsilon L\mathbf{e}-|q^T\mathbf{e}-\nu v|^2+\frac{1}{2}v^Tc^T\mathbf{e}-\mathbf{e}^T(A^TPb-vq)v+v^Te_1\\
&\quad= -\mathbf{e}^T\varepsilon L\mathbf{e}-|q^T\mathbf{e}-\nu v|^2+\frac{1}{2}v^Tc^T\mathbf{e}-\mathbf{e}^T(\frac{c}{2})v+v^Te_1\\
&\quad= -\mathbf{e}^T\varepsilon L\mathbf{e}-|q^T\mathbf{e}-\nu v|^2+v^Te_1
\end{aligned}$$

For the second term:

$$
\begin{aligned}
\tilde{\theta}^T(k & + 1)\Gamma^{-1}\tilde{\theta}(k+1) - \tilde{\theta}^T(k)\Gamma^{-1}\tilde{\theta}(k) \\
&= \tilde{\theta}^T(k+1)\Gamma^{-1}\tilde{\theta}(k+1) - \tilde{\theta}^T(k)\Gamma^{-1}\tilde{\theta}(k) + \left[\tilde{\theta}^T(k)\Gamma^{-1}\tilde{\theta}(k+1) - \tilde{\theta}^T(k)\Gamma^{-1}\tilde{\theta}(k+1)\right] \\
&= \Delta\tilde{\theta}^T(k)\Gamma^{-1}\tilde{\theta}(k+1) + \tilde{\theta}^T(k)\Gamma^{-1}\Delta\tilde{\theta}(k) \\
&= \Delta\tilde{\theta}^T(k)\Gamma^{-1}\tilde{\theta}(k+1) + \tilde{\theta}^T(k)\Gamma^{-1}\Delta\tilde{\theta}(k) + \left[-\Delta\tilde{\theta}^T(k)\Gamma^{-1}\tilde{\theta}(k) + \Delta\tilde{\theta}^T(k)\Gamma^{-1}\tilde{\theta}(k)\right] \\
&= \Delta\tilde{\theta}^T(k)\Gamma^{-1}\Delta\tilde{\theta}(k) + \tilde{\theta}^T(k)\Gamma^{-1}\Delta\tilde{\theta}(k) + \left[\Delta\tilde{\theta}^T(k)\Gamma^{-1}\tilde{\theta}(k)\right] \\
&= \Delta\tilde{\theta}^T(k)\Gamma^{-1}\Delta\tilde{\theta}(k) + \tilde{\theta}^T(k)\Gamma^{-1}\Delta\tilde{\theta}(k) + \left[\Delta\tilde{\theta}^T(k)\Gamma^{-1}\tilde{\theta}(k)\right]^T \\
&= \Delta\tilde{\theta}^T(k)\Gamma^{-1}\Delta\tilde{\theta}(k) + \tilde{\theta}^T(k)\Gamma^{-1}\Delta\tilde{\theta}(k) + \tilde{\theta}^T(k)\Gamma^{-1}\Delta\tilde{\theta}(k) \\
&= \Delta\tilde{\theta}^T(k)\Gamma^{-1}\Delta\tilde{\theta}(k) + 2\tilde{\theta}^T(k)\Gamma^{-1}\Delta\tilde{\theta}(k)
\end{aligned}
$$

Finally, combine two parts together:

$$
\begin{aligned}
\Delta V(k) &= V(k+1) - V(k) \\
&= 2\left[\mathbf{e}^T(k+1)P\mathbf{e}(k+1) - \mathbf{e}^T(k)P\mathbf{e}(k)\right] + \left[\tilde{\theta}^T(k+1)\Gamma^{-1}\tilde{\theta}(k+1) - \tilde{\theta}^T(k)\Gamma^{-1}\tilde{\theta}(k)\right] \\
&= 2\left[-\mathbf{e}^T\varepsilon L\mathbf{e} - |q^T\mathbf{e} - \nu v|^2 + v^T e_1\right] + \left[\Delta\tilde{\theta}^T(k)\Gamma^{-1}\Delta\tilde{\theta}(k) + 2\tilde{\theta}^T(k)\Gamma^{1}\Delta\tilde{\theta}(k)\right] \\
&= -2\mathbf{e}^T\varepsilon L\mathbf{e} - 2|q^T\mathbf{e} - \nu v|^2 + 2v^T e_1 + \Delta\tilde{\theta}^T(k)\Gamma^{-1}\Delta\tilde{\theta}(k) + 2\tilde{\theta}^T(k)\Gamma^{-1}\Delta\tilde{\theta}(k) \\
&= -2\mathbf{e}^T\varepsilon L\mathbf{e} - 2|q^T\mathbf{e} - \nu v|^2 + 2ve_1 + \left[\Gamma\phi e_1\right]^T\Gamma^{-1}\left[\Gamma\phi e_1\right] + 2\tilde{\theta}^T\Gamma^{-1}\left[\Gamma\phi e_1\right] \\
&= -2\mathbf{e}^T\varepsilon L\mathbf{e} - 2|q^T\mathbf{e} - \nu v|^2 + 2ve_1 + {e_1}^2\phi^T\Gamma\phi + 2\tilde{\theta}^T\phi e_1 \\
&= -2\mathbf{e}^T\varepsilon L\mathbf{e} - 2|q^T\mathbf{e} - \nu v|^2 + 2\left[-\phi^T\tilde{\theta} + \alpha\phi^T\Gamma\phi e_1\right]e_1 + 2\tilde{\theta}^T\phi e_1 + {e_1}^2\phi^T\Gamma\phi \\
&= -2\mathbf{e}^T\varepsilon L\mathbf{e} - 2|q^T\mathbf{e} - \nu v|^2 - 2\phi^T\tilde{\theta}e_1 + 2\alpha\phi^T\Gamma\phi{e_1}^2 + 2\tilde{\theta}^T\phi e_1 + \phi^T\Gamma\phi{e_1}^2 \\
&= -2\mathbf{e}^T\varepsilon L\mathbf{e} - 2|q^T\mathbf{e} - \nu v|^2 + 2\alpha\phi^T\Gamma\phi{e_1}^2 + \left[-2\phi^T\tilde{\theta}e_1 + 2\tilde{\theta}^T\phi e_1\right] + \phi^T\Gamma\phi{e_1}^2 \\
&= -2\mathbf{e}^T\varepsilon L\mathbf{e} - 2|q^T\mathbf{e} - \nu v|^2 + 2\alpha\phi^T\Gamma\phi{e_1}^2 + \phi^T\Gamma\phi{e_1}^2 \\
&= -2\mathbf{e}^T\varepsilon Le - 2|q^Te - \nu v|^2 + (2\alpha+1)\phi^T\Gamma\phi{e_1}^2 \\
&< 0 \text{ if } \alpha < -\frac{1}{2}
\end{aligned} \tag{25}
$$