Optimizing Signal Transmission in a MIMO System Influenced by Antenna Mutual Coupling

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ABSTRACT

This paper is concerned with investigations into an optimal transmission scheme for a single-user Multiple Input Multiple Output (MIMO) system influenced by the antenna mutual coupling. The concept of “coupled power” relying on the transmitting and receiving mutual impedances is introduced. This concept is used to work out an optimal MIMO transmission scheme when a non-negligible mutual coupling between antenna elements exists. Numerical results illustrate that the proposed signal transmission scheme offers extra capacity to the system. A suitable choice of antenna elements separation to obtain an improved MIMO system performance is discussed for the case of a semi-correlated uplink channel.

Keywords: Capacity, signal correlation, semi-correlated, multiple-input/multiple-output (MIMO), antenna mutual coupling, uplink channel.

1. INTRODUCTION

Wireless communication systems employing multiple antenna elements both at the transmitting and receiving sides are well known for boosting the system capacity [1, 10]. Previous studies have shown that the capacity of such Multiple Input Multiple Output (MIMO) systems can be reduced when the signal correlation at the transmitter or receiver is increased. This signal correlation can occur due to reduced scattering or an insufficient separation of antenna elements [2, 11-12]. In order to optimize the data throughput (capacity) under such conditions, an appropriate signal transmission scheme is required. The choice is dependent on the specified correlation properties of MIMO channel, which in turn is governed by a signal propagation scenario. In order to obtain an improved data throughput or transmission reliability, the availability of channel state information (CSI) is necessary. In the case the transmitter has perfect knowledge of CSI or obtains only “partial CSI”, (i.e., in the form of a covariance matrix of spatial correlation fed back from receiver), the optimum transmission scheme based on the water-filling algorithm can be adopted, [3-4]. In turn, when transmitter has no knowledge of CSI from receiver, an independent transmission scheme with the uniform power allocation is shown to be optimal, [5]. To the authors’ knowledge, the above-mentioned transmission schemes neglect antenna mutual coupling (MC) effects. In practice, the miniaturization trends in the mobile terminal design lead to small antenna element spacing. In this case, mutual coupling cannot be neglected, [6]. As a result, the commonly used assumptions of the correlation properties in signal transmission strategies ignoring the existence of MC are invalid. In this paper, we introduce the “coupled power” concept at the transmitter and the receiver and use it to modify appropriately the signal transmission scheme to optimize capacity. A suitable choice of antenna elements separation to obtain an improved MIMO system performance is devised for the case of a semi-correlated uplink channel.

2. OPTIMIZATION OF POWER ALLOCATION SCHEME IN A SINGLE-USER MIMO SYSTEM

2.1 “Coupled Power”

Our investigations commence with the introduction of the “coupled power” concept, which will be used later on to devise an optimal signal transmission scheme for a MIMO system operating under the influence of antenna mutual coupling. The configuration of the investigated system is shown in Figure.1. In this figure, the signal radiation from the input port to the air channel is represented by a coupled N port network with N terminals. The input source voltages developed at the input ports are denoted by a vector \( U_T = [U_{T1}, U_{T2}, \ldots, U_{TN}]^T \), where \( U_{Tj} \) is the uncoupled signal voltage at the \( j \)-th input terminal. The transmit voltages developed at the transmitting antennas are denoted by \( V_T = [V_{T1}, V_{T2}, \ldots, V_{TN}]^T \), where \( V_{Ti} \) is the coupled voltage at the \( j \)-th transmitting antenna. When antenna MC is present, the
transmitting mutual impedance between \( j \)-th and \( i \)-th antenna is introduced and denoted as \( Z_{ji}^T \).

**Fig.1:** A MIMO system with antenna mutual coupling effect

Here we adopt the receiving/transmitting mutual impedance concept which was described in [8]. It provides a more accurate description of the antenna MC compared with the conventional (open circuit) mutual impedance approach. Note that the self-impedances of the transmitting antennas are assumed to be of the same value.

Following [9], the total input power to the transmitting array is given by \( P_{in} = E_{Ts}[U_T^H U_T] = E_{Ts}[trace(U_T U_T^H)] \), where \( E_{Ts} \) denotes an expectation over the symbol time \( T_s \), and the superscript \( H \) stands for the Hermitian conjugate. The total transmitting power is given as

\[
P_T = E_{Ts} [V_T^H V_T] = \frac{P_{in}}{N} \text{trace}(Z_T Z_T^H) \tag{1}
\]

where \( Z_T \) is the transmitting mutual coupling matrix. The transmitting and receiving \((Z_R)\) mutual coupling matrices are given by the following expressions

\[
Z_T = \begin{bmatrix}
-Z_{11}^T & -Z_{12}^T & \cdots & -Z_{1N}^T \\
-Z_{21}^T & -Z_{22}^T & \cdots & -Z_{2N}^T \\
\vdots & \vdots & \ddots & \vdots \\
-Z_{N1}^T & -Z_{N2}^T & \cdots & -Z_{NN}^T
\end{bmatrix}
\]

\[
Z_R = \begin{bmatrix}
-Z_{11} & -Z_{12} & \cdots & -Z_{1N} \\
-Z_{21} & -Z_{22} & \cdots & -Z_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
-Z_{N1} & -Z_{N2} & \cdots & -Z_{NN}
\end{bmatrix}
\]

where \( Z_L \) is the load-impedance of the receiving antennas and \( M \) denotes the number of receiving ports.

Since the transmitting power is obtained from the input power that goes through a coupling network, the name of “(transmitting) coupled power” is used.

At the receiving end, a similar approach to defining the coupled power of output terminal yields \( P_{out} = P_R/M \text{trace}(Z_R Z_R^H) \), where \( P_R \) is the total received power.

Using the concept of “coupled power”, we first consider a special case of MIMO system in which transmitter has no knowledge of the channel. In [5], it is suggested that non-correlated data streams with equal-power allocated for each stream is a robust transmission scheme to optimize capacity of such a system. In this case the total input power is constrained to \( P_{in} \) and thus the power allocated at each input terminal is \( P_{in}/N \). However, the mutual coupling effect modifies this power distribution. From (1) and (2) we obtain the coupled power at the \( j \)-th antenna as

\[
P_{T,j} = \left[ \frac{x_{11}^T}{x} - \frac{x_{22}^T}{x} \ldots - \frac{x_{NN}^T}{x} \right] \frac{P_{in}}{N} \frac{1}{N}, \frac{P_{in}}{N} \ldots, \frac{P_{in}}{N} \cdot \frac{N}{x}(3)
\]

One can observe that in general, the actual powers transmitted by individual antennas may be not equal when the mutual coupling effect is neglected. By using the mutual coupling matrix (2), we can adjust the distribution of the input power to obtain the uniform power distribution.

### 2.2 Re-modeling the Power Allocating Algorithms

Without the loss of generality, now we consider a scenario in which both the receiving and transmitting antennas are spatially correlated and the transmitter gets a feedback about a covariance matrix. Note that the scenario concerning the transmitter having the perfect CSI can be analyzed in the same way. We assume a single-user correlated MIMO channel with \( N \) transmitting antennas and \( M \) receiving antennas and refer to it as a \((M, N)\) MIMO system. Assuming a narrow band case, the system can be modeled by equation \( y = Hx + n \), where \( x \) is a vector representing the transmitted signals, \( y \) is a vector representing the received signals and \( n \) is a vector representing noise. The channel matrix \( H \) is formed by elements \( \{H_{j,i}\} \) representing the channel gain between the \( i \)-th transmitting and the \( j \)-th receiving antenna. In general, \( \{H_{j,i}\} \) is a function of frequency. Here, we only consider a narrowband case and thus \( \{H_{j,i}\} \) is a function of a single frequency \( f \). The extension of the presented considerations to the case of wideband system can be obtained by applying an Orthogonal Frequency Division Multiplexing (OFDM) technique. However, this case is not elaborated further.

It is assumed that the channel is correlated. In this case, the channel matrix can be represented as \( H = \)
and power allocation need to be modified. This can be achieved by the antenna mutual coupling matrix, as given by $P_{\text{in}}/\text{trace}(\mathbf{Z}_T\mathbf{Z}_H^H)$, Eq. (5) also takes into account the existence of antenna mutual coupling at the receiving end of the MIMO system. The entire power at the receiver is given as $P_{\text{in}}\cdot \text{trace}(\mathbf{Z}_R\mathbf{Z}_H^H)$. These modified expressions are included in the power allocation strategy.

By using Eq. (5), individual data streams with suitable statistical properties, given by $Q = E(xx^H)$, can be chosen to reach the maximum capacity under the influence of antenna MC.

2.3 Simulation Results

Based on the above derived expressions, a computer algorithm for calculating the MIMO system capacity under the assumption of an optimal transmission scheme has been developed. The algorithm can generate the results for capacity when the effect of mutual coupling is taken into account or neglected. When the effect of mutual coupling is taken into account, the algorithm is used to analyze the performance of MIMO system with compact antenna arrays at both transmitting and receiving sides.

The last result concerns the transmitter side and is obtained by substituting Eq. (1) into Eq. (4). When the antenna mutual coupling is taken into consideration, the transmitted power radiated to the channel is not equal to the total input power allocated to the input ports of transmitting antennas. It is modified by the antenna mutual coupling matrix, as given by $P_{\text{in}}/\text{trace}(\mathbf{Z}_T\mathbf{Z}_H^H)$. When the antenna mutual coupling is taken into account, the algorithm is used to analyze the performance of MIMO system with compact antenna arrays at both transmitting and receiving sides.

**Fig. 2**: Mean capacity of (3, 3) MIMO system versus total input SNR when antenna separation is 0.2λ.

Fig. 2 shows the mean capacity of a (3, 3) MIMO system versus total input SNR, $P_{\text{in}}/\sigma_n^2$, for the range of -10dB to 20dB. The antenna separation at both the transmitting and receiving ends is fixed at 0.2λ (where λ stands for wavelength) so that MC cannot be ignored. The case when channel is identically independent distributed (i.i.d.) is also presented for the reference purposes. From the obtained results it is clear that the mean capacity increases almost linearly according to the total input SNR. With the antenna spacing being small and the antenna MC cannot be neglected, the individual data streams are allocated more power while weaker directions are allocated less or no power. The task can be regarded as equivalent to optimizing the diagonal matrix $\mathbf{A}_Q$. $\mathbf{A}_Q = \text{diag}[\lambda_1, \ldots, \lambda_N]$ and $\mathbf{A}_T = \text{diag}[\lambda_1^T, \ldots, \lambda_N^T]$ denote the eigenvalues of $\mathbf{S}_T$ and $\mathbf{S}_R$ respectively. It is shown in [3] that under these conditions the optimum transmission scheme is independent of the eigenvectors of the transmit covariance matrix. This resembles the water-filling principle, i.e., stronger directions are allocated more power while weaker directions are allocated less or no power. The task can be regarded as equivalent to optimizing the diagonal matrix $\mathbf{A}_Q$. $\mathbf{A}_Q = \text{diag}[\lambda_1, \ldots, \lambda_N]$, where $\lambda_1, \ldots, \lambda_N$ are powers allocated to individual data streams. Without taking the antenna MC into account, the optimum choice of $\mathbf{A}_Q$ is given by [3]

$$\mathbf{A}_Q^{\text{opt}} = \arg \max_{\mathbf{A}_Q} \left( I_M + \sum_{i=1}^{N} 1/\sigma_n^2 P_k \cdot \sqrt{\lambda_i^T \cdot \lambda_i^R \cdot g_{kk}} \right)$$

where $g_{kk}$ is the $k,k$th element of $\mathbf{G}$. However, when the antenna spacing is small and the antenna MC cannot be neglected, the individual data streams and power allocation need to be modified. This can be done by changing (4) to (5)

$$\mathbf{A}_Q^{\text{opt-me}} = \arg \max_{\mathbf{A}_Q} \left( I_M + \sum_{i=1}^{N} 1/\sigma_n^2 P_k \sqrt{\lambda_i^T \cdot \lambda_i^R \cdot g_{kk}} \right)$$

The last result concerns the transmitter side and is obtained by substituting Eq. (1) into Eq. (4). When the antenna mutual coupling is taken into consideration, the transmitted power radiated to the air channel is not equal to the total input power allocated to the input ports of transmitting antennas. It is modified by the antenna mutual coupling matrix, as given by $P_{\text{in}}/\text{trace}(\mathbf{Z}_T\mathbf{Z}_H^H)$. When the antenna mutual coupling is taken into account, the algorithm is used to analyze the performance of MIMO system with compact antenna arrays at both transmitting and receiving sides.
presence of antenna MC in MIMO channel, when we build the MC into power allocation scheme and modify the data streams in the input ports, the mean capacity improves approximately 1bit/s/Hz compared to the case when the existence of MC is ignored.

**Fig.3:** Mean capacity of (3, 4), (4, 3), (8, 8), (10, 10) MIMO systems versus input SNR when antenna separation is 0.2λ.

Figure 3 shows the plots for the mean capacity versus total input SNR as a function of the number of transmitting/receiving antennas $N$ (or $M$). The obtained results show two cases, one concerning the uniform power allocation scheme and the other representing the optimum water-filling scheme. It is apparent that with the presence of antenna MC, the water-filling strategy becomes advantageous when the number of antennas is small, i.e., $N$ (or $M$) = 3, 4. Nearly 0.5bit/s/Hz is gained. However, when the number of antennas increases, i.e., $N$ (or $M$) = 8, 10, the difference between the two schemes is small. This is because the presence of antenna MC affects the power distribution only to a small extent for a large size array. The reason is that most of the elements beside the two side elements experience a similar environment and thus MC does not affect much the original uniform power allocation scheme.

Another observation from Fig. 3 is that when $(M, N)$ is (4, 3) or (3, 4), the mean capacity for the two cases is almost identical.

3. ANTENNA SEPARATION SELECTION FOR CASE OF SEMI-CORRELATED UPLINK CHANNEL

3.1 Correlation Coefficients versus Antenna Separations

From the above presented analysis, it is clear that an extra capacity can be gained when a transmission scheme takes into account the antenna mutual coupling. This is demonstrated in a number of new examples, which are presented here. Fig. 4 shows the correlation coefficients $(\Sigma_{i,j}^{T(R)}, \rho_{i,j}^{MC(T)}, \rho_{i,j}^{MC(R)})$ for dipole antennas versus separation for the case of Rayleigh channel with MC taken into account.

It can be seen in Fig. 4 that the correlation coefficients $\rho_{i,j}^{MC(T)}$ and $\rho_{i,j}^{MC(R)}$ are different when antennas operate in transmitting and receiving modes. Another observation is that at large separations these coefficients are small. The point of interest is the antenna separation of approximately 0.4$\lambda$ or 0.9$\lambda$ at which minimum values are observed. One can also find that when antenna spacing is larger than 0.2$\lambda$, the correlation coefficients with antenna MC at the receiving antennas $\rho_{i,j}^{MC(R)}$ are small, but $\Sigma_{i,j}^{T(R)}$ and $\rho_{i,j}^{MC(T)}$ fluctuate as a function of antenna spacing.

3.2 Results for Semi-correlated Uplink Channel

We consider a specific but common scenario that concerns the signal transmission from an unobstructed mobile station (MS) to a compact antenna array at a base station (BS) which is surrounded by local scattering objects, as shown in Fig. 5.

**Fig.4:** The variation of the correlation coefficients with antenna separation for two dipole antennas.

**Fig.5:** Sketch of an uplink channel model with MS unobstructed and BS surrounded by local scattering objects.
For this configuration, we assume that at the receiving end the mutual coupling effect can be neglected, $\rho^{MC(R)}_{i,j}$ and the signals are uncorrelated $\Sigma_R = I_M$. However at the MS, the MC is present and the signals are correlated.

Fig. 6 shows the mean capacity of $(3,3)$ MIMO system versus antenna separation when the total input SNR is $20\text{dB}$. It can be observed that when the antenna spacing is around $0.9\lambda$ the mean capacity is optimal. The antenna separation of $0.55\lambda$ also represents a good case. The optimal spacing can also be found for other numbers of antenna elements, e.g., $M = 4$. The obtained results form guidelines for working out the proper antenna separation at the receiver for the case of semi-correlated uplink channel.

**Fig.6:** Mean capacity of $(3,3)$ MIMO system versus antenna separation for total input SNR of $20\text{dB}$.

Fig. 7 shows the variation of the average capacity with SNR for different number of transmitting and receiving antennas for a semi-correlated uplink channel.

**Fig.7:** The variation of the average capacity with SNR for different number of transmitting and receiving antennas.

The obtained results show similar trends to those seen in Fig. 3 but convey different information. We settle the antenna separation at $0.55\lambda$ at the MS and $2.0\lambda$ at the BS. Such array antenna spacing satisfies the semi-correlated uplink channel case. The optimized average capacities with and without antenna MC are compared with those obtained for the (non-optimized) equal power allocation scheme. First we observe that the optimized average capacity when the antenna MC is taken into account is much greater than that when the antenna MC is ignored, indicating a considerable error incurred in the calculated MIMO capacity when antenna MC is not taken into account. This error becomes more pronounced when the number of transmitting and receiving antennas is increased. For example, from Fig. 7 the average channel capacity for the $10 \times 10$ MIMO system at a SNR level of $5 \text{dB}$ is $18 \text{bit/s/Hz}$ when antenna MC is taken into account. In turn, when antenna MC is ignored, the capacity is only $14 \text{bit/s/Hz}$, which is $22\%$ lower compared with the case when MC is included. Using the optimum power allocation strategy, the substantial increase in average capacity is observed when antenna MC is taken into account. Fig. 7 also shows that when antenna MC is ignored while applying the optimum power allocation strategy, the result is not very different from the one based on equal-power allocation. This result is apparent for all of the MIMO cases presented in Fig. 7.

The above observations indicate that optimizing MIMO signal transmission strategy to gain extra capacity requires taking into account the presence of antenna mutual coupling. By taking the MC into account the improvement in capacity of MIMO system can be achieved via the proper choice of antenna separation.

### 4. CONCLUSION

The effect of antenna mutual coupling has been successfully included while devising an optimal signal transmission scheme in a single-user MIMO system. This has been accomplished using the concept of receiving and transmitting mutual impedances. Numerical results have shown that MC affects the signal transmission strategy that is aimed at obtaining an optimal mean capacity. An antenna spacing selection has been discussed for the case of a semi-correlated uplink channel. It has been shown that the proper antenna spacing can lead to an optimal performance of MIMO system operating under the influence of mutual coupling.

**References**


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