Robust Stability Condition for a Class of Time-Delay Plants with Uncertainty

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ABSTRACT

In this paper, we consider the robust stability condition for single-input/single-output time-delay plants with new class of uncertainties. First of all, we define a class of uncertainties to be considered. The necessary and sufficient robust stability condition for time-delay plants with such class of uncertainties is presented. The relation between the time-delay plant and the nominal time-delay plant to satisfy the robust stability condition is clarified. By using this relation, we will show the necessary and sufficient robust stability condition for the time-delay plant with varying number of right half plane poles.

Keywords: robust stability, time-delay plant, uncertainty, sensitivity function

1. INTRODUCTION

In this paper, we examine the robust stability condition for single-input/single-output time-delay plants with new class of uncertainties. Several papers have been considered on robust stabilization problem [1–6]. Doyle and Stein built the basis for this problem [1], and the condition for plants with the multiplicative uncertainty and the additive uncertainty was shown. Chen and Deseor gave the complete proof of the result by Doyle and Stein [3].Kimura [7] was considered the robust stabilizability problem for single-input/single-output plants. Vidyasagar and Kimura [8] expanded the result in [7] and considered the robust stabilizability problem for multiple-input/multiple-output plants.

According to [1, 2], in order to keep the stability for the large uncertainty, the complementary sensitivity function must be small value. In order to make the complementary sensitivity function small, brings the control systems lower performance in the meaning of disturbance attenuation property and so on. Therefore, we must make the sensitivity function small to produce the control system with high disturbance attenuation property. Since the sum of the sensitivity function and the complementary sensitivity function is equal to 1, we cannot obtain either low sensitivity or high robust stability characteristics. It is well known that low sensitivity often make the control system unstable [2]. Although, low sensitivity does not always make the control system unstable. Maeda et al. considered this problem as an infinite gain margin one [9, 10]. Nogami et al. clarified the condition that hi-gain controller does not make the control system unstable and also proposed a design method [11]. However, the method in [9–11] cannot apply for time-delay plants.

In this paper, we consider this low sensitivity control problem from another view-point; there exists a class of uncertainties that has the low sensitivity property make the control system for time-delay plants robustly stable. First of all, we define new class of uncertainties for time-delay plants. The necessary and sufficient robust stability condition for time-delay plants with such a class of uncertainties is presented. Secondary, the relation between time-delay plant and nominal time-delay plant to satisfy the robust stability condition is clarified. Using this relation between time-delay plant and nominal time-delay plant, the necessary and sufficient robust stability condition is shown in the case that the number of poles of the time-delay plant in the closed right half plane is not equal to that of the nominal time-delay plant. The robust stability condition presented in this paper is identical whether the number of poles of the nominal time-delay plant is equal to that of the time-delay plant or not. Generally, it is assumed that the number of poles of the plant in the closed right half plane is equal to that of the nominal plant [1–6] except [4]. In [4], Verma et al. considered the similar problem, but the method in [4] cannot apply for time-delay plants.

This paper is organized as follows: In Section 2., the problem considered in this paper is formulated. In Section 3., we define new class of uncertainty and give the robust stability condition for the defined class of uncertainty. Then, we find that for the defined class of uncertainty, low sensitivity makes the control system for time-delay plants robustly stable. In Section 4., we describe the relation between the nominal time-delay plant and the time-delay plant to satisfy robust stability condition clarified in Section 3. Using the result in Section 4., in Section 5., we examine the robust stability condition in the case that the number of poles of the nominal time-delay plant in the closed right half plane is not equal to that of the time-delay plant. In Section 6., a design method for robustly stabilizing controller using the parametrization of all

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stabilizing modified Smith predictor [16–18] is presented. In Section 7., we show a numerical example to illustrate the effectiveness of the proposed method.

Notations

R	The set of real numbers.
$R(s)$	The set of all real-rational transfer functions.
RH_∞	The set of stable proper real rational functions.
$\mathcal{U}$	The set of unimodular functions on RH_∞. That is, $U(s) \in \mathcal{U}$ implies both $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$.
$\|\cdot\|_\infty$	RH_∞ norm.
$F_u(P,Q)$	Upper LFT, that is $F_u(P,Q) = P_{22} + P_{21}Q(I - P_{11}Q)^{-1}P_{12}$, where $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$.
$F_l(P,Q)$	Lower LFT, that is $F_l(P,Q) = P_{11} + P_{12}Q(I - P_{22}Q)^{-1}P_{21}$.

2. PROBLEM FORMULATION

Consider the control system written by

$$\left\{ \begin{array}{lcl} y & = & G(s)e^{-sT}u + d \\ u & = & C(s)\,(r - y) \end{array} \right. \tag{1}$$

Here, $G(s)e^{-sT}$ is a single-input/single-output time-delay plant, $G(s) \in R(s)$ is strictly proper, $T > 0$ is the time-delay, $C(s)$ is the controller, $r \in R$ is the reference input, $y \in R$ is the output and $d \in R$ is the disturbance. The nominal time-delay plant of $G(s)e^{-sT}$ denotes $G_m(s)e^{-sT}$, where $G_m(s) \in R(s)$ is strictly proper. The nominal time-delay plant $G_m(s)e^{-sT}$ is not usually equal to the time-delay plant $G(s)e^{-sT}$. That is, an error between $G_m(s)e^{-sT}$ and $G(s)e^{-sT}$ exists. Let $G(s)e^{-sT}$ be denoted by using the multiplicative uncertainty like

$$G(s)e^{-sT} = G_m(s)e^{-sT}\left(1 + \Delta(s)\right). \tag{2}$$

Remarks 1: Note that from (2), in this paper, the time-delay T is assumed to have no uncertainty.

According to [1, 2], in order to keep the stability for the large uncertainty $\Delta(s)$, the complementary sensitivity function $T(s)$

$$T(s) = 1 - S(s) = \frac{C(s)G_m(s)e^{-sT}}{1 + C(s)G_m(s)e^{-sT}} \tag{3}$$

must be small value.

On the other hand, the influence of $\Delta(s)$ toward to input-output property is a tendency to decrease if the sensitivity function $S(s)$ denoted by

$$S(s) = \frac{1}{1 + C(s)G_m(s)e^{-sT}} \tag{4}$$

becomes smaller. The sensitivity function $S(s)$ is equivalent to the transfer function from the disturbance d to the output y. Therefore, the control system with low sensitivity property has nice performance. It is well known that low sensitivity will often make the control system unstable [2], since

$$S(s) + T(s) = 1. \tag{5}$$

Although, low sensitivity does not always make the control system unstable. Maeda et al. considered the this problem as an infinite gain margin problem [9, 10]. Nogami et al. clarify the condition that hi-gain controller does not make the control system unstable and also proposed a design method [11].

In this paper, we consider this low sensitivity control problem from another viewpoint; there exists a class of the uncertainty when low sensitivity property make the control system robustly stable.

3. LOW SENSITIVITY CONTROL WITH ROBUST STABILITY

In this section, we define new class of uncertainty and clarify the robust stability condition for the time-delay plant with the defined class of uncertainty.

Several classes of uncertainty $\Delta(s)$ have been considered such as

$$\|\Delta(s)\|_\infty < 1 \tag{6}$$

and

$$|\Delta(j\omega)| < |W(j\omega)| \quad \forall\omega \in R \tag{7}$$

and so on. In this paper, we adopt the following class of uncertainty

$$\left|\frac{\Delta(j\omega)}{1 + \Delta(j\omega)}\right| < |W(j\omega)| \quad \forall\omega \in R. \tag{8}$$

We will describe later, the robust stability condition for above class of uncertainty is related directly not with the complementary sensitivity function $T(s)$ but with the sensitivity function $S(s)$. That is, low sensitivity control has a tendency to guarantee the robust stability. By using such a class of uncertainty, we have following definitions.

Definition 1: $G(s)e^{-sT}$ is called the elementary of the set $\Omega_1(G_m(s)e^{-sT}, W(s))$ if following expressions hold.

- $G(s)e^{-sT}$ has the same number of poles of the nominal time-delay plant $G_m(s)e^{-sT}$ in the closed right half plane.
- $G(s)e^{-sT}$ has the same number of zeros of the nominal time-delay plant $G_m(s)e^{-sT}$ in the closed right half plane.
- $$\left|\frac{\Delta(j\omega)}{1 + \Delta(j\omega)}\right| < |W(j\omega)| \quad \forall\omega \in R, \tag{9}$$

where $W(s)$ is called the upper bound of uncertainty and is a stable rational function.

Definition 2: The controller $C(s)$ is called the robustly stabilizing controller for $\Omega_1(G_m(s)e^{-sT}, W(s))$ if $C(s)$ stabilize all of $G(s)e^{-sT} \in \Omega_1(G_m(s)e^{-sT}, W(s))$.

On the robust stability condition for time-delay plants included in $\Omega_1(G_m(s)e^{-sT}, W(s))$, next theorem is satisfied.

Theorem 1: It is assumed that $C(s)$ is a stabilizing controller for the nominal time-delay plant $G_m(s)e^{-sT}$. $C(s)$ is a robustly stabilizing controller if and only if

$$\begin{aligned}\left\| \frac{1}{1+C(s)G_m(s)e^{-sT}} W(s) \right\|_\infty &= \|S(s)W(s)\|_\infty \\ &\leq 1 \end{aligned} \quad (10)$$

holds true.

Proof: Let $P(s)$ and $\bar{\Delta}(s)$ be

$$P(s) = \begin{bmatrix} 1 & 1 \\ G_m(s)e^{-sT} & G_m(s)e^{-sT} \end{bmatrix} \quad (11)$$

and

$$\bar{\Delta}(s) = \frac{\Delta(s)}{1+\Delta(s)}, \quad (12)$$

respectively. Proof is immediately obtained by applying Theorem 3.3 in [12] to

$$F_u\left(P(s), \bar{\Delta}(s)\right) = G_m(s)e^{-sT}\left(1+\Delta(s)\right) \quad (13)$$

$$\begin{aligned} F_l\left(P(s), -C(s)\right) &= \frac{1}{1+C(s)G_m(s)e^{-sT}} \\ &= S(s). \end{aligned} \quad (14)$$

We have completed the proof of this theorem. ■

Theorem 1 shows if $G(s)e^{-sT}$ can be drawn in the form of Definition 1, the low sensitivity can be achieved with robust stability condition.

4. RELATION BETWEEN NOMINAL TIME-DELAY PLANT AND TIME-DELAY PLANT

In this section, we describe the relation between the nominal time-delay plant $G_m(s)e^{-sT}$ and the time-delay plant $G(s)e^{-sT}$ to satisfy Theorem 1.

For keeping internal stability condition, the control system in (1) must be well-posed. Therefore, the controller $C(s)$ must be causal. When the controller $C(s)$ is causal, the sensitivity function $S(s)$ in (4) has the property

$$\lim_{\omega\to\infty} |S(j\omega)| = 1 \quad (15)$$

because of (4) and the assumption that $G_m(s)$ is the strictly proper. To satisfy (10), from (15),

$$\lim_{\omega\to\infty} \left| \frac{\Delta(j\omega)}{1+\Delta(j\omega)} \right| < \lim_{\omega\to\infty} |W(j\omega)| \leq 1 \quad (16)$$

is required.

From (16), we obtain next theorem.

Theorem 2: If $G(s)e^{-sT}$ and $G_m(s)e^{-sT}$ hold (16), then the relative degree of $G(s)$ is equal to that of $G_m(s)$.

Proof: The proof is to show the contrapositive of statement. That is, we show that if the relative degree of $G_m(s)$ is not equal to that of $G(s)$, then (16) cannot be satisfied.

If the relative degree of $G_m(s)$ is greater than that of $G(s)$, from (2), we have

$$\begin{aligned} \lim_{\omega\to\infty} |W(j\omega)| &> \lim_{\omega\to\infty} \left| 1 - \frac{G_m(j\omega)}{G(j\omega)} \right| \\ &= 1. \end{aligned} \quad (17)$$

In this case, (16) is not satisfied.

Conversely, if the relative degree of $G_m(s)$ is smaller than that of $G(s)$, then

$$\begin{aligned} \lim_{\omega\to\infty} |W(j\omega)| &> \lim_{\omega\to\infty} \left| 1 - \frac{G_m(j\omega)}{G(j\omega)} \right| \\ &= \infty. \end{aligned} \quad (18)$$

This case does not satisfy (16).

We have thus completed the proof of this theorem. ■

5. MISMATCH OF THE NUMBER OF POLES IN THE CLOSED RIGHT HALF PLANE

In this section, we consider the robust stability condition in the case that the number of poles of the nominal time-delay plant $G_m(s)e^{-sT}$ in the closed right half plane is not equal to that of the time-delay plant $G(s)e^{-sT}$.

We must correct Definition 1 and Definition 2 to consider this problem with varying number of poles in the closed right half plane. Definition 1 and Definition 2 are corrected as follows:

Definition 3: $G(s)e^{-sT}$ is called the elementary of the set $\Omega_2(G_m(s)e^{-sT}, W(s))$ if following expressions hold.

- $G(s)e^{-sT}$ has the same number of zeros of the nominal time-delay plant $G_m(s)e^{-sT}$ in the closed right half plane.
- $$\left| \frac{\Delta(j\omega)}{1+\Delta(j\omega)} \right| < |W(j\omega)|, \quad (19)$$

where $W(s) \in R(s)$ is a stable rational function.

Definition 4: The controller $C(s)$ is called the robustly stabilizing controller for $\Omega_2(G_m(s)e^{-sT}, W(s))$ if $C(s)$ stabilize all of $G(s) \in \Omega_2(G_m(s)e^{-sT}, W(s))$.

The robust stability condition for time-delay plants included in $\Omega_2(G_m(s)e^{-sT}, W(s))$ is summarized as following theorem,

Theorem 3: Assume that the controller $C(s)$ stabilizes the nominal time-delay plant $G_m(s)e^{-sT}$. $C(s)$ is a robustly stabilizing controller if and only if

$$\left\| \frac{1}{1+C(s)G_m(s)e^{-sT}} W(s) \right\|_\infty = \|S(s)W(s)\|_\infty \leq 1. \quad (20)$$

In order to prove Theorem 3, necessary lemma will be shown.

Lemma 1: Assume that $W(s)$ satisfy (16). It is assumed that $G_m(s)$ has q number of zeros in the closed right half plane and p_m number of poles in the closed right half plane, and $G(s)$ has q number of zeros in the closed right half plane and p number of poles in the closed right half plane.Then the Nyquist plot of $1+\Delta(s)$ encircles the origin $(0,0)$ $p-p_m$ times in the counter-clockwise direction.

Proof: From the assumption that $W(s)$ satisfies (16) and Theorem 2, the relative degree of $G(s)$ is equivalent to that of $G_m(s)$.

From (2), $1+\Delta(s)$ is written by

$$\begin{aligned} & 1+\Delta(s) \\ = & \frac{G(s)e^{-sT}}{G_m(s)e^{-sT}} \\ = & \frac{G(s)}{G_m(s)} \end{aligned} \quad (21)$$

From an argument principle, the Nyquist plot of $1+\Delta(s)$ encircles the origin $(0,0)$ $p-q-(p_m-q) = p-p_m$ times in the counter-clockwise direction.

We have thus proved this lemma.

The proof of Theorem 3 can be done by using above lemma.

Proof: The characteristic equation of the control system in (1) is given by $1+C(s)G(s)e^{-sT}$. From the Nyquist theorem, if the Nyquist plot of $1+C(s)G(s)e^{-sT}$ encircles the origin $(0,0)$ $p+p_c$ times in the counter-clockwise direction, then the control system in (1) is stable, where p and p_c is the number of poles of $G(s)e^{-sT}$ in the closed right half plane and that of $C(s)$. This characteristic equation $1+C(s)G(s)e^{-sT}$ is rewritten by

$$\begin{aligned} & 1+C(s)G(s)e^{-sT} \\ = & 1+C(s)G_m(s)e^{-sT}(1+\Delta(s)) \\ = & \left(1+C(s)G_m(s)e^{-sT}\right) \\ & \cdot\left(1+\frac{C(s)G_m(s)e^{-sT}}{1+C(s)G_m(s)e^{-sT}}\Delta(s)\right) \\ = & \left(1+C(s)G_m(s)e^{-sT}\right) \\ & \cdot(1+\Delta(s))\left(1-\frac{1}{1+C(s)G_m(s)e^{-sT}}\frac{\Delta(s)}{1+\Delta(s)}\right) \\ = & \left(1+C(s)G_m(s)e^{-sT}\right)(1+\Delta(s)) \\ & \left(1-S(s)\frac{\Delta(s)}{1+\Delta(s)}\right). \end{aligned} \quad (22)$$

From the assumption that $C(s)$ stabilize the nominal time-delay plant $G_m(s)e^{-sT}$, the Nyquist plot of $1+C(s)G_m(s)e^{-sT}$ encircles the origin $(0,0)$ p_m+p_c times in the counter-clockwise direction. This means that $C(s)$ is a robustly stabilizing controller for $\Omega_2(G_m(s)e^{-sT}, W(s))$ if and only if the Nyquist plot of

$$(1+\Delta(s))\left(1-S(s)\frac{\Delta(s)}{1+\Delta(s)}\right)$$

for all $\Delta(s)$ encircles the origin $(0,0)$ $p-p_m$ times in the counter-clockwise direction. The Nyquist plot of $1+\Delta(s)$ encircles the origin $p-p_m$ times from Lemma 1. Therefore, the necessary and sufficient condition that $C(s)$ is a robustly stabilizing controller for $\Omega_2(G_m(s)e^{-sT}, W(s))$ is that the Nyquist plot of

$$1-S(s)\frac{\Delta(s)}{1+\Delta(s)}$$

does not encircle the origin any times.

The remaining problem is to prove the necessary and sufficient condition that the Nyquist plot of

$$1-S(s)\frac{\Delta(s)}{1+\Delta(s)}$$

does not encircle the origin any times. The condition is expressed as the same equation as (20). We adopt same procedure in [13] to prove it. Sufficient part of the proof is as follows: Assume that $\|S(s)W(s)\|_\infty < 1$. It is clear that the Nyquist plot of

$$1-S(s)\frac{\Delta(s)}{1+\Delta(s)}$$

can encircle the origin no time. Necessary part is to show if $\|S(s)W(s)\| > 1$, then there exists $\Delta(s) \in \Omega_2$ to let the Nyquist plot of

$$1-S(s)\frac{\Delta(s)}{1+\Delta(s)}$$

encircle the origin. Since $G_m(s)$ is strictly proper, some ω satisfying $|S(j\omega)W(j\omega)| = 1+\epsilon(\epsilon > 0)$ exists. If we settle

$$\frac{\Delta(j\omega)}{1+\Delta(j\omega)} = \frac{W(j\omega)e^{-j\tan^{-1}\{S(j\omega)W(j\omega)\}}}{1+\epsilon} \quad (23)$$

then we have

$$1-S(j\omega)\frac{\Delta(j\omega)}{1+\Delta(j\omega)} = 0. \quad (24)$$

This means that the Nyquist plot of $1-S(s)\Delta(s)/(1+\Delta(s))$ pass on the origin. Therefore, the control system in (1) is unstable.

We have thus proved Theorem 3.

Remarks 2: This theorem is very interesting because the robust stability condition is identical whether the number of poles of the nominal time-delay plant $G_m(s)e^{-sT}$ in the closed right half plane is equal to that of the time-delay plant $G(s)e^{-sT}$ or not.

6. DESIGN METHOD FOR ROBUSTLY STABILIZING CONTROLLER

In this section, we present a design method for robustly stabilizing controller satisfying Theorem 3 using the parametrization of all stabilizing modified Smith predictor [16–18].

6.1 Design method for stable plant

In this subsection, we present a design method for robustly stabilizing controller satisfying (20) using the parametrization of all stabilizing modified Smith predictor for stable plant $G_m(s)e^{-sT}$.

According to [16, 17], the parametrization of all stabilizing modified Smith predictor for stable plant $G_m(s)e^{-sT}$ is written by

$$C(s) = \frac{Q(s)}{1 - Q(s)G_m(s)e^{-sT}}, \tag{25}$$

where $Q(s) \in RH_\infty$ is any function. Substituting (25) for (4), we have

$$S(s) = 1 - Q(s)G_m(s)e^{-sT}. \tag{26}$$

In order to satisfy (20), we settle $Q(s)$ in (25) as

$$Q(s) = \frac{1}{\bar{G}_{mo}(s)}q(s), \tag{27}$$

where $\bar{G}_{mo}(s)$ is an outer function of $\bar{G}_m(s)$ written by

$$\begin{aligned} \bar{G}_m(s) &= G_m(s)f(s) \\ &= \bar{G}_{mi}(s)\bar{G}_{mo}(s), \end{aligned} \tag{28}$$

$\bar{G}_{mi}(s) \in RH_\infty$ is inner function of $\bar{G}_m(s)$ satisfying $G_{mi}(0) = 1$, $f(s)$ is Pade approximation of e^{-sT} and $q(s)$ is a low pass filter satisfying $q(0) = 1$, because

$$q(s) = \frac{1}{(1 + s\tau)^\alpha} \tag{29}$$

is valid, where $\tau \in R$, and α is an arbitrary positive integer to make $q(s)/\bar{G}_{mo}(s)$ proper. τ and α in (29) are settled to satisfy (20).

6.2 Design method for minimum-phase plant

In this subsection, we present a design method for robustly stabilizing controller satisfying (20) using the parametrization of all stabilizing modified Smith predictor for minimum-phase plant $G_m(s)e^{-sT}$.

According to [16, 17], the parametrization of all stabilizing modified Smith predictor for minimum-phase plant $G_m(s)e^{-sT}$ is written by

$$C(s) = \frac{C_f(s)}{1 - C_f(s)G_m(s)e^{-sT}}, \tag{30}$$

where $C_f(s)$ is given by

$$C_f(s) = \frac{\bar{G}_u(s)}{G_u(s)}\left(1 + \frac{Q(s)}{G_u(s)}\right), \tag{31}$$

$\bar{G}_u(s) \in \mathcal{U}$ is any function satisfying

$$\bar{G}_u(s_i) = \frac{1}{G_s(s_i)e^{-s_iT}}, \tag{32}$$

$s_i(i = 1, \ldots, n)$ is an unstable pole of $G_m(s)$, $G_s(s)$ is a stable minimum-phase function of $G_m(s)$, that is, when $G_m(s)$ is factorized as

$$G_m(s) = G_u(s)G_s(s), \tag{33}$$

$G_u(s)$ is the unstable biproper minimum-phase function and $G_s(s)$ is the stable minimum-phase function and $Q(s) \in RH_\infty$ is any function. Substituting (30) for (4), we have

$$\begin{aligned} S(s) &= \left(1 - \bar{G}_u(s)G_s(s)e^{-sT}\right) \\ &\quad \cdot \left\{1 - \frac{\bar{G}_u(s)G_s(s)e^{-sT}}{G_u(s)\left(1 - \bar{G}_u(s)G_s(s)e^{-sT}\right)}Q(s)\right\}. \end{aligned} \tag{34}$$

In order to satisfy (20), we settle $Q(s)$ in (31) as

$$Q(s) = \frac{1}{\bar{G}_{mm}(s)}q(s), \tag{35}$$

where $\bar{G}_{mm}(s)$ is a proper minimum-phase unstable function of $\bar{G}_m(s)$ written by

$$\begin{aligned} \bar{G}_m(s) &= \frac{\bar{G}_u(s)G_s(s)f(s)}{G_u(s)\left(1 - \bar{G}_u(s)G_s(s)f(s)\right)} \\ &= \bar{G}_{ms}(s)\bar{G}_{mm}(s), \end{aligned} \tag{36}$$

$\bar{G}_{ms}(s) \in RH_\infty$ is a biproper non-minimum-phase stable function of $\bar{G}_m(s)$ satisfying $G_{ms}(0) = 1$, $f(s)$ is Pade approximation of e^{-sT} and $q(s)$ is a low pass filter satisfying $q(0) = 1$, because

$$q(s) = \frac{1}{(1 + s\tau)^\alpha} \tag{37}$$

is valid, where $\tau \in R$, and α is an arbitrary positive integer to make $q(s)/\bar{G}_{mm}(s)$ proper. τ and α in (37) are settled to satisfy (20).

6.3 Design method for non-minimum-phase plant

In this subsection, we present a design method for robustly stabilizing controller satisfying (20) using the parametrization of all stabilizing modified Smith predictor for non-minimum-phase plant $G_m(s)e^{-sT}$.

According to [18], the parametrization of all stabilizing modified Smith predictor for non-minimum-phase plant $G_m(s)e^{-sT}$ is written by

$$C(s) = \frac{C_f(s)}{1 - C_f(s)G_m(s)e^{-sT}}, \tag{38}$$

where $C_f(s)$ is given by

$$C_f(s) = \frac{1}{G_u(s)}\left(\bar{G}_u(s) + \frac{Q(s)}{G_u(s)}\right), \tag{39}$$

$\bar{G}_u(s) \in RH_\infty$ is any function satisfying

$$\bar{G}_u(s_i) = \frac{1}{G_s(s_i)\, e^{-s_i T}}, \tag{40}$$

$s_i(i = 1, \ldots, n)$ is an unstable pole of $G_m(s)$, $G_s(s)$ is a stable minimum-phase function of $G_m(s)$, that is, when $G_m(s)$ is factorized as

$$G_m(s) = G_u(s)G_s(s), \tag{41}$$

$G_u(s)$ is the unstable biproper minimum-phase function and $G_s(s)$ is the stable minimum-phase function and $Q(s) \in RH_\infty$ is any function. Substituting (38) for (4), we have

$$\begin{aligned} S(s) &= \left(1 - \bar{G}_u(s)G_s(s)e^{-sT}\right) \\ &\quad \cdot \left\{1 - \frac{G_s(s)e^{-sT}}{G_u(s)\left(1 - \bar{G}_u(s)G_s(s)e^{-sT}\right)}Q(s)\right\}. \end{aligned} \tag{42}$$

In order to satisfy (20), we settle $Q(s)$ in (39) as

$$Q(s) = \frac{1}{\bar{G}_{mm}(s)}q(s), \tag{43}$$

where $\bar{G}_{mm}(s)$ is a proper minimum-phase unstable function of $\bar{G}_m(s)$ written by

$$\begin{aligned} \bar{G}_m(s) &= \frac{G_s(s)f(s)}{G_u(s)\left(1 - \bar{G}_u(s)G_s(s)f(s)\right)} \\ &= \bar{G}_{ms}(s)\bar{G}_{mm}(s), \end{aligned} \tag{44}$$

$\bar{G}_{ms}(s) \in RH_\infty$ is a biproper non-minimum-phase stable function of $\bar{G}_m(s)$ satisfying $G_{ms}(0) = 1$, $f(s)$ is Pade approximation of e^{-sT} and $q(s)$ is a low pass filter satisfying $q(0) = 1$, because

$$q(s) = \frac{1}{(1 + s\tau)^\alpha} \tag{45}$$

is valid, where $\tau \in R$, and α is an arbitrary positive integer to make $q(s)/\bar{G}_{mm}(s)$ proper. τ and α in (45) are settled to satisfy (20).

7. NUMERICAL EXAMPLE

In this section, we show a numerical example to illustrate the effectiveness of the proposed method.

Consider the problem to design a robustly stabilizing controller for the set $\Omega_2(G_m(s)e^{-sT}, W(s))$, where

$$G_m(s) = \frac{1}{2s + 2.5}, \tag{46}$$

$T = 0.5$[sec] and

$$W(s) = \frac{0.8s + 3}{s + 1.5}. \tag{47}$$

Since $G_m(s)$ in (46) is stable, we design a robustly stabilizing controller using the method described in Section 6.1 . From (25), a robustly stabilizing controllers for the set $\Omega_2(G_m(s)e^{-sT}, W(s))$ is written by

$$C(s) = \frac{Q(s)}{1 - Q(s)G_m(s)e^{-sT}}, \tag{48}$$

where

$$Q(s) = \frac{1}{\bar{G}_{mo}(s)}q(s), \tag{49}$$

$\bar{G}_{mo}(s)$ is an outer function of $\bar{G}(s)$ in (28) and written by

$$\bar{G}_{mo}(s) = \frac{0.5s^2 - 60s + 2400}{s^3 + 121.3s^2 + 4950s + 6000}, \tag{50}$$

$q(s)$ is a low pass filter written by

$$q(s) = \frac{1}{1 + 0.15s}. \tag{51}$$

Next we confirm designed controller $C(s)$ in (48) satisfies (20). The gain plot of $S(s) = 1/(1 + C(s)G_m(s)e^{-sT})$ and $1/W(s)$ is shown in Fig. 1. Here, the solid line shows gain plot of $S(s) = 1/(1 + C(s)G_m(s)e^{-sT})$ and the dotted line shows that of $1/W(s)$. Figure 1 shows that the designed controller satisfies (20).

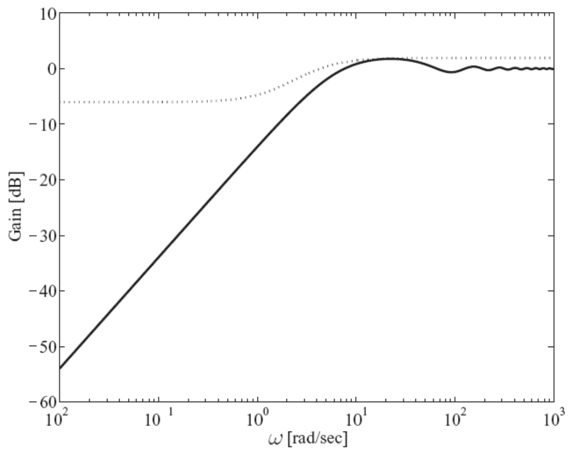

Fig.1: *Gain plot of $S(s)$ and $1/W(s)$*

Let $\Delta(s)$ be

$$\Delta(s) = \frac{s + 3.5}{s - 1}, \tag{52}$$

that is,

$$G(s) = \frac{1}{s - 1}. \tag{53}$$

From (46) and (53), the number of poles of $G(s)$ in the closed right half plane is not equal to that of $G_m(s)$. The fact that $\Delta(s)$ in (52) satisfies (19) is confirmed

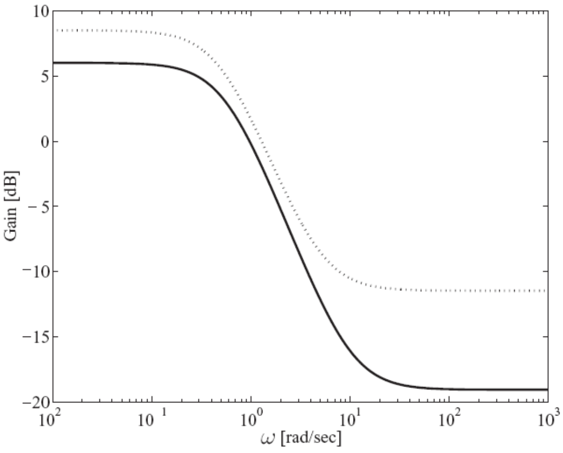

Fig.2: *Gain plot of* $W(s)$ *and* $\dfrac{\Delta(s)}{1+\Delta(s)}$

by showing gain plot of $\Delta(s)/(1+\Delta(s))$ and $W(s)$ as Fig. 2. Here, the solid line shows gain plot of $\Delta(s)/(1+\Delta(s))$ and dotted line shows that of $W(s)$. Figure 2 shows that $\Delta(s)$ in (52) satisfies (19). From (46) and (53), the number of zeros in the closed right half plane of $G_m(s)$ is equal to that of $G(s)$. In addition, the relative degree of $G_m(s)$ is equivalent to that of $G(s)$. Therefore, $G(s)$ in (53) is an element of the set $\Omega_2(G_m(s)e^{-sT}, W(s))$.

Using the controller in (48), step response for the nominal time-delay plant $G_m(s)e^{-sT}$ and that for the time-delay plant $G(s)e^{-sT}$ is shown in Fig. 3

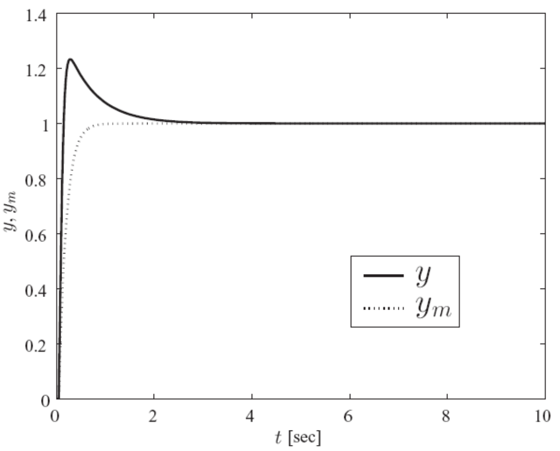

Fig.3: *Step response for* $G(s)e^{-sT}$ *and that for* $G_m(s)e^{-sT}$

Here, the solid line shows the output y of the time-delay plant $G(s)e^{-sT}$ and dotted line shows the output y_m of the nominal time-delay plant $G_m(s)e^{-sT}$. Figure 3 shows that the controller in (48) make the closed loop system in (1) stable, even if there exists uncertainty with uncertain number of poles in the closed right half plane.

8. CONCLUSION

In this paper, we considered the robust stability for single-input/single-output time-delay plants with a class of uncertainty. The necessary and sufficient robust stability condition for time-delay plants with such class of uncertainty was presented. the relation between time-delay plant and nominal time-delay plant to satisfy the robust stability condition was clarified. In addition, the necessary and sufficient robust stability condition in the case that the number of poles of the time-delay plant in the closed right half plane is not equal to that of the nominal time-delay plant, was shown. The robust stability condition presented in this paper was identical whether or not, the number of poles of the nominal time-delay plant is equal to that of the time-delay plant. In this framework, we can construct the control system with low sensitivity characteristics and robust stability for time-delay plants.

Subjects for a future study are (1) to clarify the robust stability condition for single-input/single-output time-delay plants with uncertain number of poles in the closed right half plane and uncertain time-delay, (2) to clarify the robust stability condition for multiple-input/multiple-output time-delay plants with uncertain number of poles in the closed right half plane and/or uncertain time-delay and (3) to apply proposed method for real systems, and so on.

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