A Design Method for Simple Repetitive Controllers for Time-Delay Plants

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ABSTRACT

The modified repetitive control system is a type of servomechanism for the periodic reference input. That is, the modified repetitive control system follows the periodic reference input with small steady state error, even if a periodic disturbance or uncertainty exists in the plant. Using previously proposed modified repetitive controllers, even if the plants does not includes time-delay, the transfer function from the periodic reference input to the output and that from the disturbance to the output have an infinite number of poles. When the transfer function from the periodic reference input to the output and that from the disturbance to the output have an infinite number of poles, it is difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From the practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic are easily specified. In order to specify the input-output characteristic and the disturbance attenuation characteristic easily, the transfer function from the periodic reference input to the output and that from the disturbance to the output are desirable to have a finite number of poles. Yamada et al. proposed the concept of simple repetitive control systems such that the controller works as a modified repetitive controller and the transfer function from the periodic reference input to the output and that from the disturbance to the output have a finite number of poles. In addition, Yamada et al. clarified the parametrization of all stabilizing simple repetitive controllers. However the method by Yamada et al. cannot be applied for time-delay plants. The purpose of this paper is to propose the parametrization of all stabilizing simple repetitive controllers for time-delay plants.

Keywords: repetitive control, time-delay plant, finite number of poles, parametrization

1. INTRODUCTION

The repetitive control system is a type of servomechanism for periodic reference input. That is, the repetitive control system follows the periodic reference input without steady state error, even if a periodic disturbance or uncertainty exists in the plant [1–13]. It is difficult to design stabilizing controllers for the plant, because the repetitive control system follows any periodic reference input without steady state error is a neutral type of time-delay control system [11]. In order to design a repetitive control system that follows any periodic reference input without steady state error, the plant needs to be biproper [3–11]. In practice the plant is strictly proper. Many design methods of repetitive control systems for the strictly proper plants are given [3–11]. These studies are divided into two types. One uses a low-pass filter [3–10] and the other uses an attenuator [11]. The latter is difficult to design because it uses a state variable time-delay in the repetitive controller [11]. The former has a simple structure and is easily designed. Therefore, the former type of repetitive control system is called the modified repetitive control systems [3–10].

Using the modified repetitive controllers in [3–10], even if the plants does not include time-delay, the transfer function from the periodic reference input to the output and that from the disturbance to the output have an infinite number of poles. When the transfer function from the periodic reference input to the output and that from the disturbance to the output have an infinite number of poles, it is difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From the practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic are easily specified. In order to specify the input-output characteristic and the disturbance attenuation characteristic easily, the transfer function from the periodic reference input to the output and that from the disturbance to the output are desirable to have a finite number of poles. Yamada et al. proposed the concept of simple repetitive control systems such that the controller works as a modified repetitive controller and the transfer function from the periodic reference input to the output and that from the disturbance to the output have a finite number of poles [17, 18]. In addition, Yamada et al. clarified the parametrization of all stabilizing simple repetitive controllers. However the method by Yamada et al. cannot be applied for time-delay plants.

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In this paper, we propose the concept of simple repetitive controllers for time-delay plants. In addition the parametrization of all stabilizing simple repetitive controllers for time-delay plants are clarified. This paper is organized as follows: In Section 2. , the concept of the simple repetitive controllers for time-delay plants is proposed. In addition, in Section 2., the problem considered in this paper is described. In Section 3., the controller structure of simple repetitive controllers is clarified. In Section 4. and Section 5., using the result in Section 3., the parametrization of all stabilizing simple repetitive controllers for stable time-delay plants and for unstable time-delay plants are clarified. In Section 6., the control characteristics using simple repetitive controllers for time-delay plants are written. In Section 7., a design procedure of stabilizing simple repetitive controllers for time-delay plants is presented. In Section 8., we show a numerical example to illustrate the effectiveness of the proposed method.

Notation

R	The set of real numbers.
$R(s)$	The set of real rational functions with s.
RH_∞	The set of stable proper real rational functions.

2. SIMPLE REPETITIVE CONTROL SYSTEMS AND PROBLEM FORMULATION

Consider the unity feedback control system in

$$\left\{ \begin{array}{rcl} y & = & G(s)e^{-sL}u + d \\ u & = & C(s)\,(r-y) \end{array} \right. , \quad (1)$$

where $G(s)e^{-sL}$ is the plant, $L > 0$ is the time-delay, $G(s) \in R(s)$, $C(s)$ is the controller, $u \in R$ is the control input, $y \in R$ is the output, $d \in R$ is the disturbance and $r \in R$ is the periodic reference input with period $T > 0$ satisfying

$$r(t+T) = r(t) \quad (\forall t \geq 0). \quad (2)$$

L is assumed to satisfy $T \geq L$. According to [3–10], when the plant $G(s)e^{-sL}$ has an uncertainty, in order for the output y to follow the periodic reference input r with period T with small steady state error, the controller $C(s)$ must have following structure

$$C(s) = \hat{C}(s) + \frac{\bar{C}(s)e^{-sT}}{1-q(s)e^{-sT}}, \quad (3)$$

where $\hat{C}(s)$ and $\bar{C}(s)$ are controller and $q(s) \in R(s)$ is a proper low-pass filter satisfying $q(0) = 1$. The controller in (3) is called by the modified repetitive controller [3–10]. According to [3–10], if the low-pass filter $q(s)$ satisfies

$$1 - q(j\omega_i) \simeq 0 \quad (i = 0, \ldots, N_{max}), \quad (4)$$

where $\omega_i (i = 0, \ldots, N_{max})$ is the frequency component of the periodic reference input r written by

$$\omega_i = \frac{2\pi}{T} i \quad (i = 0, \ldots, N_{max}), \quad (5)$$

N_{max} is a positive integer and $\omega_{N_{max}}$ is the maximum frequency component of the periodic reference input r, then the output y in (1) follows the periodic reference input r with a small steady-state error.

Using the modified repetitive controller $C(s)$ in (3), the transfer function from the periodic reference input r to the output y and that from the disturbance d to the output y in (1) have an infinite number of poles. When the transfer function from the periodic reference input to the output and that from the disturbance to the output have an infinite number of poles, it is difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From the practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic are easily specified. In order to specify the input-output characteristic and the disturbance attenuation characteristic easily, the transfer function from the periodic reference input to the output and that from the disturbance to the output are desirable to have a finite number of poles. From above practical requirement, we propose the concept of the simple repetitive controller for time-delay plants as follows:

Definition 1: (simple repetitive controller for time-delay plants)
We call the controller $C(s)$ the simple repetitive controller for time-delay plants, if following expressions hold true:

1. The controller $C(s)$ works as a modified repetitive controller. That is, the controller $C(s)$ is written by (3), where $q(s) \in R(s)$ satisfies $q(0) = 1$.
2. The controller $C(s)$ makes the transfer function from the periodic reference input r to the output y in (1) and that from the disturbance d to the output y in (1) have a finite number of poles.

The problem considered in this paper is to propose the parametrization of all stabilizing simple repetitive controllers for time-delay plants. That is, we find all controllers written by the form in (3) to make the transfer function from the periodic reference input r to the output y in (1) and that from the disturbance d to the output y in (1) have a finite number of poles.

3. CONTROLLER STRUCTURE OF SIMPLE REPETITIVE CONTROLLERS FOR TIME-DELAY PLANTS

In this section, the controller structure of simple repetitive controllers for time-delay plants defined in Definition 1 is clarified.

From Definition 1, since the transfer function from the periodic reference input r to the output y has a finite number of poles, the transfer function from the periodic reference input r to the output y in (1) is written by

$$\frac{y}{r} = \bar{G}_1(s)e^{-sL} + \bar{G}_2(s)e^{-sT} + \bar{G}_3(s)e^{-s(T+L)}, \quad (6)$$

where $\bar{G}_i(s) \in RH_\infty (i = 1, \ldots, 3)$. Using the controller $C(s)$ in (3), on the controller $C(s)$ in (3) to make the transfer function from the periodic reference input r to the output y be written as (6), following theorem is satisfied.

Theorem 1: In order for the controller $C(s)$ in (3) to make the transfer function from the periodic reference input r to the output y be written as (6), the controller $C(s)$ in (3) has the structure in

$$C(s) = \frac{C_1(s) + C_2(s)e^{-s(T-L)} + C_3(s)e^{-sT}}{\left(1 + C_4(s)e^{-sL}\right)\left(1 - q(s)e^{-sT}\right)}, \quad (7)$$

where $C_i(s) \in R(s)(i = 1, \ldots, 4)$.

Proof: Since the controller $C(s)$ is written by (3), the transfer function from the periodic reference input r to the error $e = r - y$ in (1) is given by

$$\begin{aligned} \frac{e}{r} &= \frac{1}{1 + C(s)G(s)e^{-sL}} \\ &= \frac{1 - q(s)e^{-sT}}{1 - q(s)e^{-sT} + \left\{\hat{C}(s)\left(1 - q(s)e^{-sT}\right) + \bar{C}(s)e^{-sT}\right\} G(s)e^{-sL}}. \end{aligned} \quad (8)$$

From the assumption that the transfer function from the periodic reference input r to the output y is written by (6), the transfer function in (8) is equal to

$$\frac{e}{r} = 1 - \bar{G}_1(s)e^{-sL} - \bar{G}_2(s)e^{-sT} - \bar{G}_3(s)e^{-s(T+L)}. \quad (9)$$

From (8) and (9), we find that (8) is rewritten as

$$\begin{aligned} \frac{e}{r} &= \frac{1}{1 + C(s)G(s)e^{-sL}} \\ &= \left(1 - q(s)e^{-sT}\right)\left(1 + H(s)e^{-sL}\right). \end{aligned} \quad (10)$$

This equation implies that $C(s)$ is written by

$$\begin{aligned} C(s) &= \frac{-\dfrac{H(s)}{G(s)} + \dfrac{q(s)}{G(s)}e^{-s(T-L)} + \dfrac{q(s)H(s)}{G(s)}e^{-sT}}{\left(1 + H(s)e^{-sL}\right)\left(1 - q(s)e^{-sT}\right)} \\ &= \frac{C_1(s) + C_2(s)e^{-s(T-L)} + C_3(s)e^{-sT}}{\left(1 + C_4(s)e^{-sL}\right)\left(1 - q(s)e^{-sT}\right)}, \end{aligned} \quad (11)$$

where

$$C_1(s) = -\frac{H(s)}{G(s)}, \quad (12)$$

$$C_2(s) = \frac{q(s)}{G(s)}, \quad (13)$$

$$C_3(s) = \frac{q(s)H(s)}{G(s)} \quad (14)$$

and

$$C_4(s) = H(s). \quad (15)$$

We have thus proved Theorem 1. ■

4. THE PARAMETRIZATION OF ALL STABILIZING SIMPLE REPETITIVE CONTROLLERS, WHEN THE TIME-DELAY PLANT IS STABLE

In this section, we propose the parametrization of all stabilizing simple repetitive controllers for time-delay plants in the case that $G(s)e^{-sL}$ is stable.

The parametrization of all stabilizing simple repetitive controllers when $G(s)e^{-sL}$ is stable is summarized in the following theorem.

Theorem 2: It is assumed that $G(s)e^{-sL}$ is stable, $q(s) \in RH_\infty$ and $q(s)/G(s) \in RH_\infty$. The parametrization of all stabilizing simple repetitive controllers $C(s)$ is written by

$$C(s) = \frac{Q(s) + \dfrac{q(s)}{G(s)}e^{-s(T-L)} - Q(s)q(s)e^{-sT}}{\left(1 - Q(s)G(s)e^{-sL}\right)\left(1 - q(s)e^{-sT}\right)}, \quad (16)$$

where $Q(s) \in RH_\infty$ is any function.

Proof: First, the necessity is shown. From Theorem 1, the controller $C(s)$ to make the transfer function from the periodic reference input r to the output y be in (6) is written by (7). Therefore, we show that if the controller $C(s)$ in (7) works as a modified repetitive controllers, then the controller $C(s)$ is written by (16). From the assumption that the controller $C(s)$ in (7) makes the transfer function from r to y of the control system in (1) have a finite number of poles,

$$\begin{aligned} &\frac{C(s)G(s)e^{-sL}}{1 + C(s)G(s)e^{-sL}} \\ &= \frac{\left(C_1(s) + C_2(s)e^{-s(T-L)} + C_3(s)e^{-sT}\right)G(s)e^{-sL}}{1 + e^{-sT}\left(G(s)C_2(s) - q(s)\right) + e^{-sL}\left(C_4(s) + C_1(s)G(s)\right) + e^{-s(T+L)}\left(G(s)C_3(s) - q(s)C_4(s)\right)} \end{aligned} \quad (17)$$

has a finite number of poles. This implies that

$$C_2(s) = \frac{q(s)}{G(s)}, \quad (18)$$

$$C_4(s) = -C_1(s)G(s) \tag{19}$$

and

$$C_3(s) = -C_1(s)q(s). \tag{20}$$

Substituting (18), (19) and (20) for (7), $C(s)$ must take the form

$$C(s) = \frac{C_1(s) + \dfrac{q(s)}{G(s)}e^{-s(T-L)} - C_1(s)q(s)e^{-sT}}{(1 - C_1(s)G(s)e^{-sL})\,(1 - q(s)e^{-sT})}. \tag{21}$$

From the assumption that $C(s)$ in (7) makes the control system in (1) stable, $C(s)G(s)e^{-sL}/(1 + C(s)G(s)e^{-sL})$, $C(s)/(1 + C(s)G(s)e^{-sL})$, $G(s)e^{-sL}/(1+C(s)G(s)e^{-sL})$ and $1/(1+C(s)G(s)e^{-sL})$ are stable. From simple manipulation and (21), we have

$$\begin{aligned}\frac{C(s)G(s)e^{-sL}}{1 + C(s)G(s)e^{-sL}} &= C_1(s)G(s)e^{-sL} + q(s)e^{-sT} \\ &\quad - C_1(s)G(s)q(s)e^{-s(T+L)},\end{aligned} \tag{22}$$

$$\begin{aligned}\frac{C(s)}{1 + C(s)G(s)e^{-sL}} &= C_1(s) + \frac{q(s)}{G(s)}e^{-s(T-L)} - C_1(s)q(s)e^{-sT},\end{aligned} \tag{23}$$

$$\begin{aligned}\frac{G(s)e^{-sL}}{1 + C(s)G(s)e^{-sL}} &= \left(1 - C_1(s)G(s)e^{-sL}\right)\left(1 - q(s)e^{-sT}\right)G(s)e^{-sL}\end{aligned} \tag{24}$$

and

$$\begin{aligned}\frac{1}{1 + C(s)G(s)e^{-sL}} &= \left(1 - C_1(s)G(s)e^{-sL}\right)\left(1 - q(s)e^{-sT}\right).\end{aligned} \tag{25}$$

It is obvious that the necessary condition that the transfer functions in (22), (23), (24) and (25) are stable is $C_1(s) \in RH_\infty$. Let $C_1(s) = Q(s)$, we have (16). Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, we show that if $C(s)$ is written by (16), then the controller $C(s)$ makes the control system in (1) stable and makes the transfer function from r to y of the control system in (1) have a finite number of poles. After simple manipulation, we have

$$\begin{aligned}\frac{C(s)G(s)e^{-sL}}{1 + C(s)G(s)e^{-sL}} &= Q(s)G(s)e^{-sL} + q(s)e^{-sT} \\ &\quad -Q(s)G(s)q(s)e^{-s(T+L)},\end{aligned} \tag{26}$$

$$\begin{aligned}\frac{C(s)}{1 + C(s)G(s)e^{-sL}} &= Q(s) + \frac{q(s)}{G(s)}e^{-s(T-L)} - Q(s)q(s)e^{-sT},\end{aligned} \tag{27}$$

$$\begin{aligned}\frac{G(s)e^{-sL}}{1 + C(s)G(s)e^{-sL}} &= \left(1 - Q(s)G(s)e^{-sL}\right)\left(1 - q(s)e^{-sT}\right)G(s)e^{-sL}\end{aligned} \tag{28}$$

and

$$\begin{aligned}\frac{1}{1 + C(s)G(s)e^{-sL}} &= \left(1 - Q(s)G(s)e^{-sL}\right)\left(1 - q(s)e^{-sT}\right).\end{aligned} \tag{29}$$

From the assumption that $G(s)$ is stable, $Q(s) \in RH_\infty$, $q(s) \in RH_\infty$ and $q(s)/G(s) \in RH_\infty$, the transfer functions in (26), (27), (28) and (29) are stable. In addition, because the transfer function from r to y of control system in (1) is given by (26), the transfer function from r to y of the control system in (1) has a finite number of poles. Thus, the sufficiency has been shown.

We have thus proved Theorem 2. ■

5. THE PARAMETRIZATION OF ALL STABILIZING SIMPLE REPETITIVE CONTROLLERS, WHEN THE TIME-DELAY PLANTS IS UNSTABLE

In this section, we propose the parametrization of all stabilizing simple repetitive controllers for time-delay plants when $G(s)e^{-sL}$ is unstable.

The parametrization of all stabilizing simple repetitive controllers for time-delay plants when $G(s)e^{-sL}$ is unstable is summarized in the following theorem.

Theorem 3: It are assumed that $G(s)e^{-sL}$ is unstable, $q(s) \in RH_\infty$ and $q(s)/G(s) \in RH_\infty$. For simplicity, the unstable poles of $G(s)e^{-sL}$ are assumed to be distinct. That is, when $s_i (i = 1, \ldots, n)$ denote unstable poles of $G(s)$, $s_i \neq s_j (i \neq j; i = 1, \ldots, n; j = 1, \ldots, n)$. Under these assumptions, the parametrization of simple repetitive controller is written by

$$C(s) = \frac{D(s)\left(\bar{G}(s) + D(s)Q(s)\right) + \dfrac{q(s)}{G(s)}e^{-s(T-L)}}{\left\{1 - \left(\bar{G}(s) + D(s)Q(s)\right)N(s)e^{-sL}\right\}} \frac{-D(s)\left(\bar{G}(s) + D(s)Q(s)\right)q(s)e^{-sT}}{\cdot\left(1 - q(s)e^{-sT}\right)}, \tag{30}$$

where $N(s) \in RH_\infty$, $D(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \frac{N(s)}{D(s)}, \tag{31}$$

$\bar{G}(s) \in RH_\infty$ is satisfying

$$\bar{G}(s_i) = \frac{1}{N(s_i)e^{-s_iL}} \quad (i = 1, 2, \ldots, n) \tag{32}$$

and $Q(s) \in RH_\infty$ is any function.
The proof of Theorem 3 requires the following Lemma.

Lemma 1: It is assumed that $G(s)e^{-sL}$ is unstable. For simplicity, the unstable poles $s_i (i = 1, \ldots, n)$ of $G(s)$ are assumed to be distinct. Under these assumptions, there exists $\bar{G}_u(s) \in RH_\infty$ satisfying (32).

Proof: For easy explanation, we denote

$$g_i = \frac{1}{N(s_i)e^{-s_i L}} \quad (i = 1, \ldots, n). \tag{33}$$

When $\bar{G}_u(s)$ is constructed by

$$\bar{G}_u(s) = H_{n-1}(s)\left(1 + \prod_{j=1}^{n-1} \frac{s - s_j}{s + s_j} \cdot f_n\right), \tag{34}$$

from easy computation, we can confirm that $\bar{G}_u(s)$ in (34) is included in RH_∞ and satisfies (32), where

$$H_1(s) = g_1, \tag{35}$$

$$H_i(s) = H_{i-1}(s)\left(1 + \prod_{j=1}^{i-1} \frac{s - s_j}{s + s_j} \cdot f_i\right) \quad (i = 2, \ldots, n-1) \tag{36}$$

and

$$f_i = \prod_{j=1}^{i-1} \frac{s_i + s_j}{s_i - s_j} \cdot \left(\frac{g_i}{H_{i-1}(s_i)} - 1\right) (i = 1, \ldots, n). \tag{37}$$

We have thus proved Lemma 1. ■

Using this Lemma, we shall show the proof of Theorem 3.

Proof: First, the necessity is shown. From Theorem 1, the controller $C(s)$ to make the transfer function from the periodic reference input r to the output y be in (6) is written by (7). Therefore, we show that if the controller $C(s)$ in (7) works as a modified repetitive controllers, then the controller $C(s)$ is written by (30). From the assumption that the controller $C(s)$ in (7) makes the transfer function from r to y of the control system in (1) have a finite number of poles,

$$\frac{C(s)G(s)e^{-sL}}{1 + C(s)G(s)e^{-sL}} = \frac{\left(C_1(s) + C_2(s)e^{-s(T-L)} + C_3(s)e^{-sT}\right) \cdot G(s)e^{-sL}}{1 + (G(s)C_2(s) - q(s))\, e^{-sT} + (C_4(s) + C_1(s)G(s))\, e^{-sL} + (G(s)C_3(s) - q(s)C_4(s))\, e^{-s(T+L)}} \tag{38}$$

has a finite number of poles. This implies that

$$C_2(s) = \frac{q(s)}{G(s)}, \tag{39}$$

$$C_4(s) = -C_1(s)G(s) \tag{40}$$

and

$$C_3(s) = -q(s)C_1(s) \tag{41}$$

are satisfied, that is, $C(s)$ is rewritten by

$$C(s) = \frac{C_1(s) + \dfrac{q(s)}{G(s)}e^{-s(T-L)} - q(s)C_1(s)e^{-sT}}{(1 - C_1(s)G(s)e^{-sL})\,(1 - q(s)e^{-sT})}. \tag{42}$$

From the assumption that the controller $C(s)$ in (7) makes the control system in (1) stable, $C(s)G(s)e^{-sL}/(1+C(s)G(s)e^{-sL})$, $C(s)/(1+C(s)G(s)e^{-sL})$, $G(s)e^{-sL}/(1+C(s)G(s)e^{-sL})$ and $1/(1+C(s)G(s)e^{-sL})$ are stable. From simple manipulation and (42), we have

$$\frac{C(s)G(s)e^{-sL}}{1 + C(s)G(s)e^{-sL}} = C_1(s)G(s)e^{-sL} + q(s)e^{-sT} - q(s)C_1(s)G(s)e^{-s(T+L)}, \tag{43}$$

$$\frac{C(s)}{1 + C(s)G(s)e^{-sL}} = C_1(s) + \frac{q(s)}{G(s)}e^{-s(T-L)} - q(s)C_1(s)e^{-sT}, \tag{44}$$

$$\frac{G(s)e^{-sL}}{1 + C(s)G(s)e^{-sL}} = \left(1 - C_1(s)G(s)e^{-sL}\right)\left(1 - q(s)e^{-sT}\right)G(s)e^{-sL} \tag{45}$$

and

$$\frac{1}{1 + C(s)G(s)e^{-sL}} = \left(1 - C_1(s)G(s)e^{-sL}\right)\left(1 - q(s)e^{-sT}\right). \tag{46}$$

Since the transfer function in (43), (44) and (46) are stable, from $q(s) \in RH_\infty$ and $q(s)/G(s) \in RH_\infty$, we have $C_1(s)G(s) \in RH_\infty$ and $C_1(s) \in RH_\infty$. This implies that $C_1(s)$ is written by

$$C_1(s) = \hat{C}_1(s)D(s), \tag{47}$$

where $\hat{C}_1(s) \in RH_\infty$. From the assumption that the transfer function in (45) is stable and from (47), for $s_i (i = 1, \ldots, n)$ which are unstable poles of $G(s)$,

$$1 - C_1(s_i)G(s_i)e^{-s_i L} = 1 - \hat{C}_1(s_i)N(s_i)e^{-s_i L} = 0 \; (i = 1, \ldots n) \tag{48}$$

must be satisfied. From Lemma 1, there exists $\bar{G}(s) \in RH_\infty$ satisfying

$$1 - \bar{G}(s_i)N(s_i)e^{-s_iL} = 0 \, (i = 1, \dots n). \qquad (49)$$

Note that (49) is equivalent to (32). From (48) and (49),

$$\hat{C}_1(s_i) - \bar{G}(s_i) = 0 \quad (i = 1, \dots, n) \qquad (50)$$

is satisfied. Equation (50) implies that $s_i \, (1, \dots, n)$, which are unstable poles of $G(s)$, are zeros of $\hat{C}_1(s) - \bar{G}(s)$, because $\hat{C}_1(s) \in RH_\infty$ and $\bar{G}(s) \in RH_\infty$. When we rewrite $\hat{C}_1(s) - \bar{G}(s)$ as

$$\hat{C}_1(s) - \bar{G}(s) = D(s)Q(s), \qquad (51)$$

then $Q(s) \in RH_\infty$, because $\hat{C}_1(s) \in RH_\infty$, $\bar{G}(s) \in RH_\infty$ and $D(s) \in RH_\infty$. From (51), (47) and (42), we have (30). In this way, it is shown that if the controller $C(s)$ in (7) makes the transfer function from r to y of the control system in (1) have a finite number of poles, then $C(s)$ is written as (30).

Next, the sufficiency is shown. That is, we show that if $C(s)$ is written by (30), then the controller $C(s)$ makes the control system in (1) stable and makes the transfer function from r to y of the control system in (1) have a finite number of poles. After simple manipulation, we have

$$\begin{aligned} &\frac{C(s)G(s)e^{-sL}}{1 + C(s)G(s)e^{-sL}} \\ &\quad = \left(\bar{G}(s) + D(s)Q(s)\right) N(s)e^{-sL} + q(s)e^{-sT} \\ &\qquad - \left(\bar{G}(s) + D(s)Q(s)\right) N(s)q(s)e^{-s(T+L)}, \end{aligned} \qquad (52)$$

$$\begin{aligned} &\frac{C(s)}{1 + C(s)G(s)e^{-sL}} \\ &\quad = D(s) \left(\bar{G}(s) + D(s)Q(s)\right) + \frac{q(s)}{G(s)} e^{-s(T-L)} \\ &\qquad - D(s) \left(\bar{G}(s) + D(s)Q(s)\right) q(s)e^{-sT}, \end{aligned} \qquad (53)$$

$$\begin{aligned} &\frac{G(s)e^{-sL}}{1 + C(s)G(s)e^{-sL}} \\ &\quad = \left\{1 - \left(\bar{G}(s) + D(s)Q(s)\right) N(s)e^{-sL}\right\} \\ &\qquad \cdot \left(1 - q(s)e^{-sT}\right) G(s)e^{-sL} \end{aligned} \qquad (54)$$

and

$$\begin{aligned} &\frac{1}{1 + C(s)G(s)e^{-sL}} \\ &\quad = \left\{1 - \left(\bar{G}(s) + D(s)Q(s)\right) N(s)e^{-sL}\right\} \\ &\qquad \cdot \left(1 - q(s)e^{-sT}\right). \end{aligned} \qquad (55)$$

Since $\bar{G}(s) \in RH_\infty$, $Q(s) \in RH_\infty$, $q(s)/G(s) \in RH_\infty$, $q(s) \in RH_\infty$, $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$, the transfer functions in (52), (53), (55) are stable. If the transfer function in (54) is unstable, unstable poles of the transfer function in (54) are that of $G(s)$. From the assumption that $\bar{G}(s)$ satisfies (32), the unstable pole of $G(s)$ are not that of $\{1-(\bar{G}(s)+D(s)Q(s))N(s)e^{-sL}\}(1-q(s)e^{-sT})G(s)e^{-sL}$. Therefore, the transfer function in (54) is stable. In addition, because the transfer function from r to y of the control system in (1) is given by (52) and $\bar{G}(s) \in RH_\infty$, $Q(s) \in RH_\infty$, $q(s) \in RH_\infty$, $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$, the transfer function from r to y of the control system in (1) has a finite number of poles.

We have thus proved Theorem 3. ■

6. CONTROL CHARACTERISTICS

In this section, we present control characteristics of simple repetitive control system using the parametrization of all stabilizing simple repetitive controllers for time-delay plants in Theorem 3.

First, the input-output characteristic is shown. Using the parametrization of all stabilizing simple repetitive controllers for time-delay plants in Theorem 3, the transfer function from the periodic reference input r to the output y and that from the periodic reference input r to the error $e = r - y$ in (1) are written by

$$\begin{aligned} \frac{y}{r} &= \left(\bar{G}(s) + D(s)Q(s)\right) N(s)e^{-sL} + q(s)e^{-sT} \\ &\quad - \left(\bar{G}(s) + D(s)Q(s)\right) N(s)q(s)e^{-s(T+L)} \end{aligned} \qquad (56)$$

and

$$\begin{aligned} \frac{e}{r} &= \left\{1 - \left(\bar{G}(s) + D(s)Q(s)\right) N(s)e^{-sL}\right\} \\ &\quad \cdot \left(1 - q(s)e^{-sT}\right), \end{aligned} \qquad (57)$$

respectively. From (57), if $q(s) \in RH_\infty$ is settled to satisfy (4), then the output y follows the periodic reference input r with small steady state error.

Next the disturbance attenuation characteristic is shown. Using the parametrization of all stabilizing simple repetitive controllers for time-delay plants in Theorem 3, the transfer function from the disturbance d to the output y in (1) is written by

$$\begin{aligned} \frac{y}{d} &= \left\{1 - \left(\bar{G}(s) + D(s)Q(s)\right) N(s)e^{-sL}\right\} \\ &\quad \cdot \left(1 - q(s)e^{-sT}\right). \end{aligned} \qquad (58)$$

From (58), if $q(s) \in RH_\infty$ is settled to satisfy (4), then the periodic disturbance d with period T is attenuated effectively. When the frequency component ω of the disturbance d is different from that of the periodic reference input r, that is $\omega \neq \omega_i$, the disturbance d cannot be attenuated even if

$$1 - q(j\omega) \simeq 0, \qquad (59)$$

because

$$e^{-j\omega T} \neq 1 \tag{60}$$

and

$$1 - q(j\omega)e^{-j\omega T} \neq 0. \tag{61}$$

In order to attenuate the frequency component ω of the disturbance d that is different from that of the periodic reference input r, we need to settle $Q(s)$, satisfying

$$1 - \left(\bar{G}(j\omega) + D(j\omega)Q(j\omega)\right) N(j\omega)e^{-j\omega L} \simeq 0. \tag{62}$$

From the above discussion, we find that the role of $q(s)$ is to specify the input-output characteristic for the periodic reference input r and to specify the disturbance attenuation characteristic for the frequency component of the periodic disturbance d with period T that is equivalent to that of the periodic reference input r and that of $Q(s)$ is to specify the disturbance attenuation characteristic for the frequency component of the disturbance d that is different from that of the periodic reference input r.

7. DESIGN PROCEDURE

In this section, a design procedure of stabilizing simple repetitive controller satisfying Theorem 2 and Theorem 3 is presented.

When $G(s)$ is stable, the procedure of stabilizing simple repetitive controller is summarized as follows.

Procedure 1

Step 1) $Q(s) \in RH_\infty$ is selected so that for the frequency component ω_d of the disturbance d,

$$|1 - Q(j\omega_d)G(j\omega_d)e^{-j\omega_d L}| \simeq 0 \tag{63}$$

is satisfied. When frequency components of disturbance d are denoted by $\omega_{di}(i = 1, \dots, n_d)$, we present two design methods of $Q(s)$ satisfying $|1 - Q(j\omega_{di})G(j\omega_{di})e^{-j\omega_{di}L}| = 0(i = 1, \dots, n_d)$ as follows:

a $Q(s)$ satisfying $|1 - Q(j\omega_{di})G(j\omega_{di})e^{-j\omega_{di}L}| = 0(i = 1, \dots, n_d)$ is constructed by

$$\begin{aligned} Q(s) &= H_{d(n_d-1)}(s) \\ &\quad \cdot \left(1 + \prod_{j=1}^{n_d-1} \frac{s - j\omega_{dj}}{s + j\omega_{dj} + \alpha_{dj}} \cdot f_{dn_d}\right), \end{aligned} \tag{64}$$

where

$$H_{d1}(s) = g_{d1}, \tag{65}$$

$$\begin{aligned} &H_{di}(s) \\ &= H_{d(i-1)}(s)\left(1 + \prod_{j=1}^{i-1} \frac{s - j\omega_{dj}}{s + j\omega_{dj} + \alpha_{dj}} \cdot f_{di}\right) \\ &\quad (i = 2, \dots, n_d - 1), \end{aligned} \tag{66}$$

$$g_{di} = \frac{1}{G(j\omega_{di})e^{-j\omega_{di}L}}(i = 1, \dots, n_d), \tag{67}$$

$$\begin{aligned} &f_{di} \\ &= \prod_{j=1}^{i-1} \frac{j\omega_{di} + \alpha_{di} + j\omega_{dj}}{j\omega_{di} - j\omega_{dj}} \\ &\quad \cdot \left(\frac{g_i}{H_{d(i-1)}(j\omega_{di})} - 1\right) \quad (i = 1, \dots, n_d) \end{aligned} \tag{68}$$

and $\alpha_{di} > 0(i = 1, \dots, n_d)$.

b $Q(s)$ satisfying $|1 - Q(j\omega_{di})G(j\omega_{di})e^{-j\omega_{di}L}| = 0(i = 1, \dots, n_d)$ is constructed as follows: A candidate of $Q(s)$ satisfying $|1-Q(j\omega_{di})G(j\omega_{di})e^{-j\omega_{di}L}| = 0(i = 1, \dots, n_d)$ is written by

$$Q(s) = \frac{\sum_{i=0}^{n_d} a_i s^i}{\prod_{i=1}^{n_d}(1 + \tau_i s)}, \tag{69}$$

where $\tau_i(i = 1, \dots, n_d)$ is arbitrary small number and $a_i(i = 0. \dots, n_d)$ is selected satisfying $|1 - Q(j\omega_{di})G(j\omega_{di})e^{-j\omega_{di}L}| = 0(i = 1, \dots, n_d)$.

Step 2) $q(s) \in RH_\infty$ is selected so that for the frequency component $\omega_i(i = 0, \dots, N_{max})$ of the periodic reference input r, $1 - q(j\omega_i) \simeq 0$ is satisfied. For easy design $q(s)$ satisfying $1 - q(j\omega_i) \simeq 0$, $q(s)$ is designed as

$$q(s) = \frac{1}{(1 + \tau s)^\alpha}, \tag{70}$$

where α is an arbitrary positive integer and $\tau > 0$ is a small number.

Step 3) Substituting above $Q(s)$ and $q(s)$ for (16), we obtain a stabilizing simple repetitive controller.

When $G(s)$ is unstable, the procedure of stabilizing simple repetitive controller is summarized as follows.

Procedure 2

Step 1) Obtain the coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ of $G(s)$ satisfying (31).

Step 2) $\bar{G}(s) \in RH_\infty$ is selected satisfying (32).

Step 3) $Q(s) \in RH_\infty$ is selected so that for the frequency component ω_d of the disturbance d, $|1 - \left(\bar{G}(j\omega_d) + D(j\omega_d)Q(j\omega_d)\right) N(j\omega_d)e^{-j\omega_d L}|$ is effectively small. When frequency components of disturbance d are denoted by $\omega_{di}(i = 1, \dots, n_d)$, $Q(s)$ satisfying $|1-\left(\bar{G}(j\omega_{di}) + D(j\omega_{di})Q(j\omega_{di})\right) N(j\omega_{di})e^{-j\omega_{di}L}| = 0(i = 1, \dots, n_d)$ is constructed by (64), where $H_{d1}(s)$, $H_{di}(s)(i = 2, \dots, n_d - 1)$, $f_{di}(i = 1, \dots, n_d)$ and $g_{di}(i = 1, \dots, n_d)$ are written by (65), (66), (68) and

$$\begin{aligned} &g_{di} \\ &= \frac{1 - \bar{G}(j\omega_{di})N(j\omega_{di})e^{-j\omega_{di}L}}{D(j\omega_{di})N(j\omega_{di})e^{-j\omega_{di}L}}(i = 1, \dots, n_d) \end{aligned} \tag{71}$$

respectively. $Q(s)$ satisfying $|1-\big(\bar{G}(j\omega_{di}) + D(j\omega_{di})Q(j\omega_{di}$ $0(i = 1, \dots, n_d)$ is also designed using the method described in Step 1) (b) in Procedure 1.

Step 4) $q(s) \in RH_\infty$ is selected so that for the frequency component $\omega_i(i = 0, \dots, N_{max})$ of the periodic reference input r, $1 - q(j\omega_i) \simeq 0$ is satisfied. For easy design $q(s)$ satisfying $1 - q(j\omega_i) \simeq 0$, $q(s)$ is designed as (70), where α is an arbitrary positive integer and $\tau > 0$ is a small number.

Step 5) Substituting above $N(s)$, $D(s)$, $\bar{G}(s)$, $Q(s)$ and $q(s)$ for (30), we obtain a stabilizing simple repetitive controller.

8. NUMERICAL EXAMPLE

In this section, a numerical example is illustrated to show the effectiveness of the proposed method.

Consider the problem to obtain the parametrization of all stabilizing simple repetitive controllers for the plant $G(s)e^{-sL}$ written by

$$G(s)e^{-sL} = \frac{s+1}{(s+2)(s+3)}e^{-0.3s}, \tag{72}$$

that follows the periodic reference input r with period $T = 1$[sec] with small steady state error, where $G(s)$ and L are

$$G(s) = \frac{s+1}{(s+2)(s+3)} \tag{73}$$

and

$$L = 0.3, \tag{74}$$

respectively.

From Theorem 2, the parametrization of all stabilizing simple repetitive controllers for $G(s)e^{-sL}$ in (72) is given by (16), where $Q(s) \in RH_\infty$ is any function.

In order for the disturbance $d = \sin(\pi t)$ for which the frequency component is different from that of the periodic reference input r, to be attenuated effectively, $Q(s)$ is selected satisfying (63) as

$$Q(s) = \frac{s^2 + 1.365s + 5.569}{(0.1s+1)(0.2s+1)}. \tag{75}$$

In order for the output y to follow the periodic reference input $r = \sin(2\pi t)$ and in order for the disturbance $d = \sin(2\pi t)$, for which the frequency component is equivalent to that of the periodic reference input r to be attenuated effectively, $q(s)$ in (16) is selected by

$$q(s) = \frac{1}{0.001s+1}. \tag{76}$$

Using above-mentioned parameters, we have the simple repetitive controller. When the designed simple repetitive controller $C(s)$ is used, the response of

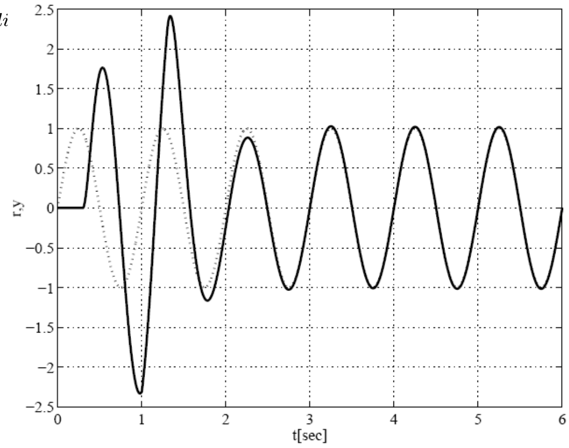

Fig.1: *The response of the output y for the periodic reference input $r = \sin(2\pi t)$*

the output y in (1) for the periodic reference input $r = \sin(2\pi t)$ is shown in Fig. 1.

Here, the dotted line shows the response of the periodic reference input $r = \sin(2\pi t)$ and the solid line shows that of the output y. Figure 1 shows that the output y follows the periodic reference input r with small steady state error.

Next, using the designed simple repetitive controller $C(s)$, the disturbance attenuation characteristic is shown. The response of the output y for the disturbance $d = \sin(2\pi t)$, for which the frequency component is equivalent to that of the periodic reference input r, is shown in Fig. 2. Here, the dotted line

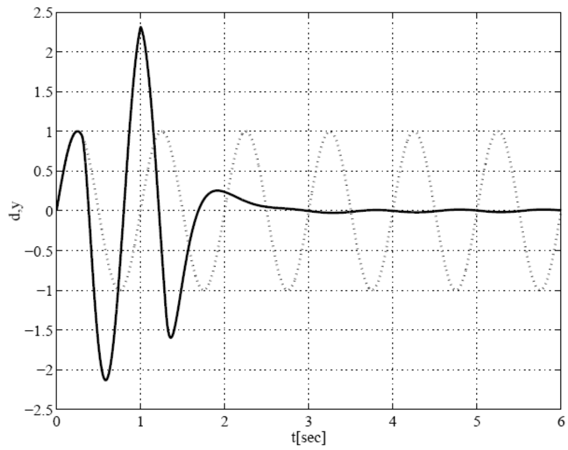

Fig.2: *The response of the output y for the disturbance $d = \sin(2\pi t)$*

shows the response of the disturbance $d = \sin(2\pi t)$ and the solid line shows that of the output y. Figure 2 shows that the disturbance d is attenuated effectively. Finally, the response of the output y for the disturbance $d = \sin(\pi t)$, for which the frequency component is different from that of the periodic reference input r, is shown in Fig. 3. Here, the dotted line

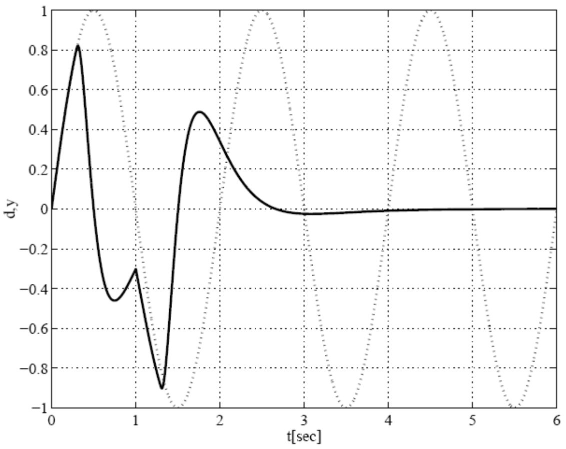

Fig.3: *The response of the output y for the disturbance $d = \sin(\pi t)$*

shows the response of the disturbance d and the solid line shows that of the output y. Figure 3 shows that the disturbance d is attenuated effectively.

A stabilizing simple repetitive controller for time-delay plant can be easily designed in the way shown here.

9. CONCLUSION

In this paper, we proposed the concept of simple repetitive controllers for time-delay plants and clarified the parametrization of all stabilizing simple repetitive controller for both stable time-delay plants and unstable time-delay plants. Control characteristics of simple repetitive control systems for time-delay plants were clarified. It is clarified that the role of $q(s)$ in (30) is to specify the input-output characteristic for the periodic reference input r and to specify the disturbance attenuation characteristic for the frequency component of the periodic disturbance d with period T that is equivalent to that of the periodic reference input r and that of $Q(s)$ in (30) is to specify the disturbance attenuation characteristic for the frequency component of the disturbance d that is different from that of the periodic reference input r. A design procedure of stabilizing simple repetitive controllers for time-delay plants was presented. Finally, a numerical example was shown to illustrate the effectiveness of the proposed method.

Note that proposed method cannot be applied for time-delay plants with uncertainty. As for a design method for robust stabilizing simple repetitive control system for time-delay plants with uncertainty, a description will be made in a separate paper.

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