Wave probabilistic functions, entanglement and quasi-non-ergodic models

Miroslav Svítek [1], Non-member

ABSTRACT

The paper presents the theory of wave probabilistic models together with their important features like inclusion-exclusion rule, product rule and entanglement. These features are mathematically described and the illustrative example is shown to demonstrate the possible applications of the theory. The presented theory can be also applied for modeling of quasi-non-ergodic probabilistic systems. First of all we show the new methodology on binary non-ergodic time series. The theory is extended into M-dimensional non-ergodic n-valued systems with linear ergodicity evolution that are called quasi-non-ergodic probabilistic systems.

1. INTRODUCTION

The problem of complex probability functions was strengthening in quantum mechanics [1,9] where the phase function was recognized as necessary for information processing and quantum system modeling. In [7] the contextual interpretation of phase functions was presented and the wave probabilistic models were introduced as necessary part of probabilistic multi-models. More rigorous introduction into wave probabilistic models was presented in [14] where phase parameters are interpreted as dependencies parameters between events. The link between wave probabilistic functions and complementarity principle was firstly introduced in [10]. The quantization principle as the consequence of phase parameters was defined in [6].

The goal of this paper is to continue in this way of thinking and more rigorously define the wave probabilistic models together with their basic features. In session 2 the mathematical theory of wave functions is presented together with their geometric interpretation. Session 3 covers the link between wave probabilities and entanglement. Session 4 describes entanglement and its impact on consciousness. Session 5 presents illustrative example. Session 6 to 11 present quasi-non-ergodic models together with illustrative example and Session 11 concludes the paper.

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[1] The author is with Faculty of Transportation Sciences, Czech Technical University in Prague Konviktska 20, 110 00 Prague 1, Czech Republic Tel: +420 224 359 631, Fax: +420 224 359 545 , E-mail: svitek@fd.cvut.cz

2. MATHEMATICAL THEORY OF WAVE PROBABILISTIC FUNCTIONS

A probability space consists of a sample space S and a probability function P(.), mapping the events of S to real numbers in [0,1], such that P(S)=1, and if A_1,A_2,... is a sequence of disjoint events, than sum rule is fulfilled:

$$P\left(\bigcup_{i\epsilon N} A_i\right) = \sum_{i\epsilon N} P(A_i) \tag{1}$$

In case the events A_1,A_2,... are not disjoint the following (product and inclusion-exclusion) rules can be defined:

$$\begin{aligned} &P\left(A_1 \cap A_2 \cap \ldots \cap A_n\right) = \\ &P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2) \cdot \\ &\ldots \cdot P(A_N|A_1 \cap \ldots \cap A_{N-1}) \end{aligned} \tag{2}$$

$$\begin{aligned} & P\left(A_1 \cup A_2 \cup \ldots \cup A_N\right) = \\ = \ & \sum_{i=1}^{N} P(A_i - \sum_{i<j}^{N} P(A_i \cap A_j) + \\ + \ & \sum_{i<j<k}^{N} P(A_i \cap A_j \cap A_k) + \ldots + \\ + \ & (-1)^{N-1} \cdot P(A_1 \cap A_2 \cap \ldots \cap A_N) \end{aligned} \tag{3}$$

Taking into consideration of the basic Laws of probability defined above we can rewrite them with help of complex representations (wave functions) summarized in Theorem 1.

<u>**Theorem1:**</u>
Let us define N events $A_i, i\epsilon\{1,2,\ldots,N\}$ of a sample space S, with defined probability functions $A_i, i\epsilon\{1,2,\ldots,N\}$, and let us define the N complex functions:

$$\psi(A_i = \alpha_1 \cdot e^{j\cdot\nu_i} = \sqrt{P(A_i)} \cdot e^{j\cdot\nu_i}, i\epsilon\{1,2,\ldots,N\} \tag{4}$$

with modules $\sqrt{P(A_i)}$ and phases ν_i where the reference phase assigned to the event A_1 is chosen as $\nu_1 = 0$, than the inclusion-exclusion rule given in (3) is resented by complex functions:

$$P(A_1 \cup A2) \cup \ldots \cup A_N = \left| \sum_{i-1}^{N} \psi(A_i) \right|^2 \quad (5)$$

where the phases ν_i are given by:

$$\nu_i =$$
$$= a\cos\left(-\frac{1}{2} \cdot \frac{P((A_1 \cup A_2 \cup \ldots \cup A_{i-1}) \cap A_i)}{\sqrt{P((A_1 \cup A_2 \cup ldots \cup A_{i-1})) \cdot P(A_i)}} \right) +$$
$$+ \nu_{1,2,\ldots,i-1} \quad (6)$$

where $\nu_{1,2,\ldots,i-1}$ is computed from[1] :

$$\psi = (A_1 \cup A_2 \cup \ldots \cup A_{i-1}) = \alpha_{1,2,\ldots,i-1} \cdot e^{j \cdot \nu_{1,2,\ldots,j-1}}$$

Proof: It is presented in [15].

Theorem 2:

Let us define N events $A_i, i\epsilon\{1, 2, \ldots, N\}$ of a sample space S, with defined probability functions $P(A_i), i\epsilon\{1, 2, \ldots, N\}$, and let us define the N complex functions:

$$\psi(A_i) = \alpha_i \cdot e^{i \cdot \nu_i} = \sqrt{P(A_i)} \cdot e^{i \cdot \nu_i}, i\epsilon\{1, 2, \ldots, N\} \quad (7)$$

with modules $\sqrt{P(A_i)}$ and phases ν_i are defined in (6) where the reference phase assigned to the event A_1 is chosen as $\nu_1 = 0$, than the inclusion-exclusion rule given for subset of events $A_r, r\epsilon\{k_1, k_2, \ldots, k_m\}$ is given as:

$$P(A_{k_1} \cup A_{k_2} \cup \ldots \cup A_{k_m}) = \lim_{\substack{\sqrt{P(A_k)} \to 0 \\ k \neq k_1, \ldots, k_m}} \left| \sum_{i=1}^{N} \psi(A_i) \right|^2 \quad (8)$$

Proof:

The proof of Theorem 2 arises directly from Theorem 1 which was proven for all probabilistic values $\sqrt{P(A_i)}$, $A_i, i\epsilon\{1, 2, \ldots, N\}$ including zeros. The zero probabilistic values have impact on phase parameters ν_i and change them in such a way that equation (8) is fulfilled.

3. WAVE PROBABILITIES AND ENTANGLEMENT

Quantum entanglement[2] is a quantum mechanical phenomenon in which the quantum states of two or more objects have to be described with reference to each other, even though the individual objects may be spatially separated. This leads to correlations between observable physical properties of the systems. For example, it is possible to prepare two particles in a single quantum state such that when one is observed to be spin-up, the other one will always be observed to be spin-down and vice versa, this despite the fact that it is impossible to predict, according to quantum mechanics, which set of measurements will be observed. As a result, measurements performed on one system seem to be instantaneously influencing other systems entangled with it.

As an example of identically entangled (two dimensional binary) events we can use the spin-up and spin-down from quantum mechanics. If two electrons with spins are entangled measurement of spin-up on first electron yields into spin-up on second one. Contrary, measurement of spin-down on first electron yields into spin-down on second one.

Theorem 3:

Let us suppose we have N discrete values $A_i, i\epsilon\{1, 2, \ldots, N\}$ with probabilities $P(A_i), i\epsilon\{1, 2, \ldots, N\}$ and complex functions $\psi(A_i)$:

$$\psi(A_i) = \sqrt{P(A_i)} \cdot e^{j \cdot \nu_i} \quad (9)$$

where the phases ν_i are given in agreement with Theorem 1, than all events $A_r\epsilon\{r_1, \ldots, r_n\}$ are fully entangled if the following form holds:

$$P(A_{r_1} \cup \ldots \cup A_{r_n}) = \lim_{\substack{\sqrt{P(A_k)} \to 0 \\ k \neq r_1, r_2, \ldots, r_n}} |\psi(A_1 + \psi A_2 + \ldots + \psi(A_N)|^2 = \quad (10)$$
$$|\tilde{\psi}(A_{r_1}) + \ldots \ldots + \tilde{\psi}(A_{r_n})|^2 \equiv 0$$

which means:

$$P(\bar{A}_{r_1} \cap \ldots \cap \bar{A}_{r_n}) = 1 \quad (11)$$

where $\bar{A}_{r_i}$ means the inversion of event A_{r_i} (for binary case $\bar{A}_{r_i}$ means that even A_{r_i} never happen).

Proof:

The Theorem 3 is coming-out directly from inclusion-exclusion rules and from definition of wave probabilities. Phase parameters assigned into wave functions can be either positive or negative ones.

The limit in (10) modifies phases for the selected subset of events $\{A_{r_1}, A_{r_2}, \ldots, A_{r_n}\}$ to comply with inclusion-exclusion rule. For special cases the inclusion-exclusion rule can yield into zero due to wave resonances within phases of events. This special case can be reached for special set up of phases.

[1] $\alpha_{1,2,\ldots,i-1}$ and $\nu_{1,2,\ldots,i-1}$ are modulus and phase of complex wave function $\psi(A_1) \cup A_2 \cup \ldots \cup A_{i-1}$

[2] We used for description of entanglement principle definition from Wikipedia available on web page: http://en.wikipedia.org/wiki/Quantum_entanglement

Zero probability (10) directly yields into equation (11) that defines that state characterized by $\bar{A}_{r_1} \cap \ldots \cap \bar{A}_{r_n}$ will surely occur. This state is not random but fully deterministic and so spatially spread within events $\{A_{r_1}, A_{r_2}, \ldots, A_{r_n}\}$.

It can be stated that entanglement is logical result of probabilistic wave functions and represents something like wave functions resonance yielding into deterministic states.

4. ENTANGLEMENT AND CONSCIOUSNESS

The entanglement has a lot of surprising features. One of them is "*entanglement swapping*". Let us have four events A_1, A_2, A_3, A_4.

Let us suppose the entanglement exists between A_1, A_2 and also between A_3, A_4. If A_1, A_4 have never interacted, the result of entanglement swapping is that A_1, A_4 will be also entangled in case there exists entanglement between events A_2, A_3. The more entangled events the more swapping states exist in our set of events. This feature can be used to explain a lot of tasks.

For connection between entanglement and consciousness, we use the arguments given at [13] describing the theory of consciousness: *carriers of consciousness are maximal coherently entangled states, and all such states are carriers of consciousness.* The consciousness carrier based on this interpretation is spatially distributed state whose separated parts are entangled.

5. ENTANGLEMENT - ILLUSTRATIVE EXAMPLE

Let us take two events A_t and A_{t+1} with wave functions ψ_0 and ψ_1. Let us define the probability:

$$P(A_t = 0 \cup A_{t+1} = 1) = \\ |\psi_o|^2 + |\psi_1|^2 + 2 \cdot |\psi_o \cdot \psi_1| \cdot \cos(\Delta\psi) \quad (12)$$

where $\Delta\psi$ is phase difference between wave functions ψ_1 and ψ_1. Suppose now with respect to Theorem 3 that:

$$P(A_t = 0 \cup A_{t+1} = 1) = 0 \quad (13)$$

This case can happen for following values of $\Delta\psi$:

$$\Delta\psi = a\cos\left(-\frac{1}{2} \cdot \frac{|\psi_0|^2 + |\psi_1|^2}{|\psi_0 \cdot \psi_1|}\right) \quad (14)$$

If, for example, $\psi_0 = \psi_1 = 1/\sqrt{2}$ than $\Delta\psi = \pi$ represents the entanglement.

As the result of entanglement we can write that following events will surely happen (there are not random values):

$$P(A_t = 1 \cap A_{t+1} = 0) = 1 \quad (15)$$

We can start also with following probability instead of (18):

$$P(A_t = 1 \cap A_{t+1} = 0) = 0 \quad (16)$$

Than entanglement yields to:

$$P(A_t = 1 \cap A_{t+1} = 1) = 1 \quad (17)$$

Both equations (16) and (17) can be written to quantum "bra-ket" representation:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \quad (18)$$

Measuring the first event A_t (probability of measuring 0 is 1/2 and probability of measuring 1 is also 1/2) fully determines the value of A_{t+1} according to (18).

6. WAVE PROBABILISTIC MODEL OF NON-ERGODIC BINARY PROCESSES

We start our discussion with probabilistic binary process and we will show the models of the zero or one occurrence (probability and structure) in form of wave probabilistic functions. The process is generally defined as the ensemble of multiple spatially distributed events realized according to predefined model (wave function). By other words it means parallel existence of spatially linked time processes.

Let us define the binary process in "bra-ket[3] " form [17]:

$$|\psi\rangle \sqrt{(1-p)} \cdot |0\rangle + \sqrt{p} \cdot e^{j \cdot \psi} \cdot |1\rangle \quad (19)$$

Parameter p defines the probability of occurrence of state $|1\rangle$ in every distributed state. The probability of occurrence of state $|0\rangle$ must be $(1-p)$. The phase ψ plays the role of "structural" parameter that expresses the rate of space ensemble randomness [16].

Process defined in (19) is time independent which means that the spatially distributed states fulfill the ergodic theorem because the time average of a series along the time trajectories exists almost everywhere and is related to the space (set of realizations) average.

In case the complex parameters of "bra-ket" model will be time dependent we speak about non-ergodic probabilistic binary process. Since the non-ergodic processes are very difficult to model we will ease our requirements only to quasi-non-ergodic processes with linear time evolution of complex "bra-ket" parameters.

If we add the time varying phase parameter we can rewrite "bra-ket" function (19) as follows:

$$|\psi\rangle = \sqrt{(1-A_\omega)} \cdot |0\rangle + \sqrt{A_\omega} \cdot e^{j\cdot\omega\cdot t} \cdot |1\rangle = \\ = \sqrt{(1-A_\omega)} \cdot |0\rangle \sqrt{A_\omega} \cdot e^{j\cdot(\omega\cdot t+\psi_\omega)} \cdot |1\rangle \quad (20)$$

where parameter A_ω defines the probability of occurrence of state $|1\rangle$. Parameter ψ_ω expresses the initial structure of studied set of (spatially) distributed states. Parameter ω represents the frequency of continual structuring and randomizing of (spatially) distributed states. Due to the time evolution of complex parameters (20) the ergodic condition is not fulfilled.

The parameters for state $|0\rangle$ can be easily computed from parameters assigned to state $|1\rangle$ because A_ω must be maximally equal to one to be probability function and the phase assigned to state $|0\rangle$ is supposed to be normalized reference phase (4) and equal to zero. The frequency ω can be interpreted as the energy spent to "structuring" or "randomizing" the set of distributed states with respect to chosen frequency ω.

We use the notation A_ω, ψ_ω as modulus and initial phase parameters assigned to frequency ω. They represent the frequency decomposition in same way as Fourier transform. The (20) can be used as one frequency component (modulus and initial phase) of non-ergodic binary time series.

The general periodic non-ergodic behavior can be expressed as the sum of different frequency components:

$$|\psi, t\rangle = \sqrt{1 - \left(\sum_{i=1}^{1} \sqrt{A_{\omega_i}} \cdot e^{j(\omega_i \cdot t+\psi_\omega)}\right)^2} \cdot |0\rangle + \\ \left(\sum_{i=1}^{N} \sqrt{A_{\omega_i}} \cdot e^{(\omega_i \cdot t+\psi_i)}\right) \cdot |1\rangle \quad (21)$$

where the final modulus and phase assigned to state $|1\rangle$ are:

$$\sum_{i=1}^{N} \sqrt{A_{\omega_i}} \cdot e^{(\omega_i \cdot t+\psi_i)} = \sqrt{\tilde{A}(t)} \cdot e^{j\cdot\tilde{\psi}(t)} \quad (22)$$

where $\tilde{A}(t)$ is time evolution of probability of state $|1\rangle$ and $\tilde{\psi}(t)$ is the evolution of the link among different spatially distributed processes (expressing structuring and randomizing). The complex parameter assigned to $|0\rangle$ is computed from the normalization and reference condition:

$$|\psi, t\rangle = \sqrt{1-\tilde{A}(t)} \cdot |0\rangle + \sqrt{\tilde{A}(t)} \cdot e^{j\cdot\tilde{\psi}(t)} \cdot |1\rangle \quad (23)$$

[3] "Bra-ket" notation is a standard notation for describing quantum states in the theory of quantum mechanics composed of angle brackets and vertical bars [9].

Equation (23) can be understood as the "bra-ket" representation of general non-ergodic binary probability process. We can see in (22) that for every state $|0\rangle, |1\rangle$ the discrete modules and phase spectrum can be defined. It means that time-evolution is modeled by periodic function. The discrete spectrum can be replaced by continuous one[4].

7. NON-ERGODIC AND QUASI-NON-ERGODIC N-VALUED PROBABILISTIC PROCESSES

Let us generalize the binary process (10). The n-valued process observed in time interval t in "bra-ket" form can be expressed:

$$\psi(I, \alpha, t) = \alpha_1(t) \cdot |I_1\rangle + \ldots + \alpha_n(t) \cdot |I_n\rangle \quad (24)$$

where $I_1, I_2, \ldots, I_n$ is the set of possible values appeared in studied process and $\alpha_1(t), \alpha_2(t), \ldots, \alpha_n(t)$ are complex parameters normalized under form:

$$|\alpha_1(t)|^2 + |\alpha_2(t)|^2 + \ldots + |\alpha_n(t)|^2 = 1 \quad (25)$$

The equation (24) can be interpreted as the existence of different spatially distributed n-valued states in each time interval t. Modules $|\alpha_1(t)|^2, \ldots, |\alpha_n(t)|^2$ represent the probabilities of falling $I_1, I_2, \ldots, I_n$ (in time interval t) in every distributed state. The phases of $\alpha_1(t), \alpha_2(t), \ldots, \alpha_n(t)$assigned to values $I_1, I_2, \ldots, I_n$ give us the order distribution (rate of order or randomness) among spatially distributed states (also in time interval t). Since the modulus and phases of complex parameters $\alpha_1(t), \alpha_2(t), \ldots, \alpha_n(t)$ are time-dependent than the equations (24) and (25) can be understood as the instrument for the non-ergodic processes description and modeling.

The time evolution of non-ergodic processes can be transformed into time evolution of complex parameters' vector $\alpha_1(t), \alpha_2(t), \ldots, \alpha_n(t)$. We define the quasi-non-ergodic n-valued process in such a way that time evolution of complex parameters $\alpha_1(t), \alpha_2(t), \ldots, \alpha_n(t)$ fulfils the linear time-invariant (LTI) evolution trajectory. Let us extend (24) and (25) into quasi-non-ergodic form:

$$\begin{bmatrix} \alpha_1(t+1) \\ \alpha_2(t+1) \\ \cdot \\ \alpha_3(t+1) \end{bmatrix} = k \cdot \left\{ \begin{bmatrix} a_{1,1} & a_{1,2} & \cdot & a_{1,n} \\ a_{2,1} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{n,1} & a_{n,2} & \cdot & a_{n,n} \end{bmatrix} \cdot \begin{bmatrix} \alpha_1(t) \\ \alpha_2(t) \\ \cdot \\ \alpha_3(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ b_n \end{bmatrix} \right\} \quad (26)$$

where complex parameters of studied probabilistic process are given as $\alpha_1(t), \alpha_2(t), \ldots, \alpha_n(t)$, the ma-

[4] In this case the sums in (21) are replaced by integrals. This replacement means the transition from Fourier series into Fourier transform.

trix **A** and vector **b** are the evolution (generally complex) parameters and constant k guarantees the normalization condition (25) in each time interval t.

8. PROCESSING OF QUASI-NON-ERGODIC SIGNALS

The evolution of complex parameters $\alpha_1(t), \alpha_2(t), \ldots, \alpha_n(t)$ presented in (26) can be further extended for more complex processing algorithms yielding into probabilistic quasi-non-ergodic circuits covering e.g. positive or negative feedback, filtering, etc.

Equation (26) can be seen as time evolution with finite response (FIR - Finite Impulse Response). On the other hand we can generally define the quasi-non-ergodic processes with infinite response (IIR - Infinite Impulse Response) as follows:

$$\begin{bmatrix} \alpha_1(t+1) \\ \alpha_2(t+1) \\ \cdot \\ \alpha_3(t+1) \end{bmatrix} = k \cdot \begin{bmatrix} a_{1,1} & a_{1,2} & \cdot & a_{1,n} \\ a_{2,1} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{n,1} & a_{n,2} & \cdot & a_{n,n} \end{bmatrix} \cdot \begin{bmatrix} \alpha_1(t) \\ \alpha_2(t) \\ \cdot \\ \alpha_3(t) \end{bmatrix} + \\ + \begin{bmatrix} b_{1,1} & b_{1,2} & \cdot & b_{1,n} \\ b_{2,1} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ b_{n,1} & b_{n,2} & \cdot & b_{n,n} \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \beta_n \end{bmatrix} \tag{27}$$

where complex parameters of studied quasi-non-ergodic process are given as $\alpha_1(t), \alpha_2(t) \ldots, \alpha_n(t)$, the matrix **A** and **B** are the evolution (generally complex) matrixes, constant k guarantees the normalization condition (25) in each time interval t and $\beta_1(t), \beta_2(t) \ldots, \beta_n(t)$ is the vector of normalized complex parameters assigned into input probabilistic process:

$$\psi(I, \beta, t) = \beta_1(t) \cdot |I_1\rangle + \ldots + \beta_n(t) \cdot |I_n\rangle \tag{28}$$

We can continue in this way of thinking and try to move from the circuit concept into more general quasi-non-ergodic system model. In next chapter we will introduce SISO (single-input, single-output) and MIMO (multi-input, multi-output) quasi-non-ergodic system models.

9. SISO QUASI-NON-ERGODIC SYSTEMS

In this chapter we will describe quasi-non-ergodic SISO systems based on the processing form (27). The presented methodology enables us to build the basic blocks in the same way as in classical signals and systems theory. The general description of processing building block of quasi-non-ergodic processes can be defined:

$$\begin{bmatrix} \gamma_1(t+1) \\ \gamma_2(t+1) \\ \cdot \\ \gamma_3(t+1) \end{bmatrix} = k_1 \cdot \left\{ \begin{bmatrix} a_{1,1} & a_{1,2} & \cdot & a_{1,n} \\ a_{2,1} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{n,1} & a_{n,2} & \cdot & a_{n,n} \end{bmatrix} \cdot \begin{bmatrix} \gamma_1(t) \\ \gamma_2(t) \\ \cdot \\ \gamma_3(t) \end{bmatrix} + \\ + \begin{bmatrix} b_{1,1} & b_{1,2} & \cdot & b_{1,n} \\ b_{2,1} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ b_{n,1} & b_{n,2} & \cdot & b_{n,n} \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \beta_n \end{bmatrix} \right\} \tag{29}$$

$$\begin{bmatrix} \alpha_1(t) \\ \alpha_2(t) \\ \cdot \\ \alpha_3(t) \end{bmatrix} = k_2 \cdot \left\{ \begin{bmatrix} c_{1,1} & c_{1,2} & \cdot & c_{1,n} \\ c_{2,1} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ c_{n,1} & c_{n,2} & \cdot & c_{n,n} \end{bmatrix} \cdot \begin{bmatrix} \gamma_1(t) \\ \gamma_2(t) \\ \cdot \\ \gamma_3(t) \end{bmatrix} + \\ + \begin{bmatrix} d_{1,1} & d_{1,2} & \cdot & d_{1,n} \\ d_{2,1} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ d_{n,1} & d_{n,2} & \cdot & d_{n,n} \end{bmatrix} \cdot \begin{bmatrix} \beta_1(t) \\ \beta_2(t) \\ \cdot \\ \beta_n(t) \end{bmatrix} \right\} \tag{30}$$

where the matrices **A**, **B**, **C**, **D** are the LTI (Linear Time Invariant) evolution $n \times n$ matrixes, constants k_1, k_2 guarantee the normalization conditions in each time interval t and complex parameters $\gamma_1(t), \gamma_2(t), \ldots, \gamma_n(t)$ represent the inner parameters of state-space process:

$$\psi(I, \gamma, t) = \gamma_1(t) \cdot |I_1\rangle + \ldots + \gamma_1(t) \cdot |I_1\rangle \tag{31}$$

The most general model for e.g. SISO systems can be defined also through time varying evolution matrixes $A(t), B(t), C(t), D(t)$ of (29) and (30). Because of the difficulty in modeling time evolution of matrices $A(t), B(t), C(t), D(t)$ we can preferably introduce the quasi-non-ergodic model and use the approach known in dynamical system theory like exponential forgetting, etc. [6]. This instruments enable us partly to model the slow evolution of matrices $A(t), B(t), C(t), D(t)$.

10. MIMO QUASI-NON-ERGODIC SYSTEMS

We can extend the SISO systems into more general MIMO ones by generalizing of the vectors used in (29) and (30) as follows:

$$\tilde{\alpha}(t) = \begin{bmatrix} {}^1\alpha_1 t \\ \cdot \\ {}^1\alpha_n t \\ {}^2\alpha_1 t \\ \cdot \\ {}^2\alpha_n t \\ \cdot \\ \cdot \\ \cdot \\ {}^M\alpha_1 t \\ \cdot \\ {}^M\alpha_n t \end{bmatrix}, \tilde{\beta}(t) = \begin{bmatrix} {}^1\beta_1 t \\ \cdot \\ {}^1\beta_n t \\ {}^2\beta_1 t \\ \cdot \\ {}^2\beta_n t \\ \cdot \\ \cdot \\ \cdot \\ {}^M\beta_1 t \\ \cdot \\ {}^M\beta_n t \end{bmatrix}, \tilde{\gamma}(t) = \begin{bmatrix} {}^1\gamma_1 t \\ \cdot \\ {}^1\gamma_n t \\ {}^2\gamma_1 t \\ \cdot \\ {}^2\gamma_n t \\ \cdot \\ \cdot \\ \cdot \\ {}^M\gamma_1 t \\ \cdot \\ {}^M\gamma\gamma_n t \end{bmatrix} \tag{32}$$

where we suppose to have M-dimensional input, output and state vectors each of which is quasi-non-ergodic n-valued probabilistic process.

The normalized[5] MIMO quasi-non-ergodic system can be described by:

$$\begin{aligned}\tilde{\gamma}(t+1) &= \tilde{A}\cdot\tilde{\gamma}(t)+\tilde{B}\cdot\tilde{\beta}(t)\\ \tilde{\alpha}(t) &= \tilde{C}\cdot\cdot\tilde{\gamma}(t)+\tilde{D}\cdot\tilde{\beta}(t)\end{aligned} \quad (33)$$

Equation (33) can be considered as the most general form of quasi-non-ergodic MIMO system. The other above presented examples can be easily derived from it. Dynamical systems described by (32) can be reduced into more dimensional systems with diagonal matrix $\tilde{A}$in accordance to reduction method presented in [6]. The set of connected one-dimensional models (in quantum way, e.g. entanglement, etc.) evokes the brain functionality.

The state-space model represented by matrices $\tilde{A},\tilde{B},\tilde{C},\tilde{D}$ can be easily transformed into transmission function S(z) as follows:

$$S(z)=\tilde{C}\cdot(z\cdot I-\tilde{A})^{-1}\cdot\tilde{\tilde{B}}+\tilde{D} \quad (34)$$

where z is the complex parameter used in z-transform [6] and I is unit matrix. The transmission functions enable to connect different blocks represented by matrices $\tilde{A},\tilde{B},\tilde{C},\tilde{D}$ into serial, parallel, feed-back, etc. ordering and build more and more complex systems. The identification method of unknown matrices $\tilde{A},\tilde{B},\tilde{C},\tilde{D}$ is not part of this paper and it will be studied in forthcoming research steps.

11. QUASI-NON-ERGODIC SYSTEMS - ILLUSTRATIVE EXAMPLE

Let us define the three valued I_1, I_2, I_3 input quantum object represented by:

$$|\xi,t\rangle_\eta=\beta_1(t)\cdot|I_1\rangle_\eta+\beta_2(t)\cdot|I_2\rangle_\eta+\beta_3(t)\cdot|I_3\rangle_\eta \quad (35)$$

Based on the above results we know that form (35) characterizes the input time series that is supposed to be the ergodic and fully random one. Than the assigned input wave functions fulfilling these requirements must yield into constants:

$$\beta_1(t)=\beta_2(t)=\beta_3(t)=\frac{1}{\sqrt{3}} \quad (36)$$

We can suppose the quasi-non-ergodic processing:

$$\begin{bmatrix}\alpha_1(t)\\ \alpha_2(t)\\ \alpha_3(t)\end{bmatrix}=k(t)\cdot\left\{\begin{bmatrix}a_{1,1}&a_{1,2}&a_{1,3}\\ a_{2,1}&a_{2,2}&a_{2,3}\\ a_{3,1}&a_{3,2}&a_{3,3}\end{bmatrix}\cdot\begin{bmatrix}\alpha_1(t-1)\\ \alpha_2(t-1)\\ \alpha_3(t-1)\end{bmatrix}+\right.$$
$$\left.+\begin{bmatrix}b_{1,1}&b_{1,2}&b_{1,3}\\ b_{2,1}&b_{2,2}&b_{2,3}\\ b_{3,1}&b_{3,2}&b_{3,3}\end{bmatrix}\cdot\begin{bmatrix}\beta_1(t)\\ \beta_2(t)\\ \beta_3(t)\end{bmatrix}\right\} \quad (37)$$

[5] Normalizing constants are parts of matrixes A,B,C,D.

with initial condition:

$$\alpha_1(1)=\alpha_2(1)=\alpha_3(1)=\frac{1}{\sqrt{3}} \quad (38)$$

and numerical values:

$$A=\begin{bmatrix}0&1&0\\ 0&0&0\\ 0.1\cdot j&0.1+0.3\cdot j&0.5+0.2\cdot j\end{bmatrix} \quad (39)$$
$$B=\begin{bmatrix}1&0&0\\ 0&1&0\\ 0&0&1\end{bmatrix}$$

The processing algorithm (37) transforms the input process (35, 36) into output process characterized by

$$|\psi,t\rangle\eta=\alpha_1(t)\cdot|I_1\rangle\eta+\alpha_1(t)\cdot|I_2\rangle\eta+\alpha_1(t)\cdot|I_3\rangle\eta \quad (40)$$

of course, under the normalization conditions:

$$|\alpha_1(t)|^2+|\alpha_2(t)|^2+|\alpha_3(t)|^2=1 \quad (41)$$

Fig. 1 shows time evolution of wave function $\alpha_1(t)$ in complex plane without normalization by parameter $k(t)$ as it is shown in (37). Other two wave functions can be presented in similar way.

Fig. 2 present time evolution of probabilities $P_1(t), P_2(t), P_3(t)$ computed from wave functions $\alpha_1(t), \alpha_2(t), \alpha_3(t)$ of output process. We can see that our approach enabled us to catch very complex time evolution of probabilities of falling/measuring states I_1, I_2, I_3.

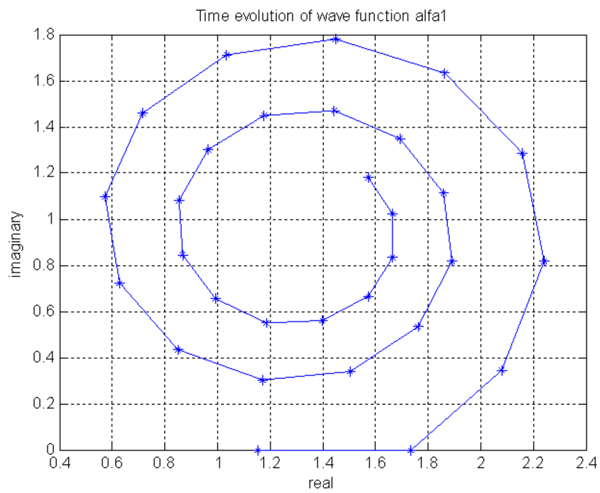

Fig.1: *Time evolution of wave function $\alpha_1(t)$ in complex plane*

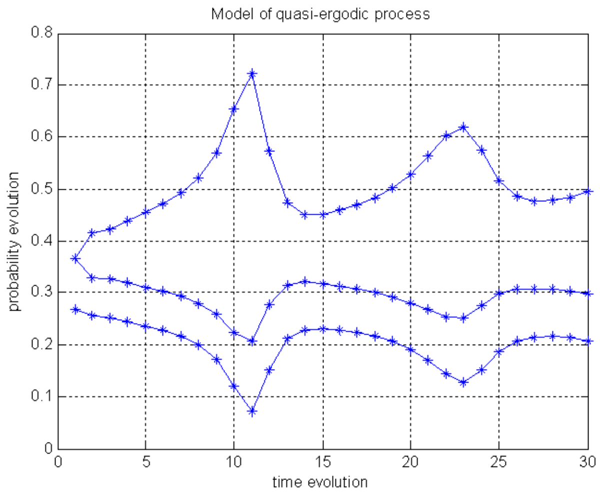

Fig.2: *Time evolution of probabilities* $p_1(t), p_2(t), p_3(t)$ *of output process*

12. CONCLUSION

In the paper the wave probabilistic models were rigorously introduced and the mathematical comparison between usually used probabilistic models and wave probabilistic models was presented. The mathematical theory point out on the applicability of wave probabilistic models and their special features. The quantum entanglement was explained as the consequence of phase parameters and it can be interpreted as the resonance principle of wave functions.

The results of wave functions resonance are fully deterministic spatially distributed states with a lot of properties. In paper the entanglement swapping was briefly discussed as the possible carriers of consciousness. The achieved entanglement interpretation can yield in the new learning algorithm of quantum systems. Based on training data the entanglement among network components could be found. Generalization principle can be understood as e.g. the entanglement swapping.

We can understand that the presented methodology yields into new models taking into account the structure of time series or by other words the time series ordering. It was shown that for time evolution models the theory of LTI dynamical systems can be used together with all instruments like serial, parallel or feed-back ordering. Due to these features very complex quasi-non-ergodic processes and systems could be modelled. We can speak about very complex circuits covering a lot of feed-backs, etc.

The application of presented methodology was shown on modelling of binary time series and three-valued quasi-ergodic process. These examples only show how the quasi-ergodicity can be represented and how the analyze can be done. Good news is that the instrument for modelling dynamical LTI systems [6] is applicable. This instrument can be used to find the stability of our system, its time response and transmission function, frequency characteristics, etc.

The inspiration for the above defined problem came from quantum physics [1, 4, 6, 7]. The analogy with quantum mechanics could be seen as very interesting and is likely to bring a lot of inspiration for the future work in statistical modelling area and wave probabilistic models.

13. ACKNOWLEDGMENT

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References

[1] Gold J.F.: *Knocking on the Devil's Door - A Naive Introduction to Quantum Mechanics*, Tachyon Publishing Company, 1995.

[2] Raymer M.G.: *Measuring the quantum mechanical wave function*, Contemporary Physics, 1997, volume 38, number 5, pages 343-355.

[3] Radon J.: *Uber die Bestimmung von Funktionen durch ihre Interalwerte langs gewisser Mannigfaltigkeiten.* Berichte Sachsiche Akademie der Wissenschaften, Leipzich, Mathematisch-Physikalische Klasse, 69:262-277,1917.

[4] Bogdanov Yu. I.: *The sixth Hilbert's problem and the principles of quantum informatics*, Institute of Physics and Technology, Russian Academy of Sciences [http://arxiv.org/ftp/quant-ph/papers/0612/0612025.pdf]

[5] Clifton R.: *Complementarity between Position and Momentum as a Consequence of Kochen-Specker Arguments* [http://eprintweb.org/S/article/quant-ph/9912108]

[6] Svítek M.: *Dynamical Systems with Reduced Dimensionality*, Monography NNW No.6, Czech Academy of Science, 161 pages, 2006, ISBN 80-903298-6-1.

[7] Svítek M.: *Probabilistic Multi-Modeling by Quantum Calculus*, ICTTA'06, IEEE Conference Damascus, Syria, 2006

[8] Deutsch D.: *Quantum Theory, the Church-Turing principle and the universal quantum computer*, Proceedings of the Royal Society of London Series A400, 97-117, 1985.

[9] Derek F. Lawden: *The Mathematical Principles of Quantum Mechanics*, Dover Publication, Inc., Mineola, New York, 1995, ISBN 0-486-44223-3.

[10] Svítek M.: *Complementary variables and its application in statistics*, Neural Network World 3/ 2007, Prague, pp. 237-253.

[11] Svítek M., Novovičová J.: *Performance Parameters Definition and Processing*, Neural Network World 6/2005, pp. 567-577, 2005.

[12] Svítek M.: *Theory and Algorithm for Time Series Compression*, Neural Network World 1/2005, pp. 53-67, 2005.

[13] Clarke Ch. J.S., *The role of Quantum Physics in the Theory of Subjective Consciousness*, Mind and Matter, Vol. 5 (1), pp. 45-81, 2007.
[14] Vedral V.: *Introduction to Quantum Information Science*, Oxford University Press, 2006.
[15] Svítek M.: *Wave probabilistic models*, Neural Network World 5/2007, pp. 469-481, 2007.
[16] Svítek M.: *Quantum System Modelling, International Journal on General Systems*, Vol. 37, No.5, pp. 603-626, 2008.
[17] Svítek, M.: *Wave probabilities and quantum entanglement*, Neural Network World 5/2008, pp. 401-406, 2008.
[18] Khrennikov A.: *Quantum nonlocality or nonergodicity? A critical study of Bell's arguments*, 2008 [http://arxiv.org/abs/quant-ph/0512178]
[19] Summhammer J.: *Neutron interferometric test of the nonergodic interpretation of quantum mechanics*, Il Nuovo Cimento, 103 B, 1989, pp. 265-280.

Miroslav Svítek was born in Rakovník, Czech Republic, in 1969. He graduated in radioelectronic from Czech Technical University in Prague, in 1992. In 1996, he received the Ph.D. degree in radioelectronic at Faculty of Electrical Engineering, Czech Technical University in Prague. Since 2002, he has been associated professor in engineering informatics at Faculty of Transportation Sciences, Czech Technical University in Prague. Since 2005, he has been nominated as the extraordinary professor in applied informatics at Faculty of Natural Sciences, University of Matej Bel in Banska Bystrica, Slovak Republic. Since 2008, he has been full professor in engineering informatics at Faculty of Transportation Sciences, Czech Technical University in Prague. He is currently teaching courses and doing research in theoretical telematics, intelligent transport systems, system theory and quantum informatics. Miroslav Svítek is president of association of transport telematics of the Czech Republic (it covers more than 70 public and private organization), Vice-Dean of Faculty of Transportation Sciences and Head of Laboratory of Telematics, Czech Technical University in Prague and the manager of more than twenty national and European R&D projects. He is author or co-author of more than 200 papers and 5 monographs.