A Design Method for Robust Stabilizing Simple Multi-Period Repetitive Controllers for Time-Delay Plants

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ABSTRACT

A multi-period repetitive control system is a type of servomechanism for a periodic reference input. Even if a plant does not include time-delay, the transfer function from the periodic reference input to the output and that from the disturbance to the output of the multi-period repetitive control system generally have an infinite number of poles. When the transfer function from the periodic reference input to the output and that from the disturbance to the output have an infinite number of poles, it is difficult to settle the input-output characteristic and the disturbance attenuation characteristic of the multi-period repetitive control system. In order to specify the input-output characteristic and the disturbance attenuation characteristic easily, Yamada and Takenaga proposed the concept of simple multi-period repetitive control systems such that the controller works as a stabilizing multi-period repetitive controller and the transfer function from the periodic reference input to the output and that from the disturbance to the output have a finite number of poles. However, the method by Yamada and Takenaga cannot be applied to time-delay plants with uncertainty. The purpose of this paper is to propose the concept of robust stabilizing simple multi-period repetitive controllers for time-delay plants with uncertainty and to clarify the parametrization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with uncertainty.

Keywords: Multi-Period Repetitive Controller, Time-Delay Plant, Finite Number of Poles, Parametrization, Robust stability

1. INTRODUCTION

A repetitive control system is a type of servomechanism for a periodic reference input for the periodic signal [1–14]. That is, the repetitive control system follows the periodic reference input without steady state error, even if there exists a periodic disturbance or the plant has uncertainty [1–14]. Because a repetitive control system that follows any periodic reference input without steady state error is a neutral type of time-delay control system, it is difficult to design stabilizing controllers for the plant [3]. To design a repetitive control system that follows any periodic reference input without steady state error, the plant needs to be biproper [1–14]. Ikeda and Takano [7] pointed out that it is physically difficult for the output to follow any periodic reference input without steady state error. In addition, they showed that the repetitive control system is L_2 stable for periodic signals that do not include infinite frequency signals if the relative degree of the plant is one. Since many actual plants are strictly proper and do not necessarily have one relative degree, many design methods are given in [1–6, 8–14]. These studies are divided into two types. One uses a low pass filter [1, 2, 4–6, 8–14] and the other uses an attenuator [3]. Since the first type of repetitive control system has a simple structure and is easy to design, this design method is used in many applications. Therefore, the first type of repetitive control system is called by a modified repetitive control system and have been studied [1, 2, 4–6, 8–14].

However, the modified repetitive control system is known to have bad disturbance attenuation characteristic [15]. In order to improve the disturbance attenuation characteristic of the modified repetitive control system, the multi-period repetitive control system was proposed [15]. Since the multi-period repetitive control system includes many controllers, it is difficult to design a stabilizing multi-period repetitive controller. If the parametrization of all stabilizing multi-period repetitive controllers is obtained, a stabilizing multi-period repetitive controller is designed easily and systematically. From this viewpoint, the parametrization of all stabilizing multi-period repetitive controllers for non-minimum phase systems was solved [16]. Since the method in [16] cannot be applied to the plant with uncertainty, Satoh, Yamada and Mei proposed the parametrization of all robust stabilizing multi-period repetitive controllers for the plant with uncertainty [17]. However, using the method in [16, 17], the transfer function from the

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periodic reference input to the output and that from the disturbance to the output have an infinite number of poles, even if the plant does not include uncertainty and time-delay. When the transfer function from the periodic reference input to the output and that from the disturbance to the output have an infinite number of poles, it is difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From a practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic are easily specified. In order to specify the input-output characteristic and the disturbance attenuation characteristic easily, the transfer function from the periodic reference input to the output and that from the disturbance to the output are desirable to have a finite number of poles. From this point of view, Yamada and Takenaga proposed the concept of a simple multi-period repetitive controller such that the controller works as a stabilizing multi-period repetitive controller and the transfer functions from the periodic reference input to the output and from the disturbance to the output have a finite number of poles [18]. However, the method in [18] cannot be applied to time-delay plants with uncertainty. Since many real systems include time-delays and uncertainties, the problem to obtain the parametrization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with uncertainty is one of important problems to solve.

In this paper, we propose the concept of robust stabilizing simple multi-period repetitive controllers for time-delay plants with uncertainty and clarify the parametrization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with uncertainty such that the controller works as a robust stabilizing multi-period repetitive controller and the transfer function from the periodic reference input to the output and that from the disturbance to the output have a finite number of poles.

Notation

R	the set of real numbers.
R_+	$R \cup \{\infty\}$.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real rational functions.
H_∞	the set of stable causal functions.
$\|\{\cdot\}\|_\infty$	H_∞ norm of $\{\cdot\}$.
$D^\perp$	orthogonal complement of D, i.e., $\begin{bmatrix} D & D^\perp \end{bmatrix}$ or $\begin{bmatrix} D \\ D^\perp \end{bmatrix}$ is unitary.
A^T	transpose of A.
$A^\dagger$	pseudo inverse of A.
$\rho(\{\cdot\})$	spectral radius of $\{\cdot\}$.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	represents the state space description $C(sI-A)^{-1}B+D$.
$\mathcal{L}\{\cdot\}$	the Laplace transformation of $\{\cdot\}$.
$\mathcal{L}^{-1}\{\cdot\}$	the Inverse Laplace transformation of $\{\cdot\}$.

2. ROBUST STABILIZING SIMPLE MULTI-PERIOD REPETITIVE CONTROL SYSTEMS AND PROBLEM FORMULATION

Consider the unity-feedback control system given by

$$\begin{cases} y &= G(s)e^{-sL}u + d \\ u &= C(s)(r-y) \end{cases}, \tag{1}$$

where $G(s)e^{-sL}$ is the time-delay plant, $L > 0$ is the time-delay, $G(s) \in R(s)$, $C(s)$ is the controller, $u \in R$ is the control input, $y \in R$ is the output, $d \in R$ is the disturbance and $r \in R$ is the periodic reference input with period $T > 0$ satisfying

$$r(t+T) = r(t) \quad (\forall t \geq 0). \tag{2}$$

The nominal time-delay plant of $G(s)e^{-sL}$ is denoted by $G_m(s)e^{-sL_m}$, where $G_m(s) \in R(s)$. Both $G(s)$ and $G_m(s)$ are assumed to have no zero or pole on the imaginary axis. In addition, it is assumed that the number of poles of $G(s)$ in the closed right half plane is equal to that of $G_m(s)$. The relation between the time-delay plant $G(s)e^{-sL}$ and the nominal time-delay plant $G_m(s)e^{-sL_m}$ is written as

$$G(s)e^{-sL} = G_m(s)(e^{-sL_m} + \Delta(s)), \tag{3}$$

where $\Delta(s)$ is an uncertainty. The set of $\Delta(s)$ is all functions satisfying

$$|\Delta(j\omega)| < |W_T(j\omega)| \quad (\forall \omega \in R_+), \tag{4}$$

where $W_T(s)$ is a stable rational function.

The robust stability condition for the time-delay plant $G(s)e^{-sL}$ with uncertainty $\Delta(s)$ satisfying (4) is given by

$$\left\| \frac{C(s)G_m(s)W_T(s)}{1 + C(s)G_m(s)e^{-sL_m}} \right\|_\infty < 1. \tag{5}$$

According to [15], the multi-period repetitive controller $C(s)$ in (1) is written by the form in

$$C(s) = C_0(s) + \frac{\sum_{i=1}^{N} C_i(s)e^{-sT_i}}{1 - \sum_{i=1}^{N} q_i(s)e^{-sT_i}}, \tag{6}$$

where N is an arbitrary positive integer, $C_0(s) \in R(s)$, $C_i(s) \neq 0 \in R(s)(i = 1, \dots, N)$, $T_i \in R(i =$

$1,\dots,N)$ and $q_i(s) \in R(s)(i = 1,\dots,N)$ are low-pass filters satisfying

$$1 - \sum_{i=1}^{N} q_i(j\omega_k) \simeq 0 \quad (k = 0,\dots,N_{max}), \tag{7}$$

where $\omega_k(k = 0,\dots,N_{max})$ is frequency components of the periodic reference input r written by

$$\omega_k = \frac{2\pi}{T}k \quad (k = 0,\dots,N_{max}) \tag{8}$$

and $\omega_{N_{max}}$ is the maximum frequency component of the periodic reference input r, then the output y in (1) follows the periodic reference input r with small steady state error.

Using the multi-period repetitive controller $C(s)$ in (6), the transfer function from the periodic reference input r to the output y and that from the disturbance d to the output y in (1) are written as

$$\begin{aligned}\frac{y}{r} &= \frac{C(s)G(s)e^{-sL}}{1+C(s)G(s)e^{-sL}} \\ &= \left\{C_0(s)G_m(s)\left(e^{-sL_m}+\Delta(s)\right) - \sum_{i=1}^{N}\left(C_0(s)\right.\right. \\ &\quad \left.\left. q_i(s) - C_i(s)\right)e^{-sT_i}G_m(s)\left(e^{-sL_m}+\Delta(s)\right)\right\} \\ &\quad \left[1 + C_0(s)G_m(s)\left(e^{-sL_m}+\Delta(s)\right)\right. \\ &\quad -\sum_{i=1}^{N}\left[q_i(s)\left\{1+C_0(s)G_m(s)\left(e^{-sL_m}+\Delta(s)\right)\right\}\right. \\ &\quad \left.\left. -C_i(s)G_m(s)\left(e^{-sL_m}+\Delta(s)\right)\right]e^{-sT_i}\right]^{-1}\end{aligned} \tag{9}$$

and

$$\begin{aligned}\frac{y}{d} &= \frac{1}{1+C(s)G(s)e^{-sL}} \\ &= \left\{1 - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right\}\left[1 + C_0(s)G_m(s)\right. \\ &\quad \left(e^{-sL_m}+\Delta(s)\right) - \sum_{i=1}^{N}\left[q_i(s)\left\{1+C_0(s)G_m(s)\right.\right. \\ &\quad \left.\left(e^{-sL_m}+\Delta(s)\right)\right\} - C_i(s)G_m(s)\left(e^{-sL_m}\right. \\ &\quad \left.\left.\left.+\Delta(s)\right)\right]e^{-sT_i}\right]^{-1},\end{aligned} \tag{10}$$

respectively. Generally, the transfer function from the periodic reference input r to the output y in (9) and that from the disturbance d to the output y in (10) have an infinite number of poles, even if $\Delta(s) = 0$. When the transfer function from the periodic reference input r to the output y and that from the disturbance d to the output y have an infinite number of poles, it is difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From a practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic are easily specified. In order to specify the input-output characteristic and the disturbance attenuation characteristic easily, the transfer function from the periodic reference input r to the output y and that from the disturbance d to the output y are desirable to have a finite number of poles.

From above practical requirement, we clarify the parametrization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with uncertainty defined as follows:

Definition 1: (robust stabilizing simple multi-period repetitive controllers for time-delay plants)
We call the controller $C(s)$ the robust stabilizing simple multi-period repetitive controllers for time-delay plants, if following expressions hold true:
1. The controller $C(s)$ works as a multi-period repetitive controller. That is, the controller $C(s)$ is written by (6), where $C_0(s) \in R(s)$, $C_i(s) \neq 0 \in R(s)(i = 1,\dots,N)$ and $q_i(s) \in R(s)(i = 1,\dots,N)$ satisfies $\sum_{i=1}^{N} q_i(0) = 1$.
2. When $\Delta(s) = 0$, the controller $C(s)$ makes the transfer function from the periodic reference input r to the output y in (1) and that from the disturbance d to the output y in (1) have a finite number of poles.
3. The controller $C(s)$ satisfies the robust stability condition in (5).

3. THE PARAMETRIZATION OF ALL ROBUST STABILIZING SIMPLE MULTI-PERIOD REPETITIVE CONTROLLERS FOR TIME-DELAY PLANTS

In this section, we give the parametrization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants defined in Definition 1.

In order to obtain the parametrization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants, we must see that the controller $C(s)$ satisfying (5). The problem of obtaining the controller $C(s)$, which is not necessarily a simple multi-period repetitive controller, satisfying (5) is equivalent to the following H_∞ problem. In order to obtain the controller $C(s)$ satisfying (5), we consider the control system shown in Fig. 1. Here, $P(s)$ is selected such that the

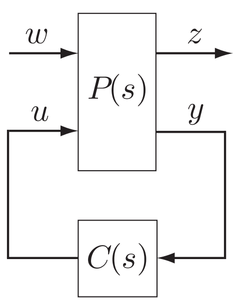

Fig.1: *Block diagram of H_∞ control problem*

transfer function from w to z in Fig. 1 is equal to

$C(s)G_m(s)W_T(s)/(1+C(s)G_m(s)e^{-sL_m})$. The state space description of $P(s)$ is, in general,

$$\left\{\begin{array}{rclll}\dot{x}(t) & = & Ax(t) & +B_1w(t) & +B_2u(t-L_m)\\ z(t) & = & C_1x(t) & & +D_{12}u(t)\\ y(t) & = & C_2x(t) & +D_{21}w(t) & \end{array}\right., \tag{11}$$

where $A\in R^{n\times n}$, $B_1\in R^n$, $B_2\in R^n$, $C_1\in R^{1\times n}$, $C_2\in R^{1\times n}$, $D_{12}\in R$, $D_{21}\in R$. $P(s)$ is called the generalized plant. $P(s)$ is assumed to satisfy the following assumptions:

1. (C_2, A) is detectable, (A, B_2) is stabilizable.
2. $D_{12}\neq 0$, $D_{21}\neq 0$.
3. rank $\begin{bmatrix} A-j\omega I & B_2\\ C_1 & D_{12}\end{bmatrix} = n+1(\forall\omega\in R)$, rank $\begin{bmatrix} A-j\omega I & B_1\\ C_2 & D_{21}\end{bmatrix} = n+1(\forall\omega\in R)$.
4. $C_1A^iB_2 = 0(i=0,1,2,\dots)$.

Under these assumptions, from [27], the following lemma holds true.

Lemma 1: There exists an H_∞ controller $C(s)$ for the generalized plant $P(s)$ in (11) if and only if there exists an H_∞ controller $C(s)$ for the generalized plant $\tilde{P}(s)$ written by:

$$\left\{\begin{array}{rclll}\dot{q}(t) & = & Aq(t) & +B_1w(t) & +\tilde{B}_2u(t)\\ \tilde{z}(t) & = & C_1q(t) & & +D_{12}u(t)\\ \tilde{y}(t) & = & C_2q(t) & +D_{21}w(t) & \end{array}\right., \tag{12}$$

where $\tilde{B}_2 = e^{-AL_m}B_2$. When $u(s) = C(s)\tilde{y}(s)$ is an H_∞ control input for (12),

$$u(t) = \mathcal{L}^{-1}\{C(s)\tilde{y}(s)\} \tag{13}$$

is an H_∞ control input for (11), where:

$$\tilde{y}(s) = \mathcal{L}\left\{y(t)+C_2\int_{-L_m}^{0} e^{-A(\tau+L_m)}B_2u(t+\tau)d\tau\right\}. \tag{14}$$

From Lemma 1 and [28], the following lemma holds true.

Lemma 2: If controllers satisfying (5) exist, both

$$X\left(A-\tilde{B}_2D_{12}^{\dagger}C_1\right)+\left(A-\tilde{B}_2D_{12}^{\dagger}C_1\right)^TX +X\left(B_1B_1^T-\tilde{B}_2\left(D_{12}^TD_{12}\right)^{-1}\tilde{B}_2^T\right)X +\left(D_{12}^{\perp}C_1^T\right)^TD_{12}^{\perp}C_1^T = 0 \tag{15}$$

and

$$Y\left(A-B_1D_{21}^{\dagger}C_2\right)^T+\left(A-B_1D_{21}^{\dagger}C_2\right)Y +Y\left(C_1^TC_1-C_2^T\left(D_{21}D_{21}^T\right)^{-1}C_2\right)Y +B_1D_{21}^{\perp}\left(B_1D_{21}^{\perp}\right)^T = 0 \tag{16}$$

have solutions $X\geq 0$ and $Y\geq 0$ such that

$$\rho(XY) < 1 \tag{17}$$

and both $A-\tilde{B}_2D_{12}^{\dagger}C_1+\{B_1B_1^T-\tilde{B}_2(D_{12}^TD_{12})^{-1}\tilde{B}_2^T\}X$ and $A-B_1D_{21}^{\dagger}C_2+Y\{C_1^TC_1-C_2(D_{21}D_{21}^T)^{-1}C_2\}$ have no eigenvalue in the closed right half plane. Using X and Y, the parameterization of all controllers satisfying (5) is given by

$$C(s) = C_{11}(s)+C_{12}(s)Q(s)(I-C_{22}(s)Q(s))^{-1}C_{21}(s), \tag{18}$$

where

$$\begin{bmatrix} C_{11}(s) & C_{12}(s)\\ C_{21}(s) & C_{22}(s)\end{bmatrix} = \left[\begin{array}{c|cc} A_c & B_{c1} & B_{c2}\\ \hline C_{c1} & D_{c11} & D_{c12}\\ C_{c2} & D_{c21} & D_{c22}\end{array}\right], \tag{19}$$

$$A_c = A+B_1B_1^TX-\tilde{B}_2\left(D_{12}^{\dagger}C_1+E_{12}^{-1}\tilde{B}_2^TX\right) -(I-XY)^{-1}\left(B_1D_{21}^{\dagger}+YC_2^TE_{21}^{-1}\right)\left(C_2+D_{21}B_1^TX\right),$$

$$B_{c1} = (I-XY)^{-1}\left(B_1D_{21}^{\dagger}+YC_2^TE_{21}^{-1}\right),$$

$$B_{c2} = (I-XY)^{-1}\left(\tilde{B}_2+YC_1^TD_{12}\right)E_{12}^{-1/2},$$

$$C_{c1} = -D_{12}^{\dagger}C_1-E_{12}^{-1}\tilde{B}_2^TX,$$

$$C_{c2} = -E_{21}^{-1/2}\left(C_2+D_{21}B_1^TX\right),$$

$$D_{c11}=0,\ D_{c12}=E_{12}^{-1/2},\ D_{c21}=E_{21}^{-1/2},\ D_{c22}=0,$$

$$E_{12}=D_{12}^TD_{12},\ E_{21}=D_{21}D_{21}^T$$

and $Q(s)\in H_\infty$ is any function satisfying $\|Q(s)\|_\infty < 1$ [28].

Using Lemma 2, the parametrization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants is given by following theorem.

Theorem 1: If simple multi-period repetitive controllers satisfying (5) exist, both (15) and (16) have solutions $X\geq 0$ and $Y\geq 0$ satisfying (17) and both $A-\tilde{B}_2D_{12}^{\dagger}C_1+\{B_1B_1^T-\tilde{B}_2(D_{12}^TD_{12})^{-1}\tilde{B}_2^T\}X$ and $A-B_1D_{21}^{\dagger}C_2+Y\{C_1^TC_1-C_2(D_{21}D_{21}^T)^{-1}C_2\}$ have no eigenvalue in the closed right half plane. Using X and Y, the parametrization of all robust stabilizing simple multi-period repetitive controllers satisfying (5) is given by

$$C(s) = C_{11}(s)+C_{12}(s)Q(s)(I-C_{22}(s)Q(s))^{-1}C_{21}(s), \tag{20}$$

where $C_{ij}(s)(i = 1, 2; j = 1, 2)$ are given by (19) and $Q(s) \in H_\infty$ is any function satisfying $\|Q(s)\|_\infty < 1$ and written by

$$Q(s) = \frac{Q_{n0}(s) + \sum_{i=1}^{N} Q_{ni}(s)e^{-sT_i}}{Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s)e^{-sT_i}}, \quad (21)$$

$$Q_{n0}(s) = \tilde{G}_d(s)\bar{Q}_0(s), \quad (22)$$

$$Q_{d0}(s) = -\frac{1}{1 + C_{11}(s)\tilde{G}_m(s)} \left(\tilde{G}_n(s)\bar{Q}_0(s) - 1\right), \quad (23)$$

$$Q_{ni}(s) = \tilde{G}_d(s)\bar{Q}_i(s) \;\; (i = 1, \dots, N) \quad (24)$$

and

$$Q_{di}(s) = -\frac{1}{1 + C_{11}(s)\tilde{G}_m(s)}\tilde{G}_n(s)\bar{Q}_i(s) \quad (i = 1, \dots, N), \quad (25)$$

where $\tilde{G}_n(s) \in RH_\infty$ and $\tilde{G}_d(s) \in RH_\infty$ are coprime factors of $-C_{22}(s) + (-C_{11}(s)C_{22}(s) + C_{12}(s)C_{21}(s))\tilde{G}_m(s)$ on RH_∞ satisfying

$$\frac{\tilde{G}_n(s)}{\tilde{G}_d(s)} = -C_{22}(s) + (-C_{11}(s)C_{22}(s) + C_{12}(s)C_{21}(s))\,\tilde{G}_m(s), \quad (26)$$

where $\tilde{G}_m(s) = C_2(sI - A)^{-1}\tilde{B}_2$, $\bar{Q}_0(s) \in RH_\infty$ is any function, $\bar{Q}_i(s) \neq 0 \in RH_\infty(i = 1, \dots, N)$ are any function and

$$\sum_{i=1}^{N} \left\{ -\frac{Q_{di}(0) - C_{22}(0)Q_{ni}(0)}{Q_{d0}(0) - C_{22}(0)Q_{n0}(0)} \right\} = 1 \quad (27)$$

holds true.

Proof: First, necessity is shown. That is, if the multi-period repetitive controller written by (6) stabilizes the control system in (1) robustly and makes the transfer function from the periodic reference input r to the output y in (9) and that from the disturbance d to the output y in (10) have a finite number of poles when $\Delta(s) = 0$, then $C(s)$ and $Q(s)$ are written by (20) and (21), respectively. From Lemma 2, the parametrization of all robust stabilizing controllers $C(s)$ for $G(s)e^{-sL}$ is written by (20), where $\|Q(s)\|_\infty < 1$. In order to prove the necessity, we will show that if $C(s)$ written by (6) stabilizes the control system in (1) robustly and makes the transfer function from the periodic reference input r to the output y in (9) and that from the disturbance d to the output y in (10) have a finite number of poles when $\Delta(s) = 0$, then $Q(s)$ in (20) is written by (21). Substituting $C(s)$ in (6) into (20), we have (21), where

$$Q_{n0}(s) = (C_{11n}(s)C_{0d}(s) - C_{11d}(s)C_{0n}(s)) C_d(s)q_d(s)C_{12d}(s)C_{22d}(s)C_{21d}(s), \quad (28)$$

$$Q_{ni}(s) = \{(C_{0n}(s)C_d(s)C_{11d}(s) - C_{0d}(s)C_d(s) C_{11n}(s))\, q_{in}(s) - C_{0d}(s)C_{in}(s)q_d(s) C_{11d}(s)\}\, C_{12d}(s)C_{22d}(s)C_{21d}(s) \quad (i = 1, \dots, N), \quad (29)$$

$$Q_{d0}(s) = (-C_{0n}(s)C_{11d}(s)C_{12d}(s)C_{22n}(s)C_{21d}(s) + C_{0d}(s)C_{11n}(s)C_{12d}(s)C_{22n}(s)C_{21d}(s) - C_{0d}(s)C_{11d}(s)C_{12n}(s)C_{22d}(s)C_{21n}(s)) C_d(s)q_d(s) \quad (30)$$

and

$$Q_{di}(s) = (C_{0n}(s)C_d(s)C_{11d}(s)C_{12d}(s)C_{22n}(s) C_{21d}(s) - C_{0d}(s)C_d(s)C_{11n}(s)C_{12d}(s) C_{22n}(s)C_{21d}(s) + C_{0d}(s)C_d(s)C_{11d}(s) C_{12n}(s)C_{22d}(s)C_{21n}(s))\, q_{in}(s) - C_{0d}(s) C_{in}(s)q_d(s)C_{11d}(s)C_{12d}(s)C_{22n}(s)C_{21d}(s) \quad (i = 1, \dots, N). \quad (31)$$

Here, $C_{0n}(s) \in RH_\infty$ and $C_{0d}(s) \in RH_\infty$ are coprime factors of $C_0(s)$ on RH_∞ satisfying

$$C_0(s) = \frac{C_{0n}(s)}{C_{0d}(s)}, \quad (32)$$

$q_{in}(s) \in RH_\infty(i = 1, \dots, N)$, $q_d(s) \in RH_\infty$, $C_{in}(s) \in RH_\infty(i = 1, \dots, N)$, $C_d(s) \in RH_\infty$, $C_{ijn}(s) \in RH_\infty$ $(i = 1, 2; j = 1, 2)$ and $C_{ijd}(s) \in RH_\infty(i = 1, 2; j = 1, 2)$ are coprime factors satisfying

$$q_i(s) = \frac{q_{in}(s)}{q_d(s)}(i = 1, \dots, N), \quad (33)$$

$$C_i(s) = \frac{C_{in}(s)}{C_d(s)}(i = 1, \dots, N) \quad (34)$$

and

$$C_{ij}(s) = \frac{C_{ijn}(s)}{C_{ijd}(s)}(i = 1, 2; j = 1, 2). \quad (35)$$

From (28)∼(31), all of $Q_{n0}(s)$, $Q_{ni}(s)(i = 1, \dots, N)$, $Q_{d0}(s)$ and $Q_{di}(s)(i = 1, \dots, N)$ are included in RH_∞. Thus, we have shown that if $C(s)$ written by (6) stabilize the control system in (1) robustly, $Q(s)$ in (20) is written by (21). Since $\sum_{i=1}^{N} q_i(0) = 1(i = 1, \dots, N)$, (27) holds true.

The rest to prove necessity is to show that when $\Delta(s) = 0$, if $C(s)$ in (6) makes the transfer function from the periodic reference input r to the output y

and that from the disturbance d to the output y have a finite number of poles, then $Q_{n0}(s)$, $Q_{d0}(s)$, $Q_{ni}(s)$ and $Q_{di}(s)$ are written by (22), (23), (24) and (25), respectively. From (6), when $\Delta(s) = 0$, the transfer function from the periodic reference input r to the output y and that from the disturbance d to the output y are written by

$$\frac{y}{r} = \frac{G_{ryn}(s)}{G_{ryd}(s)} \tag{36}$$

and

$$\frac{y}{d} = \frac{G_{dyn}(s)}{G_{dyd}(s)}, \tag{37}$$

respectively, where

$$\begin{aligned} G_{ryn}(s) &= \left\{C_{0n}(s)q_d(s)C_d(s) + \sum_{i=1}^{N}(C_{in}(s)q_d(s) \right. \\ &\quad \left. C_{0d}(s) - C_{0n}(s)q_{in}(s)C_d(s))\, e^{-sT_i}\right\} \\ &\quad \tilde{G}_{mn}(s), \end{aligned} \tag{38}$$

$$\begin{aligned} G_{ryd}(s) &= q_d(s)C_{0d}(s)\tilde{G}_{md}(s)C_d(s) + C_{0n}(s) \\ &\quad \tilde{G}_{mn}(s)q_d(s)C_d(s) - \sum_{i=1}^{N}\{(C_{0d}(s) \\ &\quad \left.\tilde{G}_{md}(s)C_d(s) + \tilde{G}_{md}(s)C_{0n}(s)C_d(s)\right) \\ &\quad q_{in}(s) - \tilde{G}_{md}(s)C_{in}(s)q_d(s) \\ &\quad C_{0d}(s)\}\, e^{-sT_i}, \end{aligned} \tag{39}$$

$$\begin{aligned} G_{dyn}(s) &= \left(q_d(s) - \sum_{i=1}^{N} q_{in}(s)e^{-sT_i}\right) C_{0d}(s) \\ &\quad \tilde{G}_{md}(s)C_d(s) \end{aligned} \tag{40}$$

and

$$\begin{aligned} G_{dyd}(s) &= \left(C_{0d}(s)\tilde{G}_{md}(s) + C_{0n}(s)\tilde{G}_{mn}(s)\right) \\ &\quad q_d(s)C_d(s) - \sum_{i=1}^{N}\left\{\left(C_{0d}(s)\tilde{G}_{md}(s)\right.\right. \\ &\quad \left. C_d(s) + \tilde{G}_{md}(s)C_{0n}(s)C_d(s)\right) q_{in}(s) \\ &\quad \left. -\tilde{G}_{md}(s)C_{in}(s)q_d(s)C_{0d}(s)\right\} e^{-sT_i}. \end{aligned} \tag{41}$$

From the assumption that the transfer function from the periodic reference input r to the output y in (36) and that from the disturbance d to the output y in (37) have a finite number of poles, (39) and (41),

$$\left(C_{0d}(s)\tilde{G}_{md}(s) + C_{0n}(s)\tilde{G}_{mn}(s)\right) q_d(s)C_d(s) = 1 \tag{42}$$

and

$$\begin{aligned} &\left(C_{0d}(s)\tilde{G}_{md}(s)C_d(s) + \tilde{G}_{md}(s)C_{0n}(s)C_d(s)\right) q_{in}(s) \\ &\quad -\tilde{G}_{md}(s)C_{in}(s)q_d(s)C_{0d}(s) = 0 \quad (i = 1, \ldots, N) \end{aligned} \tag{43}$$

are satisfied. From (28), (29), (30) and (31), (42) and (43) are rewritten by

$$\begin{aligned} &Q_{d0}(s) - C_{22}(s)Q_{n0}(s) + \{C_{11}(s)Q_{d0}(s) + (-C_{11}(s) \\ &\quad C_{22}(s) + C_{12}(s)C_{21}(s))\, Q_{n0}(s)\}\, \tilde{G}_m(s) = 1 \end{aligned} \tag{44}$$

and

$$\begin{aligned} &Q_{di}(s) - C_{22}(s)Q_{ni}(s) + \{C_{11}(s)Q_{di}(s) + (-C_{11}(s) \\ &\quad C_{22}(s) + C_{12}(s)C_{21}(s))\, Q_{ni}(s)\}\, \tilde{G}_m(s) = 0 \\ &\quad (i = 1, \ldots, N). \end{aligned} \tag{45}$$

Using (26), (44) and (45) are rewritten by

$$Q_{d0}(s) = -\frac{1}{1 + C_{11}(s)\tilde{G}_m(s)}\left(\frac{\tilde{G}_n(s)}{\tilde{G}_d(s)}Q_{n0}(s) - 1\right) \tag{46}$$

and

$$\begin{aligned} Q_{di}(s) &= -\frac{1}{1 + C_{11}(s)\tilde{G}_m(s)}\frac{\tilde{G}_n(s)}{\tilde{G}_d(s)}Q_{ni}(s) \\ &\quad (i = 1, \ldots, N). \end{aligned} \tag{47}$$

Since $Q_{n0}(s) \in RH_\infty$, $Q_{d0}(s) \in RH_\infty$, $Q_{ni}(s) \in RH_\infty(i = 1, \ldots, N)$ and $Q_{di}(s) \in RH_\infty(i = 1, \ldots, N)$, $Q_{n0}(s)$, $Q_{d0}(s)$, $Q_{ni}(s)$ and $Q_{di}(s)$ are written by (22), (23), (24) and (25), respectively, where $\bar{Q}_0(s) \in RH_\infty$ and $\bar{Q}_i(s) \in RH_\infty(i = 1, \ldots, N)$. From the assumption that $C_i(s) \neq 0(i = 1, \ldots, N)$ and from (29) and (31), $\bar{Q}_i(s) \neq 0(i = 1, \ldots, N)$ holds true. We have thus proved necessity.

Next, sufficiency is shown. That is, if $C(s)$ and $Q(s) \in H_\infty$ are settled by (20) and (21), respectively, then the controller $C(s)$ is written by the form in (6), $\sum_{i=1}^{N} q_i(0) = 1$ holds true and the transfer function from the periodic reference input r to the output y and that from the disturbance d to the output y have a finite number of poles. Substituting (21) into (20), we have (6), where, $C_0(s)$, $C_i(s)(i = 1, \ldots, N)$ and $q_i(s)(i = 1, \ldots, N)$ are denoted by

$$C_0(s) = \frac{\dfrac{C_{11}(s)Q_{d0}(s) + (C_{12}(s)C_{21}(s)}{Q_{d0}(s) - C_{22}(s)Q_{n0}(s)}}{\dfrac{-C_{11}(s)C_{22}(s))Q_{n0}(s)}{}}, \tag{48}$$

$$\begin{aligned} C_i(s) &= \frac{\dfrac{C_{11}(s)Q_{di}(s) + (-C_{11}(s)C_{22}(s)}{Q_{d0}(s) - C_{22}(s)Q_{n0}(s)}}{\dfrac{+C_{12}(s)C_{21}(s))Q_{ni}(s)}{}} + C_0(s)q_i(s) \\ &\quad (i = 1, \ldots, N) \end{aligned} \tag{49}$$

and

$$q_i(s) = -\frac{Q_{di}(s) - C_{22}(s)Q_{ni}(s)}{Q_{d0}(s) - C_{22}(s)Q_{n0}(s)} \quad (i = 1, \ldots, N). \tag{50}$$

We find that if $C(s)$ and $Q(s)$ are settled by (20) and (21), respectively, then the controller $C(s)$ is written by the form in (6). From $\bar{Q}_i(s) \neq 0 (i = 1, \ldots, N)$ and (49), $C_i(s) \neq 0 (i = 1, \ldots, N)$ holds true. Substituting (27) into (50), we have $\sum_{i=1}^{N} q_i(0) = 1$. In addition, from (22), (23), (24) and (25) and easy manipulation, we can confirm that when $\Delta(s) = 0$, the transfer function from the periodic reference input r to the output y and that from the disturbance d to the output y have a finite number of poles.

We have thus proved Theorem 1. ■

4. CONTROL CHARACTERISTICS

In this section, we describe control characteristics of control system in (1) using the robust stabilizing simple multi-period repetitive controllers for time-delay plants in (20).

First, we mention the input-output characteristic. The transfer function from the periodic reference input r to the error $e = r - y$ is written by

$$\frac{e}{r} = \frac{1}{1 + C(s)G(s)e^{-sL}} = \frac{G_{ern}(s)}{G_{erd}(s)}, \quad (51)$$

where

$$G_{ern}(s) = \left(1 + \sum_{i=1}^{N} \frac{Q_{di}(s) - C_{22}(s)Q_{ni}(s)}{Q_{d0}(s) - C_{22}(s)Q_{n0}(s)} e^{-sL}\right) (Q_{d0}(s) - C_{22}(s)Q_{n0}(s)) \quad (52)$$

and

$$\begin{aligned} G_{erd}(s) = {} & 1 + [\{C_{11}(s)Q_{d0}(s) + (-C_{11}(s)C_{22}(s) \\ & + C_{12}(s)C_{21}(s))\, Q_{n0}(s)\}\, G_m(s) \\ & + \sum_{i=1}^{N} \{C_{11}(s)Q_{di}(s) + (-C_{11}(s)C_{22}(s) \\ & + C_{12}(s)C_{21}(s))\, Q_{ni}(s)\}\, G_m(s)e^{-sT_i}]\, \Delta(s). \end{aligned} \quad (53)$$

From (51), for $\omega_k (k = 0, \ldots, N_{max})$ in (8) those are frequency components of the periodic reference input r, if

$$1 + \sum_{i=1}^{N} \frac{Q_{di}(j\omega_k) - C_{22}(j\omega_k)Q_{ni}(j\omega_k)}{Q_{d0}(j\omega_k) - C_{22}(j\omega_k)Q_{n0}(j\omega_k)} \simeq 0 \quad (k = 0, \ldots, N_{max}), \quad (54)$$

then the output y in (1) follows periodic reference input r with small steady state error.

Next, we mention the disturbance attenuation characteristic. The transfer function from the disturbance d to the output y is written by

$$\frac{y}{d} = \frac{1}{1 + C(s)G(s)e^{-sL}} = \frac{G_{ydn}(s)}{G_{ydd}(s)}, \quad (55)$$

where

$$G_{ydn}(s) = \left(1 + \sum_{i=1}^{N} \frac{Q_{di}(s) - C_{22}(s)Q_{ni}(s)}{Q_{d0}(s) - C_{22}(s)Q_{n0}(s)} e^{-sL}\right) (Q_{d0}(s) - C_{22}(s)Q_{n0}(s)) \quad (56)$$

and

$$\begin{aligned} G_{ydd}(s) = {} & 1 + [\{C_{11}(s)Q_{d0}(s) + (-C_{11}(s)C_{22}(s) \\ & + C_{12}(s)C_{21}(s))\, Q_{n0}(s)\}\, G_m(s) \\ & + \sum_{i=1}^{N} \{C_{11}(s)Q_{di}(s) + (-C_{11}(s)C_{22}(s) \\ & + C_{12}(s)C_{21}(s))\, Q_{ni}(s)\}\, G_m(s)e^{-sT_i}]\, \Delta(s). \end{aligned} \quad (57)$$

From (55), for frequency components of the disturbance d those are equivalent to frequency that of the periodic reference input r, if (54) is satisfied, then the disturbance d is attenuated effectively.

For the frequency component ω of the disturbance d that is different from that of the periodic reference input r, that is $\omega \neq \omega_k (k = 0, \ldots, N_{max})$, even if

$$1 + \sum_{i=1}^{N} \left\{ \frac{Q_{di}(j\omega) - C_{22}(j\omega)Q_{ni}(j\omega)}{Q_{d0}(j\omega) - C_{22}(j\omega)Q_{n0}(j\omega)} \right\} = 0, \quad (58)$$

the disturbance d cannot be attenuated, because

$$e^{-j\omega T_i} \neq 1 \quad (59)$$

and

$$1 + \sum_{i=1}^{N} \left\{ \frac{Q_{di}(j\omega) - C_{22}(j\omega)Q_{ni}(j\omega)}{Q_{d0}(j\omega) - C_{22}(j\omega)Q_{n0}(j\omega)} e^{-j\omega T_i} \right\} \neq 0. \quad (60)$$

In order to attenuate the frequency component ω of the disturbance d that is different from that of the periodic reference input r, if

$$Q_{d0}(j\omega) - C_{22}(j\omega)Q_{n0}(j\omega) \simeq 0, \quad (61)$$

then the disturbance d is attenuated effectively.

From the above discussion, we find that the role of $Q_{ni}(s) (i = 1, \ldots, N)$ and $Q_{di}(s) (i = 1, \ldots, N)$ is to specify the input-output characteristics for the periodic reference input r and that of $Q_{n0}(s)$ and $Q_{d0}(s)$ is to specify the disturbance attenuation characteristics for the frequency components of the disturbance d that are different from those of the periodic reference input r.

5. A DESIGN PROCEDURE

In this section, a design procedure of robust stabilizing simple multi-period repetitive controller for time-delay plants satisfying Theorem 1 is presented.

A design procedure of a robust stabilizing simple multi-period repetitive controller for time-delay plants is summarized as follows:

Procedure

Step 1) Using the result in [28] and Theorem 1, $C_{ij}(s)(i = 1, 2; j = 1, 2)$ are obtained as (19).
Step 2) $Q_{n0}(s) \in RH_\infty$ and $Q_{d0}(s) \in RH_\infty$, that is $\bar{Q}_0(s) \in RH_\infty$ in (22) and (23), is set satisfying (61) so that the disturbance d of the frequency component ω is attenuated effectively.
Step 3) $Q_{ni}(s) \in RH_\infty(i = 1, \ldots, N)$ and $Q_{di}(s) \in RH_\infty(i = 1, \ldots, N)$, that is $\bar{Q}_i(s)(i = 1, \ldots, N)$ in (24) and (25), are set satisfying (54) so that the output y to follow the periodic reference input r with small steady state error and the disturbance d of the frequency component $\omega_k(k = 0, \ldots, N_{max})$ is attenuated effectively.
Step 4) Substituting above $C_{ij}(s)(i = 1, 2; j = 1, 2)$, $Q(s)$, $Q_{n0}(s)$, $Q_{d0}(s)$, $Q_{ni}(s)$ and $Q_{di}(s)$ for (20) and (21), we obtain a robust stabilizing simple multi-period repetitive controllers for time-delay plants.

6. NUMERICAL EXAMPLE

In this section, a numerical example is illustrated to show the effectiveness of the proposed method.

Consider the problem to obtain the parametrization of all robust stabilizing simple multi-period repetitive controllers for time-delay plant $G(s)e^{-sL}$ written by

$$G(s)e^{-sL} = G_m(s)(e^{-sL_m} + \Delta(s)). \qquad (62)$$

The nominal time-delay plant of $G(s)e^{-sL}$ is given by

$$G_m(s)e^{-sL_m} = \frac{1}{(s+2)(s+3)}e^{-0.6s} \qquad (63)$$

and the upper bound $W_T(s)$ of the set of $\Delta(s)$ is given by

$$W_T(s) = \frac{3s+1}{s+10}. \qquad (64)$$

The periodic reference input r with period $T = 20$[sec] and the disturbance d are written by

$$r(t) = \sin(0.1\pi t - L_m) + \sin(0.2\pi t - L_m) + \sin(0.3\pi t - L_m), \qquad (65)$$

$$d(t) = \sin(0.1\pi t) + \sin(0.2\pi t) + \sin(0.3\pi t) \qquad (66)$$

and

$$d(t) = \sin(0.025\pi t) + \sin(0.05\pi t) + \sin(0.075\pi t). \qquad (67)$$

N in (6) and T are settled by $N = 3$ and

$$T_i = Ti, \qquad (68)$$

respectively. In order for the output y to follow the periodic reference input r in (65) with small steady state error and the disturbance d in (66) and (67) is attenuated effectively, $Q(s)$ in $Q_{n0}(s)$, $Q_{d0}(s)$, $Q_{ni}(s)(i = 1, 2, 3)$ and $Q_{di}(s)(i = 1, 2, 3)$ are settled satisfying (61) and (54).

When $\Delta(s)$ is given by

$$\Delta(s) = \frac{2s+1}{s+12}, \qquad (69)$$

the response of the output y in (1) for the periodic reference input r is shown in Fig. 2. Here, the dotted line shows the response of the periodic reference input r and the solid line shows that of the output y. Figure 2 shows that the output y follows the periodic reference input r with small steady state error.

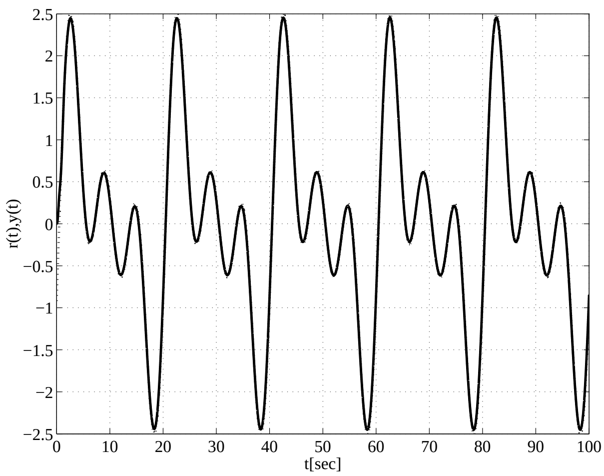

Fig.2: *The response of the output y for the periodic reference input* $r(t) = \sin(0.1\pi t - L_m) + \sin(0.2\pi t - L_m) + \sin(0.3\pi t - L_m)$

Next, using the designed robust stabilizing simple multi-period repetitive controller for time-delay plants $C(s)$, the disturbance attenuation characteristic is shown. The response of the output y for the disturbance d of which the frequency component is equivalent to that of the periodic reference input r is shown in Fig. 3. Here, the dotted line shows the response of the disturbance d and the solid line shows that of the output y. Figure 3 shows that the disturbance d is attenuated effectively. Finally, the response of the output y for the disturbance d of which the frequency component is different from that of the periodic reference input r is shown in Fig. 4. Here, the dotted line shows the response of the disturbance d and the solid line shows that of the output y. Figure 4 shows that the disturbance d is attenuated effectively.

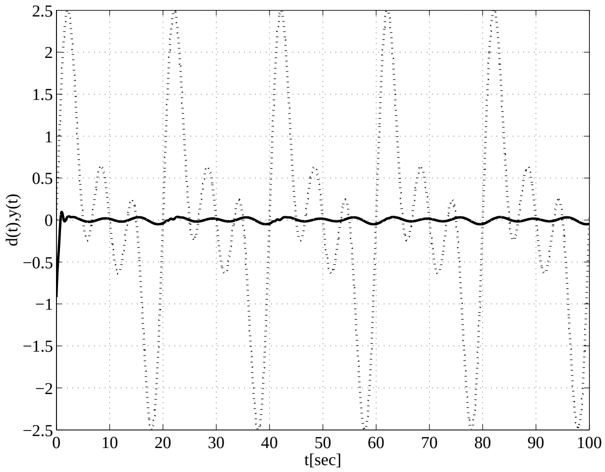

Fig.3: *The response of the output y for the disturbance* $d(t) = \sin(0.1\pi t) + \sin(0.2\pi t) + \sin(0.3\pi t)$

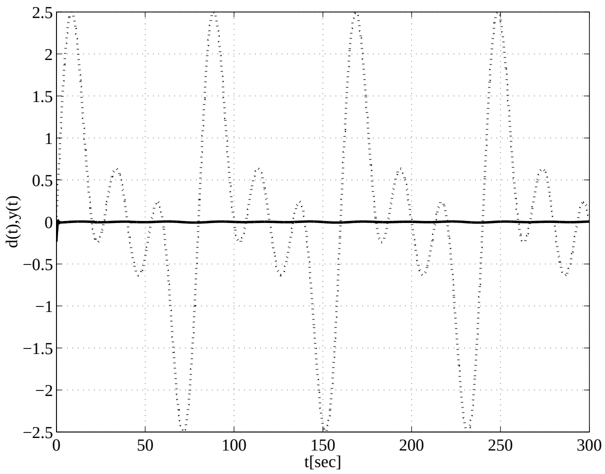

Fig.4: *The response of the output y for the disturbance* $d(t) = \sin(0.025\pi t) + \sin(0.05\pi t) + \sin(0.075\pi t)$

In this way, it is shown that using the obtained parametrization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants, we can design a robust stabilizing simple multi-period repetitive controller for time-delay plants easily.

7. CONCLUSIONS

In this paper, we proposed the parametrization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with uncertainty such that the controller works as a robust stabilizing multi-period repetitive controller and makes the transfer function from the periodic reference input to the output and that from the disturbance to the output have a finite number of poles.

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