Improved Performance Upper Bounds for Terminated Convolutional Codes in Rayleigh Fading Channels

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ABSTRACT

This paper presents improved performance upper bounds for terminated convolutional codes over Rayleigh fading channels with BPSK and QPSK modulation. The conventional union bound derived from the transfer function becomes loose when the convolutional code is terminated by a finite-length information sequence followed by additional tail bits. We use the modified trellis approach to derive tighter upper bounds for terminated convolutional codes combined with BPSK modulation over Rayleigh fading channels. Furthermore, we derive the weight enumerators for rate-1/2 terminated convolutional codes with QPSK modulation and present tighter upper bounds over Rayleigh fading channels.

Keywords: Convolutional Code, Weight Enumerator, BPSK, QPSK, Rayleigh Fading, Upper Bound

1. INTRODUCTION

The conventional union bound technique [1] for estimating the performance of Viterbi decoded convolutional codes assumes an infinitely-long input bit sequence. However, in most practical applications, such as 3G and GSM mobile communication systems, the convolutional code is terminated by a finite-length input information sequence followed by additional zero input bits. In such situations, the conventional union bound becomes loose.

A more sophisticated technique to derive performance bounds for terminated convolutional codes has been proposed by Moon and Cox in [2] [3]. They have defined a generalized weight enumerator of single error events (GWESEE) for terminated convolutional codes by modifying the trellis diagram, and presented tighter upper bounds for the AWGN channel with BPSK modulation. This technique eliminates code words composed of multiple error events and provides a tighter performance bound than the conventional union bound technique.

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In this work, we make use of the GWESEE approach to derive tighter upper bounds for Rayleigh fading channels with BPSK modulation. Furthermore, we derive the weight enumerator for rate-1/2 convolutional codes with QPSK modulation and use the new weight enumerator to derive tighter upper bounds in Rayleigh fading channels with QPSK modulation. The derived performance bounds are compared with the bounds presented in literature and simulation results for rate-1/2 convolutional codes of constraint length $K = 3$ and 5 over Rayleigh fading channels.

Section 2 describes the system under consideration while Section 3 presents the conventional upper bounds. Section 4 is devoted to the derivation of the upper bound for terminated convolutional codes in Rayleigh fading channels with BPSK modulation where as Section 5 presents the derivation of weight enumerator for QPSK and the upper bounds for QPSK in Rayleigh fading channels. The comparison of numerical results with simulated results is presented in Section 6 and the conclusion is given in Section 7.

2. SYSTEM MODEL

In the system being considered, the source bits $\{b_i\}$ are encoded by a convolutional encoder with a terminated trellis, followed by a BPSK or QPSK modulator. The communication channel is subjected to additive white Gaussian noise (AWGN) represented by independent identically distributed (i.i.d.) complex Gaussian random variables $\{n_i\}$ with zero mean and unit variance. The amplitude fading process $\{a_i\}$ is i.i.d. Rayleigh distributed with $E\{a_i^2\} = 1$ and the probability density function (pdf)

$$p_{a_i}(a) = 2a \exp(-a^2). \tag{1}$$

The hard decision demodulator is employed for BPSK modulation while soft decision demodulation [4] is applied for QPSK. At the decoder, the Viterbi algorithm [5] with soft decision decoding is employed with perfect channel state information. The i^{th} input x_i to the channel after modulation is given by,

$$x_i = \exp(j\theta_i), \tag{2}$$

where,

$$\theta_i \in \left\{ \begin{array}{ll} \{0, \pi\} & \text{for BPSK;} \\ \{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\} & \text{for QPSK.} \end{array} \right.$$

3. CONVENTIONAL TRANSFER FUNCTION BOUNDS

The performance bounds for convolutional coded BPSK systems over fading channels have been studied in [6] using the union bound derived from the transfer function while that for rate-1/2 convolutional coded QPSK systems over Rayleigh and Rician fading channels have been derived using the same approach in [7] and [8], respectively.

Accordingly, the upper bound on the bit error rate (BER) P_b for a rate-$1/n$ code is given as

$$P_b \leq \sum_{d=d_{free}} B_d P_d(X, \hat{X}), \quad (3)$$

where, $P_d(X, \hat{X})$ is the pairwise error probability when the Hamming distance between the transmitted sequence $X = \{x_1, x_2, \ldots, x_k, \ldots\}$ and the estimated sequence $\hat{X} = \{\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_k, \ldots\}$ is d, B_d is the total number of non-zero information bits on all paths of distance d and d_{free} is the free distance of the convolutional code. In the conventional approach, B_d is obtained by the transfer function of the code as

$$\left.\frac{\partial T(B, D)}{\partial B}\right|_{B=1} = \sum_{d=d_{free}}^{\infty} B_d D^d, \quad (4)$$

where, $T(B, D)$ is the transfer function of the code, the exponent of D denotes the Hamming distance of coded sequence, and the exponent of B denotes the Hamming distance of input sequence.

Since the transfer function bounds assume infinitely-long input sequences and do not consider the effect of termination, they become loose for terminated trellises with short information sequences.

4. TIGHTER UPPER BOUNDS FOR TERMINATED CODES COMBINED WITH BPSK OVER RAYLEIGH FADING CHANNELS

In this section, we derive tighter upper bounds for BER of terminated convolutional codes over Rayleigh fading channels with BPSK modulation using the GWESEE approach.

The GWESEE [2] [3] accurately represents the relationship between the Hamming distance of the coded output and the Hamming distance of the input bits of terminated trellises. This is obtained using a modified trellis approach, which eliminates the code words of multiple error events.

The new weight enumerator $W(B, D)$ for a terminated convolutional code with BPSK modulation is defined as,

$$W(B, D) = \sum_i \sum_j c_{i,j}.B^i D^j, \quad (5)$$

where $c_{i,j}$ is the number of error events composed of a single error event with input Hamming distance i and output Hamming distance j [2].

Accordingly, the new upper bound on P_b for a convolutional code terminated with L information bits over non-dispersive Rayleigh fading channel can be expressed as,

$$P_{b,Rayl} = \frac{1}{L} \sum_{j=d_{free}}^{d_{max}} \sum_i i.c_{i,j}.P_{j,Rayl}(X, \hat{X}), \quad (6)$$

where, $P_{j,Rayl}(X, \hat{X})$ is the pairwise error probability between transmitted sequence X and estimated sequence $\hat{X}$, when the Hamming distance between X and $\hat{X}$ is j, over Rayleigh fading channels and d_{max} is the maximum Hamming distance of the output code words for the particular terminated convolutional code.

For a fading channel modeled by a sequence of i.i.d. Rayleigh random variables $\{a_i\}$, and soft decision decoding with perfect channel state information, the conditional pairwise error probability $P_{d,Rayl}(X, \hat{X}|\{a_i\})$ is given by [9],

$$P_{d,Rayl}(X, \hat{X}|\{a_i\}) = Q\left(\sqrt{\frac{rE_b}{2N_0}\sum_{k=1}^{d} a_k^2 |\hat{x}_k - x_k|^2}\right), \quad (7)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{t^2}{2}}\,dt$, r is the code rate and $\frac{E_b}{N_0}$ is the bit energy to noise density ratio and d is the Hamming distance between X and $\hat{X}$.

Using the exact form of Q-function [9],

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(\frac{-x^2}{2\sin^2\theta}\right) d\theta,$$

(7) can be written as,

$$P_{d,Rayl}(X, \hat{X}|\{a_i\}) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{rE_b}{4N_0\sin^2\theta}\sum_{k=1}^{d} a_k^2|\hat{x}_k - x_k|^2\right) d\theta. \quad (8)$$

Then, the unconditional pairwise error probability $P_{d,Rayl}(X, \hat{X})$ can be written as

$$P_{d,Rayl}(X, \hat{X}) = \frac{1}{\pi}\int_0^{\pi/2} E_{a_k}\left[\exp\left(-\frac{rE_b}{N_0\sin^2\theta}\sum_{k=1}^{d} a_k^2\right)\right] d\theta \quad (9)$$

after substituting $|\hat{x}_k - x_k|^2 = 4$ for all $\hat{x}_k \neq x_k$, for BPSK signaling.

Since a_i is Rayleigh distributed with $E\{a_i^2\} = 1$, $A_d = \sum_{k=1}^{d} a_k^2$ is a χ^2-distributed random variable. Hence,

$$E[\exp(qA_d)] = \frac{1}{(1-q)^d}. \quad (10)$$

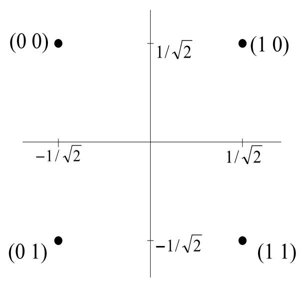

Fig.1: *QPSK signal constellation*

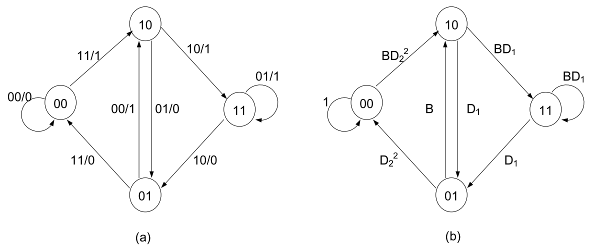

Fig.2: *(a) State diagram for code C_1 (b) Corresponding branch labels*

From that, the expectation in (9) can be evaluated as,

$$P_{d,Rayl}(X_n, \hat{X}_n) = \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{\sin^2\theta}{\sin^2\theta + \gamma_s} \right)^d d\theta, \quad (11)$$

where $\gamma_s = rE_b/N_0$.

The exact solution for (11) can be obtained as ([9], eq.(5A.8)),

$$P_{d,Rayl}(X_n, \hat{X}_n) = J_d(\gamma_s), \quad (12)$$

where,

$$J_d(\gamma_s) = \frac{1}{2} \left[1 - \mu \sum_{k=0}^{d-1} \begin{pmatrix} 2k \\ k \end{pmatrix} \left(\frac{1-\mu^2}{4} \right)^k \right]$$

and $\mu = \sqrt{\frac{\gamma_s}{1+\gamma_s}}$.

Then, the new upper bound on BER for a convolutional code terminated with L information bits over Rayleigh fading channels can be expressed as,

$$P_{b,Rayl} \leq \frac{1}{L} \sum_{j=d_{free}}^{d_{max}} \sum_i i.c_{i,j}.J_d(\gamma_s). \quad (13)$$

5. TIGHTER UPPER BOUNDS FOR TERMINATED CODES COMBINED WITH QPSK OVER RAYLEIGH FADING CHANNELS

Performance bounds for rate-1/2 convolutional codes with QPSK modulation over Rayleigh fading channels has been derived from the union bound technique in [7] for infinite length input sequences. However, as mentioned in Section 1, this bound becomes loose in this case too. In this section, we derive a new weight enumerator for rate-1/2 terminated convolutional codes with QPSK and propose a tighter upper bound over Rayleigh fading channels using the new weight enumerator. The term convolutional code refers to rate-1/2 terminated convolutional code throughout this section.

5.1 Weight Enumerator for QPSK Modulation

Consider the QPSK signal constellation shown in Fig. 1. Assume that the all zero message is transmitted ($x_i = -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$). Let $\hat{X}$ differ from X by exactly m symbols, which consist of m_1 symbols of $S_1 = \pm(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}})$ and m_2 symbols of $S_2 = (\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}})$. In this case we consider two dummy variables D_1 and D_2. For example, Fig. 2(a) shows the state diagram for the rate-1/2 convolutional code C with constraint length $K = 3$ and the generator polynomials $G = [5, 7]_8$. The resulting branch metrics for each branch in the same state diagram with the assumptions above are shown in Fig. 2(b), where the exponent of B represents the Hamming distance of the input sequence.

Accordingly, we define the new weight enumerator for QPSK as

$$W(B, D_1, D_2) = \sum_i \sum_{m_1} \sum_{m_2} c_{i,m_1,m_2} B^i D_1^{m_1} D_2^{2m_2} \quad (14)$$

where c_{i,m_1,m_2} is the number of error events consists of m_1 symbols of m_1 Hamming distance and m_2 symbols of $2m_2$ Hamming distance when the input Hamming distance is i.

In [2], the trellis diagram for convolutional codes has been modified to obtain weight enumerators for terminated convolutional codes with BPSK. The modified trellis diagram contains an additional state, denoted by 2^{K-1}, to which all branches that are merging into zero state are redirected. This eliminates the occurrence of multiple error events in the weight enumerator. The same approach can be used to obtain the weight enumerator represented by (14). The new trellis diagram for QPSK is shown in Fig. 3, for the code C_1. Here, the branches between trellis depth $t-1$ and t are labeled according to the branch metrics shown in Fig. 2(b).

Define $\underline{S}_t = (0, \cdots, 2^{K-1} - 1)$ as the set of states at depth t of the trellis diagram. Let $\underline{G}_{\underline{S}_{t-1},\underline{S}_t}(B, D_1, D_2)$ denote the label of the branch between the state $\underline{S}_{t-1}$ at depth $t-1$ and the state $\underline{S}_t$ at depth t, and $\underline{F}_{\underline{S}_t}(B, D_1, D_2)$ denote the coefficients of the polynomial of state $\underline{S}_t$ at trellis depth t. Initialize $\underline{F}_{\underline{S}_0}(B, D_1, D_2)$ as

$$\underline{F}_{\underline{S}_0}(B, D_1, D_2) = \begin{cases} 1 & \text{if } \underline{S}_0 = 0 \\ 0 & \text{otherwise.} \end{cases} \quad (15)$$

Then, for each trellis depth $t = 1$ to $(L + K - 1)$,

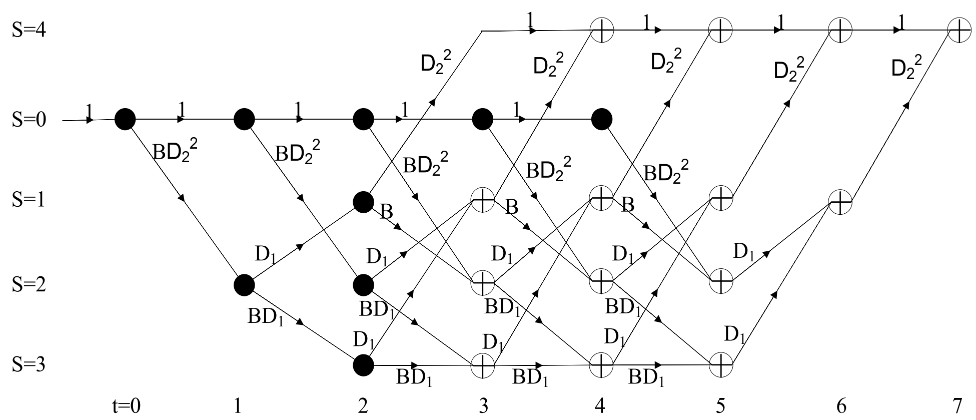

Fig.3: *Modified Trellis Diagram for QPSK*

compute

$$\underline{F}_{\underline{S}_t}(B, D_1, D_2) = \sum_{\underline{S}_{t-1}=\Lambda_{\underline{S}_t}} \underline{F}_{\underline{S}_{t-1}}(B, D_1, D_2).\underline{G}_{\underline{S}_{t-1},\underline{S}_t}(B, D_1, D_2) \quad (16)$$

for each state $\underline{S}_t$, where $\Lambda_{\underline{S}_t}$ is the set of states at depth $t-1$ from which there exists a branch merging to $\underline{S}_t$. At depth $t = L + K - 1$, the polynomial $\underline{F}_{\underline{S}_t=2^{K-1}}(B, D_1, D_2)$ is the desired weight enumerator.

The resultant weight enumerator for the code C_1 obtained from above procedure is

$$W(B, D_1, D_2) = 5BD_1D_2^4 + 7B^2D_1^2D_2^4 + 8B^3D_1^3D_2^4 + 5B^4D_1^4D_2^4 + B^5D_1^5D_2^4. \quad (17)$$

Table 1 summarizes the significant terms of weight enumerators as a function of input information length L for commonly used rate-1/2 convolutional codes of $K = 3$ to 9 with QPSK.

5.2 Tighter Upper Bound in Rayleigh Fading

Using the weight enumerator defined in (14), the upper bound on BER for rate-1/2 convolutional codes terminated with L information bits and, combined with QPSK modulation over Rayleigh fading channels $P_{b,QPSK,Rayl}$ can be expressed as,

$$P_{b,QPSK,Rayl} = \frac{1}{L}\sum_{m_2}\sum_{m_1}\sum_i i.c_{i,m_1,m_2}.P_{m_1m_2,QPSK,Rayl}(X, \hat{X}), \quad (18)$$

where $P_{m_1m_2,QPSK,Rayl}(X, \hat{X})$ is the pairwise error probability of the transmitted sequence X and the estimated sequence $\hat{X}$.

The conditional pairwise error probability can be written as,

$$P_{m_1m_2,QPSK,Rayl}(X, \hat{X}|\{a_i\}) = Q\left(\sqrt{\frac{E_s}{2N_0}\left[\sum_{l=1}^{m_1} a_l^2|\hat{x}_l - x_l|^2 + \sum_{k=1}^{m_2} a_k^2|\hat{x}_k - x_k|^2\right]}\right) \quad (19)$$

where E_s is the energy per symbol. For the rate-1/2 convolutional coded QPSK system under consideration, $E_s = E_b$, $|\hat{x}_l - x_l|^2 = 2$ for m_1 symbols of S_1 and $|\hat{x}_k - x_k|^2 = 4$ for m_2 symbols of S_2. Substituting these values in (19) and using the exact Q-function integral, $P_{m_1m_2,QPSK,Rayl}(X, \hat{X}|\{a_i\})$ can be written as,

$$\begin{aligned} &P_{m_1m_2,QPSK,Rayl}(X, \hat{X}|\{a_i\}) \\ &= Q\left(\sqrt{\frac{E_b}{N_0}\left[\sum_{l=1}^{m_1} a_l^2 + 2\sum_{k=1}^{m_2} a_k^2\right]}\right) \\ &= \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{E_b\sum_{l=1}^{m_1} a_l^2}{2N_0\sin^2\theta}\right)\exp\left(-\frac{E_b\sum_{k=1}^{m_2} a_k^2}{N_0\sin^2\theta}\right) d\theta \end{aligned} \quad (20)$$

The unconditional pairwise error probability $P_{m_1m_2,QPSK,Rayl}(X, \hat{X})$ can be obtained by taking the expected value of (20) as

$$P_{m_1m_2,QPSK,Rayl}(X, \hat{X}) = \frac{1}{\pi}\int_0^{\pi/2} E_a\left[\exp\left(-\frac{E_b\sum_{l=1}^{m_1} a_l^2}{2N_0\sin^2\theta}\right)\exp\left(-\frac{E_b\sum_{k=1}^{m_2} a_k^2}{N_0\sin^2\theta}\right)\right] d\theta. \quad (21)$$

Since the fading amplitudes are independent, the expectations in (21) can be evaluated separately. After evaluating the expectations with respect to the Rayleigh pdf in (1), (21) is expressed as,

$$P_{m_1m_2,QPSK,Rayl}(X, \hat{X}) = \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{\sin^2\theta+\gamma_1}\right)^{m_1}\left(\frac{\sin^2\theta}{\sin^2\theta+\gamma_2}\right)^{m_2} d\theta. \quad (22)$$

where, $\gamma_1 = E_b/2N_0$ and $\gamma_2 = E_b/N_0$.

The exact solution for (22) can be obtained [9] as shown in equation (23).

Equations (18) and (23), together forms the upper bound on bit error probability for rate-1/2 convolutional coded system combined with QPSK modulation over frequency non-selective Rayleigh fading channels.

6. RESULTS

In this section, the proposed upper bounds are compared with the conventional bounds and simulation results for BPSK and QPSK over Rayleigh fading

Table 1: *Weight Enumerators for QPSK*

Constraint length, K	Code	Input length, L	Weight Enumerator
3	$[5,7]_8$	≥ 12	$LBD_1D_2^4 + (2L-3)B^2D_1^2D_2^4 + (4L-12)B^3D_1^3D_2^4 + (8L-36)B^4D_1^4D_2^4 +$ $(16L-96)2B^5D_1^5D_2^4 + (32L-240)B^6D_1^6D_2^4 + (64L-576)B^7D_1^7D_2^4 + \cdots$
4	$[15,17]_8$	≥ 9	$(L-1)B^2D_1^2D_2^4 + LBD_1D_2^6 + (2L-5)B^3D_1^3D_2^4 + (L-2)B^2D_1^2D_2^6 +$ $(4L-17)B^4D_1^4D_2^4 + (3L-12)B^3D_1^3D_2^6 + (8L-48)B^5D_1^5D_2^8 +$ $(L-3)B^2D_1^2D_2^8 + (8L-44)B^4D_1^4D_2^6 + (16L-124)B^6D_1^6D_2^4 + \cdots$
5	$[23,35]_8$	≥ 10	$LBD_1^3D_2^4 + (L-3)B^3D_1^3D_2^4 + (L-4)B^2D_1^2D_2^6 + (L-7)B^6D_1^4D_2^4 +$ $(L-2)B^3D_1^3D_2^6 + (L-5)B^5D_1^3D_2^6 + (L-5)B^5D_1^5D_2^4 + (L-8)B^7D_1^5D_2^4 +$ $(L-2)B^2D_1^4D_2^6 + (3L-13)B^4D_1^4D_2^6 + (2L-14)B^6D_1^4D_2^6 + (2L-7)B^2D_1^6D_2^4 +$ $(4L-26)B^4D_1^6D_2^4 + (3L-25)B^6D_1^6D_2^4 + (L-9)B^8D_1^6D_2^4 + \cdots$
6	$[53,75]_8$	≥ 9	$(L-2)B^2D_1^4D_2^4 + LBD_1^3D_2^6 + (L-2)B^3D_1^3D_2^6 + (L-6)B^5D_1^3D_2^6 +$ $(L-4)B^3D_1^5D_2^4 + (2L-13)B^5D_1^5D_2^4 + (2L-16)B^7D_1^5D_2^4 +$ $(L-4)B^4D_1^2D_2^8 + (L-1)B^2D_1^4D_2^6 + (3L-14)B^4D_1^4D_2^6 +$ $(L-7)B^6D_1^4D_2^6 + (L-8)B^8D_1^6D_2^4 + \cdots$
7	$[133,171]_8$	≥ 11	$LBD_1^2D_2^8 + (2L-4)B^2D_1^4D_2^6 + (2L-8)B^3D_1^4D_2^6 + (2L-5)B^3D_1^6D_2^4 +$ $(2L-12)B^4D_1^6D_2^4 + (L-6)B^5D_1^6D_2^4 + (L-9)B^6D_1^6D_2^4 + (L-2)B^2D_1^4D_2^8$ $+(L-5)B^3D_1^4D_2^8 + (2L-14)B^4D_1^4D_2^8 + \cdots$
8	$[247,371]_8$	≥ 12	$(L-1)B^2D_1^4D_2^6 + (L-3)B^3D_1^3D_2^8 + LBD_1^5D_2^6 + (L-4)B^3D_1^5D_2^6 +$ $(L-3)B^3D_1^7D_2^4 + (L-8)B^5D_1^7D_2^4 + (L-7)B^7D_1^7D_2^4 + (L-2)B^2D_1^4D_2^8$ $+(3L-14)B^4D_1^6D_2^6 + (L-11)B^6D_1^6D_2^6 + (2L-12)B^4D_1^8D_2^4 + \cdots$
9	$[561,753]_8$	≥ 12	$LBD_1^4D_2^8 + (L-4)B^3D_1^4D_2^8 + (2L-3)B^2D_1^6D_2^6 + (L-2)B^3D_1^6D_2^6 +$ $(L-4)B^4D_1^6D_2^6 + (L-7)B^5D_1^67D_2^6 + (3L-12)B^3D_1^8D_2^4 + (L-5)B^4D_1^8D_2^4$ $+(3L-22)B^4D_1^6D_2^8 + (3L-20)B^5D_1^6D_2^8 + (2L-16)B^6D_1^6D_2^8 + \cdots$

$$P_{m_1 m_2,QPSK,Rayl}(X,\hat{X}) = \frac{(\gamma_1/\gamma_2)^{m_2-1}}{2(1-\gamma_1/\gamma_2)^{m_1+m_2-1}} \left[\sum_{k=0}^{m_2-1} \left(\frac{\gamma_2}{\gamma_1} - 1\right)^k B_k I_k(\gamma_2) - \frac{\gamma_1}{\gamma_2} \sum_{k=0}^{m_1-1} \left(1 - \frac{\gamma_1}{\gamma_2}\right)^k C_k I_k(\gamma_1) \right], \tag{23}$$

$$B_k = \frac{A_k}{\begin{pmatrix} m_1+m_2-1 \\ k \end{pmatrix}}, \quad C_k = \sum_{n=0}^{m_2-1} \frac{\begin{pmatrix} k \\ n \end{pmatrix}}{\begin{pmatrix} m_1+m_2-1 \\ n \end{pmatrix}} A_n,$$

$$A_k = (-1)^{m_2-1+k} \frac{\begin{pmatrix} m_2-1 \\ k \end{pmatrix}}{(m_2-1)!} \prod_{n=1, n\neq k+1}^{m_2} (m_1+m_2-n)$$

$$I_k(\gamma) = 1 - \sqrt{\frac{\gamma}{1+\gamma}} \left[1 + \sum_{n=1}^{k} \frac{(2n-1)!!}{n!2^n(1+c)^n} \right]$$

channels. Conventional bounds refer to the bounds obtained from the union bound technique using the exact Q-function integral in pairwise error probability derivation and distance spectrum of the convolutional codes with BPSK modulation and QPSK modulation. The distance spectrum has been truncated up to 7 distances from d_{free} to obtain the conventional bound.

Fig. 4 compares the proposed upper bounds for rate-1/2 convolutional codes of $K = 3$ and 5 with BPSK in Rayleigh fading channels. It can be observed that the new upper bounds are always tighter than the conventional bounds.

BER upper bounds for rate-1/2 terminated convolutional codes of $K = 3$ and 5 with QPSK in Rayleigh fading channels are shown in Fig. 5. The proposed bounds are significantly tighter than the conventional bounds.

7. CONCLUSION

In this paper, we derived tighter upper bounds for terminated convolutional codes over Rayleigh fading channels with BPSK and QPSK modulation. The new bounds are derived for BPSK case using the weight enumerator presented in [2]. A method for deriving weight enumerators for rate-1/2 terminated convolutional codes with QPSK modulation is shown and the significant terms of the weight enumerators for commonly used rate-1/2 codes of $K = 3$ to 9 are tabulated as a function of the input information length. The new bounds with both modulation techniques are tighter than the conventional upper bounds for terminated convolutional codes over Rayleigh fading channels. The comparison of the

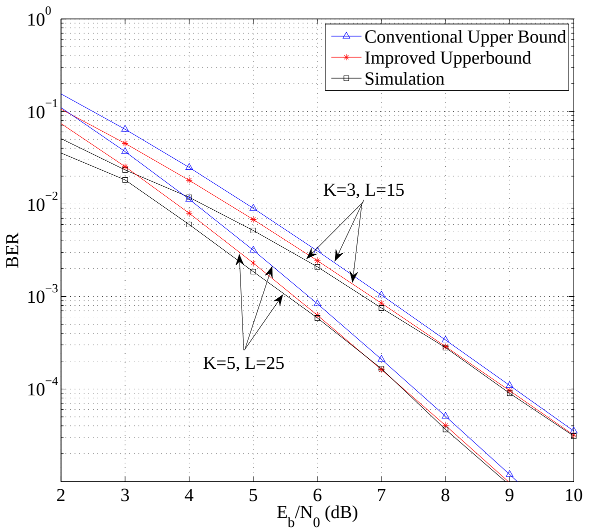

Fig.4: *BER upper bound comparison for rate-*1/2 *convolutional codes with BPSK in Rayleigh fading channel*

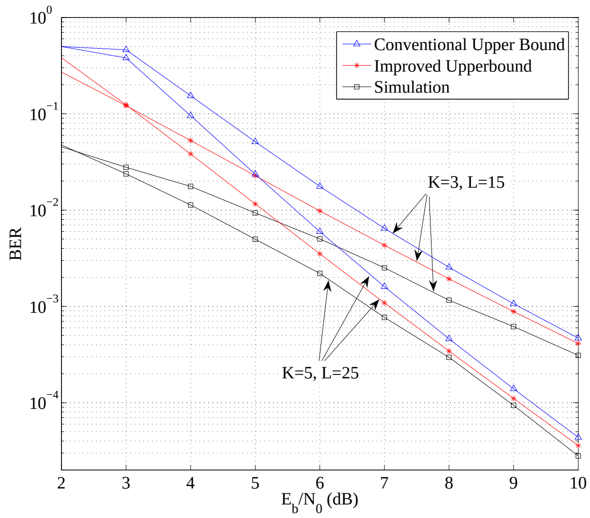

Fig.5: *BER upper bound comparison for rate-*1/2 *convolutional codes with QPSK in Rayleigh fading channel*

analytical results with the simulation results demonstrates the accuracy of the presented analysis.

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