Investigation into the Effects of Channel Properties on Training-based MIMO Channel Estimation

Xia Liu[1], **Marek E. Bialkowski**[2], and **Feng Wang**[3], Non-members

ABSTRACT

This paper reports on investigations into the effect of channel properties on training-based MIMO channel estimation. Here, the channel's properties are represented by eigenvalues of the complex channel correlation matrix. The influence of these eigenvalues is assessed for two training based channel estimation methods, Scaled Least Square (SLS) method and Minimum Mean Square Error (MMSE) method. It is shown that for a given transmitted power to noise ratio in the training mode, the performance of the two estimation methods is governed by the sum of eigenvalues of the channel correlation matrix. Simulation results support this conclusion.

Keywords: MIMO Channel Estimation, Training-Based, LS Estimation, MMSE Estimation, Channel Properties

1. INTRODUCTION

Recent works on multiple input multiple output (MIMO) techniques have shown that by using multiple element antennas with suitable signal processing algorithms the capacity of a wireless communication system can be significantly increased without the need of an extra operational bandwidth.

Of paramount importance to realization of such potential of MIMO is an accurate estimation of channel state information (CSI) at the receiver [1].

A number of channel estimation methods have been proposed to obtain CSI. The methods that are based on the use of known data both to the transmitter and receiver (also known as the training sequence) are the most popular. In [2] and [3], a number of training-based methods have been studied including the least square (LS) method, the scaled least square (SLS) method and the minimum mean square error (MMSE) method. It has been shown that the accuracy of these training-based channel estimation methods is governed by the transmitted power to noise ratio (TPNR) in the training mode and the number of antenna elements used at the transmitter and receiver. For a fixed length of training sequence the channel estimation accuracy drops when the number of transmitting or receiving antennas increases. This is one of the shortfalls of the MIMO technique.

In [2] and [3], it has been demonstrated that the SLS and MMSE methods offer better performance over the LS method. The reason is that the SLS and MMSE estimators utilize the channel correlation to reduce estimation errors while the LS method neglects channel's properties.

Channel properties are influenced by a signal propagation environment and a resulting spatial correlation (SC) that is present in antenna arrays. In this paper, the effect of channel properties on the SLS and MMSE channel estimation methods are investigated. It is shown that at a fixed transmitted power to noise ratio (TPNR) the accuracy of SLS and MMSE methods is governed by the sum of eigenvalues of the channel correlation matrix.

The rest of the paper is organized as follows. In section 2, a MIMO system model is introduced followed by the description of LS, SLS and MMSE channel estimation methods. The channel estimation analysis for these selected methods is given in section 3. The simulation set up and results are presented in section 4. Section 5 concludes the paper.

2. SYSTEM MODEL & CHANNEL ESTIMATION

2.1 MIMO system model

Here, a narrow-band flat block-fading MIMO system with M_t antenna elements at transmitter and M_r antenna elements at receiver is considered. The relationship between the received signals and the transmitted signals is given by (1):

$$Y_s = HS + V \tag{1}$$

where Y_s is the $M_r \times N$ complex matrix representing the received signals; S is the $M_t \times N$ complex matrix representing transmitted signals; H is the $M_r \times M_t$ complex channel matrix and V is the $M_r \times N$ complex zero-mean white noise matrix. N is the length of transmitted signal sequence.

2.2 Training-based MIMO channel estimation

For a training based channel estimation method, the relationship between the received signal and the training sequence is given by (2):

Manuscript received on November 8, 2008 ; revised on January 7, 2009.
[1,2,3] The authors are with The Group of MOC, School of ITEE, The University of Queensland, E-mail:

$$Y = HP + V \tag{2}$$

where the transmitted signal S in (1) is replaced by P representing the $M_t \times L$ complex training matrix. L is the length of the training sequence. The goal is to estimate the complex channel matrix H from the knowledge of Y and P.

The transmitted power in the training mode is constrained by the following expression

$$\|P\|_F^2 = \mathrm{P} \tag{3}$$

where P is a given power constant and $\|.\|_F^2$ stands for Frobenius norm.

The training sequences are assumed to be orthogonal [2] [3] and the transmitted power to noise ratio (TPNR) in the training mode is set to the value of ρ.

In the LS method, the estimated channel matrix can be written as [4],

$$\hat{H}_{LS} = YP^{\dagger} \tag{4}$$

where $\{.\}^{\dagger}$ stands for pseudo-inverse.

The mean square error (MSE) of the estimated channel matrix in the LS method is given as

$$MSE_{LS} = E\{\|H - \hat{H}_{LS}\|_F^2\} \tag{5}$$

in which $E\{.\}$ denotes a statistical expectation. According to [2] and [3], the minimum value of MSE for the LS method is given as

$$MSE_{LS} = \frac{M_t^2 M_r}{\rho} \tag{6}$$

From equation (6) one can see that the optimal performance of the LS estimator is influenced by the number of antenna elements is at the transmitter and the receiver. The channel matrix has no effect on the minimum value of MSE.

The SLS method further reduces the estimation error that is obtained in the LS method. The improvement is given by a scaling factor γ which is defined as [2][3]

$$\gamma = \frac{tr\{R_H\}}{MSE_{LS} + tr\{R_H\}} \tag{7}$$

The SLS estimation mean square error is given as [2][3]

$$MSE_{SLS} = (1-\gamma)^2 tr\{R_H\} + \gamma^2 MSE_{LS} \tag{8}$$

Here, R_H is the channel correlation matrix defined as $R_H = E\{H^H H\}$ and $tr\{.\}$ implies the trace operation. The minimized value of MSE can be written as [2][3]

$$MSE_{SLS} = \frac{MSE_{LS} tr\{R_H\}}{MSE_{LS} + tr\{R_H\}} \tag{9}$$

In the MMSE method, the estimated channel matrix and the minimized MSE are given as (10) and (11) [2][3], respectively.

$$\hat{H}_{MMSE} = Y(P^H R_H P + \sigma_n^2 M_r I)^{-1} P^H R_H \tag{10}$$

$$MSE_{MMSE} = tr\{(\Lambda^{-1} + \sigma_n^2 M_r^{-1} Q^H P P^H Q)^{-1}\} \tag{11}$$

In equation (10) and (11), σ_n^2 is the noise power; Q is a unitary eigenvector matrix of R_H and Λ is a diagonal matrix with eigenvalues of R_H, which are given through the eigenvalue decomposition of R_H as

$$R^H = Q\Lambda Q^H \tag{12}$$

3. CHANNEL ESTIMATION ACCURACY ANALYSIS

3.1 SLS method

By taking into account expression (6), the minimized MSE of the SLS method (9) can be rewritten as

$$\begin{aligned} MSE_{SLS} &= [(tr\{R_H\})^{-1} + \frac{\rho}{M_t^2 M_r}]^{-1} \\ &= [(tr\{\Lambda\})^{-1} + \frac{\rho}{M_t^2 M_r}]^{-1} \\ &= [(\sum_i^n \lambda_i)^{-1} + \frac{\rho}{M_t^2 M_r}]^{-1} \end{aligned} \tag{13}$$

where $n = min(M_r, M_t)$ and λ_i is the i-th eigenvalue of the channel correlation matrix R_H.

If ρ is fixed then the following relationship can be derived

$$MSE_{SLS} = [(\sum_i^n \lambda_i)^{-1} + \frac{\rho}{M_t^2 M_r}]^{-1} < \sum_i^n \lambda_i \tag{14}$$

It can be seen from (14) that MSE_{SLS} is smaller than the sum of the eigenvalues of the channel correlation matrix R_H. Therefore, MSE decreases when the sum of eignvalues of R_H decreases.

This property shows that the MSE in the SLS method is influenced by the number of antenna elements at both transmitter and receiver. The MSE becomes smaller and the estimation accuracy is improved for a reduced number of transmitting and receiving antenna elements. When the number of antenna elements on the two sides of communication link drops to one, the system becomes the conventional SISO system. In this case, the channel estimation using a fixed-length training sequence becomes most accurate. This confirms the expectation that it is easier to estimate the SISO channel which is characterized by a single transfer coefficient between two antennas than the MIMO channel which is described by a matrix of transfer coefficients between many antenna elements.

The derived expression shows that when the number of transmit and receive antennas is fixed, MSE can be minimized by minimizing the sum of eigenvalues of R_H.

3.2 MMSE method

The expression (11) for the minimized value of MSE can be rewritten using the orthogonality properties of a training sequence P and the unitary matrix Q, as shown by [2][3](15):

$$MSE_{MMSE} = tr\{(\Lambda^{-1} + \rho M_r^{-1} I)^{-1}\}$$

$$= tr\begin{bmatrix} (\lambda_1^1 + \rho M_r^{-1})^{-1} & 0 & \cdots & 0 \\ 0 & (\lambda_2^1 + \rho M_r^{-1})^{-1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & (\lambda_n^1 + \rho M_r^{-1})^{-1} \end{bmatrix} \quad (15)$$

$$= \sum_i^n ((\lambda_i^{-1} + \rho M_r^{-1})^{-1})$$

Assuming that ρ in (15) is fixed, the upper bound for MSE can be expressed by (16)

$$MSE_{MMSE} = \sum_i^n (\lambda_i^{-1} + \rho M_r^{-1})^{-1} < \sum_i^n \lambda_i \quad (16)$$

The expression (16) shows that similarly as in the SLS method, a smaller sum of eigenvalues of the channel correlation matrix R_H leads to a smaller estimation error in the MMSE method. Also, as in the SLS method, the MSE is influenced by the number of antenna elements. The smaller sum of eigenvalues of the channel correlation matrix leads to the more accurate channel estimation in the MMSE method.

Through the above mathematical analysis, one can see that if ρ is fixed, the accuracy of a training-based MIMO channel estimation is governed by the sum of eigenvalues of the channel correlation matrix R_H.

It has to be noted that the eigenvalues of R_H represent the channel properties that are governed by a signal propagation environment and antenna spatial correlation.

The next section presents the simulation results that support the findings obtained from expressions (14) and (16). A suitable signal propagation environment model is introduced. The effects of channel properties on the estimation accuracy are assessed in terms of angle spread (AS) and Rician factor K, which are shown to be related to the sum of eigenvalues of the channel correlation matrix.

4. SIMULATION

4.1 Simulation settings

In undertaken computer simulations, the signal propagation environment is considered to be represented as a combination of line of sight (LOS) and non-line of sight (NLOS) conditions. Therefore the MIMO channel matrix is represented by two components, as shown in (17)[5].

$$H = \sqrt{\frac{1}{1+K}} H_{NLOS} + \sqrt{\frac{K}{1+K}} H_{LOS} \quad (17)$$

where H_{LOS} denotes the LOS part and H_{NLOS} denotes the NLOS part. K is the Rician factor that is given as the ratio of power in LOS and the mean power in the NLOS signal component. The elements of H_{LOS} matrix can be written as [5]

$$H_{LOS}^{rt} = \exp(-j\frac{2\pi}{\lambda} D_{rt}) \quad (18)$$

where D_{rt} is the distance between the t-th transmit antenna and the r-th receive antenna. Assuming that the components of NLOS are jointly Gaussian, H_{NLOS} can be written as

$$H_{NLOS} = R_R^{1/2} H_g R_T^{1/2} \quad (19)$$

where H_g represents a matrix with i.i.d. Gaussian entries.

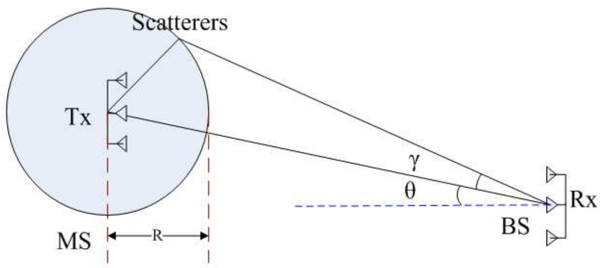


Fig.1: *Jakes model: an uplink case*

In the undertaken simulations, the Jakes fading model [7][8] is applied to represent the signal propagation conditions. An uplink case between a base station (BS) and a mobile station (MS) is assumed. The model is illustrated in Figure 1. This model is used to obtain the receive R_R and transmit R_T spatial correlation matrices that are required to obtain the NLOS component of the channel matrix, as given by (19). As seen in Fig.1, the BS antennas are positioned at a large height above the ground where there are no scattering objects. In turn, MS is assumed to be surrounded by many scatterers distributed within a "circle of influence". For this case, the signal correlation coefficients at the receiver BS and transmitter MS, ρ_R^{BS} and ρ_T^{MS}, can be obtained from [8] and are given as:

$$\rho_T^{MS}(\delta_{ij}^{MS}) = J_0[2\pi\delta_{ij}^{MS}/\lambda]$$

$$\rho_R^{BS}(\delta_{ij}^{BS}) = J_0[\frac{2\pi}{\lambda}\gamma_{max}\cos(\theta)]\exp(-j\frac{2\pi}{ij}\sin(\theta)) \quad (20)$$

where, δ_{ij}^{MS} and δ_{ij}^{BS} are the spacing distances between i-th and j-th antenna at MS and BS, respectively; λ is the wavelength of the carrier; γ_{max} is the maximum angular spread (AS); θ is the AoA of LOS and J_0 is the Bessel function of 0-th order. Using $\rho_R^{BS}(\delta_{ij}^{BS})$ and $\rho_T^{MS}(\delta_{ij}^{MS})$, the correlation matrices R_R and R_T for BS and MS links can be generated, as shown in (20) and (21).

$$R_R^{BS} = \begin{bmatrix} \rho_r^{BS}(\delta_{11}^{bs}) & \cdots & \rho_R^{BS}(\delta_{1M_r}^{BS}) \\ \vdots & \ddots & \vdots \\ \rho_r^{BS}(\delta_{M_r1}^{bs}) & \cdots & \rho_R^{BS}(\delta_{M_rM_r}^{BS}) \end{bmatrix} \quad (21)$$

$$R_T^{BS} = \begin{bmatrix} \rho_T^{BS}(\delta_{11}^{bs}) & \cdots & \rho_t^{BS}(\delta_{1M_t}^{BS}) \\ \vdots & \ddots & \vdots \\ \rho_T^{BS}(\delta_{M_t1}^{bs}) & \cdots & \rho_R^{BS}(\delta_{M_tM_t}^{BS}) \end{bmatrix} \quad (22)$$

4.2 Simulation results

Using the above described signal propagation model, a 4 × 4 MIMO system including 4-element linear array antennas at the transmitter and receiver sides is simulated. The distance between the transmitter and the receiver is 1000λ, where λ is the signal wavelength. An Angle of Arrival (AoA) of LOS is set to 0°. The training sequence length L is assumed to be 4. The default antenna element spacing at both BS and MS is set to be 0.5λ. This assumption allows for neglecting mutual coupling between the antenna elements.

Figure 2 presents a relationship between MSE and the sum of eigenvalues of the channel correlation matrix for both MMSE and SLS methods. Both sub-figure A and sub-figure B show that for MMSE and SLS methods channel estimation errors are smaller for smaller sums of eigenvalues. When the sum of eigenvalues increases, estimation accuracy becomes worse. This result confirms that it is easier to estimate a more correlated MIMO channel than a less correlated MIMO channel.

Figure 3 presents the effects of maximum angle spread (AS), antenna spacing and Rician factor K on the sum of eigenvalues of the channel correlation matrix. The obtained results are given in three sub-figures A, B and C.

Sub-figure A reveals the relationship between the sum of eigenvalues of R_H and the maximum AS, which is linked to the level of spatial correlation. It can be observed that the sum of eigenvalues increases

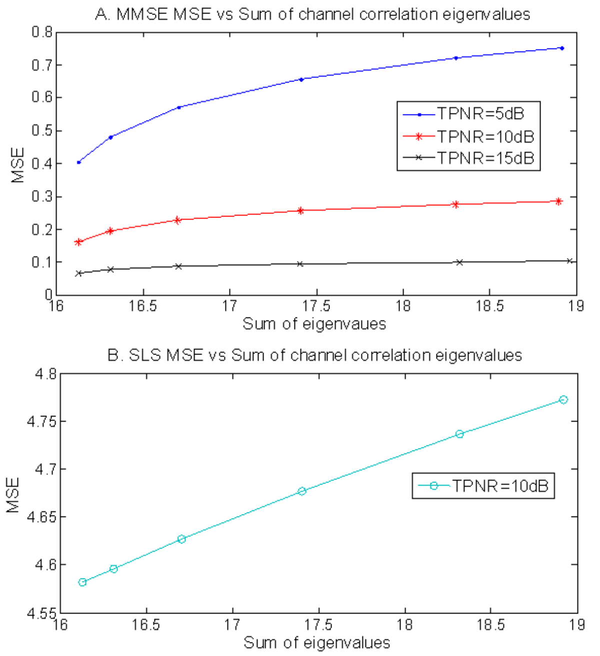


Fig.2: *Relationship between MSE and Sum of eigenvalues of R_H for MMSE and SLS methods*

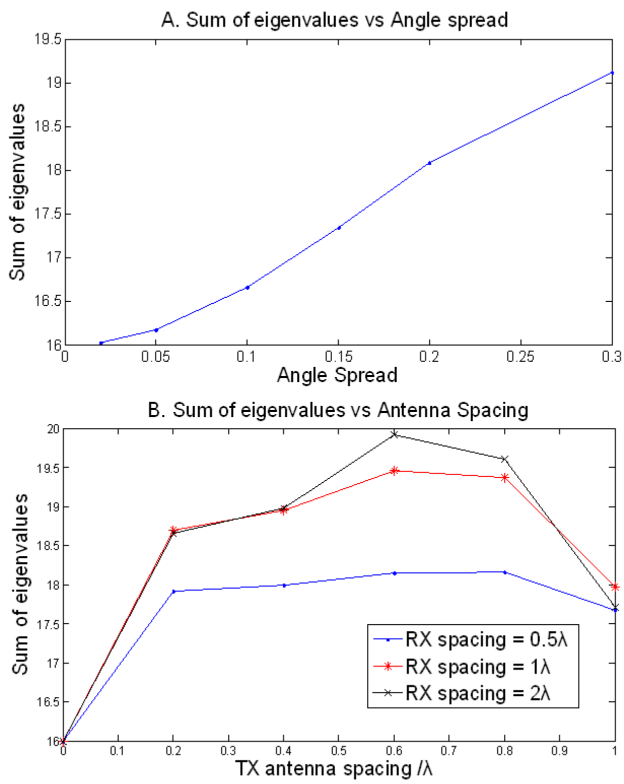


Fig.3: *Relationship between angle spread (AS), antenna spacing, Rician factor K and Sum of eigenvalues of RH showing the effect of AS, antenna spacing and K.*

when AS becomes larger. Larger values of AS correspond to a lower level correlation while smaller AS corresponds to higher levels of correlation. Therefore, a higher spatial correlation leads to a smaller sum of eigenvalues of R_H. This helps to improve the accuracy of the training based MIMO channel estimation. The obtained result agrees well with the finding presented in [9].

Sub-figure B gives the relationship between the sum of eigenvalues and the MS transmitter antenna spacing. One can see that the sum of eigenvalues becomes smaller when the spacing distance is less than 0.2λ. When the transmitter and receiver antenna array spacing drops to 0λ the MIMO system collapses to the SISO system with only one transmitting and one receiving antenna. The obtained result proves again that for a fixed-length training sequence sent over a noisy channel it is easier to estimate a SISO channel than a MIMO channel. This is because there are more unknowns that have to be determined.

The relationship between the sum of eigenvalues and the Rician factor K is illustrated in sub-figure C. It is apparent that the sum of eigenvalues is smaller for higher values of K. This case occurs when a LOS component dominates. In this case, the sum of eigenvalues becomes reduced and the estimation accuracy becomes improved.

Figure 4 and 5 are plotted in three dimensions (3D) to provide a further insight into the results shown in Figure 3. The relationship between MSE, ρ and K for both MMSE and SLS methods are presented at three different values of maximum AS. One can see that when ρ is increased to 30dB the estimation error decreases almost to zero. When the value of Rician factor K is increased, the MSE decreases. This is consistent with the trend observed in Figure 3 that a stronger LOS component results in better estimation accuracy. The presented results also show that for both SLS and MMSE methods MSE provides the best performance at the smallest AS, which corresponds to the highest spatial correlation level.

The simulation results in this section demonstrate that it is easier to estimate a more correlated MIMO channel. This finding reveals a trade off met in designing of MIMO systems. Uncorrelated channels offer higher capacity for MIMO. However, the improvement in capacity is obtained only under the condition that MIMO channel is estimated accurately. This poses the challenge to the MIMO system designer. They have to pay a special attention to the accurate estimation of MIMO channel to obtain the promised benefits of increased capacity or signal transmission quality offered by the MIMO technique.

5. CONCLUSIONS

In this paper, the effect of channel properties on a training-based MIMO channel estimation has been investigated. The presented mathematical analysis and simulation results have shown that for a fixed transmitted power to noise ratio assumed in the training mode, the accuracy of the training based MIMO channel estimation is governed by the sum of eigenvalues of the channel correlation matrix. Specifically, a smaller value of the sum of eigenvalues leads to a more accurate channel matrix estimation. It has been shown that large values of Rician factor K and a higher level of spatial correlation lead to a reduced value of the sum of channel correlation eigenvalues. Therefore, the channel estimation accuracy is improved for such conditions. The obtained findings clearly show that the MIMO system designer faces an increased challenge of accurate estimation of uncorrelated channels before the benefits of MIMO technique such as the improved capacity or signal transmission quality are achieved.

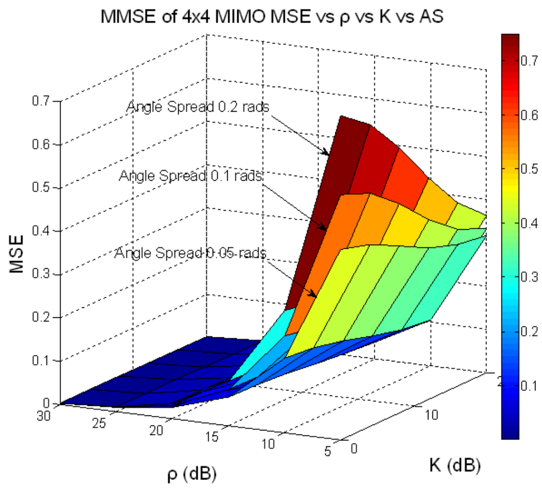


Fig.4: *3D plot of MSE vs ρ vs K for different values of AS for MMSE method*

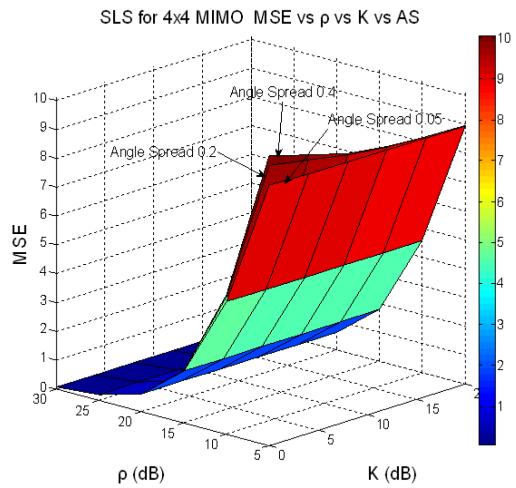


Fig.5: *3D plot of MSE vs ρ vs K at different values of AS for SLS method*

References

[1] C. Budianu and L. Tong, "Channel estimation for space-time orthogonal block codes," *IEEE Trans. Signal Process.*, Vol. 50, pp. 2515-2528, Oct, 2002.

[2] M. Biguesh and A. B. Gershman, "MIMO channel estimation: optimal training and tradeoffs between estimation techniques," *Proc. ICC'04*, Paris, France, June 2004.

[3] M. Biguesh and A. B. Gershman, "Training-based MIMO channel estimation: a study of estimator tradeoffs and optimal training signals," *IEEE Trans. Signal Processing*, Vol. 54, No. 3, Mar. 2006.

[4] S. M. Kay, *Fundamentals of Statistic Signal Processing: Estimation Theory*, Prentice-Hall, Inc., 1993

[5] P. Uthansakul, M. E. Bialkowski, S. Durrani, K. Bialkowski, and A. Postula, "Effect of line of sight propagation on capacity of an indoor MIMO system," *IEEE Int. Antennas Propagat. Symp.*, Washington, DC, 2005, pp. 707-710.

[6] E. G. Larsson and P. Stoica, *Space-time block coding for wireless communication*, Cambridge University Press, 2003.

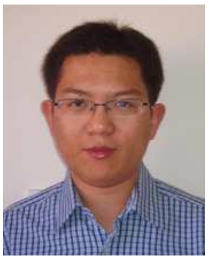
[7] W. C. Jakes, *Microwave Mobile Communications*, New York: John Wiley & Sons, 1974.

[8] T. L. Fulghum, K. J. Molnar and A. Duel-Hallen, "The Jakes fading model for antenna arrays incorporating azimuth spread," *IEEE Trans. on Vehicular Techn.*, Vol.51, No.5, Sep 2002.

[9] X. Liu, S. Lu, M. E. Bialkowski and H.T. Hui, "MMSE Channel estimation for MIMO system with receiver equipped with a circular array antenna," *Proc IEEE APMC2007*, Thailand 2007.

Xia Liu received his Bachelor degree in Communication Engineering from Zhejiang University of Technology (ZJUT), China in June 2005. In July 2006, he obtained his Master degree in Telecommunication Engineering from the University of Queensland, Australia. At present, he is a PhD candidate in the School of Information Technology and Electrical Engineering, University of Queensland. He is a recipient of the University of Queensland Research Scholarship (UQRS) award. His research interests are channel estimation methods for MIMO systems.

Marek E. Bialkowski received the M.Eng.Sc. degree (1974) in applied mathematics and the Ph.D. degree (1979) in electrical engineering both from the Warsaw University of Technology and a higher doctorate (D.Sc. Eng.) in computer science and electrical engineering from the University of Queensland (2000). He held teaching and research appointments at universities in Poland, Ireland, Australia, UK, Canada, Singapore, Hong Kong and Switzerland. At present, he is a Chair Professor in the School of Information Technology and Electrical Engineering at the University of Queensland. His research interests include technologies and signal processing techniques for smart antennas and MIMO systems, antennas for mobile cellular and satellite communications, low profile antennas for reception of satellite broadcast TV pro-grams, conventional and spatial power combining techniques, six-port vector network analysers, and medical and industrial applications of microwaves. He has published over 550 technical papers, several book chapters and one book. His contributions earned him the IEEE Fellow award in 2002.



Feng Wang received the B.E and M.E degrees from Xidian University in 1998 and 2004, respectively. Since March 2008, he has been with the University of Queensland, where he is studying towards his Ph.D. degree. His research interests include information theory and wireless communications, signal processing techniques for MIMO systems, and antenna technologies for future wireless communications.