A new Tool for Input to State Stabilization of Delayed System via FDE Dissipativity

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ABSTRACT

In this paper, we derive a new tool for Input to State Stabilization (ISS) of a delayed systems. The considered system is in a very general form of Functional Differential Equation (FDE). Firstly, a notion of FDE Dissipativity that is applicable to a class of delayed nonlinear systems is introduced and explored. It will be shown that FDE-Dissipativity will imply Input to State Stability (ISS) for time delay systems. Moreover it has equivalent characteristics with the non-delayed nonlinear systems, for instance, feedback interconnection of two FDE-Dissipative systems is also FDE-Dissipative and it is possible to reconstruct loop-transformation with properties similar to the non-delayed passive system. Based on this, we construct a Lyapunov-Krasovskii like functional as the main tool for the IS-Stabilization of the FDE systems. Finally, a numerical studies are presented in order to verify the effectiveness of the proposed tool.

Keywords: Function Differential Equation (FDE), FDE-Dissipativity, Input to State Stability (ISS), Feedback Connection

1. INTRODUCTION

In almost parallel to the non-delayed case, the stability theory for delayed nonlinear systems undergoes a long time history. The foundation was long establised since 1950s by series of distinguished researchers, for instance, Krasovskii [12], Hale [5] , Driver [3], Burton [2] and many more. As summarized by Burton [1] and references therein, the main tools for analyzing the stability of delayed nonlinear systems come in the form of Functional Differential Equation (FDE) namely Lyapunov-Krasovskii type theorem and Lyapunov-Razumikhin type theorem. Unfortunately, so far there is no systematic way to find Lyapunov-Krasovskii Functional or Lyapunov-Razumikhin Function. Therefore, the search for both of tools can be a time consuming task.

One of natural way to find the stabilizing controller for delayed nonlinear systems is by extending the control ideas from the non-delayed one. For instance, [9] extended the concept of Control Lyapunov Function (CLF) in the non-delayed case into delayed case known as Control Lyapunov-Razumikhin Function (CLRF). However, it is often more difficult to find CLRF than the solution itself. Other works from [14], [10], [13] utilize backstepping tool in accordance with Lyapunov-Krasovskii functional or Lyapunov-Razumikhin function for delayed-nonlinear systems. While the other famous approaches use the extension of Input to State Stability (ISS) concept into delayed case using the two tools above [18], [16].

In contrast to the above approaches, although hardly discussed in the literature, dissipativity of solution of FDE gave promising results. Series of works from Huang [6], Tian [19], and recently generalized by Wen and Li [20] show that if the solution of FDE exists and moreover satisfies some simple assumptions, then its solution will be bounded for all time. Therefore in this work, their results are utilized and reformulated into FDE-Dissipativity as mentioned in the next session. It is surprisingly interesting as properties similar with the ISS system for non-delayed case are inherited for FDE-dissipative systems. In special cases, FDE-Dissipativity will also imply Input to State Stability (ISS) for time delay systems.

In this paper, a term of FDE-Dissipativity in delayed nonlinear system is introduced and explored. It is shown that the FDE-Dissipativity is a sufficient condition for output to be bounded. Moreover it is also shown that it has equivalent characteristics with non-delayed nonlinear systems, for instance, feedback interconnection of two FDE-Dissipative systems is also FDE-Dissipative and it is possible to reconstruct loop-transformation with property similar to non-delayed passive system. To achieve this goal an approach from dissipativity of functional differential equation is employed. In this extended abstract version, in order to keep the space minimal we are going to omit all of the proofs. Moreover, no verifications either from simulation or experimental results are given so far.

Throughout this paper the following notation is used. Suppose there is a given constant $\tau \geq 0$, $\mathbb{R}$ and $\mathbb{R}^{\ltimes}$ a real number and n-dimensional vector space over $\mathbb{R}$, respectively. Define $C([a,b],\mathbb{R}^{\ltimes})$ a function that maps the interval $[a,b]$ into $\mathbb{R}^{\ltimes}$ with norm $\|x\|_\infty = \sup_{t\in[a,b]} \|x(t)\|$, where $\|\cdot\|$ is Euclidean

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Manuscript received on June 29, 2009 ; revised on April 4, 2010.

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norm. In this paper we also use class K and class KL function defined respectivelly as follows: if

2. DISSIPATIVITY OF FUNCTIONAL DIFFERENTIAL EQUATIONS (FDE)

In this paper, we are going to work with the following delayed nonlinear system that is in the form of FDE

$$\Sigma : \begin{cases} x'(t) = f(t, x(t), x(\cdot), u(t)), & t \in [0, +\infty) \\ x(t) = \varphi, & t \in [-\tau, 0) \end{cases} \tag{1}$$

where $f \in C([-\tau, +\infty), \mathbb{R}^{\ltimes})$ is locally Lipschitz continuous function on $\mathbb{R}^{\ltimes}$ and $u(t)$ is a continuous system input on $\mathbb{R}^{\urcorner}$, $\urcorner \leq \ltimes$. The above delayed nonlinear system (1) is more general than the one used by Wen and Li, (2007) in the sense that in the above equation the input is considered.

Lemma 1 (Weak generalized Halanay inequality) Suppose that

$$u' \leq -\alpha(t)u(t) + \beta(t) \sup_{-\tau \leq \delta < t} u(\delta) \tag{2}$$

where $t \geq t_0$, $\alpha(t)$, $\beta(t)$ are positive continous functions such that $\beta \leq q\alpha$ with $0 < q \leq 1$ then

$$u(t) \leq k e^{-\mu(t-t_0)} \quad \forall t \geq t_0. \tag{3}$$

Here the gain k is defined as $k = \sup_{-\tau \leq t < t_0} |u(t)|$ and $\mu = \{\inf_{t \geq t_0} s(t) : s(t) - \alpha(t) + \beta(t)e^{s(t)\tau} = 0\}$
The word 'weak' in the above lemma refers to the fact that one term is missing from the generalized version of Halanay inequality from [19], [20].

The following is the famous definition of dissipative FDE that is often found in the literature.

Definition 1: The above dynamical system (1) with zero input is said to be dissipative if there exists a bounded set of ball $B \subset \mathbb{R}^{\ltimes}$, such that for any given bounded set $\Phi \subset \mathbb{R}^{\ltimes}$, there is time $t_0 = t_0(\Phi)$, such that for any given set $\phi \subset C([-\tau, 0], \mathbb{R}^{\ltimes})$ that is contained in Φ for all $t \in [-\tau, 0]$, the corresponding solution $y(t)$ are contained in the ball B for all $t \geq 0$. If the solution goes to zero as $t \to \infty$ then it is called strictly dissipative.
The above definition will also imply that the system is stable in the sense of Lyapunov by noting that there exists $\epsilon \in B$ and such that for $x(-\tau) \leq \delta \in \Phi \Rightarrow x(t) \leq \epsilon$.

Starting from this definition, by considering the input in (1) the following definition is introduced.

Definition 2 (FDE-Dissipativity) The above dynamical system (1) with zero input is called FDE-dissipative if for zero input u, the system is dissipative while for bounded input u, the solution is bounded.

By defining an output equation as

$$y(t) = h(x(t), x(\cdot), u(t)) \tag{4}$$

where $y(t)$ is a class K function then the above definition will also valid if we change the word 'solution' with 'output'. It is supposed that the class of FDE mentioned in this paper satisfies the following condition

$$\begin{aligned} 2\langle z, f(z, \psi(\cdot), \chi(t))\rangle \leq & \alpha(t)\|z\|^2 + \beta(t)\|\psi(\zeta)\|^2 \\ & + \kappa(t)\|\chi(t)\|_\infty. \end{aligned} \tag{5}$$

For the system represented in the affine form, *i.e.,*

$$\dot{x} = f(x, x(\cdot)) + gu(t) \tag{6}$$

the above condition can be relaxed to be independent of the input since

$$gu(t) \leq \|g\| \max_{s \geq 0} u(s) := \Gamma \tag{7}$$

thus by setting $\kappa(t) = \Gamma$ will solve the last term requirement.

Proposition 1: The system (1) satisfying (5) implies $\alpha(t) \leq 0$, $\beta(t) \geq 0$ and $\kappa(t) \geq 0$.
The following lemma is the extension of Wen and Li, (2007) for the FDE system with input. In our lemma three additional features appear: first, $u(t)$ will be explicitly shown; second, the bound is in a very explicit form; and third, the condition (5) relaxes their restrictions.

Lemma 2: Suppose

$$Y'(t) \leq \alpha(t)Y(t) + \beta(t)Y(\cdot) + \kappa(t)U(t) \tag{8}$$

where $U \geq 0$ is an input. If

$$\alpha(\zeta) + \beta(\zeta) \leq 0 \tag{9}$$

then

$$\begin{aligned} Y(t) \leq & \Bigg(\exp(-\int_0^t \alpha(\eta)d\eta) + \\ & + \int_0^t (\beta(\zeta)\exp(-\int_\zeta^t \alpha(\eta)d\eta)d\zeta) \Bigg) \max_{-\tau \leq \zeta \leq 0} Y(\zeta) \\ & + \int_0^t \kappa(\zeta)\exp(-\int_\zeta^t \alpha(\eta)d\eta)d\zeta \max_{0 \leq \zeta \leq t} U(\zeta). \end{aligned} \tag{10}$$

Proof: Let us set a function $Q(t)$ defined as follows

$$Q(t) = p(t)(Y(t) + \delta_1 q(t) + \delta_2 s(t)) \tag{11}$$

where

$$\begin{aligned} p(t) = & \exp(-\int_0^t \alpha(\eta)d\eta) \\ q(t) = & -[p(t)]^{-1}\exp(-\int_0^t \beta(\zeta)p(\zeta)d\zeta) \\ s(t) = & -[p(t)]^{-1}\exp(-\int_0^t \kappa(\zeta)p(\zeta)d\zeta) \end{aligned}$$

while δ_1 and δ_2 are constant that have to be determined later. Differentiation of $p(t), q(t)$ and $s(t)$ will give

$$\begin{aligned} p'(t) =& -\alpha(t)p(t) \\ q'(t) =& \alpha(t)q(t) - \beta(t) \\ s'(t) =& \alpha(t)w(t) - \kappa(t). \end{aligned}$$

Considering (8) then we get

$$\begin{aligned} Q'(t) =& p(t)(-\alpha(t)(Y(t) + \delta_1 q(t)) + \delta_2 s(t)) \\ &+ 2Y'(t) + \delta_1(\alpha(t)q(t) - \beta(t)) \\ &+ \delta_2(\alpha(t)s(t) - \kappa(t)) \\ =& p(t)(\beta(t)(Y(t-\tau) - \delta_1) + \kappa(t)(U - \delta_2)). \end{aligned}$$

Choosing δ_1 and δ_2 as

$$\delta_1 = \max_{-\tau \le \zeta \le t} Y(\zeta)$$

$$\delta_2 = \max_{0 \le \zeta \le \infty} U(\zeta)$$

will make $Q'(t) \le 0$, thus by rearranging $Q(t_2) - Q(t_1)$ where $-\tau \le t_1 < t_2$ the following bound applies

$$\begin{aligned} Y(t_2) \le& \exp(-\int_{t_1}^{t_2} \alpha(\eta)d\eta)Y(t_1) \\ &+ \left(\int_{t_1}^{t_2} \beta(\zeta)\exp(-\int_{\zeta}^{t_2} \alpha(\eta)d\eta d\zeta\right)\delta_1 \\ &+ \left(\int_{t_1}^{t_2} \kappa(\zeta)\exp(-\int_{\zeta}^{t_2} \alpha(\eta)d\eta)\right)d\zeta\delta_2 \\ \le& \Bigg(\exp(-\int_{0}^{t_2} \alpha(\eta)d\eta) + \\ &+ \left(\int_{0}^{t_2} \beta(\zeta)\exp(-\int_{\zeta}^{t_2} \alpha(\eta)d\eta d\zeta\right)\delta_1 \\ &+ \left(\int_{0}^{t_2} \kappa(\zeta)\exp(-\int_{\zeta}^{t_2} \alpha(\eta)d\eta d\zeta\right)\delta_2. \end{aligned} \tag{12}$$

In order to finalize the proof we need to show that for all $0 \le t_1 < t_2$, $Y(t_2) \le Y(t_1)$. We are going to prove this by contradiction. Let us assume that there is $\hat{t}_2 > t_1$ such that $Y(\hat{t}_2) > Y(t_1)$, which means that $Y'(\hat{t}_2) > 0$. However, from (9) it is easy to show that by a direct differentiation on (12) $Y'(t) \le 0$ for all t. Therefore the inequality bound (10) follows. ■

Theorem 1: Suppose that the system (1) with solution $x(t)$ satisfies (5) then it is FDE-dissipative. For zero input the system is strictly dissipative in the large, i.e., the absorbing set is all $x(-\tau) \in \mathbb{R}^{\ltimes}$.

Proof: Let us take $Y(t)$ and $U(t)$ as follows

$$\begin{aligned} Y(t) = \|x(t)\|^2 \\ U(t) = \|u(t)\|_\infty \end{aligned} \tag{13}$$

then

$$\begin{aligned} Y'(t) =& 2\langle x(t), f(x(t), x(\cdot), u(t))\rangle \\ \le& \alpha(t)\|x(t)\|^2 + \beta(t)\|x(\zeta)\|^2 + \kappa(t)\|u(t)\|_\infty. \end{aligned}$$

Then lemma 2 follows. From (9) and (10) it is seen that if $u(t)$ is a constant the solution is decreasing for all x, which means that the solution will tend to zero. To be precise, let us proceed straightforward. By differentiating (10), using (5) and considering the fact from Proposition 1 then the above relation is non-positive. ■

The first two terms in (10) belongs to class K_∞, *i.e.*, the function is continuous and zero at zero. For fixed t the function is increasing and unbounded while for fixed $\max_{-\tau \le \zeta \le 0} Y(\zeta)$ the function is decreasing to zero. With an additional property that

$$\int_0^t \kappa(\zeta)exp(\int_\zeta^t \alpha(\eta)d\eta)d\zeta \le \overline{\kappa}exp(\overline{\alpha}t) \tag{14}$$

where $\overline{\alpha}$ and $\overline{\kappa}$ are, respectively, the maximum of $\alpha(\zeta)$ and $\kappa(\zeta)$ over $0 \le \zeta \le t$ then the last term in (10) belongs to class K. Therefore, the following corollary holds.

Corollary 1: If $\kappa(t)$ is an energy signal, *i.e.*, $\lim_{0 \le s \le \infty} \kappa(s)$ is finite then the system that is mentioned in Theorem 1 will also inherit ISS property with ISS gain $\overline{\kappa}(t)$.

Proof: From (9) and (14) the equation bound (10) can be restated as

$$\begin{aligned} y(t) \le& \Bigg(exp(\int_0^t \alpha(\eta)d\eta) + \\ &+ \int_0^t \beta(\zeta)exp(\int_\zeta^t \alpha(\eta)d\eta)d\zeta\Bigg)^{\frac{1}{2}} \|y(\zeta)\|_\infty \\ &+ \left(\int_0^t \kappa(\zeta)exp(\int_\zeta^t \alpha(\eta)d\eta)d\zeta\right)^{\frac{1}{2}} \|u(t)\|_\infty \\ \le& \pi(t, \|y(t)\|_\infty) + \rho(\|u(t)\|_\infty). \end{aligned} \tag{15}$$

where we have defined

$$\begin{aligned} \pi(t, \|y(t)\|_\infty) =& \Bigg(\exp(\int_0^t \alpha(\eta)d\eta) + \\ &+ \int_0^t \beta(\zeta)\exp(\int_\zeta^t \alpha(\eta)d\eta)d\zeta\Bigg)^{\frac{1}{2}} \|y(\zeta)\|_\infty \end{aligned}$$

and $\rho(\|u(t)\|_\infty) = \overline{\kappa}\|u(t)\|_\infty \ge \overline{\kappa}\exp(\overline{\alpha}t)\|u(t)\|_\infty$. ■

3. SOME SIMILARITIES TO ISS FOR NON-DELAYED SYSTEMS

We are going to explore some similarities of the dissipativity and FDE-dissipativity of delayed nonlinear systems to the ones owned by non-delayed nonlinear systems, *i.e.*, assymptotic stability and ISS property.

Consider now connection of two nonlinear delayed systems represented by

$$\Sigma_i : \begin{cases} x_i'(t) = f(t, x_i(t), x_i(\cdot), w_i(t)), & t \in [0, +\infty) \\ x_i(t) = \varphi_i, \quad t \in [-\tau, 0) & i \in \{1, 2\} \end{cases} \tag{16}$$

A cascade connection of the above system can be obtained by coupling the input $w_2(t)$ of second system to the first system via $w_2(t) = h_1(x_1(t))$. It is assumed that the output functions $h_i(x_i, x_i(\cdot))$ satisfy

$$\|h_i(x_i, x_i(\cdot))\| \le a_i(t)\|x_i\|^2 + b_i(t)\|x_i(\cdot)\|^2. \quad (17)$$

Theorem 2: The cascade connection of FDE-dissipative system and FDE-dissipative system is FDE-dissipative if

$$\begin{aligned} &\max\{\alpha_1(t) + \kappa_2(t)a_1(t), \alpha_2(t)\} \\ &+ \max\{\beta_1(t) + \kappa_2(t)b_1(t), \beta_2(t)\} \\ &+\kappa_1(t) \le 0. \end{aligned} \quad (18)$$

Proof: Suppose that

$$\begin{aligned} 2\langle x_1, f_1(x_1, x_1(\cdot), w_1)\rangle \le &\alpha_1(t)\|x_1\|^2 + \beta_1(t)\|x_1(\cdot)\|^2 \\ &+ \kappa_1(t)\|w_1\|_\infty \\ 2\langle x_2, f_2(x_2, x_2(\cdot), w_2)\rangle \le &\alpha_2(t)\|x_2\|^2 + \beta_2(t)\|x_2(\cdot)\|^2 \\ &+ \kappa_2(t)\|w_2\|_\infty. \end{aligned}$$

Take $x = [x_1\ x_2]^T$, $f = [f_1\ f_2]^T$ and $w = [w_1\ w_2]^T = [w_1\ h_1(x_1(t), x_1(\cdot))]^T$ then

$$\begin{aligned} \langle x, f(x, x(\cdot), w)\rangle = &\langle x_1, f_1(x_1, x_1(\cdot), w_1)\rangle \\ &+ \langle x_2, f_2(x_2, x_2(\cdot), w_2)\rangle \\ \le &\alpha(t)\|x\|^2 + \beta(t)\|x(\cdot)\|^2 + \kappa_1(t)\|w\|_\infty. \end{aligned}$$

with $\alpha(t) = \max\{\alpha_1(t) + \kappa_2(t)a_1(t), \alpha_2(t)\}$ and $\beta(t) = \max\{\beta_1(t) + \kappa_2(t)b_1(t), \beta_2(t)\}$. ■

Corollary 2: The cascade connection of strictly dissipative system with FDE-dissipative system satisfying (18) is strictly dissipative

Consider now a feedback connection as illustrated by Fig.1 where each block is represented by (16).

$$w_1 = v_1 - h_2(x_2, x_2(\cdot)) \quad (19a)$$
$$w_2 = v_1 + h_2(x_2, x_2(\cdot)). \quad (19b)$$

It is also assumed that the output $h_i(x_i, x_i(\cdot))$ satisfies (17).

Theorem 3: The feedback connection of two FDE-dissipative systems is FDE-dissipative if

$$\begin{aligned} \max\{\alpha_i(t) + \kappa_i(t)a_i(t)\} + \max\{\beta_i(t) + \kappa_i(t)b_i(t)\} \\ + \max\{\kappa_i(t)\} \le 0 \end{aligned} \quad (20)$$

Proof: As before we define $x = [x_1,\ x_2]^T$, $\hat{f} = [\hat{f}_1,\ \hat{f}_2]^T$, $w = [w_1,\ w_2]^T$ and $v = [v_1,\ v_2]^T$ where

$$\hat{f}_i(x, x(\cdot), v_i) = f_i(x_i, x_i(\cdot), w_i) \quad (21)$$

then

$$\langle x_i, f(x_i, x_i(\cdot), w_i)\rangle = \left\langle x_i, \hat{f}(x, x(\cdot), v_i)\right\rangle.$$

Therefore the following relation holds

$$\left\langle x, \hat{f}(x, x(\cdot), v)\right\rangle \le \alpha(t)\|x\|^2 + \beta(t)\|x(\cdot)\|^2 + \kappa(t)\|v(t)\|^2$$

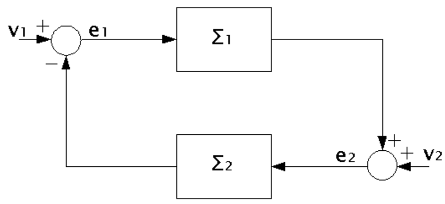

Fig.1: *Feedback connection of two FDE systems.*

where

$$\begin{aligned} \alpha(t) &= \max\{\alpha_i(t) + \kappa_i(t)a_i(t)\} \\ \beta(t) &= \max\{\beta_i(t) + \kappa_i(t)b_i(t)\} \\ \kappa(t) &= \max\{\kappa_i(t)\}. \end{aligned}$$

Therefore, if the condition (20) is satisfied, then the FDE-dissipativity follows. ■

In [8], Theorem 1, the input for the feedback connection is restricted to $w_i(t) \le \chi_i(\|x(t)\|)$ where $\chi_i(\cdot) \in K$. Therefore, Theorem 1 in this paper will be applicable in more general sense.

3.1 FDE-disipativity, Lyapunov stability and IS-Stabilization

In this section, firstly we are going to prove a less general dissipative condition for system (1). Using our main tool, i.e., Theorem 1 we can prove exponential stability solution of system (1) easily.

Theorem 4 (Lyapunov exponential stability) If there exists a functional $V : R^1 \times R^n \to R^1$ such that

- $W_1(x) \le V(t, x) \le W_2(x)$
- $W_3(x(\cdot)) \le V(t, x(\cdot))$
- $\dot{V}(t, x) \le \alpha(t)W_2(x) + \beta(t)W_3(x(\cdot))$

where W_1, W_2 and W_2 belong to class K function, then the system is globally exponentially stable.

Proof: By a direct differentiation we get

$$\begin{aligned} \dot{V}(t, x) \le &\alpha(t)W_2(x) + \beta(t)W_3(x(\cdot)) \\ &\alpha(t)V(t, x) + \beta(t)V(t, x(\cdot)) \end{aligned}$$

Then we can apply Lemma 2 in order to arrive at

$$\begin{aligned} V(t, x) \le &\left(\exp(-\int_0^t \alpha(\eta)d\eta) + \right. \\ &\left. + \int_0^t (\beta(\zeta)\exp(-\int_\zeta^t \alpha(\eta)d\eta)d\zeta)\right) \max_{-\tau \le \zeta \le 0} V(t, \zeta) \\ &:= V_0 \exp(\rho t). \end{aligned}$$

Therefore, we conclude that $x(t) \le W_3^{-1} V_0 \exp(\rho t)$ ■

Finally we consider the following control problem as:

For a given delayed nonlinear system, find a (state) feedback controller to render the close-loop system FDE-dissipative

Then we propose the following step:

1. Transform the original system in form of system (1).

2. Consider $\langle x, f(x, x(\cdot), u(t))\rangle$ then set our condition of interest satisfying (5).
3. Set $u(t)$ until the condition is met.
4. (Extra step) make $\kappa(t)$ satisfies Theorem 1 in order to make our closed-loop system ISS.

4. ILLUSTRATIVE EXAMPLE

4.1 Control law comparison

In order to make everything clearer let us take an example from [16] as follows

$$\begin{aligned}\dot{x}_1(t) =& \frac{x_2(t)}{1+x_1^2(t-\Delta)} + \delta x_2(t-\Delta)\\ \dot{x}_2(t) =& x_1(t)x_2(t-\Delta) + u(t) + w(t)\end{aligned} \tag{22}$$

where $x(t) = [x_1(t),\ x_2(t)]^T$, $u(t)$ is the control input and $w(t)$ is a locally essentially measurable disturbance, δ is a bounded constant that represents uncertain parameter and Δ is a positive constant that represents an arbitrarily long time delay in the system. In [16] by utilizing Lyapunov-Krasovskii Theorem, the control law that is able to render the system (22) to be ISS is as follows:

$$\begin{aligned}u(t) = -x_1(t)x_2(t-\Delta) + (1+x_1^2(t-\Delta))\cdot\\ \left(-k^T \begin{bmatrix} x_1(t) \\ \frac{x_2(t)}{1+x_1^2(t)} + \delta x_2(t-\Delta)\end{bmatrix}\right.\\ -\delta x_1(t-\Delta)x_2(t-2\Delta)+\\ \left.+\frac{2x_2(t)x_1(t-\Delta)}{(1+x_1^2(t-\Delta))^2}\left(\frac{x_2(t-\Delta)}{1+x_1^2(t-2\Delta)}\right)\right)\\ -\delta(1+x_1^2(t-\Delta))u(t-\Delta).\end{aligned} \tag{23}$$

From the above equation ones can see how complicated the control law is. At first, this control law has the factor of double delayed state information $x(t-2\Delta)$. Due to its dependence with $x(t-\Delta)$ the above control law is only applicable at $t \geq \Delta$. In real applications, this will imply that some buffer has to be used in order to store the delayed variable information. Moreover, the inclusion of uncertain parameter δ and the delayed control law $u(t-\Delta)$ can make the control law more complicated than the system that has to be stabilized. This will also imply that that the control law is impossible to be applied in a real system.

Let us compare with our proposed tool. We are going to use another scenario utilizing the above outcomes. Let us start with (5). By defining $z = x$, $x(t-\Delta) = x(\cdot)$ and $f(x, x(\cdot), [u, w]^T) = \dot{(x)}$ then we arrive at

$$\begin{aligned}\langle z, f(z, \psi(\cdot), u(t))\rangle \leq x_1(t)\frac{x_2(t)}{1+x_1^2(t-\Delta)}\\ +\delta x_1(t)x_2(t-\Delta)\\ +x_1(t)x_2(t)x_2(t-\Delta) + x_2(t)u(t).\end{aligned} \tag{24}$$

In order to simplify the problem let us set $u(t) = u_1(t) + u_2(t)$ where $x_1(t)$ is set to

$$u_1 = -\frac{x_1(t)}{1+x_1^2(t-\Delta)} - x_2(t-\Delta). \tag{25}$$

Therefore (24) can be reduced to

$$\langle z, f(z, \psi(\cdot), u(t))\rangle \leq x_2(t)u_2(t) + \delta x_1(t)x_2(t-\Delta). \tag{26}$$

By assumption that the bound of the uncertain parameter δ is known, we can set

$$u_2 = \frac{x_2(t)}{1+x_2^2(t)}(-x_1^2(t) - \frac{1}{\sqrt{2}}\sqrt(\max\|\delta\|)x_2^2(t) + v) \tag{27}$$

in order to arrive at

$$\langle z, f(z, \psi(\cdot), u(t))\rangle \leq v. \tag{28}$$

Therefore, we have freedoms in choosing v, for instance,

$$v = -ax_1^2(t) - bx_2^2(t) + cx_1^2(t-\Delta) + dx_2^2(t-\Delta)$$

where a, b, c and d are positive constants. One can see that the proposed controller is easy to be obtained. Moreover, it is simpler than (23). Numerical verifications have been done in order to verify the effectiveness of the controller. The initial conditions are $x(0) = [0.1,\ -0.1]^T$ and controller constants a, b, c and d are set to 100, 100, 10 and 10, respectively. The results shown in Fig. 2 and Fig. 3 show the ISS property of the stabilized system. In Fig. 2 when no input is applied, in spite of being slowly converged, the system is shown to be stable. Finally Fig. 3 show the boundedness of the solution when a disturbance (see Fig. 4) is given to the system.

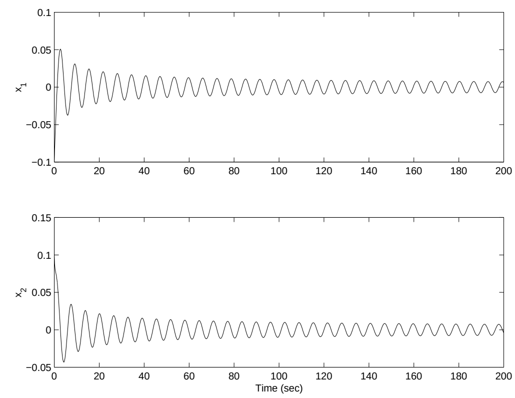

Fig.2: *Responses without input*

4.2 Electrohydraulic motor with long inlet hose

A hydraulic motor using servo valve dynamics with long inlet hose can be modelled as follows

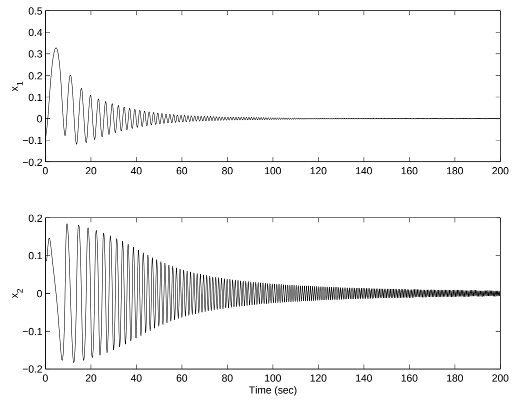

Fig.3: *Responses with input*

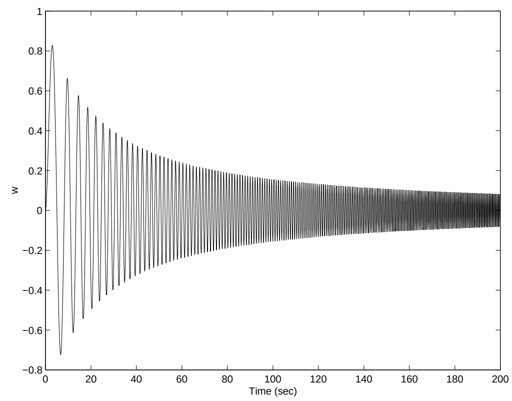

Fig.4: *Disturbance applied to the system*

$$\dot{x} = \begin{bmatrix} c_1 & 0 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ c_2 \end{bmatrix} u(t-\Delta) \quad (29)$$

$$y = [0 \;\; 1]^T x \quad (30)$$

where c_1 and c_2 are positive constant parameters of the hydraulic servosystem,Δ represents the uncertain delay, x and $\cot x$ are the angular displacement and angular velocity, respectively. It is seen that the eignvalues of the above system are $\lambda_1 = 0$ and $\lambda_2 = -c_1$, means that without delay, the above system is stable. While there is no instability theorem that able show the stability behavior of the system, we are interested to find a control law that make the system FDE-Dissipative. In order to make the problem more general, let us consider the following system

$$\dot{x} = Ax + Bu(t-\Delta) \quad (31)$$

$$y = cx \quad (32)$$

$$x = \varphi, \quad t \in [-\tau, 0) \quad (33)$$

Let us perform the similar steps as before. Using (5) by defining $z = x$, $x(t-\Delta) = x(\cdot)$ and $f(x, x(\cdot), [u, w]^T) = \dot{(x)}$ then we arrive at

$$\langle z, f(z, \psi(\cdot), u(t)) \rangle \leq z^T Az + z^T Bu(t-\Delta).$$

In case of $\mathrm{Dim}(u) \leq \mathrm{Dim}(x)$, u and B can be reshaped by zero padding

$$\hat{B} = \begin{bmatrix} 0 & B \\ 0 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} B & 0 \\ 0 & 0 \end{bmatrix}$$

If u can accept non-delayed state x, the stabilization problem can be solved by

$$u = -B^T x - B^T x \mathrm{sign}(x) x^T (t-\Delta) x(t-\Delta)$$

Proposition 2: The delayed system (31) is not FDE-stabilizable.

Proof: We are going to prove by contradiction. Assume that there exists control law $u(x(t-\Delta))$ such that

$$\begin{aligned} \langle x, \dot{x} \rangle &= x^T Ax + x^T Bu(t-\Delta) \\ &\leq -\mathrm{mineig}(A)\|x\|^2 + \beta\|x(t-\Delta)\| \end{aligned}$$

where β is any positive constant. We have two conditions:

- If $\mathrm{eig}(a) > 0$, it will lead to a direct contradiction
- If $\mathrm{eig}(a)/leq0$, we have to assure that

$$x^T Bu(x(t-\Delta)) \leq \beta\|x(t-\Delta)\|^2$$

however it is impossible to find such $u(x(t-\Delta))$. This is also a contradiction.

■

Although we fail to find the stabilizing control law, we are still able to propose control law that technically applicable by only using the knowledge of the $\mathrm{sign}(x)$.

Proposition 3: The delayed linear system (31) with $\mathrm{eig}(A) \leq 0$ is stabilizable by

$$u = \mathrm{sgn}(x) B^T x^T (t-\Delta) x(t-\Delta)$$

Proof: Substitute u into (5). ■

5. CONCLUSION

The similarities between the delayed FDE-Passive system and ISS system for non-delayed system is presented in this paper. Firstly, FDE-Dissipativity term for delayed nonlinear system is introduced and explored. Besides its sufficiency for output to be bounded, FDE-Dissipativity will also imply Input to State Stability (ISS) for time delay systems in special cases. Moreover it is also shown that it has equivalent characteristics with non-delayed nonlinear systems, for instance, feedback interconnection of two FDE-Dissipative systems is also FDE-Dissipative. This paper is finalized with two examples for demonstrating the effectiveness of the proposed tool via numerical verifications.

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