The Application of Duffing Oscillator in Weak Signal Detection

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ABSTRACT

In this paper, a method for identifying the chaotic state of duffing oscillator is proposed where the duffing oscillator is used for weak signal detection. This method is based on frequency spectrum analysis and filtering. Some relative aspects of this method for practical using are studied in details too. The proposed method has three properties; reasonable calculation complexity, robustness to moderate noise amount, and capability of detection with short signal sequence. The proposed approach has good robustness, which is successfully shown in this paper.

Keywords: Duffing Oscillator, Weak Signal, Chaos, Noise

1. INTRODUCTION

Weak signal detection is a challenging task in signal detection and also in fault detection cases. In the fault detection cases, there will be some abnormal signals before a fault breaks out but the signal to noise (SNR) is very low, due to relatively weak characteristic signals as well as effects of transmission path, transmission media, reflection, refraction, etc. It is of great value to study how to detect harmonic signal of a given frequency in noise.

Chaos has potential application outlook in weak signal detection, because of the properties which are sensitive to certain signal and immune to noise at same time. Because of a tiny perturbation of a parameter might cause an essential change of the state in a non-linear chaotic system, a large number of researchers used chaotic oscillations to detect weak signal in a noisy environment. The duffing oscillator is frequently used to detect weak signals [1-3]. In this method the key is to identifying the state of the oscillator. There are several methods for identifying the chaotic character. Common theoretical indications of the duffing state are Lyapunov coefficients. However, Lyapunov coefficients are not practically applicable since their evaluation requires very large signal sequence [4]. In addition Lyapunov coefficients are very sensitive to noise influence [5-6]. These drawbacks force the recent advance in development of reliable chaos detection measures some of them are observation of time- history, phase plane, Fourier spectrum and autocorrelation, Poincare maps, fractal dimensions and so on [5]. However, with these methods, either the complex calculation is necessary, or it is not convenient for automatic identification by computer, or it is can not be used when the system involves noise.

In this paper, we use a method based on the analysis of the frequency spectrum of the duffing oscillator. The main advantages of this approach are reasonable calculation complexity and robustness to moderate noise amount. Also its application for evaluation of the measure requires a short signal sequence.

2. FUNDAMENTAL PRINCIPLES FOR USING DUFFING OSCILLATOR IN SIGNAL DETECTION

Generally, a nonlinear dynamic system has four states: The fixed point, the small periodic motion, the chaotic motion and the quasi-periodic motion (large periodic motion). When the system is in the critical state, a small perturbation of the system parameters may lead to the qualitative change of the system state [1]. The basic idea of the signal detection scheme based on chaotic oscillator is that a small periodic signal in noise can be detected by duffing oscillator via a transition from chaotic motion to periodic motion as a classic nonlinear system [2]. Generally, the chaotic system is constructed by the duffing oscillator. The normal form of the duffing equation is shown as:

$$\frac{d^2x}{dt^2} + \delta\frac{dx}{dt} - x + x^3 = \gamma\cos(t) \quad (1)$$

Where δ is the ratio of damping, $\gamma\cos(t)$ is the periodic driving force and $-x + x^3$ is the nonlinear restoring force. Assuming $y = \frac{dx}{dt} = \dot{x}$, then we have:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -\delta y + x - x^3 + \gamma\cos(t) \end{aligned} \quad (2)$$

If we keep δ fixed then as γ varies from small to big, the system state varies from small periodic motion (Fig. 1), to chaotic motion (Fig. 2), and, at last, to great periodic motion (Fig. 3).

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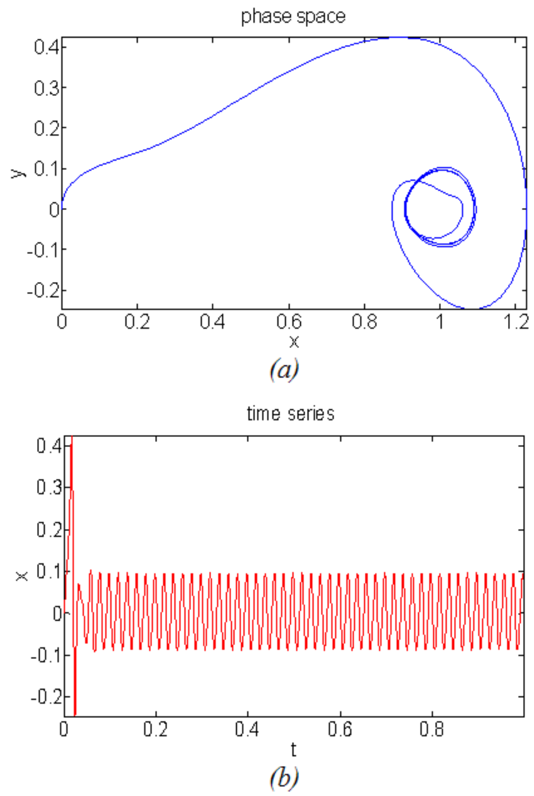

Fig.1: *(a) phase plane diagram of small periodic motion (b) time series diagram of small periodic motion*

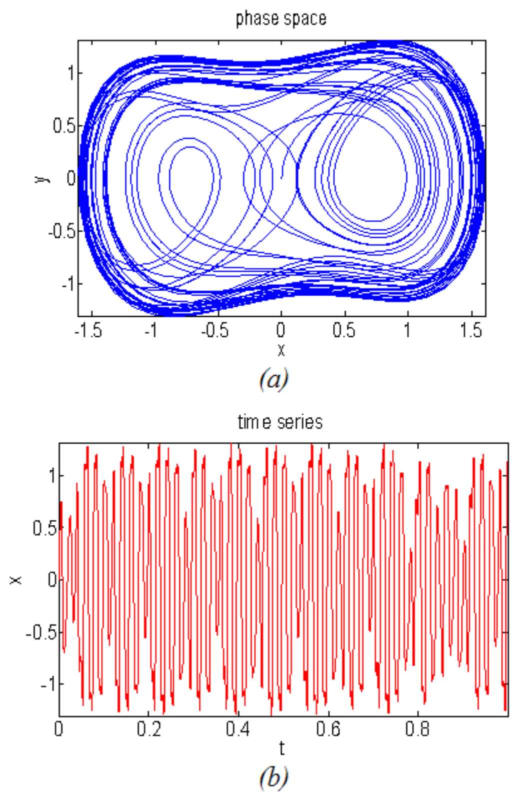

Fig.2: *(a) phase plane diagram of chaotic motion (b) time series diagram of chaotic motion*

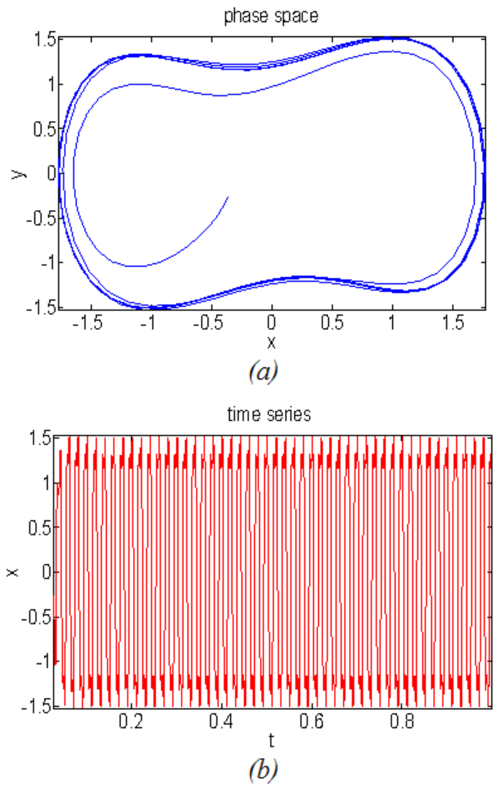

Fig.3: *(a) phase plane diagram of great periodic motion (b) time series diagram of great periodic motion*

If we fix $\gamma = \gamma_c$ (γ_c refers to the critical value), so the system is put into the critical state (chaos, but on the verge of changing to the periodic motion). The to-be-detected signal can be viewed as a perturbation of the main sinusoidal deriving force $\gamma \cos(t)$ (the reference signal). Although noise may be intensive, it can only affect the local trajectory on phase plane diagram, without causing any phase transition. To detect weak signals with different frequencies by applying (2), we must do some frequency transformation. Defining $t = \omega\tau$, we obtain:

$$\begin{aligned} x(t) &= x(\omega t) = x^*(t) \\ \frac{dx(t)}{dt} &= \frac{1}{\omega}\frac{dx(\omega\tau)}{d\tau} = \frac{1}{\omega}\frac{dx^*(\tau)}{d\tau} \\ \frac{d^2x(t)}{dt^2} &= \frac{1}{\omega^2}\frac{d^2x(\omega\tau)}{d\tau^2} = \frac{1}{\omega^2}\frac{d^2x^*(\tau)}{d\tau^2} \end{aligned} \quad (3)$$

Substituting (3) into (2), omitting the superscript * of x^* , and adding the input signal we obtain:

$$\begin{aligned} \dot{x} &= \omega y \\ \dot{y} &= \omega(\delta y + x - x^3 + \gamma \cos(\omega t) + Input) \end{aligned} \quad (4)$$

where $Input = s(\tau) + \sigma(\tau) + a\cos((\omega + \Delta\omega)\tau + \varphi) + \sigma(\tau)$,$\sigma(\tau)$is the Gaussian noise, $\Delta\omega$ is the frequency difference and φ is the primary phase difference. By changing the value of ω in (4) the weak signal with different frequencies can be detected using above mentioned principles.

In this paper, we used the fourth-order Runge-Kutta algorithm to solve the duffing equation. Therefore, the system is a discrete dynamic system by nature, slightly different from the original continues system based on the chosen step size. As we know, there is truncation error (also known as discrimination error) involved in a Runge-Kutta algorithm [1]. Truncation error depends on the step size used, and the dependence is especially distinct when the system is strongly non-linear. As far as our system is concerned, if the step size used is different, the truncation error will bring about a distinct discrepancy of the critical value γ_c. Whatever the value of step size is, the phase transition itself is clear and distinct; what makes the different is just the value of γ_c. The truncation error dose not means that the step size is required to be very small to detect chaos onset accurately.

In this paper, we choose $\delta = 0.5$ and $h = 0.0002$ (step size) fixed. The value of γ_c is different, depends on the system conditions and the reference signal frequency.

3. QUANTITATIVE DESCRIPTION OF DUFFING OSCILLATOR STATE

The duffing oscillator state has been described by observing the trajectory (phase plane diagram) which is not suitable for automatic recognition. Hence, we need to quantitatively judge the duffing oscillator state using the method based on frequency spectrum and filtering.

Analyzing the frequency spectrum of the output of the duffing system, we found that in periodic motion, it only includes the fundamental wave and its odd harmonics, while in chaotic motion; various components are in the spectra of the output of the duffing system, especially the components with the frequency lower than fundamental component. The frequency spectrum of the output of the duffing system $0.825\cos(100\pi t)$ is shown in Fig.4.

As Fig. 4 indicates, information on the system state can be extracted from the frequency spectrum of the output of the duffing system. Therefore, we designed a low pass filter (its cut-off frequency is lower than the reference frequency) to filter the output of the duffing oscillator, and then with the use of the Root Mean Square (RMS) value of the remaining components; we estimate the state of the duffing oscillator. The specifications of this filter for above oscillator are $F_{pass} = 35Hz$ and $F_{stop} = 45Hz$. After using the desired filter and calculating the RMS value of the output of the duffing oscillator in a specific range ($RMS_{f<50}$), we can see, in chaotic motion, $RMS_{f<50} = 0.6055$ and in periodic motion $RMS_{f<50} = 0.0662$. Therefore $RMS_{f<50}$ in the output of the oscillator can be considered as a distinct criterion for identifying the state of the duffing oscillator.

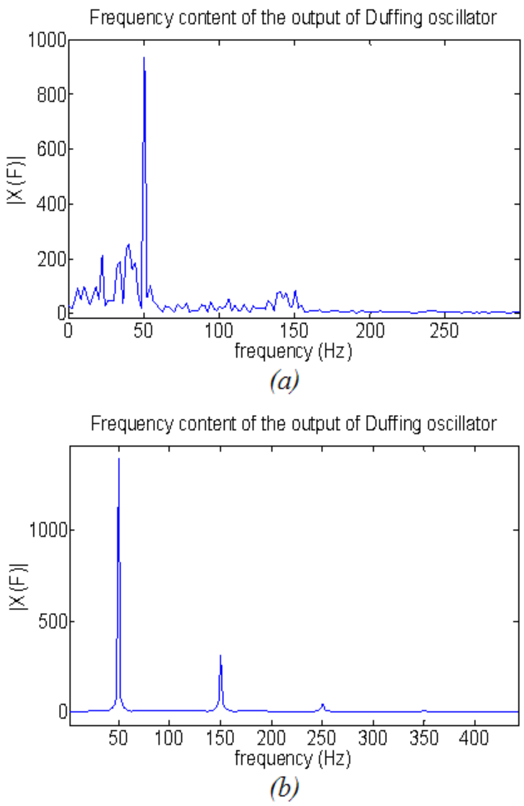

Fig.4: *(a) frequency spectrum of chaotic motion (b) frequency spectrum of periodic motion*

4. FURTHER DISCUSSION

In this section we discuss problems related to solutions of the duffing equation and propose a method for detecting the weak signal based on the characteristics of the solutions. Assume that our purpose is the detection of a weak signal with frequency by duffing oscillator. Then we obtain:

$$\ddot{x} + 0.5\dot{x} - x + x^3 = 0.825\cos(\omega\tau) \tag{5}$$

At this moment the system is in critical state. Now if an external weak signal (for example $s(\tau) = 0.001 cos(\omega t)$), with the same frequency as the reference signal, is merged into duffing oscillator, we obtain:

$$\ddot{x} + 0.5\dot{x} - x + x^3 = 0.825\cos(\omega\tau) + 0.001\cos(\omega\tau) \tag{6}$$

The ultimate state of the oscillator is the periodic (Fig. 5). Although the weak signal (such as $0.001\cos(100\pi t)$) can be detected but in practice we must first solve a series of problems.

4.1 The Influence of the Noise

If the external exciting signal is Gaussian noise $S(t) = \varepsilon(t)$, then the solutions of Eq. (6) will be as shown in Fig. 6. The orbits are chaotic, means the orbits can keep the state of the motion steadily under the influence of the noise.

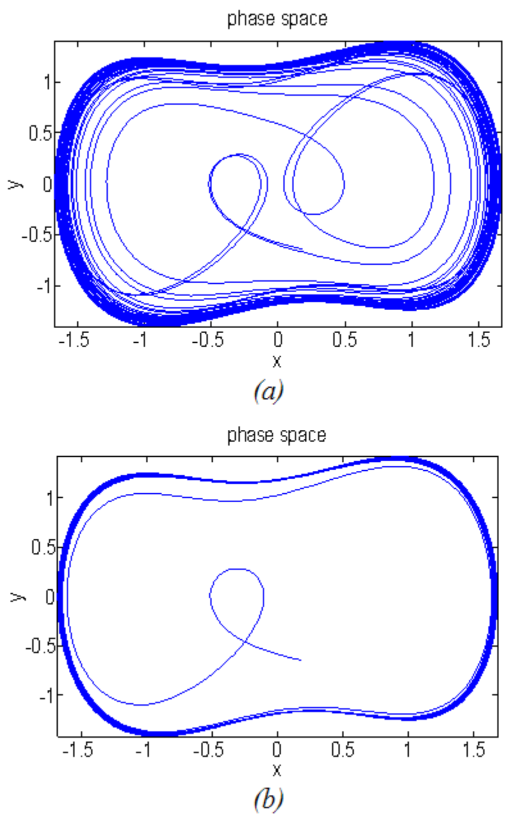

Fig.5: *(a) phase plane diagram before merging the input signal (b) time series diagram after merging the input signal*

4.2 Influence of to-be-Detected Signal Padded Noise

It is assumed that the exciting signal consists of the noise and weak signal with the same frequency as the reference signal ($S(t) = 0.01cos(100\pi\tau) + \sigma\varepsilon(t)$), where σ is 0.03, 0.05, 0.08, 0.1and 0.2 respectively. The results show that only when $\sigma \leq 0.08$ the weak signal is identified reliably (Fig.7). Therefore, we can conclude that the threshold of the signal to noise ratio of the weak signal should be greater than:

$$\begin{aligned} SNR &= 10\log\left(0.5\frac{a^2}{\sigma^2}\right) = 10\log\left(0.5\frac{0.01^2}{0.08^2}\right) \quad (7) \\ &= -21.0721db \end{aligned}$$

Otherwise, it is not detectable by the mentioned method.

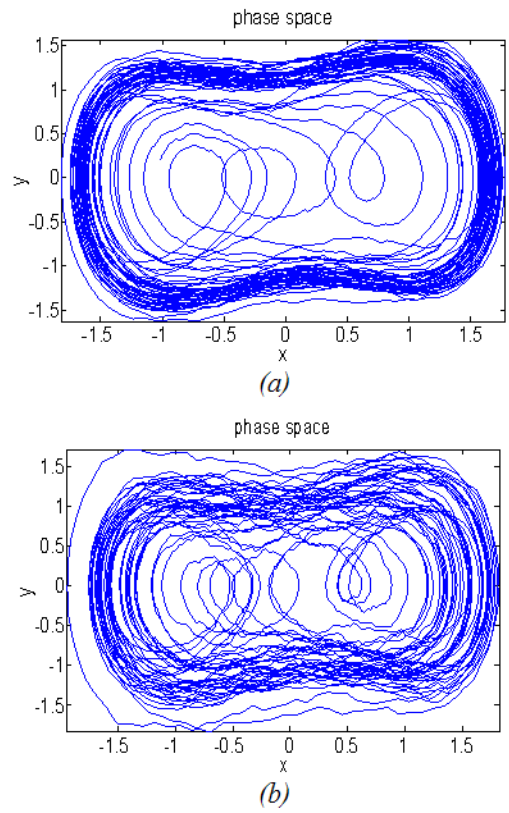

Fig.6: *(a) phase plane diagram* $\sigma = 1$ *(b) phase plane diagram* $\sigma = 4$

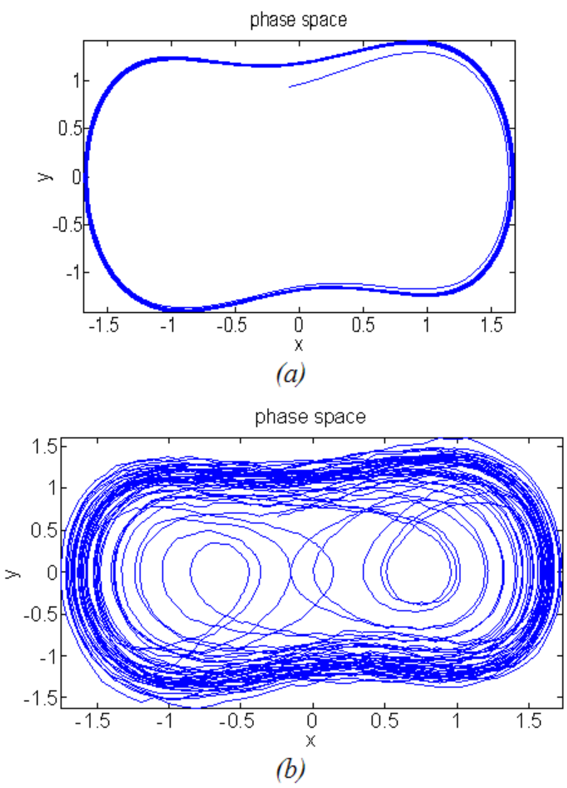

Fig.7: *(a) phase plane diagram* $\sigma = 0.08$ *(b) phase plane diagram* $\sigma = 0.02$

4.3 Influence of the Different Frequency Signal

When the frequency of the external signal is different from that of the reference signal, for example $S(t) = 0.8\cos((\omega \pm \Delta\omega)\tau)$ and $\Delta\omega = [3.1416\ 6.2832\ 62.8319\ 94.2478]$ we again solve Eq (6). The result shows that the orbits keep chaotic state and the state of motion isn't shifted (Fig.8).

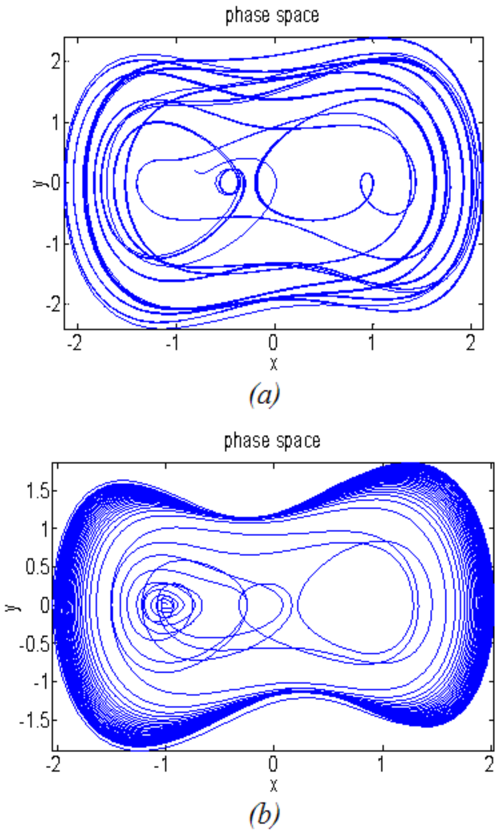

Fig.8: *(a) phase plane diagram* $\Delta\omega = 94.2498$ *(b) phase plane diagram* $\Delta\omega = 3.1416$

4.4 Influence of the Initial Phase

To consider the difference of initial phase between the external signal and the reference signal, assuming the reference signal be $\gamma_c\, a\cos(\omega\tau+\theta))$ and the weak signal be $a\cos(\omega\tau+\varphi))$, then we can write the total periodic exciting force as:

$$\begin{aligned} A(t) &= \gamma_c\cos(\omega\tau+\theta) + a\cos(\omega\tau+\varphi) \\ &= \gamma_c[\cos(\omega\tau)\cos(\theta) - \sin(\omega\tau)\sin(\theta)] \quad (8) \\ &+ a[\cos(\omega\tau)\cos(\varphi) - \sin(\omega\tau)\sin(\varphi)] \end{aligned}$$

(1): if $\theta = 0$ then:

$$\begin{aligned} A(t) &= [\gamma_c + a\cos(\varphi)]\cos(\omega\tau) - a\sin(\varphi)\sin(\omega\tau) \\ &= \gamma(\tau)\cos(\omega\tau + \varphi(\tau)) \quad (9) \end{aligned}$$

$$\gamma(\tau) = \sqrt{\gamma_c^2 + a^2 + 2\gamma_c a\cos(\varphi)} \quad (10)$$

$$\varphi(\tau) = \arctan\left(\frac{a\sin(\varphi)}{a\cos(\varphi)+\gamma_c}\right) \quad (11)$$

It can be seen that the phase shift is related to the difference of phase between the external signal and the reference signal.

When $\pi - \arccos\left(\frac{a}{2\gamma_c}\right) \le 4 \le \pi + \arccos\left(\frac{a}{2\gamma_c}\right)$,$\gamma_c \ge \gamma(\tau)$ the orbit transition will not occur. (2): if then:

$$\begin{aligned} A(t) &= [-\gamma_c + a\cos(\varphi)]\cos(\omega\tau) - a\sin(\varphi)\sin(\omega\tau) \\ &= \gamma(\tau)\cos(\omega\tau + \varphi(\tau)) \quad (12) \end{aligned}$$

$$\gamma(\tau) = \sqrt{\gamma_c^2 + a^2 - 2\gamma_c a\cos(\varphi)} \quad (13)$$

$$\varphi(\tau) = \arctan\left(\frac{a\sin(\varphi)}{a\cos(\varphi)-\gamma_c}\right) \quad (14)$$

When $\pi - \arccos\left(\frac{a}{2\gamma_c}\right) \le 4 \le \pi + \arccos\left(\frac{a}{2\gamma_c}\right)$,$\gamma_c \ge \gamma(\tau)$ the orbit transition will occur. The results of theoretical computation show that if $S(\tau) = 0.01\cos(\omega\tau+\varphi)$ and the referenced signal is $0.825\cos(\omega\tau)$, $\omega = 100\pi[rad/sec]$, then the phase transition from chaos to orbit will occur when $\varphi\epsilon[1.5769, 4.7063$. In practice, because of the discretization error, range of the φ, which in it the phase transition will occur, is narrowed ($\varphi\epsilon[1.67, 4.61]$).

5. CONCLUSION

This paper presented a simple method for signal detection, based on identifying chaos in duffing oscillator. The advantages of this method are: evaluation of the measure requires short interval (window width); reasonable calculation complexity; robustness to moderate noise amount. Furthermore the related problems that may occur when we use the duffing oscillator for weak signal detection have been discussed.

References

[1] W.Guanyu, G.Dajun, and X.Chen, "The application of chaotic oscillator to weak signal detection," *IEEE Transaction on Industrial Electronics*, vo1. 46 No. 2, 1999, pp. 440-444.

[2] W.Guanyu and S.He, "A Quantitative study on detection and estimation of weak signals by using chaotic duffing oscillator," *IEEE Trans. Circuits and systems-I: Fundamental theory and applications*, vo1. 50, No. 7, 1999, pp. 945-953.

[3] L.Yue, Y. Baojun and S.Y.Wu, "Chaos- based weak signal sinusoidal signal detection approach

under colored noise background," *acta physica sinica* 52 (3), 2003, pp. 440-444.

[4] V. Rubezic, I. Djurovic and M. Dakovic, "Time-frequency representations-based detector of chaos in oscillatory circuits," *Elsevier Signal Processing*, 86(9), 2006, pp. 2255-2270.

[5] C. S. Poon, M. Barahona, "Titration of chaos with added noise," *Proc. Natl. Acta. Sci*, Vol 98, No. 13, 2001, pp. 7107-7112.

[6] M. I. Rosenstein, J. J. Collins and C.J. Deluca, "A practical method for calculating largest Lyapunov exponents for small data sets," *Physica D*, 65, 1993, PP. 117-134.

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