The Parameterization of All Disturbance Observers for Time-Delay Plants

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ABSTRACT

We examine the parameterization of all disturbance observers for time-delay plants. Disturbance observers have been used to estimate unknown disturbance in plants. There have been several works on design methods of disturbance observers for time-delay plants but no published work on the parameterization of all disturbance observers for time-delay plants. We propose the parameterization of all disturbance observers for time-delay plants and that of all linear functional disturbance observers for time-delay plants.

Keywords: Disturbance Observers, Parameterization, Time-Delay System

1. INTRODUCTION

We examine the parameterization of all disturbance observers for time-delay plants with any disturbance. A disturbance observer is used in the motion-control field to cancel the disturbance or to make a closed-loop system robustly stable [1–8]. Generally, the disturbance observer includes a disturbance signal generator and observer. The disturbance, which is usually assumed to be a step disturbance, is then estimated using the observer. Since the disturbance observer has a simple structure and is easy to understand, it is applied to many applications [1–6, 8]. However, Mita et al. pointed out that the disturbance observer is nothing more than an alternative design of an integral controller [7]. That is, a control system with a disturbance observer does not guarantee robust stability. In addition, in [7], an extended H_∞ control is proposed as a robust motion-control method that achieves disturbance cancellation. This implies that a control system with a disturbance observer can be designed using the method in [7] to guarantee robust stability. From another viewpoint, Kobayashi et al. considered the robust stability of a control system with a disturbance observer and analyzed parameter variations of the disturbance observer [8]. In this way, robustness analysis of a control system with a disturbance observer has been conducted.

Another important control problem is that of parameterization; i.e., the problem of finding all stabilizing controllers for a plant [9–14]. If the parameterization of all disturbance observers for plants with any disturbance could be obtained, we could express the results of previous studies of the disturbance observer in a uniform manner. In addition, the disturbance observer for plants with any disturbance could be designed systematically. From this viewpoint, the parameterization of all disturbance observers for plants with any disturbance is considered [15].

In many dynamical systems such as biological systems, chemical systems, metallurgical processing systems, nuclear reactors, pneumatic and hydraulic systems with long transmission lines, and electrical networks, the presence of a time delay is quite common and often leads to poor performance and instability of a control system [16–25]. This implies that if we design a control system that considers the time delay in the plant, then in some cases, the control system is unstable and has poor performance. That is, design methods for nondelay plants cannot be applied to a time-delay plant while guaranteeing the stability of the control system and good performance. In particular, it is difficult to attenuate the unknown disturbances for a time-delay plant effectively. The disturbance observer for time-delay plants with any disturbance works well to attenuate unknown disturbances for a time-delay plant. However, no paper has examined a design method for disturbance observers for time-delay plants with any disturbance while considering the parameterization of all disturbance observers for time-delay plants with any disturbance.

In this paper, we propose the parameterization of all disturbance observers for time-delay plants with any disturbance and that of all linear functional disturbance observers for time-delay plants with any disturbance. First, the structure and necessary characteristics of disturbance observers for time-delay plants with any disturbance are defined. Next, the parameterization of all disturbance observers for time-delay plants with any disturbance and that of all linear functional disturbance observers for time-delay plants with any disturbance are clarified. Finally, numerical examples are presented to show the effectiveness of

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the proposed parameterization.

This paper is organized as follows. In Section 2. , a disturbance observer for time-delay plants with any disturbance is introduced briefly and the problem considered in this paper is explained. In Sections 3. and 4., the parameterization of all disturbance observers for time-delay plants with any disturbance and that of all linear functional disturbance observers for time-delay plants with any disturbance are clarified, respectively. Simple numerical examples are given in Section 5. Section 6. presents an application of the proposed method for estimating unknown disturbances in a heat-flow experiment. Section 7. gives concluding remarks.

Notation

R	the set of real numbers.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real rational functions.
$\mathcal{U}$	the unimodular procession in RH_∞. That is, $U(s) \in \mathcal{U}$ means that $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	represents the state space description $C(sI-A)^{-1}B + D$.
$\mathcal{L}^{-1}\{\cdot\}$	inverse Laplace transformation of $\{\cdot\}$.

2. DISTURBANCE OBSERVER AND PROBLEM FORMULATION

Consider the time-delay plant written as

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bu(t-L) \\ y(t) & = & Cx(t) + d(t) \end{array} \right. , \tag{1}$$

where $x \in R^n$ is the state variable, $u \in R^p$ is the control input, $y \in R^m$ is the output, $d \in R^m$ is the disturbance, $A \in R^{n\times n}$, $B \in R^{n\times p}$, $C \in R^{m\times n}$, and $L > 0$ is the time-delay. It is assumed that (A, B) is stabilizable, (C, A) is detectable, $u(t-L)$ and $y(t)$ are available, but $d(t)$ is unavailable. The transfer function of the output y in (1) is denoted as

$$y(s) = G(s)e^{-sL}u(s) + d(s), \tag{2}$$

where

$$G(s) = C\left(sI - A\right)^{-1} B \quad \in R^{m\times p}(s). \tag{3}$$

When the disturbance $d(t)$ is not available, in many cases, the disturbance estimator referred to as the disturbance observer is used. The disturbance observer estimates the disturbance $d(t)$ in (1) using available measurements $u(t-L)$ and $y(t)$. Since the available measurements of the plant in (1) are $u(t-L)$ and $y(t)$, the general form of the disturbance observer $\tilde{d}(s)$ for time-delay plants in (1) is written as

$$\tilde{d}(s) = F_1(s)y(s) + F_2(s)e^{-sL}u(s), \tag{4}$$

where $F_1(s) \in R^{m\times m}(s)$, $F_2(s) \in R^{m\times p}(s)$, $\tilde{d}(s) = \mathcal{L}(\tilde{d}(t))$, and $\tilde{d}(t) \in R^m(t)$. In the following, we refer to the system $\tilde{d}(s)$ in (4) as a disturbance observer for time-delay plants with any disturbance if

$$\lim_{t\to\infty}\left(d(t) - \tilde{d}(t)\right) = 0 \tag{5}$$

is satisfied for any $x(0)$, $u(t)$, $y(t)$, and $d(t)$.

The problem considered in this paper is the parameterization of all disturbance observers $\tilde{d}(s)$ in (4) for time-delay plants with any disturbance.

3. PARAMETERIZATION OF ALL DISTURBANCE OBSERVERS FOR TIME-DELAY PLANTS WITH ANY DISTURBANCE

In this section, we propose the parameterization of all disturbance observers for time-delay plants with any disturbance.

The parameterization of all disturbance observers $\tilde{d}(s)$ in (4) for time-delay plants with any disturbance is summarized in the following theorem.

Theorem 1: The system $\tilde{d}(s)$ in (4) is a disturbance observer for time-delay plants with any disturbance if and only if $F_1(s)$ and $F_2(s)$ can be written as

$$F_1(s) \quad = \quad I \tag{6}$$

and

$$F_2(s) \quad = \quad -G(s) \in RH_\infty, \tag{7}$$

respectively.

Proof: First, necessity is shown. That is, we show that if the system $\tilde{d}(s)$ in (4) satisfies (5), then $F_1(s)$ and $F_2(s)$ in (4) are written as (6) and (7), respectively. The control input $u(s)$ is written as

$$u(s)e^{-sL} = D(s)\xi(s), \tag{8}$$

where $\xi(s)$ is the pseudo state variable and $D(s) \in RH_\infty$ and $N(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = N(s)D^{-1}(s). \tag{9}$$

From (8) and (9), (2) is rewritten as

$$y(s) = N(s)\xi(s) + d(s). \tag{10}$$

Substituting (8) and (10) into (4), we have

$$\tilde{d}(s) \quad = \quad \left(F_1(s)N(s) + F_2(s)D(s)\right)\xi(s) + F_1(s)d(s). \tag{11}$$

From (11),

$$\begin{array}{rl} & d(s) - \tilde{d}(s) \\ = & \left(I - F_1(s)\right)d(s) - \left(F_1(s)N(s)\right. \\ & \left. +F_2(s)D(s)\right)\xi(s) \end{array} \tag{12}$$

is satisfied. From the assumption that (5) is satisfied, $F_1(s) \in RH_\infty$ and $F_2(s) \in RH_\infty$ are written as (6) and (7), respectively. In this way, necessity has been proved.

Next, sufficiency is shown. That is, we show that if $F_1(s)$ and $F_2(s)$ are written as (6) and (7), then the system $\tilde{d}(s)$ in (4) satisfies (5). Substituting (6) and (7) into (4), $\tilde{d}(s)$ is written as

$$\tilde{d}(s) = y(s) - G(s)e^{-sL}u(s). \tag{13}$$

From (13) and (2), $d(s) - \tilde{d}(s)$ satisfies

$$\begin{aligned} & d(s) - \tilde{d}(s) \\ &= y(s) - G(s)e^{-sL}u(s) - (y(s) - G(s)e^{-sL}u(s)) \\ &= 0. \end{aligned} \tag{14}$$

This yields

$$\lim_{t\to\infty}\left(d(t) - \tilde{d}(t)\right) = 0. \tag{15}$$

In this way, sufficiency has been proved.

We have thus proved Theorem 1. ■

Note that from Theorem 1, if the plant $G(s)e^{-sL}$ is unstable, there exists no disturbance observer for time-delay plants with any disturbance satisfying (5). Since almost all plants in the motion-control field are unstable, a problem of obtaining a disturbance observer for time-delay plants with any disturbance is important. When a disturbance observer for time-delay plants with any disturbance is used to attenuate disturbances such as in [1–6], even if $\tilde{d}(s)$ satisfying (5) cannot be designed, a control system can be designed to attenuate disturbance effectively. That is, to attenuate disturbances, it is enough to estimate $(I - F(s))d(s)$, where $F(s) \in RH_\infty$. From this point of view, in the next section, when $G(s)e^{-sL}$ is unstable, we define a linear functional disturbance observer for time-delay plants with any disturbance and clarify the parameterization of all linear functional disturbance observers for time-delay plants with any disturbance.

4. PARAMETERIZATION OF ALL LINEAR FUNCTIONAL DISTURBANCE OBSERVERS FOR TIME-DELAY PLANTS WITH ANY DISTURBANCE

In this section, we define the linear functional disturbance observer and clarify the parameterization of all linear functional disturbance observers for time-delay plants with any disturbance.

For any $d(s)$, $x(0)$, $u(t)$ and $y(s)$, we refer to $\tilde{d}(s)$ as a linear functional disturbance observer for time-delay plants with any disturbance if

$$d(s) - \tilde{d}(s) = F(s)d(s) \tag{16}$$

is satisfied, where $F(s) \in RH_\infty$. Since the available measurements of the plant in (1) are the control input $u(t-L)$ and the output $y(t)$, the general form of the linear functional disturbance observer for time-delay plants with any disturbance is written as (4), where $F_1(s) \in R^{m\times m}(s)$ and $F_2(s) \in R^{m\times p}(s)$.

Next, we clarify the parameterization of all linear functional disturbance observers for time-delay plants with any disturbance. The parameterization of all linear functional disturbance observers for time-delay plants with any disturbance is summarized in the following theorem.

Theorem 2: The system $\tilde{d}(s)$ in (4) is a linear functional disturbance observer for time-delay plants with any disturbance if and only if $F_1(s)$, $F_2(s)$ and $F(s)$ are written as

$$F_1(s) = \tilde{D}(s) + Q(s)\tilde{D}(s), \tag{17}$$

$$F_2(s) = -\tilde{N}(s) - Q(s)\tilde{N}(s) \tag{18}$$

and

$$F(s) = I - F_1(s), \tag{19}$$

respectively, where $N(s) \in RH_\infty^{m\times p}$, $D(s) \in RH_\infty^{p\times p}$, $\tilde{N}(s) \in RH_\infty^{m\times p}$, and $\tilde{D}(s) \in RH_\infty^{m\times m}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \tilde{D}^{-1}(s)\tilde{N}(s) = N(s)D^{-1}(s) \tag{20}$$

and

$$\tilde{D}(s)N(s) - \tilde{N}(s)D(s) = 0. \tag{21}$$

$Q(s) \in RH_\infty^{m\times m}$ is any function.

Proof of this theorem requires the following lemma.

Lemma 1: Suppose that $A(s) \in RH_\infty^{n\times m}$, $B(s) \in RH_\infty^{q\times m}$, $C(s) \in RH_\infty^{p\times m}$ and

$$\text{rank}\begin{bmatrix} A(s) \\ B(s) \end{bmatrix} = r. \tag{22}$$

The equation

$$X(s)A(s) + Y(s)B(s) = C(s) \tag{23}$$

has a solution $X(s)$ and $Y(s)$ if and only if there exists $U(s) \in \mathcal{U}$ to satisfy

$$\begin{bmatrix} A(s) \\ B(s) \\ C(s) \end{bmatrix} = U(s)\begin{bmatrix} A(s) \\ B(s) \\ 0 \end{bmatrix}. \tag{24}$$

When a pair of $X_0(s)$ and $Y_0(s)$ is a solution to (23), all solutions are given by

$$\begin{aligned} & \begin{bmatrix} X(s) & Y(s) \end{bmatrix} \\ &= \begin{bmatrix} X_0(s) & Y_0(s) \end{bmatrix} + Q(s)\begin{bmatrix} W_1(s) & W_2(s) \end{bmatrix}, \end{aligned} \tag{25}$$

where $W_1(s) \in RH_\infty$ and $W_2(s) \in RH_\infty$ are functions satisfying

$$W_1(s)A(s) + W_2(s)B(s) = 0 \tag{26}$$

and

$$\mathrm{rank}\left[\begin{array}{cc} W_1(s) & W_2(s) \end{array}\right] = n+q-r. \quad (27)$$

$Q(s) \in RH_\infty^{p\times(n+q-r)}$ is any function [12].
Using the above Lemma 1, Theorem 2 is proved.
Proof: First, necessity is shown. That is, we show that if the system $\tilde{d}(s)$ in (4) satisfies (16), then $F_1(s)$ and $F_2(s)$ in (4) and $F(s)$ are written as (17), (18), and (19), respectively.

From (20), the control input $u(s)e^{-sL}$ is written as

$$u(s)e^{-sL} = D(s)\xi(s), \quad (28)$$

where $\xi(s)$ is the pseudo state variable. Using the pseudo state variable $\xi(s)$, (4) is rewritten as

$$\tilde{d}(s) = \left(F_1(s)N(s) + F_2(s)D(s)\right)\xi(s) + F_1(s)d(s). \quad (29)$$

$d(s) - \tilde{d}(s)$ is then written as

$$\begin{aligned} & d(s) - \tilde{d}(s) \\ = \; & (I - F_1(s))\, d(s) - (F_1(s)N(s) \\ & + F_2(s)D(s))\, \xi(s). \end{aligned} \quad (30)$$

From the assumption that (16) is satisfied,

$$I - F_1(s) = F(s) \quad (31)$$

and

$$F_1(s)N(s) + F_2(s)D(s) = 0 \quad (32)$$

hold true. Equation (31) corresponds to (19).

From (21) and Lemma 1, a solution pair for (32) is given as

$$F_1(s) = \tilde{D}(s) \quad (33)$$

and

$$F_2(s) = -\tilde{N}(s). \quad (34)$$

Since $N(s)$ and $D(s)$ are right coprime,

$$\mathrm{rank}\left[\begin{array}{c} N(s) \\ D(s) \end{array}\right] = p. \quad (35)$$

From (35) and (21), a pair of $W_1(s)$ and $W_2(s)$ satisfying

$$W_1(s)N(s) + W_2(s)D(s) = 0 \quad (36)$$

and

$$\begin{aligned} & \mathrm{rank}\left[\begin{array}{cc} W_1(s) & W_2(s) \end{array}\right] \\ = \; & m + p - \mathrm{rank}\left[\begin{array}{c} N(s) \\ D(s) \end{array}\right] \\ = \; & m \end{aligned} \quad (37)$$

is

$$W_1(s) = \tilde{D}(s) \quad (38)$$

and

$$W_2(s) = -\tilde{N}(s). \quad (39)$$

From Lemma 1, all solutions $F_1(s)$ and $F_2(s)$ to (32) are written as

$$F_1(s) = \tilde{D}(s) + Q(s)\tilde{D}(s) \quad (40)$$

and

$$F_2(s) = -\tilde{N}(s) - Q(s)\tilde{N}(s) \quad (41)$$

respectively, where $Q(s) \in RH_\infty^{m\times m}$ is any function. In this way, necessity has been proved.

Next, sufficiency is shown. That is, we show that if $F_1(s)$, $F_2(s)$ and $F(s)$ are written as (17), (18), and (19), respectively, then (4) satisfies (16). Substituting (17) and (18) into (4), we have

$$\tilde{d}(s) = \left(\tilde{D}(s) + Q(s)\tilde{D}(s)\right) d(s) = F_1(s)d(s). \quad (42)$$

From (42), $d(s) - \tilde{d}(s)$ is written as

$$\begin{aligned} d(s) - \tilde{d}(s) &= (I - F_1(s))\, d(s) \\ &= F(s)d(s). \end{aligned} \quad (43)$$

In this way, sufficiency has been proved.

We have thus proved Theorem 2. ■

5. NUMERICAL EXAMPLE

In this section, numerical examples are presented to show the effectiveness of the proposed parameterizations of all disturbance observers for time-delay plants with any disturbance and of all linear functional disturbance observers for time-delay plants with any disturbance.

5.1 Numerical example of a disturbance observer

Consider the problem of parameterizing all disturbance observers for a stable time-delay plant $G(s)e^{-sL}$ written as

$$\begin{aligned} & G(s)e^{-sL} \\ = \; & \left[\begin{array}{cc} \frac{2}{(s+1)(s+2)} & \frac{s+12}{(s+1)(s+2)} \\ \frac{s+11}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \end{array}\right] e^{-0.5s} \\ = \; & \left[\begin{array}{cccc|cc} -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 1 \\ \hline 2 & 11 & -2 & -10 & 0 & 0 \\ 10 & 1 & -9 & -1 & 0 & 0 \end{array}\right] e^{-0.5s}. \end{aligned} \quad (44)$$

From Theorem 1, the parameterization of all disturbance observers for time-delay plants $G(s)e^{-sL}$ in (44) with any disturbance is given by

$$\tilde{d}(s) = y(s) - G(s)e^{-sL}u(s). \tag{45}$$

If the control input $u(t)$, the disturbance $d(t)$ and the initial state $x(0)$ are given by

$$\begin{aligned} u(t) &= \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \end{aligned} \tag{46}$$

$$\begin{aligned} d(t) &= \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} \\ &= \begin{bmatrix} \sin t \\ \sin 0.5t \end{bmatrix} \end{aligned} \tag{47}$$

and

$$x(0) = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}^T, \tag{48}$$

respectively. The response of the error

$$\begin{aligned} e(t) &= \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} \\ &= d(t) - \tilde{d}(t) \\ &= \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} - \begin{bmatrix} \tilde{d}_1(t) \\ \tilde{d}_2(t) \end{bmatrix} \end{aligned} \tag{49}$$

is shown in Fig. 1. Here, the solid line shows the

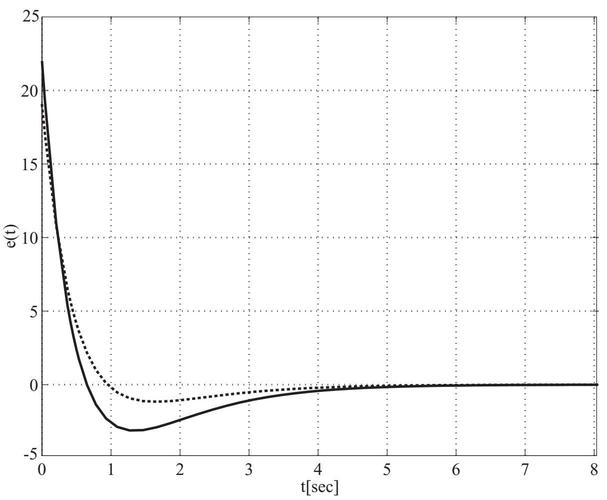

Fig.1: *The response of the error* $e(t) = d(t) - \tilde{d}(t)$

response of $e_1(t)$ and the dotted line shows that of $e_2(t)$. Figure 1 shows that the disturbance observer $\tilde{d}(s)$ in (4) for time-delay plants with any disturbance can estimate the disturbance $d(t)$ effectively.

In this way, it is shown that using the obtained parameterization of all disturbance observers for time-delay plants with any disturbance, we can easily design the disturbance observer for time-delay plants with any disturbance.

5.2 Numerical example for a linear functional disturbance observer

Consider the problem of parameterizing all linear functional disturbance observers for an unstable time-delay plant $G(s)e^{-sL}$ written as

$$\begin{aligned} G(s)e^{-sL} &= \begin{bmatrix} \dfrac{2}{(s-1)(s+2)} & \dfrac{s-3}{(s-1)(s+2)} \\ \dfrac{s-6}{(s-1)(s+2)} & \dfrac{-1}{(s-1)(s+2)} \end{bmatrix} e^{-0.5s} \\ &= \left[\begin{array}{ccc|cc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \\ \hline 0.6667 & -0.6667 & -0.6667 \\ -1.6667 & -0.3333 & 2.6667 \end{array}\right. \\ &\qquad \left.\begin{array}{c|cc} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \\ \hline 1.6667 & 0 & 0 \\ 0.3333 & 0 & 0 \end{array}\right] e^{-0.5s}. \end{aligned} \tag{50}$$

Using the method in [26], state space descriptions of $\tilde{N}(s)$ and $\tilde{D}(s)$ satisfying (20) and (21) are given by

$$\begin{aligned} \tilde{N}(s) &= \left[\begin{array}{ccc} -3.3834 & 0.7002 & 6.2251 \\ 1.5121 & -2.2833 & -0.6265 \\ 0.0716 & 0.0802 & -2.1474 \\ 0.0818 & 0.0917 & -0.1686 \\ \hline 0.6667 & -0.6667 & -0.6667 \\ -1.6667 & -0.3333 & 2.6667 \end{array}\right. \\ &\qquad \left.\begin{array}{c|cc} -2.6712 & 1 & 0 \\ 7.7654 & 0 & 1 \\ -0.1626 & 1 & 0 \\ -2.1859 & 0 & 1 \\ \hline 1.6667 & 0 & 0 \\ 0.3333 & 0 & 0 \end{array}\right] \end{aligned} \tag{51}$$

and

$$\begin{aligned} \tilde{D}(s) &= \left[\begin{array}{ccc} -3.3834 & 0.7002 & 6.2251 \\ 1.5121 & -2.2833 & -0.6265 \\ 0.0716 & 0.0802 & -2.1474 \\ 0.0818 & 0.0917 & -0.1686 \\ \hline -0.6667 & 0.6667 & 0.6667 \\ 1.6667 & 0.3333 & -2.6667 \end{array}\right. \\ &\qquad \left.\begin{array}{c|cc} -2.6712 & 1.9711 & -1.8416 \\ 7.7654 & -4.4821 & -0.8856 \\ -0.1626 & 0.0824 & 0.0759 \\ -2.1859 & 0.0942 & 0.0868 \\ \hline -1.6667 & 1 & 0 \\ -0.3333 & 0 & 1 \end{array}\right], \end{aligned} \tag{52}$$

respectively. From Theorem 2, the parameterization of all linear functional disturbance observers for time-delay plants $G(s)e^{-sL}$ in (50) with any disturbance is given by (4) with (17), (18), and (19).

$Q(s)$ in (17) and (18) is set as

$$\begin{aligned} Q(s) &= \begin{bmatrix} \dfrac{1}{s+1} & 0 \\ 0 & \dfrac{2}{s+2} \end{bmatrix} \\ &= \left[\begin{array}{cc|cc} -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{array}\right]. \end{aligned} \quad (53)$$

Substituting the above-mentioned parameters into (17) and (18), the linear functional disturbance observer $\tilde{d}(s)$ for time-delay plants $G(s)e^{-sL}$ in (50) with any disturbance is designed as (4), where

$$\begin{aligned} F_1(s) &= \begin{bmatrix} \dfrac{s^4+5.596s^3+7.094s^2-4.694s-8.996}{(s+1)^2(s+3)(s+4)} & \dfrac{0.5434s^3+1.912s^2+0.282s-2.737}{(s+1)^2(s+3)(s+4)} \\ \dfrac{1.54s^2+2.293s-6.502}{(s+1)(s+2)(s+3)} & \dfrac{s^3+4.404s^2+1.098s-6.502}{(s+1)(s+2)(s+3)} \end{bmatrix} \\ &= \left[\begin{array}{cccc|ccc} -2 & 1.2581 & -0.1213 & 1.3633 & 0.0351 & 0 & 1 \\ 0 & -3.2763 & -0.0173 & -2.5232 & 0.9428 & 2.1568 & -2.7121 \\ 0 & -0.4042 & -1.0399 & 2.0491 & 0.6999 & 2.2032 & 0.5430 \\ 0 & -0.0306 & -0.0041 & -2.3847 & -0.1734 & -0.5032 & -0.0644 \\ 0 & 1.2758 & -0.0315 & -4.8991 & -2.2991 & -6.2627 & -0.8508 \\ \hline 0 & -0.2595 & 0.9182 & 0.5766 & 0.7309 & 1 & 0 \\ 2 & 1.2581 & -0.1214 & 1.3633 & 0.0351 & 0 & 1 \end{array}\right], \end{aligned} \quad (54)$$

$$\begin{aligned} F_2(s) &= \begin{bmatrix} \dfrac{2.543s^4+17.47s^3+40.16s^2+32.34s+3.134}{(s+1)^2(s+2)^2(s+3)(s+4)} & \dfrac{s^5+5.596s^4+0.5504s^3-47.81s^2-98.78s-59.45}{(s+1)^1(s+2)^2(s+3)(s+4)} \\ \dfrac{s^3-0.5958s^2-22.84s-31.35}{(s+1)(s+2)^2(s+3)} & \dfrac{0.5401s^2-6.191s-18}{(s+1)(s+2)^2(s+3)} \end{bmatrix} \\ &= \left[\begin{array}{cccccc|cc} -3.8716 & 0 & 1.1162 & -2.9145 & -1.7114 & 0.6547 & 0.2718 & 0.3911 \\ 1.7872 & -2 & 0.0193 & 1.2574 & 0.4793 & -0.2241 & 0 & 0 \\ -3.215 & 0 & -0.3537 & -3.6046 & 2.4233 & 1.8537 & 1.6314 & -0.2637 \\ -0.5782 & 0 & 0.2560 & -1.8561 & 0.8893 & 0.4903 & -1.0594 & 0.2205 \\ 0.0310 & 0 & -0.1202 & 0.2811 & -2.1256 & -0.1401 & -0.3769 & -1.4087 \\ 1.5231 & 0 & -0.1369 & 2.51 & -7.1936 & -2.793 & 0.0200 & 1.3207 \\ \hline 0.8833 & 0 & -0.4785 & -0.1115 & -1.1645 & -0.8234 & 0 & 0 \\ -1.7872 & -2 & -0.0193 & -1.2574 & -0.4793 & 0.2241 & 0 & 0 \end{array}\right] \end{aligned} \quad (55)$$

and

$$\begin{aligned} F(s) &= \begin{bmatrix} \dfrac{3.404s^3+19.91s^2+35.69s+21}{(s+1)^2(s+3)(s+4)} & \dfrac{-0.5434s^3-1.912s^2-0.282s+2.737}{(s+1)^2(s+3)(s+4)} \\ \dfrac{-1.54s^2-2.293s+3.833}{(s+1)(s+2)(s+3)} & \dfrac{1.596s^2+9.902s+12.5}{(s+1)(s+2)(s+3)} \end{bmatrix} \\ &= \left[\begin{array}{cccc|ccc} -2 & 1.2581 & -0.1213 & 1.3633 & 0.0351 & 0 & 1 \\ 0 & -3.2763 & -0.0173 & -2.5232 & 0.9428 & 2.1568 & -2.7121 \\ 0 & -0.4042 & -1.0399 & 2.0491 & 0.6999 & 2.2032 & 0.5430 \\ 0 & -0.0306 & -0.0041 & -2.3847 & -0.1734 & -0.5032 & -0.0644 \\ 0 & 1.2758 & -0.0315 & -4.8991 & -2.2991 & -6.2627 & -0.8508 \\ \hline 0 & 0.2595 & -0.9182 & -0.5766 & -0.7309 & 0 & 0 \\ -2 & -1.2581 & 0.1213 & -1.3633 & -0.0351 & 0 & 0 \end{array}\right]. \end{aligned} \quad (56)$$

Note that the designed disturbance observer $\tilde{d}(s)$ for time-delay plants with any disturbance is the system for estimating $d(t) - \mathcal{L}^{-1}(F(s)d(s))$.

If the control input $u(t)$, the disturbance $d(t)$ and the initial state $x(0)$ are given by

$$\begin{aligned} u(t) &= \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \end{aligned} \quad (57)$$

$$\begin{aligned} d(t) &= \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} \\ &= \begin{bmatrix} \sin t \\ \sin 0.5t \end{bmatrix} \end{aligned} \quad (58)$$

and

$$x(0) = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}^T, \quad (59)$$

respectively. The response of the error

$$\begin{aligned} e(t) &= \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} \\ &= d(t) - \tilde{d}(t) - \mathcal{L}^{-1}(F(s)d(s)) \end{aligned} \quad (60)$$

is shown in Fig. 2. Here, the solid line shows the response of $e_1(t)$ and the dotted line shows that of $e_2(t)$. Figure 2 shows that the designed linear functional disturbance observer $\tilde{d}(s)$ in (4) for time-delay plants $G(s)e^{-sL}$ in (50) with any disturbance can estimate $d(t) - \mathcal{L}^{-1}(F(s)d(s))$ effectively.

In this way, it is shown that using the obtained parameterization of all linear functional disturbance observers for time-delay plants with any disturbance, we can easily design the linear functional disturbance observer for time-delay plants with any disturbance.

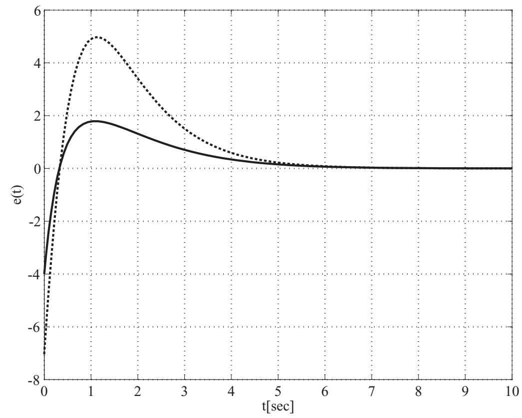

Fig.2: *The response of the error* $e(t) = d(t) - \tilde{d}(t) - \mathcal{L}^{-1}(F(s)d(s))$

6. APPLICATION IN A HEAT-FLOW EXPERIMENT

In this section, we apply the proposed method to estimating unknown disturbances in a heat-flow experiment.

The heat-flow experiment is illustrated in Fig. 3. The heat-flow experiment consists of a duct equipped

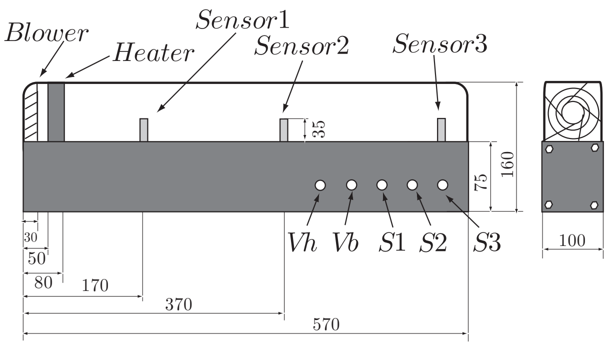

Fig.3: *Illustration of the heat-flow experiment*

with a heater and blower at one end and three temperature sensors located along the duct as shown in Fig. 3. V_h and V_b denote the voltage supplied to the heater and that supplied to the blower, respectively. S_1, S_2 and S_3 are terminals for the measurement of temperature at Sensor 1, Sensor 2 and Sensor 3. T_i[deg] denotes the measurement of temperature at Sensor $i(i = 1, 2, 3)$. $V_b = 5$[V] is a constant, V_h is considered a control input $u(s)$ and the available voltage of V_h is $0 \leq V_h \leq 5$[V]. When T_3 is considered as an output $y(s)$, the transfer function from the control input $u(s)$ to the output $y(s)$ is given by

$$\begin{aligned} y(s) &= G(s)e^{-sL}u(s) \\ &= \frac{5.16}{20.15s+1}e^{-0.68s}u(s). \end{aligned} \quad (61)$$

Setting $u(t) = V_h = 5$[V], the response of the output $y(t)$ is shown in Fig. 4. Figure 4 shows that

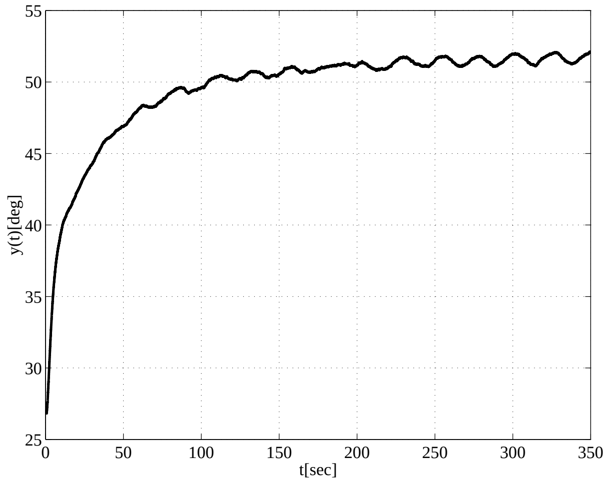

Fig.4: *Response of the output* $y(t)$ *when the control input* $u(t) = V_h = 5$[V]

unknown disturbances exist.

Using the method described in Section 3., the response of the disturbance observer is shown in Fig. 5. Figure 5 shows that the disturbance observer can

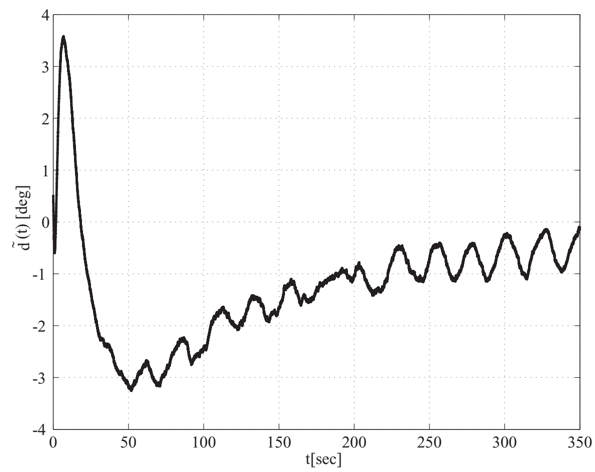

Fig.5: *Response of the disturbance observer* $\tilde{d}(t)$

estimate the unknown disturbance.

The proposed disturbance observer can be easily applied to a real plant in the way described here.

7. CONCLUSIONS

We proposed parameterizations of all disturbance observers for time-delay plants with any disturbance and of all linear functional disturbance observers for time-delay plants with any disturbance. The results of this paper are summarized as follows.

1. We clarified that for stable time-delay plants, an observer that estimates disturbance exactly for time-delay plants with any disturbance can be designed.
2. The parameterization of all disturbance ob-

servers fortime-delay plants with any disturbance was proposed.

3. The linear functional disturbance observer for time-delay plants with any disturbance was defined.
4. The parameterization of all linear functional disturbance observers for time-delay plants with any disturbance was proposed.
5. Numerical examples were presented to illustrate the effectiveness of the proposed parameterizations of all disturbance observers for time-delay plants with any disturbance and of all linear functional disturbance observers for time-delay plants with any disturbance.
6. An application of the proposed method for estimating unknown disturbances in a heat-flow experiment was presented. It was shown that the proposed disturbance the proposed disturbance observer could be easily applied to a real plant.

A design method of a control system that uses the obtained parameterizations of all disturbance observers for time-delay plants with any disturbance and of all linear functional disturbance observers for time-delay plants with any disturbance and a design method for robust disturbance observers for time-delay plants will be described in another article.

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