The Parameterization of All Stabilizing Multi-Period Repetitive Controllers for Multiple-Input/Multiple-Output Plants With the Specified Input-Output Frequency Characteristic

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ABSTRACT

In this paper, we examine the parameterization of all stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with the specified input-output frequency characteristic. The parameterization of all stabilizing multi-period repetitive controllers, those are used to improve the disturbance attenuation characteristics of the repetitive controller, for non-minimum phase systems was solved by Yamada et al. However, when we design a stabilizing multi-period repetitive controller using the parameterization by Yamada et al., the input-output frequency characteristic of the control system cannot be settled so easily. From the practical point of view, the input-output frequency characteristic of the control systems is required to be easily settled. This problem is solved by obtaining the parameterization of all stabilizing multi-period repetitive controllers with the specified input-output frequency characteristic. However, no paper has proposed the parameterization of all stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with the specified input-output frequency characteristic. In this paper, we propose the parameterization of all stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with the specified input-output frequency characteristic.

Keywords: Periodic Signal, Multi-Period Repetitive Controller, Parameterization, Low-Pass Filter, Multiple-Input/Multiple-Output Plants

1. INTRODUCTION

In this paper, we investigate the parameterization of all stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with the specified input-output frequency characteristic. The parameterization problem is to find all stabilizing controllers [1–5]. First, the parameterization of all stabilizing modified repetitive controllers that follows the periodic reference input with small steady state error even if there exists a periodic disturbance or the uncertainty of the plant was studied by [6]. In [6], since the stability sufficient condition of repetitive control system is decided as H_∞ norm problem, the parameterization for repetitive control system is given by resolving into the interpolation problem of Nevanlinna-Pick. Katoh and Funahashi gave the parameterization of all stabilizing repetitive controllers for minimum phase systems by solving exactly Bezout equation [7]. However, Katoh and Funahashi [7] assumed the plant is stable. This implies that they gave the parameterization of all causal repetitive controllers for a stable and minimum phase plant. That is, they do not give the explicit parameterization for minimum phase systems [7]. In addition, in [7], it is assumed that the relative degree of low-pass filter in the repetitive compensator is equal to that of the plant. Extending the results in [7], Yamada and Okuyama gave the parameterization of all stabilizing repetitive controllers for minimum phase systems [8]. Yamada et al. expanded the result in [8] and gave the parameterization of all stabilizing repetitive controllers for a certain class of non-minimum phase systems [9]. They obtained the parameterization of all stabilizing repetitive controllers using fusion of the parallel compensation technique and the solution of Bezout equation. However, they gave the parameterization of all stabilizing repetitive controllers for limited class of non-minimum phase systems. Yamada et al. gave the complete parameterization of all stabilizing modified repetitive controllers for non-minimum phase single-input/single-output plants [10]. In addition, the parameterization of all stabilizing repetitive controllers for non-minimum phase multivariable systems was considered in [11]. In this way, the parameterization of all stabilizing repetitive controllers has been established.

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However, the repetitive control system has a bad effect on the disturbance attenuation characteristics [12], in that at certain frequencies, the sensitivity to disturbances of a control system with a conventional repetitive controller becomes twice as bad as that of a control system without a repetitive controller. Gotou et al. overcame this problem by proposing a multi-period repetitive control system [12]. However, the phase angle of the low-pass filter in a multi-period repetitive controller has a bad effect on the disturbance attenuation characteristics [13, 14]. Yamada et al. overcame this problem and proposed a design method for multi-period repetitive controllers to attenuate disturbances effectively [15, 16] using the time advance compensation described in [13, 14, 17]. Using this multi-period repetitive control structure, Steinbuch proposed a design method for repetitive control systems with uncertain period time [18].

The parameterization of all stabilizing multi-period repetitive controllers for non-minimum phase systems was solved in [19]. However, when we design a stabilizing multi-period repetitive controller using the parameterization in [19], the input-output frequency characteristic of the control system cannot be settled so easily. From the practical point of view, the input-output frequency characteristic of the control systems is required to be easily settled. Yamada et al. [20] solved this problem by obtaining the parameterization of all stabilizing multi-period repetitive controllers with the specified input-output frequency characteristic. However, the method by Yamada et al. [20] cannot be applied to multiple-input/multiple-output plants. Because, the method by Yamada et al. [20] uses the characteristic of single-input/single-output system.

In this paper, we expand the result in [20] and propose the parameterization of all stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with the specified input-output frequency characteristic. The basic idea to obtain the parameterization of all stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with the specified input-output frequency characteristic is very simple. If the multi-period repetitive controller stabilizes the plant, then the multi-period repetitive controller must be included in the class of all stabilizing controllers for the plant. The parameterization of all stabilizing controllers for the plant is obtained using the method in [4, 5]. The parameterization of all stabilizing controllers includes the free parameter, which is an element of the set of stable causal function. That is, the parameterization of all stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with the specified input-output frequency characteristic can be designed using the free parameter in the parameterization. Using this idea, we obtain the parameterization of all stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with the specified input-output frequency characteristic.

Notation

R	the set of real numbers.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real coefficient rational functions.
H_∞	the set of stable causal functions.
$\bar{\sigma}(\{\cdot\})$	largest singular value of $\{\cdot\}$.
$\mathcal{U}$	the unimodular procession in H_∞. That is, $U(s) \in \mathcal{U}$ means that $U(s) \in H_\infty$ and $U^{-1}(s) \in H_\infty$.
diag $\{a_1, \dots, a_n\}$	an $n \times n$ diagonal matrix with a_i its i-th diagonal element.
A^T	transposed matrix of A.

2. PROBLEM FORMULATION

Consider the unity feedback control system in

$$\begin{cases} y = G(s)u \\ u = C(s)(r - y), \end{cases} \tag{1}$$

where $G(s) \in R^{p\times p}(s)$ is the multiple-input/multiple-output plant satisfying rank $G(s) = p$. $G(s)$ is assumed to be coprime. $C(s)$ is the multi-period repetitive controller defined later, $u \in R^p$ is the control input, $y \in R^p$ is the output and $r \in R^p$ is the periodic reference input with period $T > 0$ satisfying

$$r(t + T) = r(t) \ \ (\forall t \geq 0). \tag{2}$$

According to [12], the multi-period repetitive controller $C(s)$ in (1) is written by the form in

$$C(s) = C_0(s) + \left(\sum_{i=1}^{N} C_i(s) q_i(s) e^{-sT_i}\right) \left(I - \sum_{i=1}^{N} q_i(s) e^{-sT_i}\right)^{-1}, \tag{3}$$

where N is an arbitrary positive integer, $C_0(s) \in R^{p\times p}(s)$, $C_i(s) \in R^{p\times p}(s)(i = 1, \dots, N)$, rank $C_i(s) = p(i = 1, \dots, N)$, $q_i(s) \in R^{p\times p}(s)(i = 1, \dots, N)$ are low-pass filters satisfying $\sum_{i=1}^{N} q_i(0) = I$ and $T_i > 0(i = 1, \dots, N)$ and rank $q_i(s) = p(i = 1, \dots, N)$. From [12], if low-pass filters $q_i(s) \in RH_\infty(i = 1, \dots, N)$ satisfy

$$I - \sum_{i=1}^{N} q_i(j\omega_k) e^{-j\omega_k T_i} = 0, \tag{4}$$

where $\omega_k(k = 0, \dots, n)$ are frequency components of the periodic reference input written by

$$\omega_k = \frac{2\pi k}{T}(k = 0, \dots, n) \tag{5}$$

and ω_n is the maximum frequency component of the periodic reference input r, then the output y in (1) follows the periodic reference input r with small steady state error. In order for low-pass filters $q_i(s) \in RH_\infty (i = 1, \ldots, N)$ to satisfy (4) in wide frequency range, low-pass filters $q_i(s) \in RH_\infty (i = 1, \ldots, N)$ must be stable and of minimum phase. Using result in [19], it is difficult to settle low-pass filters $q_i(s) \in RH_\infty (i = 1, \ldots, N)$ to be stable and of minimum phase. If we obtain the parameterization of all stabilizing multi-period repetitive controllers such that low-pass filters $q_i(s) \in RH_\infty (i = 1, \ldots, N)$ in (3) are settled beforehand, we can easily design the multi-period repetitive controller in (3) satisfying (4). Since low-pass filters $q_i(s) \in RH_\infty (i = 1, \ldots, N)$ work to specify the input-output frequency characteristic of the control system in (1), we call the parameterization of all stabilizing multi-period repetitive controllers such that low-pass filters $q_i(s) \in RH_\infty (i = 1, \ldots, N)$ in (3) are settled beforehand the parameterization with the specified input-output frequency characteristic.

The problem considered in this paper is that when low-pass filters $q_i(s) \in RH_\infty (i = 1, \ldots, N)$ in (3) are settled beforehand, we obtain the parameterization of all stabilizing multi-period repetitive controllers with the specified input-output frequency characteristic written in (3).

3. THE PARAMETERIZATION OF ALL STABILIZING MULTI-PERIOD REPETITIVE CONTROLLERS WITH THE SPECIFIED INPUT-OUTPUT FREQUENCY CHARACTERISTIC

In this section, we give the parameterization of all stabilizing multi-period repetitive controllers with the specified input-output frequency characteristic.

In order to obtain the parameterization of all stabilizing multi-period repetitive controllers with the specified input-output frequency characteristic, $q_i(s) \in RH_\infty (i = 1, \ldots, N)$ are assumed to be settled beforehand. The parameterization of all stabilizing multi-period repetitive controllers with the specified input-output frequency characteristic written by the form in (3) is given by following theorem.

Theorem 1: The parameterization of all stabilizing multi-period repetitive controllers written by the form in (3) if and only if $C(s)$ is written by

$$\begin{aligned} C(s) &= \left(\tilde{X}(s) + D(s)Q(s)\right)\left(\tilde{Y}(s) - N(s)Q(s)\right)^{-1} \\ &= \left(Y(s) - Q(s)\tilde{N}(s)\right)^{-1}\left(X(s) + Q(s)\tilde{D}(s)\right), \end{aligned} \tag{6}$$

where $N(s) \in RH_\infty$, $D(s) \in RH_\infty$, $\tilde{N}(s) \in RH_\infty$ and $\tilde{D}(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = N(s)D^{-1}(s) = \tilde{D}^{-1}(s)\tilde{N}(s). \tag{7}$$

$X(s)$, $Y(s)$, $\tilde{X}(s)$ and $\tilde{Y}(s)$ are RH_∞ functions satisfying

$$\begin{aligned} &\begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{D}(s) \end{bmatrix}\begin{bmatrix} D(s) & -\tilde{X}(s) \\ N(s) & \tilde{Y}(s) \end{bmatrix} \\ &= \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \\ &= \begin{bmatrix} D(s) & -\tilde{X}(s) \\ N(s) & \tilde{Y}(s) \end{bmatrix}\begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{D}(s) \end{bmatrix}. \end{aligned} \tag{8}$$

$Q(s) \in H_\infty$ is written by

$$\begin{aligned} Q(s) &= \left\{Q_n(s) + \sum_{i=1}^{N}\left(Y(s)\bar{Q}_i(s) - Q_n(s)\right) q_i(s)e^{-sT_i}\right\} \\ &\quad \left\{Q_d(s) + \sum_{i=1}^{N}\left(\tilde{N}(s)\bar{Q}_i(s) - Q_d(s)\right) q_i(s)e^{-sT_i}\right\}^{-1}, \end{aligned} \tag{9}$$

where $Q_n(s) \in RH_\infty$, $Q_d(s) \in RH_\infty$, rank $Q_d(s) = p$, $\bar{Q}_i(s) \in RH_\infty (i = 1, \ldots, N)$ and rank $\bar{Q}_i(s) = p (i = 1, \ldots, N)$.

Proof of this theorem requires following lemma.

Lemma 1: Unity feedback control system in

$$\begin{cases} y = G(s)u \\ u = C(s)(r - y) \end{cases} \tag{10}$$

is stable if and only if $C(s)$ is written by

$$\begin{aligned} C(s) &= \left(\tilde{X}(s) + D(s)Q(s)\right)\left(\tilde{Y}(s) - N(s)Q(s)\right)^{-1} \\ &= \left(Y(s) - Q(s)\tilde{N}(s)\right)^{-1}\left(X(s) + Q(s)\tilde{D}(s)\right), \end{aligned} \tag{11}$$

where $N(s) \in RH_\infty$, $D(s) \in RH_\infty$, $\tilde{N}(s) \in RH_\infty$ and $\tilde{D}(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying (7). $X(s)$, $Y(s)$, $\tilde{X}(s)$ and $\tilde{Y}(s)$ are RH_∞ functions satisfying (8) and $Q(s) \in H_\infty$ is any function [5].

Using above Lemma 1, we shall show the proof of Theorem 1.

Proof: First, the necessity is shown. That is, we show that if the controller written by (3) stabilizes the control system in (1), then $C(s)$ and $Q(s)$ are written by (6) and (9), respectively. From Lemma 1, the parameterization of all stabilizing controllers $C(s)$ for $G(s)$ is written by (6), where $Q(s) \in H_\infty$ is any function. In order to prove necessity, we will show that if $C(s)$ written by (3) stabilizes the control system in (1), then the free parameter $Q(s) \in H_\infty$ is written by (9). Substituting $C(s)$ in (3) into (6), we have (9), where

$$Q_n(s) = (Y(s)C_{0n}(s) - X(s)C_{0d}(s))\, C_d(s), \quad (12)$$

$$Q_d(s) = \left(\tilde{N}(s)C_{0n}(s) + \tilde{D}(s)C_{0d}(s)\right) C_d(s) \quad (13)$$

and

$$\bar{Q}_i(s) = C_{in}(s)(i = 1, \dots, N). \quad (14)$$

Here, $C_{0n}(s) \in RH_\infty$ and $C_{0d}(s) \in RH_\infty$ are coprime factors of $C_0(s)$ on RH_∞ satisfying

$$C_0(s) = C_{0n}(s)C_{0d}^{-1}(s). \quad (15)$$

$C_{in}(s) \in RH_\infty(i = 1, \dots, N)$ and $C_d(s) \in RH_\infty$ are coprime factors of $C_i(s)C_{0d}(s)$ on RH_∞ written by

$$C_i(s)C_{0d}(s) = C_{in}(s)C_d^{-1}(s)(i = 1, \dots, N)(16)$$

Since $\tilde{N}(s) \in RH_\infty$, $\tilde{D}(s) \in RH_\infty$, $X(s) \in RH_\infty$, $Y(s) \in RH_\infty$, $C_{0n}(s) \in RH_\infty$, $C_{0d}(s) \in RH_\infty$, $C_{in}(s) \in RH_\infty(i = 1, \dots, N)$ and $C_d(s) \in RH_\infty$, we find that $Q_n(s) \in RH_\infty$, $Q_d(s) \in RH_\infty$ and $\bar{Q}_i(s) \in RH_\infty(i = 1, \dots, N)$ in (9). From the assumption of $\operatorname{rank} C_i(s) = p \quad (i = 1, \dots, N)$, $\operatorname{rank} \bar{Q}_i(s) = p(i = 1, \dots, N)$ holds true. Therefore, it was proved the necessity.

Next the sufficiency is shown. That is, if $Q(s)$ in (6) is written by (9), then the controller $C(s)$ is written by (3). Substituting (9) into (6), we have

$$C(s) = C_0(s) + \left(\sum_{i=1}^{N} C_i(s)q_i(s)e^{-sT_i}\right) \left(I - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right)^{-1}, \quad (17)$$

where $C_0(s)$ and $C_i(s)(i = 1, \cdots, N)$ are denoted by

$$C_0(s) = \left(\tilde{X}(s)Q_d(s) + D(s)Q_n(s)\right) \left(\tilde{Y}(s)Q_d(s) - N(s)Q_n(s)\right)^{-1} \quad (18)$$

and

$$C_i(s) = \bar{Q}_i(s)\left(\tilde{Y}(s)Q_d(s) - N(s)Q_n(s)\right)^{-1} (i = 1, \dots, N), \quad (19)$$

respectively. We find that if $C(s)$ and $Q(s)$ are settled by (6) and (9), then the controller $C(s)$ is written by the form in (3). From (19) and the assumption of $\operatorname{rank} \bar{Q}_i(s) = p(i = 1, \dots, N)$, $\operatorname{rank} C_i(s) = p(i = 1, \dots, N)$ holds true. Therefore, it was proved the sufficiency.

We have thus proved Theorem 1. ∎

4. CONTROL CHARACTERISTICS

In this section, we explain control characteristics of the control system in (1) using the parameterization of all stabilizing multi-period repetitive controllers $C(s)$ for multiple-input/multiple-output plants with the specified input-output frequency characteristic. In addition, roles of $Q_n(s)$, $Q_d(s)$ and $\bar{Q}_i(s)(i = 1, \dots, N)$ in (9) are clarified.

From Theorem 1, $Q(s)$ in (9) must be included in H_∞. Since $Q_n(s) \in RH_\infty$ and $\bar{Q}_i(s) \in RH_\infty(i = 1, \dots, N)$ in (9), if

$$\left\{Q_d(s) + \sum_{i=1}^{N}(\tilde{N}(s)\bar{Q}_i(s) - Q_d(s))q_i(s)e^{-sT_i}\right\}^{-1} \in H_\infty, (20)$$

then $Q(s) \in H_\infty$. That is, the role of $Q_d(s)$ and $\bar{Q}_i(s)(i = 1, \dots, N)$ is to assure the stability of the control system in (1).

Next, the input-output characteristic of the control system in (1) is shown. The transfer function $S(s)$ from the periodic reference input $r(s)$ to the error $e(s) = r(s) - y(s)$ of the control system in (1) is written by

$$S(s) = \left(\tilde{Y}(s)Q_d(s) - N(s)Q_n(s)\right)\left(I - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right) \left\{Q_d(s) + \sum_{i=1}^{N}\left(\tilde{N}(s)\bar{Q}_i(s) - Q_d(s)\right) q_i(s)e^{-sT_i}\right\}^{-1} \tilde{D}(s). \quad (21)$$

Since $q_i(s)(i = 1, \dots, N)$ are settled beforehand to satisfy (4), the output $y(s)$ follows the reference input $r(s)$ with small steady state error. If the gain of $\bar{Q}_i(s)(i = 1, \dots, N)$ increases, then steady state error decrease. That is, the role of $q_i(s)(i = 1, \dots, N)$ is to specify the input-output characteristic of the control system in (1). In addition, $\bar{Q}_i(s)(i = 1, \dots, N)$ play an auxiliary role to specify the input-output characteristic of the control system in (1).

Next, the disturbance attenuation characteristic of the control system in (1) is shown. The transfer function from the disturbance $d(s)$ to the output $y(s)$ of the control system in (1) is written by (21). From (4) and (21), for the disturbance $d(s)$ with same frequency components $\omega_k(k = 0, \dots, n)$ of the periodic reference input $r(s)$, $S(j\omega_k)$ satisfy $S(j\omega_k) \simeq 0(k = 0, \dots, n)$ independent from $Q_d(s)$ and $Q_n(s)$. This implies that the disturbance with same frequency components $\omega_k(k = 0, \dots, n)$ of the periodic reference input $r(s)$ is attenuated effectively. That is, the role of $q_i(s)(i = 1, \dots, N)$ is to specify the disturbance attenuation characteristic for the disturbance with same frequency components $\omega_k(k = 0, \dots, n)$ of the periodic reference input $r(s)$. For the disturbance $d(s)$ with frequency component $\omega \neq \omega_k(k=0,\ldots,n)$, even if $q_i(s)(i=1,\ldots,N)$ satisfy (4), $S(j\omega)$ satisfies $S(j\omega)\neq 0$, since $e^{-j\omega T_i}\neq 0(i=1,\ldots,N)$. If $Q_n(s)$ is settled satisfying $\bar\sigma(\tilde Y(j\omega)Q_d(j\omega)-N(j\omega)Q_n(j\omega))\simeq 0$, then the disturbance with frequency component $\omega\neq\omega_k(k=0,\ldots,n)$ is attenuated. That is, the role of $Q_n(s)$ is to specify the disturbance attenuation characteristic for the disturbance with frequency component $\omega\neq\omega_k(k=0,\ldots,n)$.

Next, a design method for $Q_n(s)$ to reduce influence of the disturbance $d(s)$ with frequency component $\omega\neq\omega_k(k=0,\ldots,n)$ is shown. When $G(s)$ is of minimum phase, that is, $N(s)$ is of minimum phase, $Q_n(s)$ is chosen by

$$Q_n(s)=N^{-1}(s)\bar q(s)\tilde Y(s)Q_d(s), \tag{22}$$

where

$$\begin{aligned}&\bar q(s)\\ &= \mathrm{diag}\left\{\frac{1}{(1+s\tau_1)^{\alpha_1}} \quad \cdots \quad \frac{1}{(1+s\tau_p)^{\alpha_p}}\right\}\\ &\quad \in RH_\infty.\end{aligned} \tag{23}$$

$\tau_i>0(i=1,\ldots,p)$ and $\alpha_i(i=1,\ldots,p)$ is a positive integer to make $Q_n(s)$ proper. When $Q_n(s)$ is chosen as (22), $\tilde Y(j\omega)Q_d(j\omega)-N(j\omega)Q_n(j\omega)$ satisfy

$$\begin{aligned}&\tilde Y(j\omega)Q_d(j\omega)-N(j\omega)Q_n(j\omega)\\ &\quad = (I-\bar q(j\omega))\tilde Y(j\omega)Q_d(j\omega).\end{aligned} \tag{24}$$

Therefore, when $\bar q(s)$ in (23) is set satisfying

$$\bar\sigma\{I-\bar q(j\omega)\}\simeq 0, \tag{25}$$

the disturbance $d(s)$ with frequency component $\omega\neq\omega_k(k=0,\ldots,n)$ is attenuated effectively, since

$$\begin{aligned}&\bar\sigma\{S(j\omega)\}\\ &= \bar\sigma\left\{\left(\tilde Y(j\omega)Q_d(j\omega)-N(j\omega)Q_n(j\omega)\right)\right.\\ &\qquad\left.\left(I-\sum_{i=1}^N q_i(j\omega)e^{-j\omega T_i}\right)S_d^{-1}(j\omega)\tilde D(j\omega)\right\}\\ &= \bar\sigma\left\{(I-\bar q(j\omega))\tilde Y(j\omega)Q_d(j\omega)\right.\\ &\qquad\left.\left(I-\sum_{i=1}^N q_i(j\omega)e^{-j\omega T_i}\right)S_d^{-1}(j\omega)\tilde D(j\omega)\right\}\\ &\simeq 0,\end{aligned} \tag{26}$$

where

$$\begin{aligned}&S_d(s)\\ &= Q_d(s)+\sum_{i=1}^N\left(\tilde N(s)\bar Q_i(s)-Q_d(s)\right)q_i(s)e^{-sT_i}\end{aligned} \tag{27}$$

On the other hand, when $G(s)$ is of non-minimum phase, that is, $N(s)$ is of non-minimum phase, $Q(s)$ is chosen by

$$Q_n(s)=N_o^{-1}(s)\bar q(s)\tilde Y(s)Q_d(s). \tag{28}$$

Here, $N_o(s)\in RH_\infty$ is an outer function of $N(s)$ satisfying

$$N(s)=N_i(s)N_o(s), \tag{29}$$

where $N_i(s)\in RH_\infty$ is an inner function of $N(s)$ satisfying $N_i(0)=I$. $\bar q(s)$ is set as (23). When $Q_n(s)$ is chosen as (28), $\tilde Y(j\omega)Q_d(j\omega)-N(j\omega)Q_n(j\omega)$ satisfy

$$\begin{aligned}&\tilde Y(j\omega)Q_d(j\omega)-N(j\omega)Q_n(j\omega)\\ &\quad = (I-N_i(j\omega)\bar q(j\omega))\tilde Y(j\omega)Q_d(j\omega).\end{aligned} \tag{30}$$

Therefore, when $\bar q(s)$ in (23) is set satisfying

$$\bar\sigma\{I-N_i(j\omega)\bar q(j\omega)\}\simeq 0, \tag{31}$$

the disturbance $d(s)$ with frequency component $\omega\neq\omega_k(k=0,\ldots,n)$ is attenuated effectively, since

$$\begin{aligned}&\bar\sigma\{S(j\omega)\}\\ &= \bar\sigma\left\{\left(\tilde Y(j\omega)Q_d(j\omega)-N(j\omega)Q_n(j\omega)\right)\right.\\ &\qquad\left.\left(I-\sum_{i=1}^N q_i(j\omega)e^{-j\omega T_i}\right)S_d^{-1}(j\omega)\tilde D(j\omega)\right\}\\ &= \bar\sigma\left\{(I-N_i(j\omega)\bar q(j\omega))\tilde Y(j\omega)Q_d(j\omega)\right.\\ &\qquad\left.\left(I-\sum_{i=1}^N q_i(j\omega)e^{-j\omega T_i}\right)S_d^{-1}(j\omega)\tilde D(j\omega)\right\}\\ &\simeq 0.\end{aligned} \tag{32}$$

In summary, the role of $Q_d(s)$ and $\bar Q_i(s)(i=1,\ldots,N)$ is to assure the stability of the control system in (1) by satisfying $Q(s)\in H_\infty$. The role of $q_i(s)(i=1,\ldots,N)$ is to specify the input-output characteristic of the control system in (1) and to specify the disturbance attenuation characteristic for disturbance $d(s)$ with same frequency components $\omega_k(k=0,\ldots,n)$ of the periodic reference input $r(s)$. The role of $\bar Q_i(s)(i=1,\ldots,N)$ plays an auxiliary role to specify the input-output characteristic of the control system in (1). The role of $Q_n(s)$ is to specify the disturbance attenuation characteristic for disturbance $d(s)$ with frequency components $\omega\neq\omega_k(k=0,\ldots,n)$.

5. NUMERICAL EXAMPLE

In this section, a numerical example is shown to illustrate the effectiveness of the proposed parameterization.

Consider the problem to obtain the parameterization of all stabilizing multi-period repetitive controllers with the specified input-output frequency characteristic for the plant $G(s)$ written by

$$G(s) = \begin{bmatrix} \dfrac{s+3}{(s-2)(s+9)} & \dfrac{2}{(s-2)(s+9)} \\ \dfrac{s+3}{(s-2)(s+9)} & \dfrac{s+4}{(s-2)(s+9)} \end{bmatrix}. \quad (33)$$

Here, the period T of the reference input r is $T = 5$[sec], N in (3) is $N = 5$. $T_i(i = 1, \ldots, 5)$ and $q_i(s)(i = 1, \ldots, 5)$ in (3) are settled by

$$T_i = 5 \cdot i(i = 1, \ldots, 5) \quad (34)$$

and

$$q_i(s) = \frac{1}{5} \begin{bmatrix} \dfrac{100}{s+100} & 0 \\ 0 & \dfrac{100}{s+100} \end{bmatrix} (i = 1, \ldots, 5), \quad (35)$$

respectively. A pair of coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ of $G(s)$ in (33) satisfying (7) is given by

$$D(s) = \begin{bmatrix} \dfrac{s^2+7s-18}{s^2+209+10920} & 0 \\ 0 & \dfrac{s^2+7s-18}{s^2+203s+10920} \end{bmatrix} \quad (36)$$

and

$$N(s) = \begin{bmatrix} \dfrac{s+3}{s^2+209s+10300} & \dfrac{0.6s+1.9}{s^2+203s+10300} \\ \dfrac{s+3}{s^2+209s+10920} & \dfrac{s+4}{s^2+203s+10300} \end{bmatrix}. \quad (37)$$

$\tilde{X}(s) \in RH_\infty$ and $\tilde{Y}(s) \in RH_\infty$ are settled to satisfy (8). From Theorem 1, the parameterization of all stabilizing multi-period repetitive controllers with the specified input-output frequency characteristic is written by (6) and (9).

In order to satisfy $Q(s) \in H_\infty$, $Q_n(s)$, $Q_d(s)$, $\bar{Q}_i(s)(i = 1, \ldots, 5)$ in (9) are settled by

$$Q_n(s) = N^{-1}(s)\bar{q}(s)\tilde{Y}(s)Q_d(s), \quad (38)$$

$$Q_d(s) = \tilde{N}(s)\bar{Q}_1(s) + I, \quad (39)$$

$$\bar{Q}_i(s) = \begin{bmatrix} \dfrac{s+51}{s+5} & \dfrac{107}{s+5} \\ \dfrac{-1202}{s+5} & \dfrac{s+1210}{s+5} \end{bmatrix} (i = 1, \ldots, 5), \quad (40)$$

where

$$\bar{q}(s) = \begin{bmatrix} \dfrac{100}{s+100} & 0 \\ 0 & \dfrac{100}{s+100} \end{bmatrix}. \quad (41)$$

From the Nyquist theorem, $Q_d(s) \in RH_\infty$ and $\bar{Q}_i(s) \in RH_\infty$, if the Nyquist plot of $Q_d(s) + \sum_{i=1}^{N}(\tilde{N}(s)\bar{Q}_i(s) - Q_d(s))q_i(s)e^{-sT_i}$ does not encircle the origin, then $Q(s) \in H_\infty$. Using $Q_d(s)$ in (39) and $\bar{Q}_i(s)(i = 1, \ldots, 5)$ in (40), the Nyquist plot of $Q_d(s) + \sum_{i=1}^{N}(\tilde{N}(s)\bar{Q}_i(s) - Q_d(s))q_i(s)e^{-sT_i}$ is shown in Fig. 1. A specially magnified detail drawing showing the origin $(0, 0)$ of Fig. 1 is shown in Fig. 2.

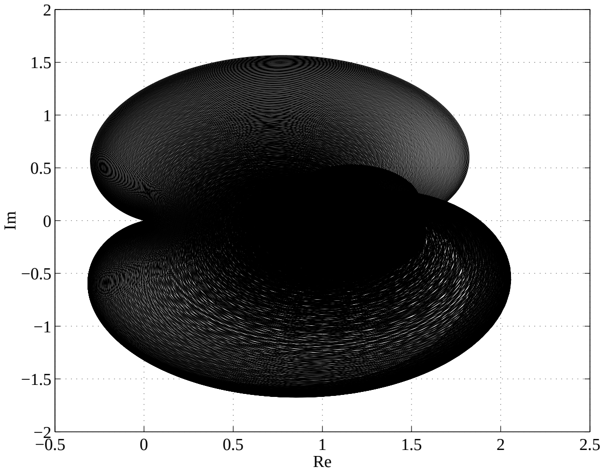

Fig.1: *The Nyquist plot of* $Q_d(s) + \sum_{i=1}^{N}\left(\tilde{N}(s)\bar{Q}_i(s) - Q_d(s)\right)q_i(s)e^{-sT_i}$

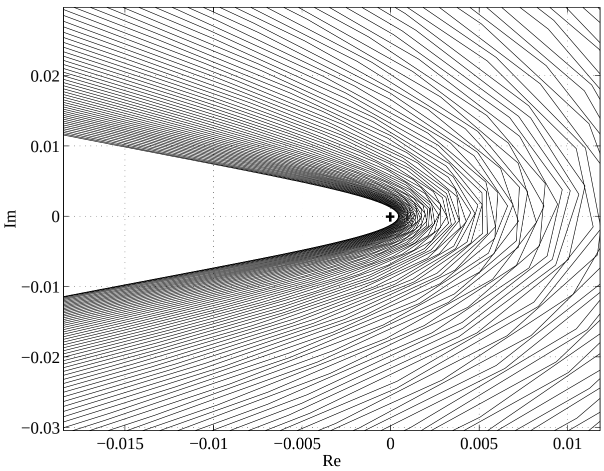

Fig.2: *Specially magnified detail drawing showing the origin* $(0, 0)$ *of the Nyquist plot of* $Q_d(s) + \sum_{i=1}^{N}\left(\tilde{N}(s)\bar{Q}_i(s) - Q_d(s)\right)q_i(s)e^{-sT_i}$

From Fig. 1 and Fig. 2, since the Nyquist plot of $Q_d(s) + \sum_{i=1}^{N}(\tilde{N}(s)\bar{Q}_i(s) - Q_d(s))q_i(s)e^{-sT_i}$ does not encircle the origin, we find that $Q(s) \in H_\infty$.

Using above-mentioned parameters, we have a stabilizing multi-period repetitive controller with the specified input-output frequency characteristic. When the designed multi-period repetitive controller $C(s)$ is used, the response of the error $e = r - y$ in

(1) written by

$$
\begin{aligned}
e &= \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \\
&= r - y \\
&= \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} - \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \\
&= \begin{bmatrix} r_1 - y_1 \\ r_2 - y_2 \end{bmatrix} \qquad (42)
\end{aligned}
$$

for the periodic reference input r

$$
\begin{aligned}
r &= \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \\
&= \begin{bmatrix} \sin\left(\frac{2\pi}{T}t\right) \\ 2\sin\left(\frac{2\pi}{T}t\right) \end{bmatrix} \qquad (43)
\end{aligned}
$$

is shown in Fig. 3. Here, the broken line shows the response of the periodic reference input r_1, the dotted line shows that of the periodic reference input r_2, the solid line shows that of the error e_1, and the dotted and broken line shows that of the error e_2. Figure 3 shows that the output y follows the periodic reference input r with small steady state error.

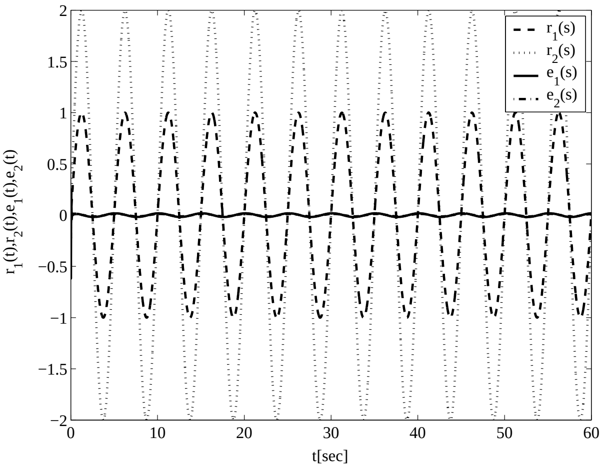

Fig.3: *Response of the error e for the periodic reference input $r = \left[\sin\left(\frac{2\pi}{T}t\right), 2\sin\left(\frac{2\pi}{T}t\right)\right]^T$*

Next, using the designed the multi-period repetitive controller $C(s)$, the disturbance attenuation characteristic is shown. The response of the output y for the disturbance

$$
\begin{aligned}
d &= \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \\
&= \begin{bmatrix} \sin\left(\frac{4\pi}{T}t\right) \\ 2\sin\left(\frac{4\pi}{T}t\right) \end{bmatrix} \qquad (44)
\end{aligned}
$$

of which the frequency component is equivalent to that of the periodic reference input r is shown in Fig. 4. Here, the broken line shows the response of the disturbance d_1, the dotted line shows that of the disturbance d_2, the solid line shows that of the output y_1 and the dotted and broken line shows that of the output y_2. Figure 4 shows that the disturbance d is attenuated effectively.

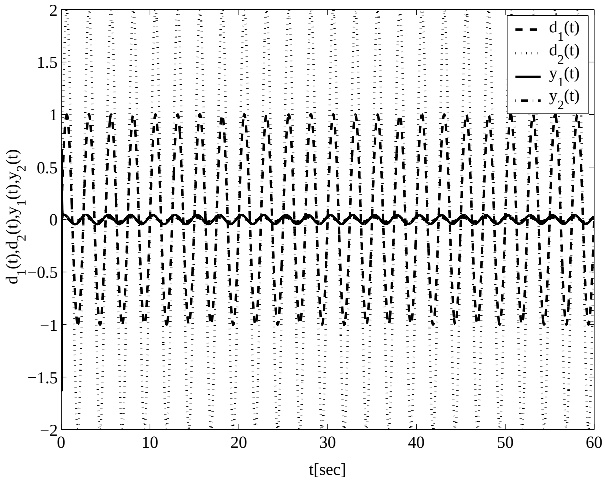

Fig.4: *Response of the output y for the disturbance $d = \left[\sin\left(\frac{4\pi}{T}t\right), 2\sin\left(\frac{4\pi}{T}t\right)\right]^T$*

Finally, the response of the output y for the disturbance

$$
\begin{aligned}
d &= \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \\
&= \begin{bmatrix} \sin\left(\frac{\pi}{T}t\right) \\ 2\sin\left(\frac{\pi}{T}t\right) \end{bmatrix} \qquad (45)
\end{aligned}
$$

of which the frequency component is different from that of the periodic reference input r is shown in Fig. 5. Here, the broken line shows the response of the

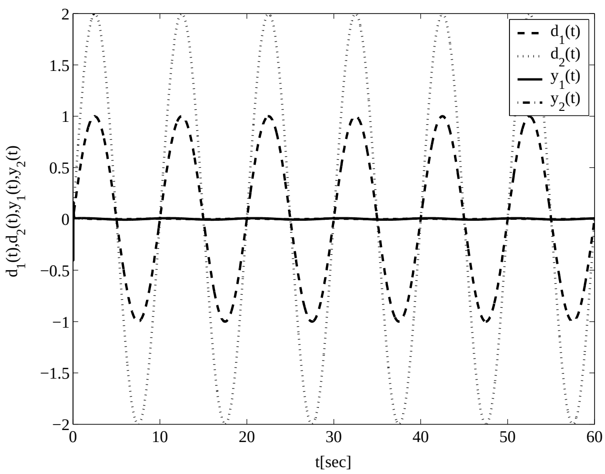

Fig.5: *Response of the output y for the disturbance $d = \left[\sin\left(\frac{\pi}{T}t\right), 2\sin\left(\frac{\pi}{T}t\right)\right]^T$*

disturbance d_1, the dotted line shows that of the disturbance d_2, the solid line shows that of the output y_1 and the dotted and broken line shows that of the output y_2. Figure 4 shows that the disturbance d is attenuated effectively. In this way, we can easily design a stabilizing multi-period repetitive controller with the specified input-output frequency characteristic using Theorem 1.

6. CONCLUSION

In this paper, we proposed the parameterization of all stabilizing multi-period repetitive controllers for multiple-input/multiple-output plants with the specified input-output frequency characteristic. Using the result in this paper, we can easily design a stabilizing multi-period repetitive controller.

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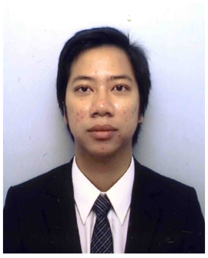

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