

# A Design Method for Modified Smith Predictive Control System to Attenuate Periodic Disturbances

Kou Yamada<sup>1</sup>, Nghia Thi Mai<sup>2</sup>, Takaaki Hagiwara<sup>3</sup>,  
Iwanori Murakami<sup>4</sup>, and Tatsuya Hoshikawa<sup>5</sup>, Non-members

## ABSTRACT

The modified Smith predictor is well known as an effective time-delay compensator for a plant with large time-delays, and several papers on the modified Smith predictor have been published. Recently, the parameterization of all stabilizing modified Smith predictors for minimum-phase time-delay plants is obtained by Yamada and Matsushima. In addition, Yamada et al. expanded the result by Yamada and Matsushima and proposed the parameterization of all stabilizing modified Smith predictors for non-minimum-phase time-delay plants. However, they do not examine a design method for modified Smith predictive control system using the parameterization of all stabilizing modified Smith predictors to achieve desirable control specification. In this paper, we propose a design method for modified Smith predictive control system to attenuate periodic disturbances that often appear real time-delay plants using the parameterization of all stabilizing modified Smith predictors.

**Keywords:** Time-Delay System, Smith Predictor, Periodic Disturbances

## 1. INTRODUCTION

In this paper, we examine a design method for modified Smith predictive control system to attenuate periodic disturbances. A Smith predictor is proposed by Smith to overcome time-delay [1]. It is well known as an effective time-delay compensator for a stable plant with large time-delays [1–12]. The Smith predictor in [1] cannot be used for time-delay plants having an integral mode, because a step disturbance will result in a steady state error [2–4]. To overcome this problem, Watanabe and Ito [4], Astrom et al. [9] and Matusek and Micic [10] proposed a design method for modified Smith predictor for time-delay plants with an integrator. Watanabe and Sato expanded the result in [4] and proposed a design method for modified

Smith predictors for multivariable systems with multiple time-delays in inputs and outputs [5].

Because the modified Smith predictor cannot be used for unstable time-delay plants [2–11], De Paor [6], De Paor and Egan [8] and Kwak et al. [12] proposed a design method for modified Smith predictors for unstable time-delay plants. Thus, several design methods for modified Smith predictors have been published.

On the other hand, another important control problem is the parameterization problem, the problem of finding all stabilizing controllers for plant [14–22]. The parameterization of all stabilizing controllers for time-delay plants was considered in [21, 22], but that of all stabilizing modified Smith predictors has not been obtained. Yamada and Matsushima gave the parameterization of all stabilizing modified Smith predictors for minimum-phase time-delay plants [23, 24]. Since the parameterization of all stabilizing modified Smith predictors for minimum-phase time-delay plants was obtained, we could express previous studies of modified Smith predictors for minimum-phase time-delay plants in a uniform manner and could design modified Smith predictors for minimum-phase time-delay plants systematically. Yamada et al. expanded the result in [23, 24] and proposed the parameterization of all stabilizing modified Smith predictors for non-minimum-phase time-delay plants [25]. However, they do not examine a design method for modified Smith predictive control system using the parameterization of all stabilizing modified Smith predictors to achieve desirable control specification.

In this paper, we propose a design method for modified Smith predictive control system to attenuate periodic disturbances that often appear real time-delay plants using the parameterization of all stabilizing modified Smith predictors. This paper is organized as follows: In Section 2., modified Smith predictors are introduced briefly. In addition, the problem considered in this paper is described. In Section 3., we propose a design method for modified Smith predictive control system to attenuate periodic disturbances effectively. The modified Smith predictive control system proposed in Section 3. cannot specify the input-output characteristic and the disturbance

Manuscript received on July 15, 2010 ; revised on August 29, 2010.

This paper is extended from the paper presented in ECTI-CON 2010.

<sup>1,2,3,4,5</sup> The author is with The Department of Mechanical System Engineering, Gunma University 1-5-1 Tenjincho, Kiryu 376-8515, Japan, E-mail: yamada@me.gunma-u.ac.jp

attenuation characteristic separately. In Section 4., we present a design method for two-degree-of-freedom modified Smith predictive control to attenuate periodic disturbances effectively and to specify the input-output characteristic and the disturbance attenuation characteristic separately. In Section 5., a numerical example is illustrated to show the effectiveness of the proposed design method. Section 6. gives concluding remarks.

#### Notations

$R$	The set of real numbers.
$R(s)$	The set of real rational functions with $s$ .
$RH_\infty$	The set of stable proper real rational functions.
$H_\infty$	The set of stable causal functions.
$\mathcal{U}$	The set of unimodular functions on $RH_\infty$ . That is, $U(s) \in \mathcal{U}$ implies both $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$ .
$\Re\{\cdot\}$	The real part of $\{\cdot\}$ .

## 2. MODIFIED SMITH PREDICTOR AND PROBLEM FORMULATION

Consider the control system:

$$\begin{cases} y(s) &= G(s)e^{-sT}u(s) + d_2(s) \\ u(s) &= C(s)(r(s) - y(s)) + d_1(s) \end{cases}, \quad (1)$$

where  $G(s)e^{-sT}$  is the time-delay plant,  $G(s) \in R(s)$ ,  $T > 0$  is the time-delay,  $C(s)$  is the controller,  $y(s) \in R(s)$  is the output,  $u(s) \in R(s)$  is the control input,  $d_1(s) \in R(s)$  and  $d_2(s) \in R(s)$  are periodic disturbances with period  $L > 0$  satisfying

$$d_1(t+L) = d_1(t) \quad (\forall t \geq 0) \quad (2)$$

and

$$d_2(t+L) = d_2(t) \quad (\forall t \geq 0), \quad (3)$$

$r(s) \in R(s)$  is the reference input. It is assumed that  $G(s)$  is coprime,  $u(t)$  and  $y(t)$  are available,  $d(t)$  is unavailable.

According to [2–13], the modified Smith predictor  $C(s)$  is decided by the form of

$$C(s) = \frac{C_1(s)}{1 + C_2(s)e^{-sT}}, \quad (4)$$

where  $C_1(s) \in R(s)$  and  $C_2(s) \in R(s)$ . In addition, using the modified Smith predictor in [2–13], the transfer function from  $r(s)$  to  $y(s)$  of the control system in (1), written as

$$y(s) = \frac{C(s)G(s)e^{-sT}}{1 + C(s)G(s)e^{-sT}}r(s) \quad (5)$$

has finite numbers of poles. That is, the transfer function from  $r(s)$  to  $y(s)$  of the control system in (1) is written as

$$y(s) = \bar{G}(s)e^{-sT}r(s), \quad (6)$$

where  $\bar{G}(s) \in RH_\infty$ . Therefore, we call  $C(s)$  the modified Smith predictor if  $C(s)$  takes the form of (4) and the transfer function from  $r(s)$  to  $y(s)$  of the control system in (1) has finite numbers of poles.

From [23–25], the parameterization of all stabilizing modified Smith predictors  $C(s)$  is written by

$$C(s) = \frac{C_f(s)}{1 - C_f(s)G(s)e^{-sT}}, \quad (7)$$

where  $C_f(s)$  is given by

$$C_f(s) = \frac{1}{G_u(s)} \left( \bar{G}_u(s) + \frac{Q(s)}{G_u(s)} \right), \quad (8)$$

$\bar{G}_u(s) \in RH_\infty$  is satisfying

$$\bar{G}_u(s_i) = \frac{1}{G_s(s_i)e^{-s_iT}} \quad (\forall i = 1, \dots, n), \quad (9)$$

$G_s(s)$  is a stable non-minimum-phase function of  $G(s)$ , that is, when  $G(s)$  is factorized as

$$G(s) = G_u(s)G_s(s), \quad (10)$$

$G_u(s) \in R(s)$  is an unstable biproper minimum-phase function,  $G_s(s) \in R(s)$  is a stable non-minimum-phase function,  $s_i (i = 1, \dots, n)$  denote unstable poles of  $G(s)$  and  $Q(s) \in RH_\infty$  is any function.

The problem considered in this paper is to propose a design method for modified Smith predictive control system to attenuate periodic disturbances effectively.

## 3. A DESIGN METHOD FOR MODIFIED SMITH PREDICTIVE CONTROL SYSTEM TO ATTENUATE PERIODIC DISTURBANCES

In this section, we propose a design method for modified Smith predictive control system to attenuate periodic disturbances effectively.

Using the parameterization of all stabilizing modified Smith predictor  $C(s)$  in (7), transfer functions from the disturbance  $d_1(s)$  to the output  $y(s)$  and from the disturbance  $d_2(s)$  to the output  $y(s)$  of the control system in (1) are given by

$$\begin{aligned} y(s) &= \left\{ 1 - \left( \bar{G}_u(s) + \frac{Q(s)}{G_u(s)} \right) G_s(s)e^{-sT} \right\} \\ &\quad G(s)e^{-sT}d_1(s) \end{aligned} \quad (11)$$

and

$$\begin{aligned} y(s) &= \left\{ 1 - \left( \bar{G}_u(s) + \frac{Q(s)}{G_u(s)} \right) G_s(s)e^{-sT} \right\} d_2(s). \end{aligned} \quad (12)$$

Therefore, in order to attenuate the disturbance  $d_1(s)$  and  $d_2(s)$  effectively,  $Q(s)$  is settled satisfying

$$\left( \bar{G}_u(j\omega_{di}) + \frac{Q(j\omega_{di})}{G_u(j\omega_{di})} \right) G_s(j\omega_{di}) e^{-j\omega_{di}T} = 1 \quad (\forall i = 0, \dots, n_d), \quad (13)$$

where  $\omega_{di} (i = 0, \dots, n_d)$  are frequency components of periodic disturbances  $d_1(s)$  and  $d_2(s)$  with period  $L$  written by

$$\omega_{di} = \frac{2\pi i}{L} \quad (\forall i = 0, \dots, n_d), \quad (14)$$

and  $\omega_{dn_d}$  is the maximum frequency component of periodic disturbances  $d_1(s)$  and  $d_2(s)$  with period  $L$ .

Next, we derive a design method for  $Q(s)$  in (8) satisfying (13). Since  $1/G_u(s) \in RH_\infty$  and  $G_s(s) \in RH_\infty$  in (10), there exists  $\tilde{Q}(s) \in RH_\infty$  satisfying

$$\frac{G_s(s)}{G_u(s)} \tilde{Q}(s) = G_i(s) q(s), \quad (15)$$

where  $G_i(s)$  is an inner function of  $G_s(s)/G_u(s)$  satisfying  $G_i(0) = 1$  and  $q(s)$  is written by

$$q(s) = \frac{1}{(1 + s\tau)^\alpha}, \quad (16)$$

$\tau \in R$  is an arbitrary positive real number and  $\alpha$  is an arbitrary positive integer to make  $\tilde{Q}(s)$  proper.  $\tau$  and  $\alpha$  are chosen to satisfy

$$1 - G_i(j\omega_{di}) q(j\omega_{di}) = 1 - G_i(j\omega_{di}) \frac{1}{(1 + j\omega_{di}\tau)^\alpha} \simeq 0 \quad (\forall i = 0, \dots, n_d). \quad (17)$$

Using  $\tilde{Q}(s) \in RH_\infty$  satisfying (15),  $Q(s)$  is selected as

$$Q(s) = \tilde{Q}(s) (1 - \bar{G}_u(s) G_s(s) e^{-sT}) e^{-s\bar{T}}, \quad (18)$$

where  $\bar{T} \geq 0$  is chosen as

$$\bar{T} = mL - T, \quad (19)$$

and  $m$  is the smallest positive integer that makes  $\bar{T}$  in (19) nonnegative. The fact that using  $Q(s)$  in (18), periodic disturbances  $d_1(s)$  and  $d_2(s)$  are attenuated is confirmed as follows: Since

$$\begin{aligned} 1 - \left( \bar{G}_u(j\omega_{di}) + \frac{Q(j\omega_{di})}{G_u(j\omega_{di})} \right) G_s(j\omega_{di}) e^{-j\omega_{di}T} \\ = (1 - \bar{G}_u(j\omega_{di}) G_s(j\omega_{di}) e^{-j\omega_{di}T}) \\ \quad \left( 1 - G_i(j\omega_{di}) q(j\omega_{di}) e^{-j\omega_{di}(T+\bar{T})} \right) \\ = (1 - \bar{G}_u(j\omega_{di}) G_s(j\omega_{di}) e^{-j\omega_{di}T}) \\ \quad \left( 1 - G_i(j\omega_{di}) q(j\omega_{di}) e^{-j\omega_{di}mL} \right) \\ \simeq 0 \quad (\forall i = 0, \dots, n_d), \end{aligned} \quad (20)$$

$$e^{-j\omega_{di}mL} = 1 \quad (\forall i = 0, \dots, n_d) \quad (21)$$

and (17), (11) and (12) are rewritten by

$$y(j\omega_{di}) \simeq 0 \quad (\forall i = 0, \dots, n_d). \quad (22)$$

Note that even if  $Q(s)$  in (8) is chosen as (18), the transfer function from  $r(s)$  to  $y(s)$  has finite numbers of poles, since the transfer function from  $r(s)$  to  $y(s)$  is written by

$$\begin{aligned} y(s) &= \{ \bar{G}_u(s) G_s(s) + G_i(s) q(s) \\ &\quad (1 - \bar{G}_u(s) G_s(s) e^{-sT}) e^{-s\bar{T}} \} e^{-sT} r(s) \end{aligned} \quad (23)$$

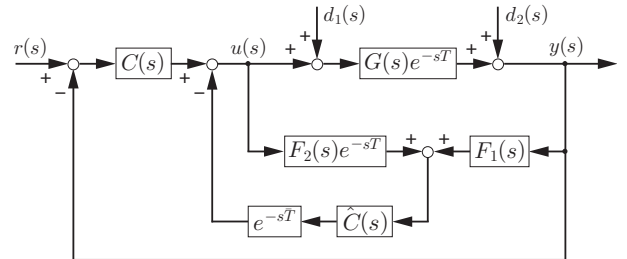
$\bar{G}_u(s) \in RH_\infty$ ,  $G_s(s) \in RH_\infty$ ,  $G_i(s) \in RH_\infty$  and  $q(s) \in RH_\infty$ .

In this section, we proposed a design method for modified Smith predictive control system to attenuate periodic disturbances  $d_1(s)$  and  $d_2(s)$  effectively. However, the modified Smith predictive control system in (1) cannot specify the input-output characteristic and the disturbance attenuation characteristic separately. From the practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic separately. In the next section, we present a design method for two-degree-of-freedom modified Smith predictive control system to attenuate periodic disturbances effectively and to specify the input-output characteristic and the disturbance attenuation characteristic separately.

#### 4. A DESIGN METHOD FOR TWO-DEGREE-OF-FREEDOM MODIFIED SMITH PREDICTIVE CONTROL SYSTEM

In this section, we present a design method for two-degree-of-freedom modified Smith predictive control system to attenuate periodic disturbances effectively and to specify the input-output characteristic and the disturbance attenuation characteristic separately.

In order to attenuate periodic disturbances effectively and to specify the input-output characteristic and the disturbance attenuation characteristic separately, we present a two-degree-of-freedom modified Smith predictive control system shown in Fig. 1. Here,  $C(s) \in R(s)$  is the modified Smith predic-



**Fig. 1:** Two-degree-of-freedom modified Smith predictive control system to attenuate periodic disturbances

tor written by (7),  $\hat{C}(s) \in RH(s)$  is the controller to attenuate periodic disturbances,  $F_1(s) \in RH(s)$  and  $F_2(s) \in RH(s)$  are written by

$$F_1(s) = G_u^{-1}(s) + Q_f(s)G_u^{-1}(s) \quad (24)$$

and

$$F_2(s) = -G_s(s) - Q_f(s)G_s(s), \quad (25)$$

respectively,  $Q_f(s) \in RH_\infty$  is any function,  $\bar{T} \geq 0$  is chosen as (19) and  $m$  is the smallest positive integer that makes  $\bar{T}$  in (19) nonnegative.

Next, we clarify control characteristics of control system in Fig. 1. First, the input-output characteristic of control system in Fig. 1 is shown. The transfer function from the reference input  $r(s)$  to the output  $y(s)$  is written by

$$y(s) = \left( \bar{G}_u(s) + \frac{Q(s)}{G_u(s)} \right) G_s(s) e^{-sT} r(s). \quad (26)$$

Therefore, the input-output characteristic is specified using  $Q(s)$  in (8). That is, it is specified using the modified Smith predictor  $C(s)$  in (7).

Next, the disturbance attenuation characteristic is shown. The transfer function from the periodic disturbance  $d_1(s)$  to the output  $y(s)$  and that from the periodic disturbance  $d_2(s)$  to the output  $y(s)$  of the control system in Fig. 1 are given by

$$\begin{aligned} y(s) &= \left\{ 1 - \left( \bar{G}_u(s) + \frac{Q(s)}{G_u(s)} \right) G_s(s) e^{-sT} \right\} G_s(s) e^{-sT} \\ &\quad \left\{ 1 - G_s(s) \hat{C}(s) e^{-smL} (1 + Q_f(s)) \right\} d_1(s) \end{aligned} \quad (27)$$

and

$$\begin{aligned} y(s) &= \left\{ 1 - \left( \bar{G}_u(s) + \frac{Q(s)}{G_u(s)} \right) G_s(s) e^{-sT} \right\} \\ &\quad \left\{ 1 - G_s(s) \hat{C}(s) e^{-smL} (1 + Q_f(s)) \right\} d_2(s). \end{aligned} \quad (28)$$

Therefore, the disturbance characteristic is specified using  $Q_f(s)$  in (24) and (25) and the controller  $\hat{C}(s)$ .

From (26), (27) and (28), the role of the modified Smith predictor  $C(s)$  is different from that of the controller  $\hat{C}(s)$ . The role of the modified Smith predictor  $C(s)$  is to specify the input-output characteristic and that of the controller  $\hat{C}(s)$  is to specify the disturbance attenuation characteristic.

Finally, the condition that the control system in Fig. 1 is stable is clarified. From (26), (27) and (28), it is obvious that the control system in Fig. 1 is stable if and only if following expressions hold true.

1. The modified Smith predictor  $C(s)$  makes the control system in (1) stable.

2.  $\hat{C}(s) \in RH_\infty$ .

Next, we describe a design method for the controller  $\hat{C}(s)$  to attenuate periodic disturbances  $d_1(s)$  and  $d_2(s)$ . From (27) and (28), in order to attenuate periodic disturbances  $d_1(s)$  and  $d_2(s)$  effectively,  $\hat{C}(s)$  is settled satisfying

$$1 - G_s(j\omega_{di}) \hat{C}(j\omega_{di}) e^{-j\omega_{di}mL} (1 + Q_f(j\omega_{di})) = 0 \quad (\forall i = 0, \dots, n_d), \quad (29)$$

where  $\omega_{di} (i = 0, \dots, n_d)$  is the frequency component of periodic disturbances  $d_1(s)$  and  $d_2(s)$  with period  $L$  written by (14). A design method for  $\hat{C}(s)$  satisfying (29) is summarized as follows:  $Q_f(s) \in RH_\infty$  in (24) and (25) is selected to satisfy  $1 + Q_f(s) \in \mathcal{U}$ . Using the method in [26], there exists  $\tilde{G}_s(s) \in RH_\infty$  satisfying

$$G_s(s) \tilde{G}_s(s) = G_{si}(s) q(s), \quad (30)$$

where  $G_{si}(s)$  is an inner function of  $G_s(s)$  satisfying  $G_{si}(0) = 1$ , and  $q(s)$  is written by (16),  $\tau \in R$  is an arbitrary positive real number and  $\alpha$  is an arbitrary positive integer to make  $\tilde{G}_s(s)$  proper.  $\tau$  and  $\alpha$  are chosen to satisfy

$$\begin{aligned} 1 - G_{si}(j\omega_{di}) q(j\omega_{di}) &= 1 - G_{si}(j\omega_{di}) \frac{1}{(1 + j\omega_{di}\tau)^\alpha} \\ &\simeq 0 \quad (\forall i = 0, \dots, n_d). \end{aligned} \quad (31)$$

Using  $\tilde{G}_s(s)$ ,  $\hat{C}(s)$  is selected as

$$\hat{C}(s) = \frac{\tilde{G}_s(s)}{1 + Q_f(s)}. \quad (32)$$

The fact that using  $\hat{C}(s)$  in (32), periodic disturbances  $d_1(s)$  and  $d_2(s)$  are attenuated is confirmed as follows: Since

$$\begin{aligned} 1 - G_s(j\omega_{di}) \hat{C}(j\omega_{di}) e^{-j\omega_{di}mL} (1 + Q_f(j\omega_{di})) \\ = 1 - G_{si}(j\omega_{di}) \frac{1}{(1 + j\omega_{di}\tau)^\alpha} e^{-j\omega_{di}mL} \\ \simeq 0 \quad (\forall i = 0, \dots, n_d), \end{aligned} \quad (33)$$

(31) and (21), (27) and (28) are rewritten by

$$y(j\omega_{di}) \simeq 0 \quad (\forall i = 0, \dots, n_d). \quad (34)$$

Note that even if  $\hat{C}(s)$  in Fig. 1 is chosen as (32), the transfer function from  $r(s)$  to  $y(s)$  has finite numbers of poles, since  $\bar{G}_u(s) \in RH_\infty$ ,  $G_s(s) \in RH_\infty$ ,  $G_{si}(s) \in RH_\infty$  and  $q(s) \in RH_\infty$ .

## 5. NUMERICAL EXAMPLE

In this section, a numerical example is illustrated to show the effectiveness of the proposed method.

Consider the problem to design a two-degree-of-freedom modified Smith predictive control system to attenuate periodic disturbances with period  $L =$

2[sec] in Fig. 1 for the unstable minimum-phase time-delay plant  $G(s)e^{-sT}$  written by

$$G(s)e^{-sT} = \frac{s+10}{(s+20)(2s-1)}e^{-0.5s}, \quad (35)$$

where

$$G(s) = \frac{s+10}{(s+20)(2s-1)} \quad (36)$$

and  $T = 0.5$ [sec].

First, we design a modified Smith predictor  $C(s)$  in Fig. 1.  $G(s)$  is factorized by (10) as

$$G_u(s) = \frac{2s+1}{2s-1} \quad (37)$$

and

$$G_s(s) = \frac{s+10}{(s+20)(2s+1)}. \quad (38)$$

One of  $\bar{G}_u(s)$  in (8) satisfying (9) is given by

$$\bar{G}_u(s) = \frac{12.84(s+40.5)}{s+104.5}. \quad (39)$$

From (7), the parameterization of all stabilizing modified Smith predictors  $C(s)$  for the unstable minimum-phase time-delay plant  $G(s)e^{-sT}$  in (36) is given by

$$C(s) = \frac{C_f(s)}{1 - C_f(s) \frac{s+10}{(s+20)(2s-1)}e^{-0.5s}}, \quad (40)$$

where  $C_f(s)$  is given by

$$C_f(s) = \frac{2s-1}{2s+1} \left( \frac{12.84s+520}{s+104.5} + \frac{2s-1}{2s+1} Q(s) \right) \quad (41)$$

and  $Q(s) \in RH_\infty$  is any function. In order for the output  $y(s)$  to follow the step reference input  $r(s) = 1/s$  without steady state error,  $Q(s) \in RH_\infty$  is settled by

$$Q(s) = \frac{s+59.53}{s+20}. \quad (42)$$

Substituting (42) for (41), we have

$$C_f(s) = \frac{13.84(s+0.1243)}{(s+104.5)(s+20)(s+0.5)^2} \cdot (s^2 + 68.28s + 1215)(s-0.5). \quad (43)$$

Next,  $\bar{T}$  is designed.  $\bar{T} \geq 0$  in Fig. 1 is set by (19) as  $\bar{T} = 1.5$ [sec].

Next,  $F_1(s)$  and  $F_2(s)$  in Fig. 1 are designed.  $Q_f(s) \in RH_\infty$  in (24) and (25) satisfying  $1+Q_f(s) \in \mathcal{U}$  is set as

$$Q_f(s) = 1. \quad (44)$$

Substituting (37), (38) and (44) for (24) and (25),  $F_1(s)$  and  $F_2(s)$  in (24) and (25) are written by

$$F_1(s) = \frac{2s-1}{s+0.5} \quad (45)$$

and

$$F_2(s) = \frac{-s-10}{(s+20)(s+0.5)}. \quad (46)$$

Next,  $\hat{C}(s)$  in Fig. 1 is designed in order to attenuate periodic disturbances  $d_1(s)$  and  $d_2(s)$  with period  $L$  effectively.  $\hat{C}(s)$  in Fig. 1 is settled by (32) as

$$\hat{C}(s) = \frac{(s+20)(2s+1)}{2(s+10)(0.001s+1)}, \quad (47)$$

where  $\tilde{G}_s(s)$  satisfying (30) is given by

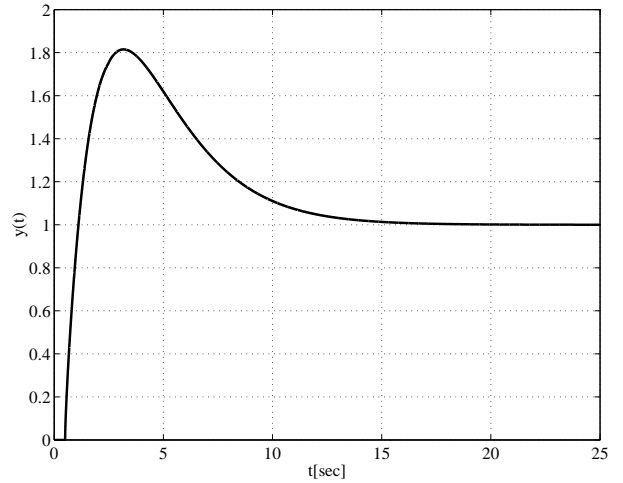
$$\tilde{G}_s(s) = \frac{(s+20)(2s+1)}{(s+10)(0.001s+1)}, \quad (48)$$

$$G_{si}(s) = 1 \quad (49)$$

and

$$q(s) = \frac{1}{0.001s+1}. \quad (50)$$

Using the obtained control system in Fig. 1, the response of the output  $y(t)$  for the step reference input  $r(t) = 1$  is shown in Fig. 2. Figure 2 shows that



**Fig.2:** Response of the output  $y(t)$  for the step reference input  $r(t) = 1$

the control system in Fig. 1 is stable and the output  $y(t)$  follows the step reference input  $r(t) = 1$  without steady state error.

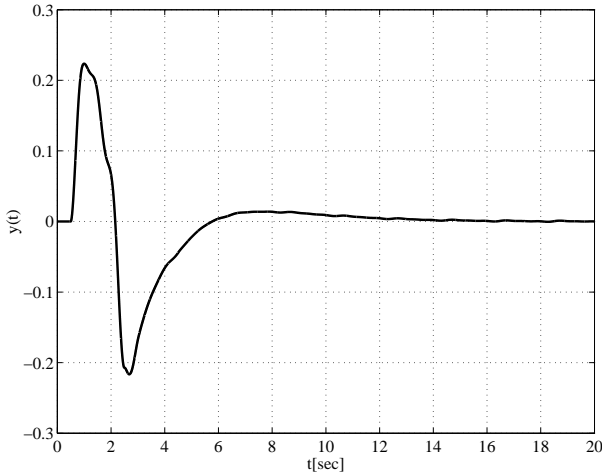
Next, the disturbance attenuation characteristic is shown. Responses of the output  $y(t)$  for periodic disturbances

$$d_1(t) = \sin \pi t \quad (51)$$

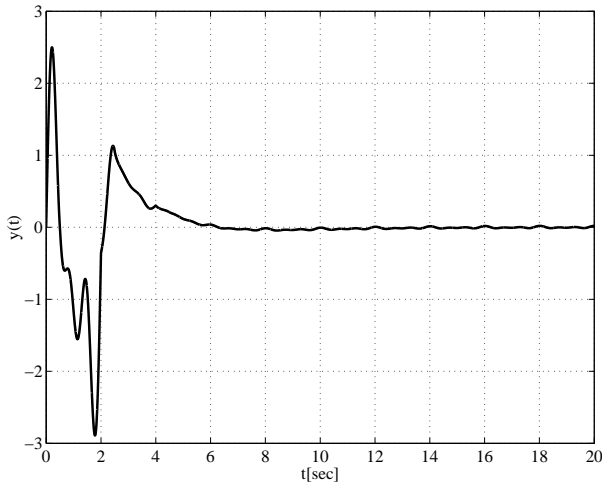
and

$$d_2(t) = \sin \pi t \quad (52)$$

in Fig. 1 are shown in Fig. 3 and Fig. 4, respectively. Figure 3 and Fig. 4 show that periodic disturbances



**Fig.3:** Response of the output  $y(t)$  for the periodic disturbance  $d_1(t) = \sin \pi t$



**Fig.4:** Response of the output  $y(t)$  for the periodic disturbance  $d_2(t) = \sin \pi t$

$d_1(t)$  and  $d_2(t)$  are attenuated effectively.

In this way, we find that by using the result in this paper, we can easily design a two-degree-of-freedom modified Smith predictive control system to attenuate periodic disturbances.

## 6. CONCLUSIONS

In this paper, we proposed a design method for modified Smith predictive control system to attenuate periodic disturbances. First, we proposed a design method using the parameterization of all stabilizing modified Smith predictors in [23–25]. Since pro-

posed method cannot specify the input-output characteristic and the disturbance attenuation characteristic separately, we presented a design method for two-degree-of-freedom modified Smith predictive control system to attenuate periodic disturbances and to specify the input-output characteristic and the disturbance attenuation characteristic separately. Finally, we showed a numerical example to illustrate the effectiveness of the proposed design method.

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**Kou Yamada** was born in Akita, Japan, in 1964. He received B.S. and M.S. degrees from Yamagata University, Yamagata, Japan in 1987 and 1989, respectively, and a Dr. Eng. degree from Osaka University, Osaka, Japan in 1997. From 1991 to 2000, he was with the Department of Electrical and Information Engineering, Yamagata University, Yamagata, Japan as a research associate. From 2000 to 2008, he was an associate professor in the Department of Mechanical System Engineering, Gunma University, Gunma, Japan. Since 2008, he has been a professor in the Department of Mechanical System Engineering, Gunma University, Gunma, Japan. His research interests include robust control, repetitive control, process control, and control theory for inverse systems and infinite-dimensional systems. Dr. Yamada received the 2005 Yokoyama Award in Science and Technology, the 2005 Electrical Engineering/Electronics, Computer, Telecommunication, and Information Technology International Conference (ECTI-CON2005) Best Paper Award, the Japanese Ergonomics Society Encouragement Award for an Academic Paper in 2007, the 2008 Electrical Engineering/Electronics, Computer, Telecommunication, and Information Technology International Conference (ECTI-CON2008) Best Paper Award, and the Fourth International Conference on Innovative Computing, Information and Control Best Paper Award in 2009.



**Nghia Thi Mai** was born in Bim Son, Thanh Hoa, Vietnam, in 1985. She received a B.S. degree in Mechanical System Engineering from Gunma University, Gunma, Japan, in 2009. She is currently M.S. candidate in Mechanical System Engineering at Gunma University. Her research interest includes Smith predictor.



**Takaaki Hagiwara** was born in Gunma, Japan, in 1982. He received the B.S. and M.S. degrees in Mechanical System Engineering from Gunma University, Gunma Japan, in 2006 and 2008, respectively. He is currently a doctor candidate in Mechanical System Engineering at Gunma University. His research interests include process control and PID control. He received Fourth International Conference on Innovative Computing, Information and Control Best Paper Award in 2009.



**Iwanori Murakami** was born in Hokkaido, Japan, in 1968. He received the B.S., M.S and Dr. Eng. degrees from Gunma University, Gunma, Japan in 1992, 1994 and 1997, respectively. Since 1997, he has been an assistant professor in the Department of Mechanical System Engineering, Gunma University, Gunma, Japan. His research interests include applied electronics, magnetics, mechanics and robotics. He received

Fourth International Conference on Innovative Computing, Information and Control Best Paper Award in 2009.



**Tatsuya Hoshikawa** was born in Gunma, Japan, in 1988. He received a B.S. degree in Mechanical System Engineering from Gunma University, Gunma, Japan, in 2010. He is currently M.S. candidate in Mechanical System Engineering at Gunma University. His research interest includes Smith predictor.