Position Control of a Linear Variable Reluctance Motor with Magnetically Coupled Phases

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ABSTRACT

The objective of this paper is to design and implement a direct-drive position control based on a simplified sinusoidal flux model for a linear variable reluctance motor. The motor under consideration is a three-phase linear reluctance motor with strong magnetic coupling between phases that has the advantages of simple structure, compactness and low cost with no permanent magnet. The experimental results show overall good performance indicating that the developed system can be considered as a strong candidate for high precision manufacturing automation applications.

Keywords: Linear Motor, Variable Reluctance Motor, Position Control

1. INTRODUCTION

In recent years, linear motors have gained considerable attention in applications for linear motion such as modern laser cutting, robotic assembly systems and transportation [1]. The direct-drive linear motor systems have many advantages compared with traditional (i.e. indirect) rotary motor drive systems coupled with lead screws or toothed belts. The electromagnetic force from linear motor can be applied directly to the payload without any mechanical transmission that usually imposes mechanical limitations on velocity. As a result, the system can operate with higher acceleration and velocity to achieve higher accuracy that is now only limited by the bandwidth of the position measurement system or by the power electronics system. In addition, the linear motor drive systems provide less friction, no backlash, low mechanical maintenance and longer lifetime.

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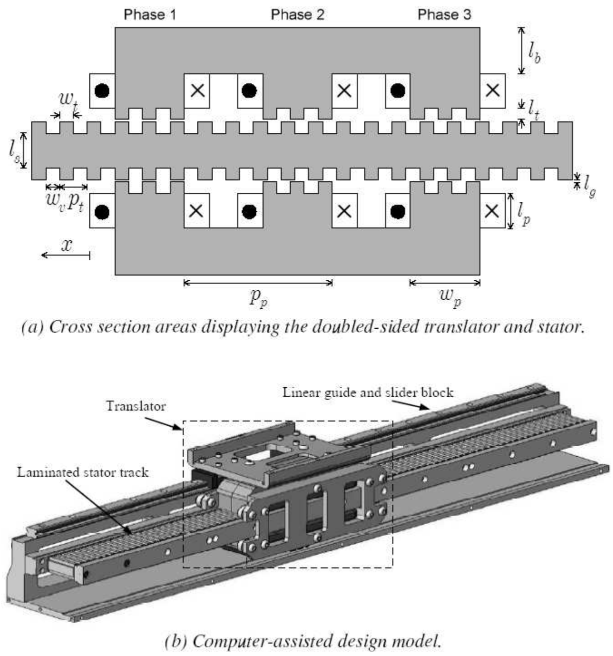

(a) Cross section areas displaying the doubled-sided translator and stator.

(b) Computer-assisted design model.

Fig.1: *The LVR motor with magnetically coupled between phases.*

This paper considers a specific type of linear motors, namely a linear variable reluctance (LVR) motor with magnetically coupled phases, as shown in Fig. 1. The translator consists of double-sided E-cores moving along the laminated stator track. Motor windings are installed on each side of the translator with series connection between two concentrated coils placed on opposing poles of the two E-cores. The present linear motor configuration has advantages of simple structures, compactness and low cost with no permanent magnet. The structure exhibits strong magnetic coupling between phases.

The early research on modeling and controls of the LVR motor of the present configuration were reported in [2-4], which only consider operating the motor as a stepping motor.

The modern literature on the LVR motor is quite limited and does not target experimental implementation. In [5, 6], an analysis and magnetic circuit model of the LVR motor are discussed. As for control system development of the LVR motor, optimal excitation current control, force control and position control implemented on simulation software are presented in [7-9]. Note that recent research on linear switched reluctance motors, such as [10-12], focus on a different type of linear motors with no magnetically coupled geometry where each phase of such motor may be controlled independently with higher degree of freedom.

The objective of this paper is to design and implement a position control system based on simplified sinusoidal flux model for the LVR motor [7, 8]. The actual implementation with continuous control mode has not been investigated as the motor was implemented only as a linear stepping motor. The proposed control method optimizes current commands for minimum copper losses resulting in continuous excitation for all phases. The proposed method has the advantages of simplicity and computational efficient. The experiment is conducted to exhibit the effectiveness for short- and long- distance motions.

2. DYNAMIC MODEL OF THE LVR MOTOR

According to [13, 14], the phase voltage equation of the LVR motor is given by

$$u_{123} = Ri_{123} + \frac{d(\lambda_{123})}{dt} \tag{1}$$

where the phase voltage $u_{123} = [u_1 \ u_2 \ u_3]^T$, phase current $i_{123} = [i_1 \ i_2 \ i_3]^T$ and flux linkage $\lambda_{123} = [\lambda_1 \ \lambda_2 \ \lambda_3]^T$, with the sub-script 1, 2 and 3 denote the phase number; R is the phase resistance. Assuming magnetic linearity, the flux linkage is given by

$$\lambda_{123} = L(x)i_{123} \tag{2}$$

where x is the position of the translator with position direction shown in Fig. 1. and L is the position dependent inductance matrix, which is described by

$$L(x)=\begin{bmatrix} L_s & -M_s & -M_s \\ -M_s & L_s & -M_s \\ -M_s & -M_s & L_s \end{bmatrix}+L_m\begin{bmatrix} \cos\theta_1 & \cos\theta_2 & \cos\theta_3 \\ \cos\theta_2 & \cos\theta_3 & \cos\theta_1 \\ \cos\theta_3 & \cos\theta_1 & \cos\theta_2 \end{bmatrix} \tag{3}$$

where $\theta_j = \frac{2\pi}{p_i}x + (j-1)\frac{2\pi}{3}, j = 1, 2, 3;$
p_t is the tooth pitch, as defined in Fig. 1; L_s is the average self inductance; L_m is the variation in inductance due to air gap variation and $M_s = 1/2L_s$ is the average mutual inductance.

The winding connection of the LVR motor is wye connection (i.e. $i_1 + i_2 + i_3 = 0$). The dq0 reference frame transformation is used to eliminate the dependence variables on position to simplify the model for control design. The original LVR motor model (1) can be expressed by those written in dq0 reference frame by employing mathematical transformation

$$u_{dq0} = Su_{123}, i_{dq0} = Si_{123}, \lambda_{dq0} = S\lambda_{123}, \tag{4}$$

where $u_{dq0} = [u_d \ u_q \ u0]^T$, $i_{dq0} = [id \ i_q \ i_0]^T$, $\lambda_{dq0} = [\lambda_d \ \lambda_q \ \lambda_0]^T$, with the subscript d, q, 0 denote the d-axis, q-axis and zero-axis in the dq0 reference frame, respectively. The orthonormal transformation matrix S is given by

$$S(x)=\sqrt{\frac{2}{3}}\begin{bmatrix} \cos\left(\frac{\pi}{p_t}x\right) & \cos\left(\frac{\pi}{p_t}x+\frac{2\pi}{3}\right) & \cos\left(\frac{\pi}{p_t}x-\frac{2\pi}{3}\right) \\ -\sin\left(\frac{\pi}{p_t}x\right) & -\sin\left(\frac{\pi}{p_t}x+\frac{2\pi}{3}\right) & -\sin\left(\frac{\pi}{p_t}x-\frac{2\pi}{3}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{5}$$

The phase voltage equation of the LVR motor in dq0 reference frame can be derived by substituting (4) into (1) to obtain

$$\begin{aligned} u_d &= Ri_d - \alpha L_q i_q \frac{dx}{dt} + L_d \frac{di_d}{dt} \\ u_q &= Ri_q - \alpha L_d i_d \frac{dx}{dt} + L_d \frac{di_q}{dt} \\ u_0 &= Ri_0 \end{aligned} \tag{6}$$

where the d-axis inductance L_d and q-axis inductance L_q are given by

$$\begin{aligned} L_d &= L_s + M_s + \frac{3}{2}L_m \\ L_q &= L_s + M_s - \frac{3}{2}L_m \end{aligned}$$

and the constant parameter α is

$$\alpha = \frac{\pi}{p_t}$$

The force function of the LVR motor is given by

$$F = \alpha(L_d - L_q)i_q i_d \tag{7}$$

With wye connection, the current i_0 and voltage u_0 in (6) equal zero. Note that this force model, which would be used for control design, assumes sinusoidal flux model as in (3) and it is essentially equivalent to that of idealized rotary synchronous motors [10]. In practice, it is more convenient to determine the aligned inductance L_a and unaligned inductance L_u as

$$\begin{aligned} L_a &= \frac{1}{2}(L_d + L_q) \\ L_u &= \frac{1}{2}(L_d - L_q) \end{aligned}$$

Since the position movement of LVR motor depends on the force produced by the motor, F is chosen as a control variable for the position control system.

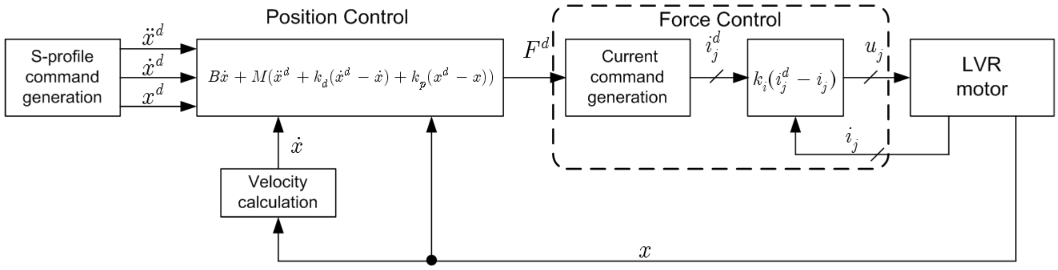

Fig.2: *Control block diagram for the LVR motor.*

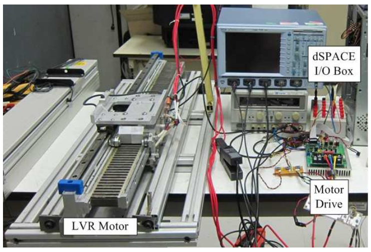

Fig.3: *Experimental setup.*

Table 1: *Motor Mechanical and Electrical Parameters*

Parameter	Value
Device depth (d)	50mm
Tooth width (w_t)	6 mm
Valley width (w_v)	6 mm
Tooth pitch ($p_t = w_v + w_t$)	12 mm
Pole pitch ($p_p = (3+2+1/3)p_t$)	64 mm
Tooth length (l_t)	5 mm
Pole length (l_p)	15 mm
Back length (l_b)	20 mm
Air gap length (l_g)	0.45 mm (each side)
Stator length (l_s)	15 mm
Number of teeth per pole (n)	3
Turns per phase (N)	200(100 each side)
SWG(AWG)	20(19)
Moving mass(M)	8 kg
Phase resistance (R)	1.4Ω
Aligned inductance (L_a)	31.12 mH
Unaligned inductance (L_u)	29.90 mH
Peak force generation per phase	100 N
Max travel distance	80 cm
Encoder accuracy	$\pm 1\mu$m

3. POSITION CONTROLLER DESIGN

The control algorithm presented in this section is based on the dq0 theory of classical synchronous reluctance motors; in particular, it makes use of sinusoidal reluctance and ideal material approximation [7, 8] which is optimized for minimum copper losses and has the advantage of simplicity. The overall control structure is shown in Fig. 2.

3.1 Force Control

The desired phase voltages are chosen to be proportional to measured current errors, i.e.

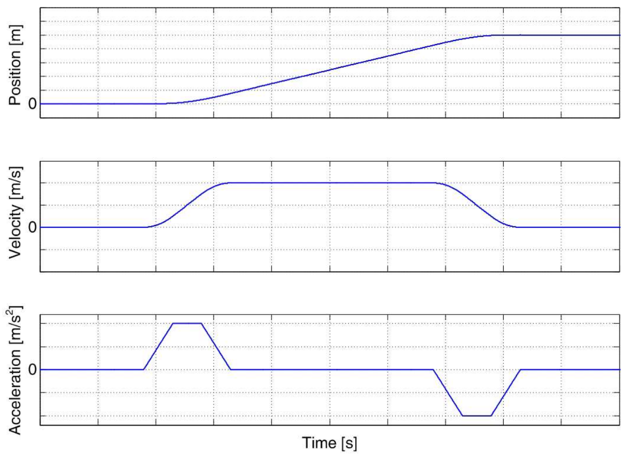

Fig.4: *Example of the S-curve profile for a desired trajectory.*

$$u_j = k_i(i_j^d - i_j) \tag{8}$$

where i_j^d is the desired phase current and k_i is a positive feed back gain. This high current feedback gain is used to decouple the electrical dynamics from the mechanical dynamics of the LVR motor to obtain the reduced order design model, which is a common practice in control engineering [15].

According to [7] the desired phase currents are chosen according to the commutation strategy

$$\begin{bmatrix} i_1^d \\ i_2^d \\ i_3^d \end{bmatrix} = \sqrt{\frac{|F^d|}{\gamma}} \begin{bmatrix} \cos x_1 & -\sin x_1 \\ \cos x_2 & -\sin x_2 \\ \cos x_3 & -\sin x_3 \end{bmatrix} \begin{bmatrix} 1 \\ sgn(F^d) \end{bmatrix} \tag{9}$$

where F^d is the desired force,

$$x_j = \frac{\pi}{p_t}x + (j-1)\frac{2\pi}{3} \tag{10}$$

is the electrical position of phase J,

$$\gamma = \frac{3}{2}\alpha L_d - L_q \tag{11}$$

is a control parameter. This choice of current commands minimizes instantaneous copper loss of each

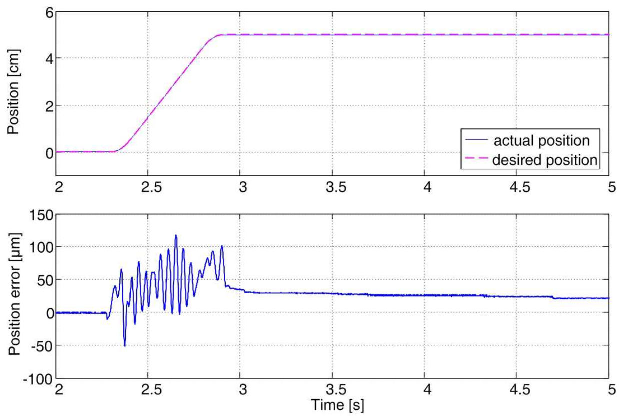

Fig.5: *Position response for the short-distance profile.*

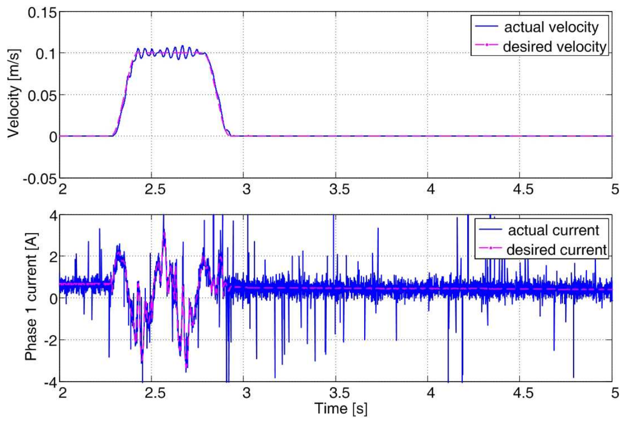

Fig.6: *Velocity and phase 1 current responses for the short-distance profile.*

phase [7]. The force control is shown in Fig. 2, as the inner loop of the position control system.

3.2 Position Control

The mechanical dynamic equation of the LVR motor is given by

$$F = M\ddot{x} + B\ddot{x} + F_L \tag{12}$$

where M is the moving mass, B is the viscous friction coefficient, $\ddot{x}$ is the acceleration, $\dot{x}$ is the velocity and F_L is the external force. Since the position movement of the LVR motor depends on the force, F is chosen as a desired control signal for force control loop.
The position controller is based on a input-output linearization control structure according to

$$F^d = B\dot{x} + M(\ddot{x} + k_d(\dot{x}^d - \dot{x}) + k_p(x^d - x)) \tag{13}$$

where $(x^d, \dot{x}^d, \ddot{x}^d)$ is the desired trajectory and (k_p, k_d) are positive feedback gains. The control system block for the proposed system is shown in Fig. 2.

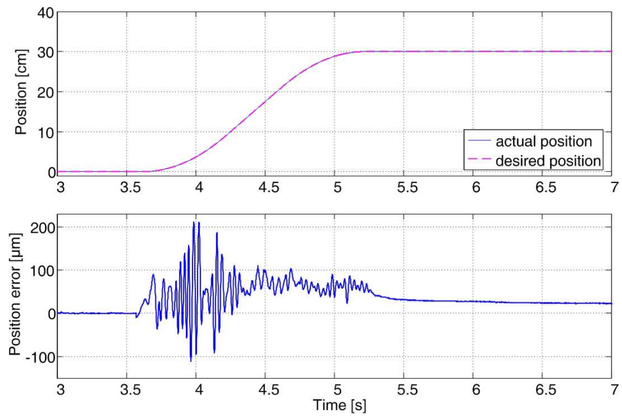

Fig.7: *Position response for the long-distance profile.*

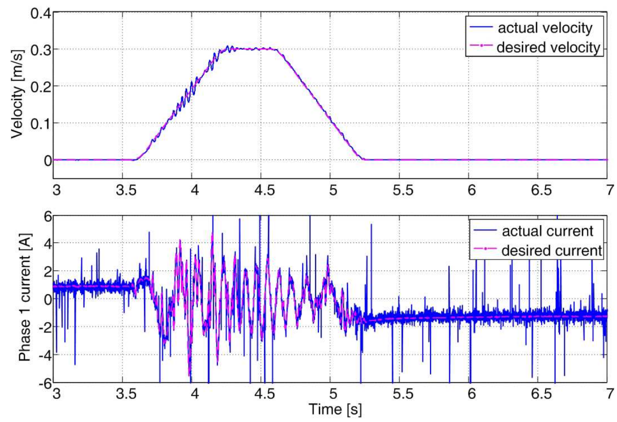

Fig.8: *Velocity and phase 1 current response for the long-distance profile.*

The control design proposed here is based on the following justification: If $F = F^d$, then position error $e = x^d - x$ satisfies $\ddot{e} + k_d\dot{e} + k_p e = 0$, and hence $x \to x^d$ as $t \to \infty$. The use of large k_i also leads to $i_j \approx i_j^d$, but this alone does not guarantee accurate force production. The proposed commutation strategy is based on assumptions of sinusoidal reluctance and magnetic linearity, so spatial harmonics will lead to force ripple and material nonlinearity will lead to force saturation. Even though the proposed commutation strategy may not very well matched to the motor's magnetic characteristics, the average force does have the correct sign and exhibits monotonic shape. Hence, the control design should intuitively be feasible and actual implementation is conducted to make definitive conclusions.

4. EXPERIMENTAL RESULTS

The experiment setup is shown in Fig. 3, whose mechanical and electrical parameters are listed in Table 1. The controller is implemented using a DSP-based DS1104 dSPACE controller card. Three-phase power inverters are employed to drive the LVR motor with 40 V dc bus voltage, 10 kHz PWM switching frequency and 2 μs dead time for short circuit protection. Two ACS712 current sensing ICs are used to sense the phase currents with two second order analog active filters to filter out the high-frequency components of the current signals. The controller is executed with 1 kHz sampling frequency.

The position is measured by a differential optical liner encoder with $\pm$ 1μm resolution. To test the effectiveness of the proposed position controller, a third-order S-curve profile is chosen as the desired trajectory command. The example trajectory plots are shown in Fig. 4 and the mathematical expressions are given in Appendix A. Two desired trajectories are use for the experiments: short-distance of 5 cm and long-distance of 30 cm. The control parameters used for both short- and long-distance profiles are $k_i = 170$, $k_d = 57$ and $k_p = 13296$.

The experimental results for the position tracking and position error of the short-distance profile with 14 kg payload are shown in Fig. 5. The actual position tracks the desired position profile closely with the maximum dynamic error approximately 120 ?m. The steady state error is less than 30 μm. The responses of the velocity and phase 1 current for the short-distance profile are shown in Fig. 6. The controller can track the velocity profile closely with velocity error around 0.01 m/s when the LVR motor reaches 0.1 m/s. The maximum phase current for the short-distance profile is around 3 A (neglecting measurement noises).

The experimental results of the position tracking and position error of the long-distance profile with 14 kg payload are shown in Fig. 7. The actual position also tracks the desired position profile closely with the maximum dynamic error approximately 210 μm. The steady state error is also less than 30 μm. The responses of the velocity and phase 1 current for the long-distance profile are shown in Fig. 8. The controller can track the velocity profile closely with velocity error around 0.01 m/s when the LVR motor reaches 0.3 m/s. The maximum phase current for the long-distance profile is around 5 A (neglecting measurement noises).

The expected force ripples due to spatial harmonics have some influence on the final accuracy of the system. However, the experimental results show overall satisfactory performance indicating that the proposed simple structure controller is feasible and the developed linear motor system can be considered as a strong candidate for manufacturing automation applications with moderately high accuracy.

5. CONCLUSION

The simple direct-drive position control of a linear variable reluctance motor based on a sinusoidal flux model has been successfully implemented. The proposed method has the advantages of simplicity and computational efficient. The force ripples due to spatial harmonics influence the final accuracy of the system. However, the proposed simple structure control system provides good performance for the developed linear motor system. The possible future work is to develop a higher performance but yet simple control system to achieve higher accuracy as well as to reduce the vibration due to force ripples.

6. ACKNOWLEDGMENT

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APPENDIX A

Mathematical expressions for S-curve profile for desired trajectory
According to [16] the conditions of the S-curve profile considered in this paper are

$$v_{max} \geq (a_{max}^2)/(j_{max})$$
$$x^r \geq (v_{max}^2)/(a_{max}) + (v_{max}a_{max})/(J_{max})$$

where v_{max} is the maximum velocity, a_{max} is the maximum acceleration and J_{max} is the maximum jerk. Let x^r be the desired travel distance, then the mathematical expression of the desired acceleration/deceleration trajectory is given by

$$\ddot{x}^d(t) = \begin{cases} J_{max}t & \text{if} 0 \leq t < t_1 \\ a_{max} & \text{if} t_1 \leq t < t_2 \\ -J_{max}(t-t_3) & \text{if} t_2 \leq t < t_3 \\ 0 & \text{if} t_3 \leq t < t_4 \\ -J_{max}(t-t_4) & \text{if} t_4 \leq t < t_5 \\ -a_{max} & \text{if} t_5 \leq t < t_6 \\ J_{max}(t-t_f) & \text{if} t_6 \leq t < t_f \end{cases} \quad (A.1)$$

where the time for each sub-interval are

$$t_1 = \frac{a_{max}}{J_{max}}$$
$$t_2 = t_1 + \frac{v_{max}}{a_{max}} - \frac{a_{max}}{J_{max}}$$
$$t_3 = t_2 + \frac{a_{max}}{J_{max}}$$
$$t_4 = t_3 + \frac{x^r}{v_{max}} - \frac{v_{max}}{a_{max}} - \frac{a_{max}}{J_{max}}$$
$$t_5 = t_4 + \frac{a_{max}}{J_{max}}$$
$$t_6 = t_5 + \frac{v_{max}}{a_{max}} - \frac{a_{max}}{J_{max}}$$
$$t_f = t_6 + \frac{a_{max}}{J_{max}}$$

The desired velocity and desired position then can be calculated accordingly from (A.1), i.e.

$$\dot{x}^d(t) = \int \ddot{x}^d(t)dt$$
$$x^d(t) = \int \dot{x}^d(t)dt$$

With boundary conditions $x^d(0) = 0$, $\dot{x}^d(0) = 0$, $x^d(t_f) = x^r$ and $\dot{x}^d(t_f) = 0$.
$(x^d(0), \dot{x}^d(0)) = (0, 0)$ and $(x^d(t_f), \dot{x}^d(t_f)) = (x^r, 0)$.