A Design Method for Simple Multi-Period Repetitive Controllers for Multiple-Input/Multiple-Output Plants

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ABSTRACT

The multi-period repetitive control system is a type of servomechanism for a periodic reference input. Even if a plant does not include time-delays, using multi-period repetitive controllers, the transfer function from the periodic reference input to the output and that from the disturbance to the output of the multi-period repetitive control system generally have infinite numbers of poles. To specify the input-output characteristic and the disturbance attenuation characteristic easily, Yamada and Takenaga proposed the concept of simple multi-period repetitive control systems, such that the controller works as a stabilizing multi-period repetitive controller and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. In addition, the parameterization of all stabilizing simple multi-period repetitive controllers was clarified. However, Yamada and Takenaga did not clarify the parameterization of all stabilizing simple multi-period repetitive controllers for multiple-input/multiple-output plants. The purpose of this paper is to propose the parameterization of all stabilizing simple multi-period repetitive controllers for multiple-input/multiple-output plants.

Keywords: Periodic Signal, Multi-Period Repetitive Controller, Finite Number of Poles, Parameterization, Multiple-input/multiple-output Plant

1. INTRODUCTION

A modified repetitive control system is a type of servomechanism for a periodic reference input, i.e., it follows a periodic reference input without steady state error, even when there exists a periodic disturbance or an uncertainty of a plant [1–10].

However, the modified repetitive control system has a bad effect on the disturbance attenuation characteristic [11], in that at certain frequencies, the sensitivity to disturbances of a control system with a conventional repetitive controller becomes twice as worse as that of a control system without a repetitive controller. Gotou et al. overcame this problem by proposing a multi-period repetitive control system [11]. However, the phase angle of the low-pass filter in a multi-period repetitive controller has a bad effect on the disturbance attenuation characteristics [12, 13]. Yamada et al. overcame this problem and proposed a design method for multi-period repetitive controllers to attenuate disturbances effectively [14, 15] using the time advance compensation described in [12, 13, 16]. Using this multi-period repetitive control structure, Steinbuch proposed a design method for repetitive control systems with uncertain period time [17].

On the other hand, there is an important control problem of finding all stabilizing controllers, named the parameterization problem [18–22]. The parameterization of all stabilizing multi-period repetitive controllers was solved in [23, 24]. However, when we design stabilizing multi-period repetitive controllers using the parameterization in [23, 24], the input-output frequency characteristics of the control system cannot be determined easily. From a practical point of view, the input-output frequency characteristics of a control system must be determined easily. Satoh et al. proposed the parameterization of all stabilizing multi-period repetitive controllers with specified input-output frequency characteristics [25, 26].

Using the multi-period repetitive controllers in [11, 14, 15, 23–26], even if the plant does not include time delays, the transfer function from the periodic reference input to the output and that from the disturbance to the output have infinite numbers of poles. In this situation, it is difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From a practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic are easy to determine. To do this, the transfer function from the periodic reference input to the output and that from the disturbance to the output should have finite numbers of poles. If we can design multi-period repetitive control systems where these transfer func-

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tions have finite numbers of poles, then they will become more widely used controller structures, like the Smith predictor [27] for time-delay plants. From this viewpoint, Yamada and Takenaga [28] proposed such multi-period repetitive controller, named simple multi-period repetitive controller, and clarified the parameterization of all stabilizing simple multi-period repetitive controllers for single-input/single-output plants. However, the method in [28] cannot be applied for multiple-input/multiple-output plants, since it uses the characteristic of single-input/single-output systems. Almost plants have multiple-input and multiple-output. In addition, the parameterization is effective for designing a stabilizing controller. Therefore the problem to obtain the parameterization of all stabilizing simple multi-period repetitive controllers for multiple-input/multiple-output plants is one of considerable problems.

In this paper, we propose the concept of a simple multi-period repetitive control system for multiple-input/multiple-output plants, such that the controller works as a multi-period repetitive controller and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. In addition, the parameterization of all stabilizing simple multi-period repetitive controllers for multiple-input/multiple-output plants is clarified. This paper is organized as follows. In Section 2., the concept of the simple multi-period repetitive controller for multiple-input/multiple-output plants is proposed and the problem considered in this paper is described. In Section 3, the parameterization of all stabilizing simple multi-period repetitive controllers for multiple-input/multiple-output plants is clarified. In Section 4, control characteristics using the parameterization of all stabilizing simple multi-period repetitive controllers for multiple-input/multiple-output plants are derived. In Section 5., we present a design procedure for stabilizing simple multi-period repetitive controllers for multiple-input/multiple-output plants. In Section 6., we show a numerical example to illustrate the effectiveness of our proposed method. Section 7gives concluding remarks.

Notation

R	the set of real numbers.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real rational functions.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	represents the state space description $C(sI-A)^{-1}B+D$.
diag $\{\ a_1 \ \ \cdots \ \ a_n\ \}$	an $n \times n$ diagonal matrix with a_i as its i-th diagonal element.
$\bar{\sigma}\{\cdot\}$	the maximum singular value of $\{\cdot\}$.
A^T	transposed matrix of A.

2. SIMPLE MULTI-PERIOD REPETITIVE CONTROL SYSTEMS FOR MULTIPLE-INPUT/MULTIPLE-OUTPUT PLANTS AND PROBLEM FORMULATION

Consider the unity-feedback control system given by

$$\left\{\begin{array}{ll} y & = G(s)u + d \\ u & = C(s)(r-y) \end{array}\right. , \tag{1}$$

where $G(s) \in R^{p\times p}(s)$ is the strictly proper plant satisfying

$$\text{rank } G(s) = p, \tag{2}$$

$C(s)$ is the controller, $u \in R^p$ is the control input, $y \in R^p$ is the output, $d \in R^p$ is the disturbance and $r \in R^p$ is the periodic reference input with period $T > 0$ satisfying

$$r(t+T) = r(t) \ \ (\forall t \geq 0). \tag{3}$$

According to [11, 14, 15, 23–26], the multi-period repetitive controller $C(s)$ in (1) is written in the form

$$\begin{aligned} C(s) \ \ = \ \ & C_0(s) + \sum_{i=1}^{N} C_i(s) e^{-sT_i} \\ & \left(I - \sum_{i=1}^{N} q_i(s) e^{-sT_i}\right)^{-1}, \end{aligned} \tag{4}$$

where N is an arbitrary positive integer, $C_0(s) \in R^{p\times p}(s)$, $C_i(s) \in R^{p\times p}(s)(i=1,\ldots,N)$, $q_i(s) \in R^{p\times p}(s)(i=1,\ldots,N)$ are low-pass filters satisfying

$$\sum_{i=1}^{N} q_i(0) = I \tag{5}$$

and $T_i > 0 \in R(i=1,\ldots,N)$. Without loss of generality, we assume that

$$\text{rank } C_i(s) = p(i=1,\ldots,N) \tag{6}$$

and

$$\text{rank } q_i(s) = p(i=1,\ldots,N). \tag{7}$$

If the low-pass filters $q_i(s)(i=1,\ldots,N)$ satisfy

$$\bar{\sigma}\left\{I - \sum_{i=1}^{N} q_i(j\omega_k)\right\} \simeq 0 \quad (k=0,\ldots,N_{max}), \tag{8}$$

where $\omega_k(k=0,\ldots,N_{max})$ are frequency components of the periodic reference input r written by

$$\omega_k = \frac{2\pi}{T}k \quad (k=0,\ldots,N_{max}), \tag{9}$$

and $\omega_{N_{max}}$ is the maximum frequency component of the periodic reference input r, then the output y in (1) follows the periodic reference input r with a small steady state error.

Using the multi-period repetitive controller $C(s)$ in (4), the transfer function from the periodic reference input r to the output y and that from the disturbance d to the output y in (1) are written as

$$\begin{aligned} y &= (I + G(s)C(s))^{-1} G(s)C(s)r \\ &= G(s)\left\{C_0(s) + \sum_{i=1}^{N} (C_i(s) - C_0(s)q_i(s))\, e^{-sT_i}\right\} \\ &\quad \left[I + G(s)C_0(s) - \sum_{i=1}^{N} \{(I + G(s)C_0(s))\, q_i(s) \right. \\ &\quad \left. -G(s)C_i(s)\}\, e^{-sT_i}\right]^{-1} r \end{aligned} \tag{10}$$

and

$$\begin{aligned} y &= (I + G(s)C(s))^{-1} d \\ &= \left(I - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right) \\ &\quad \left[I + G(s)C_0(s) - \sum_{i=1}^{N} \{(I + G(s)C_0(s))\, q_i(s) \right. \\ &\quad \left. -G(s)C_i(s)\}\, e^{-sT_i}\right]^{-1} d, \end{aligned} \tag{11}$$

respectively. In general, transfer functions from the periodic reference input r to the output y in (10) and from the disturbance d to the output y in (11) have infinite numbers of poles. In this situation, it is difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From a practical point of view, it is desirable that these characteristics are easily specified. To do so, the transfer function from the periodic reference input r to the output y and that from the disturbance d to the output y should have finite numbers of poles.

From the above practical requirement, we propose the concept of a simple multi-period repetitive controller for multiple-input/multiple-output plants, defined as follows.

Definition 1. *(simple multi-period repetitive controller for multiple-input/multiple-output plants)*
We call the controller $C(s)$ a "simple multi-period repetitive controller for multiple-input/multiple-output plants" if the following expressions hold true:
1. The controller $C(s)$ works as a multi-period repetitive controller for multiple-input/multiple-output plants. That is, the controller $C(s)$ is written by (4), where $C_0(s) \in R^{p\times p}(s)$, $C_i(s) \in R^{p\times p}(i = 1,\dots,N)$, $\text{rank}\, C_i(s) = p(i = 1,\dots,N)$ and $q_i(s) \in R^{p\times p}(s)(i = 1,\dots,N)$ satisfies $\sum_{i=1}^{N} q_i(0) = I$ and $\text{rank}\, q_i(s) = p(i = 1,\dots,N)$.
2. The controller $C(s)$ defines transfer functions from the periodic reference input r to the output y in (1) and from the disturbance d to the output y in (1) have finite numbers of poles.

The problem considered in this paper is to clarify the parameterization of all stabilizing simple multi-period repetitive controllers for multiple-input/multiple-output plants. That is, we find all controllers $C(s)$ written in the form in (4) that allow the required transfer functions to be written as

$$y = \left(\bar{G}_{r0}(s) + \sum_{i=1}^{N} \bar{G}_{ri}(s)e^{-sT_i}\right) r \tag{12}$$

and

$$y = \left(\bar{G}_{d0}(s) + \sum_{i=1}^{N} \bar{G}_{di}(s)e^{-sT_i}\right) d, \tag{13}$$

respectively, where $\bar{G}_{ri}(s) \in RH_\infty(i = 0,\dots,N)$ and $\bar{G}_{di}(s) \in RH_\infty(i = 0,\dots,N)$.

3. THE PARAMETERIZATION

In this section, we clarify the parameterization of all stabilizing simple multi-period repetitive controllers for multiple-input/multiple-output plants.

The parameterization of all stabilizing simple multi-period repetitive controllers for multiple-input/multiple-output plants is summarized in the following theorem.

Theorem 1. *The controller $C(s)$ is a stabilizing simple multi-period repetitive controller if and only if $C(s)$ is written by*

$$\begin{aligned} C(s) &\\ = &\left\{\tilde{X}(s) + D(s)\left(Q(s) + \sum_{i=1}^{N} \bar{Q}_i(s)e^{-sT_i}\right)\right\} \\ &\left\{\tilde{Y}(s) - N(s)\left(Q(s) + \sum_{i=1}^{N} \bar{Q}_i(s)e^{-sT_i}\right)\right\}^{-1}. \end{aligned} \tag{14}$$

Here $N(s) \in RH_\infty$, $D(s) \in RH_\infty$, $\tilde{N}(s) \in RH_\infty$ and $\tilde{D}(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = N(s)D^{-1}(s) = \tilde{D}^{-1}(s)\tilde{N}(s). \tag{15}$$

$X(s) \in RH_\infty$, $Y(s) \in RH_\infty$, $\tilde{X}(s) \in RH_\infty$ and $\tilde{Y}(s) \in RH_\infty$ are functions satisfying

$$\begin{aligned} &\begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{D}(s) \end{bmatrix} \begin{bmatrix} D(s) & -\tilde{X}(s) \\ N(s) & \tilde{Y}(s) \end{bmatrix} \\ &= I \\ &= \begin{bmatrix} D(s) & -\tilde{X}(s) \\ N(s) & \tilde{Y}(s) \end{bmatrix} \begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{D}(s) \end{bmatrix}. \end{aligned} \tag{16}$$

$Q(s) \in RH_\infty$ and $\bar{Q}_i(s) \in RH_\infty (i = 1, \ldots, N)$ are any functions satisfying

$$\sum_{i=1}^{N} N(0)\bar{Q}_i(0)\left(\tilde{Y}(0) - N(0)Q(0)\right)^{-1} = I \qquad (17)$$

and

$$\mathrm{rank}\ \bar{Q}_i(s) = p(i = 1, \ldots, N). \qquad (18)$$

Note 1. *The coprime factorization of $G(s)$ satisfying (16) is called doubly coprime factorization. According to [22], any $G(s)$ has doubly coprime factorization.*

Proof of this theorem requires the following lemma.

Lemma 1. *Unity feedback control system in (1) is stable if and only if the controller $C(s)$ is written by*

$$\begin{aligned} C(s) &= \left(\tilde{X}(s) + D(s)Q(s)\right)\left(\tilde{Y}(s) - N(s)Q(s)\right)^{-1} \\ &= \left(Y(s) - Q(s)\tilde{N}(s)\right)^{-1}\left(X(s) + Q(s)\tilde{D}(s)\right), \end{aligned} \qquad (19)$$

where $N(s) \in RH_\infty$, $D(s) \in RH_\infty$, $\tilde{N}(s) \in RH_\infty$ and $\tilde{D}(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying (15), $X(s) \in RH_\infty$, $Y(s) \in RH_\infty$, $\tilde{X}(s) \in RH_\infty$ and $\tilde{Y}(s) \in RH_\infty$ are functions satisfying (16). $Q(s) \in RH_\infty$ is any function [22].

Using Lemma 1, we shall show the proof of Theorem 1.

Proof: First, the necessity is shown. That is, we show that if the controller $C(s)$ in (4) makes the control system in (1) stable and ensures that the transfer function from r to y of the control system in (1) has finite numbers of poles, then $C(s)$ takes the form (14). From the assumption that the controller $C(s)$ in (4) ensures that the transfer function from r to y of the control system in (1) has finite numbers of poles,

$$\begin{aligned} &(I + G(s)C(s))^{-1} G(s)C(s) \\ &= G(s)\left\{C_0(s) + \sum_{i=1}^{N}\left(C_i(s) - C_0(s)q_i(s)\right)e^{-sT_i}\right\} \\ &\quad \left[I + G(s)C_0(s) - \sum_{i=1}^{N}\left\{\left(I + G(s)C_0(s)\right)q_i(s)\right.\right. \\ &\quad \left.\left. - G(s)C_i(s)\right\}e^{-sT_i}\right]^{-1} \end{aligned} \qquad (20)$$

has finite numbers of poles. This implies that

$$\begin{aligned} C_i(s) &= G^{-1}(s)\left(I + G(s)C_0(s)\right)q_i(s) \\ &\quad (i = 1, \ldots, N) \end{aligned} \qquad (21)$$

are satisfied, that is, $C(s)$ is necessarily

$$\begin{aligned} C(s) &= G^{-1}(s)\left(G(s)C_0(s) + \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right) \\ &\quad \left(I - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right)^{-1}. \end{aligned} \qquad (22)$$

From the assumption that $C(s)$ in (4) makes the control system in (1) stable, $(I + G(s)C(s))^{-1}G(s)C(s)$, $(I + C(s)G(s))^{-1}C(s)$, $(I + G(s)C(s))^{-1}G(s)$ and $(I + G(s)C(s))^{-1}$ are stable. From simple manipulations and (22), we have

$$\begin{aligned} &(I + G(s)C(s))^{-1} G(s)C(s) \\ &= \left(G(s)C_0(s) + \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right) \\ &\quad (I + G(s)C_0(s))^{-1}, \end{aligned} \qquad (23)$$

$$\begin{aligned} &(I + C(s)G(s))^{-1} C(s) \\ &= G^{-1}(s)\left(G(s)C_0(s) + \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right) \\ &\quad (I + G(s)C_0(s))^{-1}, \end{aligned} \qquad (24)$$

$$\begin{aligned} &(I + G(s)C(s))^{-1} G(s) \\ &= \left(I - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right) G(s)\left(I + C_0(s)G(s)\right)^{-1} \end{aligned} \qquad (25)$$

and

$$\begin{aligned} &(I + G(s)C(s))^{-1} \\ &= \left(I - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right)\left(I + G(s)C_0(s)\right)^{-1}. \end{aligned} \qquad (26)$$

From the assumption that all transfer functions in (23), (24), (25) and (26) are stable, $(I + G(s)C_0(s))^{-1}G(s)C_0(s)$, $(I + C_0(s)G(s))^{-1}C_0(s)$, $(I+G(s)C_0(s))^{-1}G(s)$ and $(I+G(s)C_0(s))^{-1}$ are stable. This means that $C_0(s)$ is a stabilizing controller for $G(s)$. From Lemma 1, $C_0(s)$ must take the form

$$\begin{aligned} C_0(s) &= \left(\tilde{X}(s) + D(s)Q(s)\right)\left(\tilde{Y}(s) - N(s)Q(s)\right)^{-1} \\ &= \left(Y(s) - Q(s)\tilde{N}(s)\right)^{-1}\left(X(s) + Q(s)\tilde{D}(s)\right), \end{aligned} \qquad (27)$$

where $Q(s) \in RH_\infty$.

From the assumption that transfer functions in (23), (25) and (26) are stable, $q_i(s)(I + G(s)C_0(s))^{-1}(i = 1, \ldots, N)$ are stable. This implies that unstable poles of $q_i(s)$ are included in those of $C_0(s)$. That is, $q_i(s)(i = 1, \ldots, N)$ are written by

$$\begin{aligned} q_i(s) &= \hat{Q}_i(s)\left(\tilde{Y}(s) - N(s)Q(s)\right)^{-1} \\ &\quad (i = 1, \ldots, N), \end{aligned} \qquad (28)$$

where $\hat{Q}_i(s) \in RH_\infty(i = 1, \ldots, N)$ and rank $\hat{Q}_i(s) = p(i = 1, \ldots, N)$, because $q_i(s) \in RH_\infty(i = 1, \ldots, N)$ and rank $q_i(s) = p$.

Since the transfer function in (24) is stable, $G^{-1}(s)q_i(s)\left(I + G(s)C_0(s)\right)^{-1}$ $(i = 1, \ldots, N)$ are stable. From (27) and (28), we have

$$\begin{aligned} & G^{-1}(s)q_i(s)\left(I + G(s)C_0(s)\right)^{-1} \\ &= D(s)N^{-1}(s)\hat{Q}_i(s)\tilde{D}(s). \end{aligned} \tag{29}$$

From the assumption that $N(s)$, $D(s)$ and $\tilde{D}(s)$ are coprime and $G^{-1}(s)q_i(s)\left(I + G(s)C_0(s)\right)^{-1}$ $(i = 1, \ldots, N)$ in (29) are stable, $\hat{Q}_i(s)(i = 1, \ldots, N)$ is written in the form

$$\hat{Q}_i(s) = N(s)\bar{Q}_i(s) \ \ (i = 1, \ldots, N), \tag{30}$$

where $\bar{Q}_i(s) \in RH_\infty(i = 1, \ldots, N)$ and rank $\bar{Q}_i(s) = p(i = 1, \ldots, N)$, because rank $\hat{Q}_i(s) = p(i = 1, \ldots, N)$ in (28). Substituting (21), (27), (28) and (30) for (4), we have (14). In this way, it is shown that if the controller $C(s)$ in (4) makes the control system in (1) stable, and ensures that the transfer function from r to y of the control system in (1) has finite numbers of poles, then $C(s)$ is written as (14). In addition, because $\sum_{i=1}^{N} q_i(0) = I$ holds true, from (28) and (30), (17) is satisfied. Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, it is shown that if $C(s)$ takes the form (14), then the controller $C(s)$ makes the control system in (1) stable and ensures the transfer function from r to y of the control system in (1) has finite numbers of poles, works as a stabilizing multi-period repetitive controller and $q_i(s)(i = 1, \ldots, N)$ in (5) satisfy $\sum_{i=1}^{N} q_i(0) = I$. After simple manipulations, we have

$$\begin{aligned} & \left(I + G(s)C(s)\right)^{-1} G(s)C(s) \\ &= I + \left(N(s)Q(s) - \tilde{Y}(s) + N(s)\sum_{i=1}^{N}\bar{Q}_i(s)e^{-sT_i}\right) \\ & \quad \tilde{D}(s), \end{aligned} \tag{31}$$

$$\begin{aligned} & \left(I + C(s)G(s)\right)^{-1} C(s) \\ &= \left(\tilde{X}(s) + D(s)Q(s) + D(s)\sum_{i=1}^{N}\bar{Q}_i(s)e^{-sT_i}\right) \\ & \quad \tilde{D}(s), \end{aligned} \tag{32}$$

$$\begin{aligned} & \left(I + G(s)C(s)\right)^{-1} G(s) \\ &= \left(\tilde{Y}(s) - N(s)Q(s) - N(s)\sum_{i=1}^{N}\bar{Q}_i(s)e^{-sT_i}\right) \\ & \quad \tilde{N}(s) \end{aligned} \tag{33}$$

and

$$\begin{aligned} & \left(I + G(s)C(s)\right)^{-1} \\ &= \left(\tilde{Y}(s) - N(s)Q(s) - N(s)\sum_{i=1}^{N}\bar{Q}_i(s)e^{-sT_i}\right) \\ & \quad \tilde{D}(s). \end{aligned} \tag{34}$$

Since $\tilde{X}(s) \in RH_\infty$, $\tilde{Y}(s) \in RH_\infty$, $N(s) \in RH_\infty$, $D(s) \in RH_\infty$, $\tilde{N}(s) \in RH_\infty$, $\tilde{D}(s) \in RH_\infty$, $Q(s) \in RH_\infty$ and $\bar{Q}_i(s) \in RH_\infty(i = 1, \ldots, N)$, transfer functions in (31), (32), (33) and (34) are stable. In addition, from the same reason, the transfer function from r to y of the control system in (1) has finite numbers of poles.

Next we show that the controller $C(s)$ in (14) works as a multi-period repetitive controller. The controller $C(s)$ in (14) is rewritten by the form in (4), where

$$\begin{aligned} C_0(s) &= \left(\tilde{X}(s) + D(s)Q(s)\right) \\ & \quad \left(\tilde{Y}(s) - N(s)Q(s)\right)^{-1}, \end{aligned} \tag{35}$$

$$\begin{aligned} C_i(s) &= \left(Y(s) - Q(s)\tilde{N}(s)\right)^{-1}\bar{Q}_i(s) \\ & \quad \left(\tilde{Y}(s) - N(s)Q(s)\right)^{-1} \\ & \quad (i = 1, \ldots, N) \end{aligned} \tag{36}$$

and

$$\begin{aligned} q_i(s) &= N(s)\bar{Q}_i(s)\left(\tilde{Y}(s) - N(s)Q(s)\right)^{-1} \\ & \quad (i = 1, \ldots, N). \end{aligned} \tag{37}$$

From the assumption that rank $\bar{Q}_i(s) = p(i = 1, \ldots, N)$ and (36), rank $C_i(s) = p(i = 1, \ldots, N)$ hold true. In addition, from (37) and the assumption in (17), $\sum_{i=1}^{N} q_i(0) = I$ is satisfied. These expressions imply that the controller $C(s)$ in (14) works as a multi-period repetitive controller. Thus, the sufficiency has been shown.

We have thus proved Theorem 1.

4. CONTROL CHARACTERISTICS

In this section, we describe control characteristics of control system in (1) using the stabilizing simple multi-period repetitive controller for multiple-input/multiple-output plants in (14).

First, we mention the input-output characteristic. The transfer function from the periodic reference input r to the error $e = r - y$ is written by

$$\begin{aligned} e &= \left\{I - \sum_{i=1}^{N} N(s)\bar{Q}_i(s)\right. \\ & \quad \left.\left(\tilde{Y}(s) - N(s)Q(s)\right)^{-1} e^{-sT_i}\right\} \\ & \quad \left(\tilde{Y}(s) - N(s)Q(s)\right)\tilde{D}(s)r. \end{aligned} \tag{38}$$

From (38), for $\omega_k (k = 0, \ldots, N_{max})$ in (9) those are frequency components of the periodic reference input r, if

$$\bar{\sigma}\left\{ I - \sum_{i=1}^{N} N(j\omega_k)\bar{Q}_i(j\omega_k) \left(\tilde{Y}(j\omega_k) - N(j\omega_k)Q(j\omega_k)\right)^{-1} e^{-j\omega_k T_i} \right\} \simeq 0 \ \ (k = 0, \ldots, N_{max}), \quad (39)$$

then the output y follows the periodic reference input r with small steady state error.

Next, we mention the disturbance attenuation characteristic. The transfer function from the disturbance d to the output y is written by

$$y = \left\{ I - \sum_{i=1}^{N} N(s)\bar{Q}_i(s) \left(\tilde{Y}(s) - N(s)Q(s)\right)^{-1} e^{-sT_i} \right\} \left(\tilde{Y}(s) - N(s)Q(s)\right)\tilde{D}(s)d. \quad (40)$$

From (40), for frequency components $\omega_k (k = 0, \ldots, N_{max})$ in (9) of the disturbance d those are same to those of the periodic reference input r, if (39) is satisfied, then the disturbance d is attenuated effectively. For the frequency component ω of the disturbance d that is different from that of the periodic reference input r, that is $\omega \neq \omega_k (\forall k = 0, \ldots, N_{max})$, even if

$$\bar{\sigma}\left\{ I - \sum_{i=1}^{N} N(j\omega)\bar{Q}_i(j\omega) \left(\tilde{Y}(j\omega) - N(j\omega)Q(j\omega)\right)^{-1} \right\} \simeq 0, \quad (41)$$

the disturbance d cannot be attenuated, because

$$e^{-j\omega T_i} \neq 1 \ \ (i = 1, \ldots, N) \quad (42)$$

and

$$\bar{\sigma}\left\{ I - \sum_{i=1}^{N} N(j\omega)\bar{Q}_i(j\omega) \left(\tilde{Y}(j\omega) - N(j\omega)Q(j\omega)\right)^{-1} e^{-j\omega T_i} \right\} \neq 0. \quad (43)$$

In order to attenuate the frequency component ω of the disturbance d that is different from that of the periodic reference input r, we need to settle $Q(s)$ satisfying

$$\bar{\sigma}\left\{ \tilde{Y}(j\omega) - N(j\omega)Q(j\omega) \right\} \simeq 0. \quad (44)$$

From above discussion, we find that the role of $\bar{Q}_i(s) (i = 1, \ldots, N)$ is to specify the input-output characteristic for the periodic reference input r and to specify the disturbance attenuation characteristic for the frequency component of the disturbance d that is equivalent to that of the periodic reference input r and that of $Q(s)$ is to specify the disturbance attenuation characteristic for the frequency component of the disturbance d that is different from that of the periodic reference input r.

5. DESIGN PROCEDURE

In this section, a design procedure of a stabilizing simple multi-period repetitive controller for multiple-input/multiple-output plants is presented.

A design procedure of simple multi-period repetitive controllers satisfying Theorem 1 is summarized as follows:

<u>Procedure</u>

Step 1) Obtain coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ of $G(s)$ satisfying (15).
Step 2) $\tilde{X}(s) \in RH_\infty$ and $\tilde{Y}(s) \in RH_\infty$ are settled satisfying (16).
Step 3) $Q(s) \in RH_\infty$ is settled so that for the frequency component ω of the disturbance d, $\bar{\sigma}\{(\tilde{Y}(j\omega) - N(j\omega)Q(j\omega))\tilde{D}(j\omega)\}$ is effectively small. In order to design $Q(s)$ to make $\bar{\sigma}\{(\tilde{Y}(j\omega) - N(j\omega)Q(j\omega))\tilde{D}(j\omega)\}$ is effectively small, $Q(s)$ is settled by

$$Q(s) = N_o^{-1}(s)\bar{q}_d(s)\tilde{Y}(s), \quad (45)$$

where $N_o(s) \in RH_\infty$ is an outer function of $N(s)$ satisfying

$$N(s) = N_i(s)N_o(s), \quad (46)$$

$N_i(s) \in RH_\infty$ is an inner function satisfying $N_i(0) = I$ and $\bar{\sigma}\{N_i(j\omega)\} = 1 (\forall \omega \in R_+)$, $\bar{q}_d(s)$ is a low-pass filter satisfying $\bar{q}_d(0) = I$, as

$$\bar{q}_d(s) = \mathrm{diag}\left\{ \frac{1}{(1 + s\tau_{d1})^{\alpha_{d1}}}, \cdots, \frac{1}{(1 + s\tau_{dp})^{\alpha_{dp}}} \right\} \quad (47)$$

is valid, $\alpha_{di} (i = 1, \ldots, p)$ are arbitrary positive integers to make $N_o^{-1}(s)\bar{q}_d(s)$ proper and $\tau_{di} \in R (i = 1, \ldots, p)$ are any positive real numbers satisfying

$$\bar{\sigma}\left[I - N_i(j\omega)\mathrm{diag}\left\{ \frac{1}{(1 + j\omega\tau_{d1})^{\alpha_{d1}}}, \cdots, \frac{1}{(1 + j\omega\tau_{dp})^{\alpha_{dp}}} \right\} \right] \simeq 0. \quad (48)$$

Step 4) $\bar{Q}_i(s) \in RH_\infty (i = 1, \ldots, N)$ are settled so that for frequency components $\omega_k (k = 0, \ldots, N_{max})$

of the periodic reference input r,

$$\bar{\sigma}\left\{I-\sum_{i=1}^{N}N(j\omega_k)\bar{Q}_i(j\omega_k)\left(\tilde{Y}(j\omega_k)-N(j\omega_k)Q(j\omega_k)\right)^{-1}\right\}\simeq 0 \quad (k=0,\dots,N_{max}) \tag{49}$$

is satisfied. In order to satisfy (49), $\bar{Q}_i(s)\in RH_\infty(i=1,\dots,N)$ are settled by

$$\bar{Q}_i(s) = N_o^{-1}(s)\bar{q}_{ri}(s)\left(\tilde{Y}(s)-N(s)Q(s)\right) \quad (i=1,\dots,N), \tag{50}$$

where $\bar{q}_{ri}(s)(i=1,\dots,N)$ are low-pass filters satisfying $\sum_{i=1}^{N}\bar{q}_{ri}(0)=I$, as

$$\bar{q}_{ri}(s) = \frac{1}{N}\text{diag}\left\{\frac{1}{(1+s\tau_{r1})^{\alpha_{r1}}},\cdots,\frac{1}{(1+s\tau_{rp})^{\alpha_{rp}}}\right\} \quad (i=1,\dots,N) \tag{51}$$

are valid, $\alpha_{ri}(i=1,\dots,p)$ are arbitrary positive integers to make $N_o^{-1}(s)\bar{q}_{ri}(s)$ proper and $\tau_{ri}\in R(i=1,\dots,p)$ are any positive real numbers satisfying

$$\bar{\sigma}\left[I-N_i(j\omega_k)\sum_{i=1}^{N}\frac{1}{N}\text{diag}\left\{\frac{1}{(1+j\omega_k\tau_{r1})^{\alpha_{r1}}},\cdots,\frac{1}{(1+j\omega_k\tau_{rp})^{\alpha_{rp}}}\right\}\right]\simeq 0(k=0,\dots,N_{max}). \tag{52}$$

6. NUMERICAL EXAMPLE

In this section, a numerical example is illustrated to show the effectiveness of the proposed method.

Consider the problem to obtain the parameterization of all stabilizing simple multi-period repetitive controllers for the plant $G(s)$ written by

$$G(s)=\begin{bmatrix}\frac{s+60}{(s-2)(s-3)} & \frac{10}{(s-2)(s-3)}\\ \frac{s+100}{(s-2)(s-3)} & \frac{s+70}{(s-2)(s-3)}\end{bmatrix} \tag{53}$$

that follows the periodic reference input $r=[r_1,r_2]^T$ with period $T=2$[sec]. N in (4) and $T_i(i=1,\dots,N)$ are chosen as $N=3$ and

$$T_i=T\cdot i \quad (i=1,2,3), \tag{54}$$

respectively. A pair of coprime factors $N(s)\in RH_\infty$ and $D(s)\in RH_\infty$ of $G(s)$ in (53) satisfying (15) is given by

$$N(s)=\begin{bmatrix}\frac{s+50}{s^2+105s+2750} & \frac{10}{s^2+85s+1800}\\ \frac{s+100}{s^2+105s+2750} & \frac{s+60}{s^2+85s+1800}\end{bmatrix} \tag{55}$$

and

$$D(s)=\begin{bmatrix}\frac{s^2-5s+6}{s^2+105s+2750} & 0\\ 0 & \frac{s^2-5s+6}{s^2+85s+1800}\end{bmatrix}. \tag{56}$$

$\tilde{X}(s)\in RH_\infty$ and $\tilde{Y}(s)\in RH_\infty$ are settled to satisfy (16).

From Theorem 1, the parameterization of all stabilizing simple multi-period repetitive controllers for $G(s)$ in (53) is given by (14), where $Q(s)\in RH_\infty$ and $\bar{Q}_i(s)\in RH_\infty(i=1,2,3)$ are any functions satisfying (17).

So that the disturbances

$$d=\begin{bmatrix}d_1\\ d_2\end{bmatrix}=\begin{bmatrix}\sin(2\pi t)+\sin(4\pi t)+\sin(6\pi t)\\ 2\sin(2\pi t)+2\sin(4\pi t)+2\sin(6\pi t)\end{bmatrix} \tag{57}$$

and

$$d=\begin{bmatrix}d_1\\ d_2\end{bmatrix}=\begin{bmatrix}\sin\left(\frac{\pi t}{4}\right)+\sin\left(\frac{\pi t}{2}\right)+\sin\left(\frac{3\pi t}{4}\right)\\ 2\sin\left(\frac{\pi t}{4}\right)+2\sin\left(\frac{\pi t}{2}\right)+2\sin\left(\frac{3\pi t}{4}\right)\end{bmatrix} \tag{58}$$

can be attenuated effectively, and for the output $y=[y_1,y_2]^T$ to follow the periodic reference input

$$r=\begin{bmatrix}r_1\\ r_2\end{bmatrix}=\begin{bmatrix}\sin(\pi t)+\sin(2\pi t)+\sin(3\pi t)\\ 2\sin(\pi t)+2\sin(2\pi t)+2\sin(3\pi t)\end{bmatrix} \tag{59}$$

with a small steady state error, $Q(s)$ and $\bar{Q}_i(s)(i=1,2,3)$ are designed using (45) and (50), respectively, where

$$\bar{q}_d(s)=\begin{bmatrix}\frac{1}{0.001s+1} & 0\\ 0 & \frac{1}{0.001s+1}\end{bmatrix}, \tag{60}$$

$$\bar{q}_{ri}(s)=\begin{bmatrix}\frac{1}{3(0.001s+1)} & 0\\ 0 & \frac{1}{3(0.001s+1)}\end{bmatrix} \quad (i=1,2,3), \tag{61}$$

$$N_i(s)=I \tag{62}$$

and

$$N_o(s)=N(s). \tag{63}$$

Using above-mentioned parameters, we have a stabilizing simple multi-period repetitive controller for the

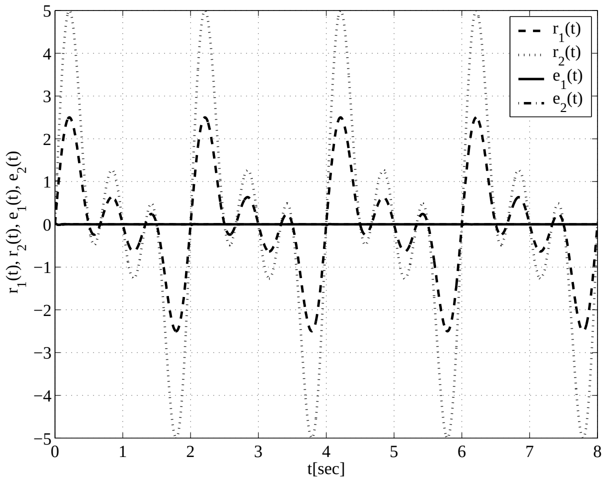

Fig.1: *Response of the error e for the periodic reference input r in (59)*

multiple-input/multiple-output plant. Using the designed stabilizing simple multi-period repetitive controller, the response of the error $e = r - y = [e_1, e_2]^T$ in (1) for the periodic reference input r in (59) is shown in Fig. 1. Here, the broken line shows the response of the periodic reference input r_1, the dotted line shows that of the periodic reference input r_2, the solid line shows that of the error e_1 and the dotted and broken line shows that of the error e_2. Figure 1 shows that the output y follows the periodic reference input r in (59) with small steady state error.

Next, using the designed stabilizing simple multi-period repetitive controller $C(s)$, the disturbance attenuation characteristic is shown. The response of the output $y = [y_1, y_2]^T$ for the disturbance d in (57) of which the frequency component is equivalent to that of the periodic reference input r is shown in Fig. 2. Here, the broken line shows the response of the disturbance d_1, the dotted line shows that of the disturbance d_2, the solid line shows that of the output y_1 and the dotted and broken line shows that of the output y_2. Figure 2 shows that the disturbance d in (57) is attenuated effectively. Finally, the response of the output y for the disturbance d in (58) of which the frequency component is different from that of the periodic reference input r is shown in Fig. 3. Here, the broken line shows the response of the disturbance d_1, the dotted line shows that of the disturbance d_2, the solid line shows that of the output y_1 and the dotted and broken line shows that of the output y_2. Figure 3 shows that the disturbance d in (58) is attenuated effectively.

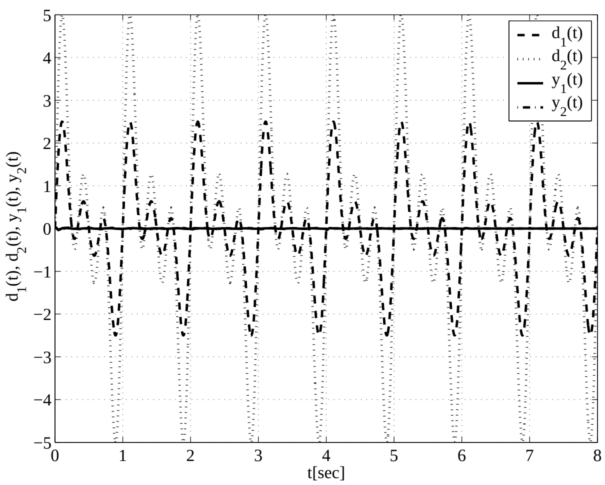

Fig.2: *Response of the output y for the disturbance d in (57)*

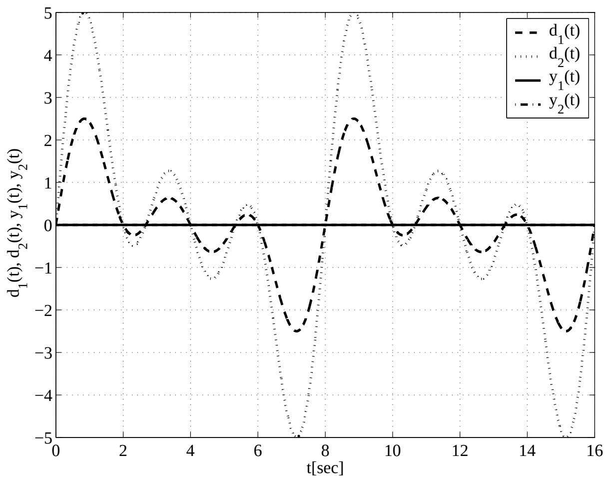

Fig.3: *Response of the output y for the disturbance d in (58)*

In this way, it is shown that we can design a stabilizing simple multi-period repetitive controller easily.

7. CONCLUSIONS

We have proposed the concept of simple multi-period repetitive control systems for multiple-input/multiple-output plants, such that the controller works as a multi-period repetitive controller and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. In addition, the parameterization of all stabilizing simple multi-period repetitive controllers for multiple-input/multiple-output plants was clarified. Design methods for conventional multi-period repetitive control systems [11, 14, 15, 23–26] cannot give control systems where transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles, but our proposed method can. Control characteristics of simple multi-period repetitive control systems and a design procedure for such controllers were also presented. Finally, a numerical example showed the effectiveness of our proposed method. Because the simple multi-period repetitive control system has merits, such as the transfer function from the periodic reference input to the output having finite numbers of poles and it is easy to design, practical applications of this control system are expected.

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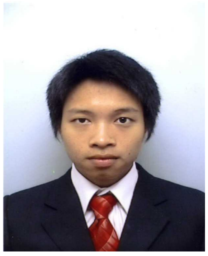

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