Simulation of Surface Mesh Deformation in Orthodontics by Mass-Spring Model

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ABSTRACT

The mass-spring model has been used to describe elastically deformable models such as skin, textiles, and soft tissue in computer graphics. A mass-spring mesh is composed of a network of masses and springs, in which each edge is a spring. We apply the mass-spring system to mesh deformation in 3D orthodontic simulation, the movement of which is evaluated using the numerical integration of the fundamental law of dynamics based on the 4th-order Runge-Kutta method. Computational quantity and accuracy is demonstrated on test and dental cast model examples. The experimental results show that it can simulate the deformation change in real time and display the results vividly.

Keywords: Mass-Spring Model, Model Deformation, Orthodontic Simulation, STL Dental Cast Model

1. INTRODUCTION

Traditionally, orthodontists deal with correcting malocclusion in young patients. The “typical” orthodontic patient is 12 to 14 years old while adults seeking orthodontic treatment for aesthetic reasons alone is very rare. However, it is becoming common nowadays for appearance-conscious adults to place great personal value on the appearance of their smile, and cosmetic dentistry has become increasingly popular among older patients [1].

Orthodontists use cephalometric projections to plan their treatments [2]. Cephalometric projections are x-rays taken of the side of head. However, it is not convenient to store and compare orthodontic patient records of each patient for each treatment period. Therefore, computer technology is used in orthodontics to simulate treatment changes in a 3D space.

Soft tissue holds a large proportion of the whole individual structure, including a great deal of important organs, for example, the heart, skin, muscles and so on. The simulation of soft tissue deformation relates to the fields of medicine, mechanics, biology, computer graphics, and robot vision. In addition, the simulation of soft tissue deformation is widely used in the medicinal simulation system including plastic and musculoskeletal surgery. Modeling of deformable object has two models which are considered the most popular methods. The methods are Finite Element Method (FEM) and Mass-Spring. However, an ideal soft tissue deforming simulation is still a challenging task due to the complex internal structure and surface appearance of the deformable objects. FEM is the most accurate method for simulation, but it hardly satisfies the real-time requirement because of its high complexity and large numbers of parameter definition. Mass-Spring model could have acceptable response-time but results in low accuracy.

This paper develops models of gingival tissue deformation in orthodontics based on Mass-Spring model because we need simulate deformation of tissue in real-time. By simulating physical properties such as tension and rigidity, we can model static shapes exhibited by a wide range of deformation objects. Furthermore, by including physical properties such as mass and damping, we can simulate the dynamics of these objects. The simulation involves numerically solving the partial differential equations that govern the evolving shape of the deformable object and its motion through space. Solving partial differential equations is important to describe the behavior of gingival deformation brought about by tooth movement in the process of rectification that can be simulated by computer. Gingival deformation is the change of gingival shape (soft tissue) caused by the change of tooth (rigid body) position under external force in the process of orthodontic treatment. Realistic simulation of gingival tissue deformation requires an appropriate physics-based model of soft tissue deformation.

Section 2 describes our model for tissue deformation. The model is composed of a network of masses and springs, which can be considered as a variant of elastic models. We give differential equations of motion describing the dynamics behavior of deformable models under the influence of external forces. Section 3 presents implementation mass-spring model to simulate soft tissue deformation. We end with our results, discussion and conclusions.

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2. MASS-SPRING MODEL

2.1 Mesh Representation

In our model, elastically soft tissue is represented by a triangle mesh of m masses, each mass being linked to its neighbors by mass springs of natural length not equal to zero (Figure 1). Each spring's natural length is the distance between masses in the rest position.

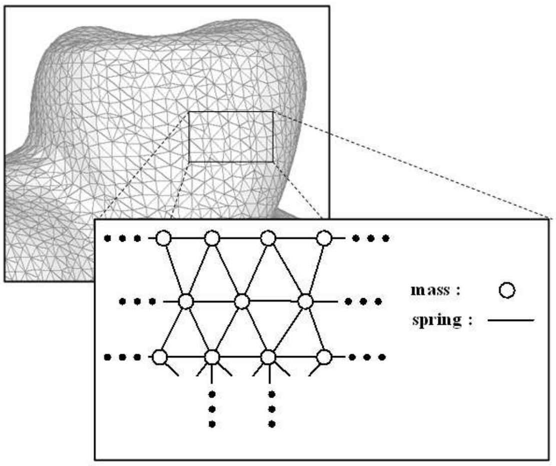

Fig.1: *Mesh of masses and springs used for our model*

2.2 Point Mass

In our mass-spring models, masses are allocated at the vertices of the triangle mesh and damped springs along the edges. To accurately distribute the total mass of the mesh, we compute the mass m_i of each vertex i according to the area $\mathbf{A}_{N(i)}$, where $N(i)$ is the set of indices of triangles adjacent to vertex i. If D is the material density [4], then:

$$m_i = \frac{D \sum A_{N(i)}}{3} \qquad (1)$$

2.3 Spring Stiffness

The easiest way to calculate spring stiffness is to use a constant value for the stiffness (k). More commonly, k is computed as $k = 1/L$, where L is the length of the spring at rest. In [3], Van Gelder suggested a formula to compute spring stiffness for a 3D mesh that is the closest to elastic continuous representation. Let E be the Young's modulus, then:

$$k_{ij} = \frac{E \sum A_{N(i)}}{{l_{ij}}^2} \qquad (2)$$

where:
- k_{ij} is the stiffness of the spring linking point i and point j
- l_{ij} is the length between point i and point j

2.4 Spring Damping

The question of how to assign different damping (c) values to the various springs in a mass-spring system has been largely ignored in the literature. Traditionally, c is treated as a constant throughout the system. We performed the same simulation as before using (1) and (2) to calculate m and k. To simulate the best behavior of our models, we compute spring damping (c) of each spring e followed by equation (3) [4].

$$c_{ij} = \frac{2\sqrt{k(m_i + m_j)}}{l_{ij}} \qquad (3)$$

where:
- c_{ij} is spring damping of the spring linking point i and point j
- l_{ij} is the length between point i and point j

2.5 Applied Forces

The system under study is the triangle mesh of the m masses, each mass being positioned at time t on the point i, where $i = 1, \ldots, m$. The position of each point, at any time t, can be derived through the fundamental law of dynamics. The internal forces are due to the tensions of the springs [5, 6]. The overall internal force applied at the point i at time t is a result of the stiffness of all the springs linking this point to its neighbors:

$$F_i^{int}(t) = -\sum_{j \in R_i} k_{ij} \left(\mathbf{P}_{ij}(t) - l_{ij}^0 \frac{\mathbf{P}_{ij}(t)}{||\mathbf{P}_{ij}(t)||} \right) \qquad (4)$$

where:
- R_i is the set of all couples (j) such as point j is linked by a spring to point i,
- $\mathbf{P}_{ij}(t)$ is position of the spring linking point i and point j which $\mathbf{P}_{ij}(t)$=$\mathbf{P}_i(t)$-$\mathbf{P}_j(t)$,
- $\mathbf{P}_i(t)$ is position of point i at time t,
- l_{ij}^0 is the natural length of the spring linking point i and point j which $l_{ij}^0 = ||\mathbf{P}_{ij}(0)||$,
- k_{ij} is the stiffness of the spring linking point i and point j

The external forces (F_{ext}) are of various natures according to the load to which we wish the model to be exposed. Omnipresent loads will include gravity, a viscous damping and a viscous interaction with an air stream (or wind), but we only applied damping forces (F_{dis}) at time t to simulate our models. The viscous damping will be given by:

$$F_i^{ext} = F_i^{dis}(t) = -c_i v_i(t) \qquad (5)$$

where:
- c_i is all damping coefficients that are computed by equation (3) $c_i = \sum_{j \in R_i} c_{ij}$,

- $v_i(t)$ is its velocity of point i at time t which can be computed following equation (6).

The role of this damping is in fact to model the first approximate dissipation of the mechanical energy of our model.

3. IMPLEMENTATION

In orthodontic treatment simulation, we can extract teeth structure from the gingival surface before teeth alignment. To analyze gingival surface deformation, a deformable model (gingival surface) must be created by animation and the differential equations of motion are simulated numerically. The first step is defining the point force at a point on the gingival surface which is the applied force when teeth are aligned. Finally, the new position of points at time are computed by solving the differential equations.

3.1 Point Force

Point forces are applied forces when teeth are moved. We can define point forces with boundary points of gingival surface connecting to the moved tooth (Figure 2).

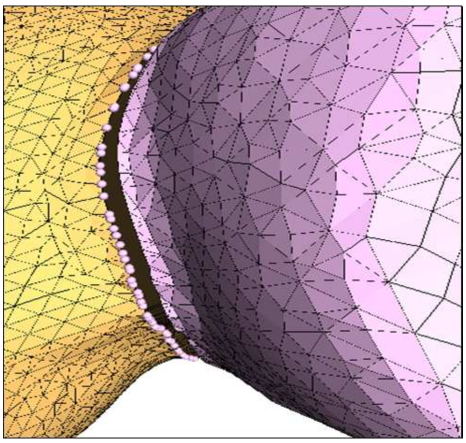

Fig.2: *Point Forces (balls) are contributed on the gingival surface boundary connecting to the moved tooth.*

3.2 Gingival Surface Motivation

The movement of gingival surface is caused by forces of teeth movement applied to each point of the model. The position of each point, at any time t, can be derived through Newton's second law of motion. All the above formulations make it possible to compute the force $F_i(t)$ applied on point P_i at any time t. The fundamental equation of dynamics can therefore be explicitly integrated through time by the simple Euler's method [5,6]:

$$\begin{cases} m_i \dfrac{\partial^2 P_i t}{\partial t^2} = F_i^{int}(t) + F_i^{ext}(t) \\ a_i(t) = \dfrac{F_i^{int}(t) + F_i^{ext}(t)}{m_i} = \dfrac{F_i^{int}(t) - c_i v_i}{m_i} \\ v_i(t) = \dfrac{\partial P_i(t)}{\partial t} \end{cases} \quad (6)$$

We can solve motion equation 6 to evaluate a new position at any time $P_i(t)$. The positions and velocities of the points are computed based on 4^{th} order Runge-Kutta method as follows:

$$\begin{aligned} r_1 &= hf(P_0, t_0) \\ r_2 &= hf(P_0 + k_1/2, t_0 + h/2) \\ r_3 &= hf(P_0 + k_1/2, t_0 + h/2) \\ r_4 &= hf(P_0 + k_3, t_0 + h) \\ P(t_0 + h) &= P_0 + (r_1 + 2r_2 + 2r_3 + r_4)/6 \end{aligned} \quad (7)$$

where:

- r are Runge-Kutta functions,
- $P_i(t)$ is position of point i at any time t,
- $f(P, t)$ are the forces at point at any time t,
- h is the minimum time step.

4. EXPERIMENTAL RESULTS AND DISCUSSION

The algorithm of the technique was implemented under Windows with Borland C++ Builder, using the OpenGL Library for rendering the 3D images. A set of patient's pretreatment models from the Orthodontic Clinic, Advanced Dental Technology Center, was selected. Two different sets were used during the experiments - a mandible model (Figure 3) and a maxilla model (Figure 5). The triangle meshes of two models were generated from the CT scanner. First step, we extract a tooth from gingival surface and then we move the tooth to simulate the gingival deformation. The simulation results of the gingival deformation are shown in the following figures. Figures 4 and 6 are the deformation simulation result for translation and rotation of tooth by arrow direction, respectively. Iterations of each deformable model are shown by Figures 7 and 8. Lastly, the simulated times that include the processing and rendering times are shown in Table 1.

After simulation at 100 iterations, large deformation appears near the point forces, and constraints are sometime not met. But after simulation at 500 iterations, we ensure both a more natural look of the gingival surface and the enforcement of constraints. The simulation takes rather a long time since the 4th order Runge-Kutta method requires many iterations in order to achieve the final result.

5. CONCLUSION

The soft tissue deformation needs to be consistent with the physical properties. In this paper, we apply

Table 1: *Simulation Time*

Num. of Triangles	Frame Rate (msec.)	Num. of Frames/sec
29108	0.56	1769
26198	0.53	1904
23286	0.45	2210
11452	0.29	3395
11642	0.26	3832
5820	0.16	6101

mass-spring system to soft tissue deformation in 3D orthodontic simulation. The process of dynamic deformation can be described effectively after the time variable is introduced. The dynamic motion rule adopts the differential equation form; the numerical method can be carried out for the real-time computation of the dynamic system. The experimental results show that this method satisfies the demand for the computational real-time and the gingival deformation 3D in virtual orthodontics.

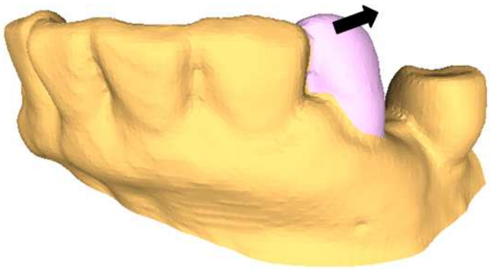

Fig.3: *Original mandible model.*

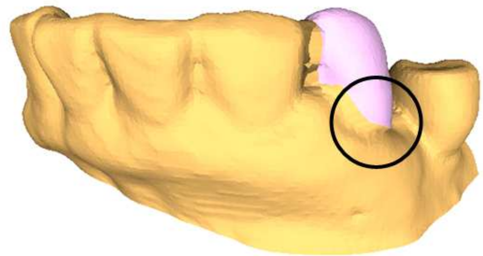

Fig.4: *Gingival deformation of mandible model.*

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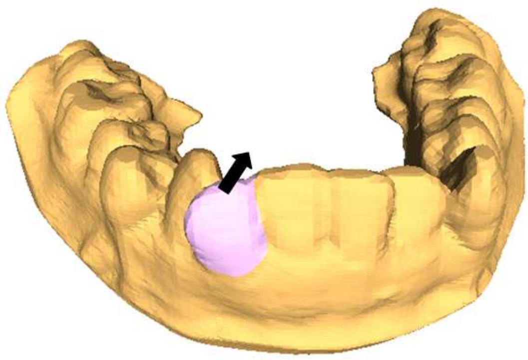

Fig.5: *Original maxilla model.*

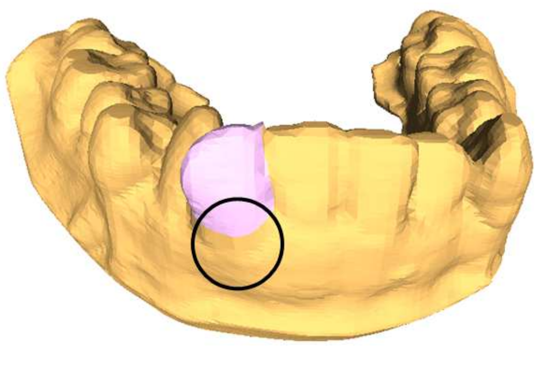

Fig.6: *Gingival deformation of maxilla model.*

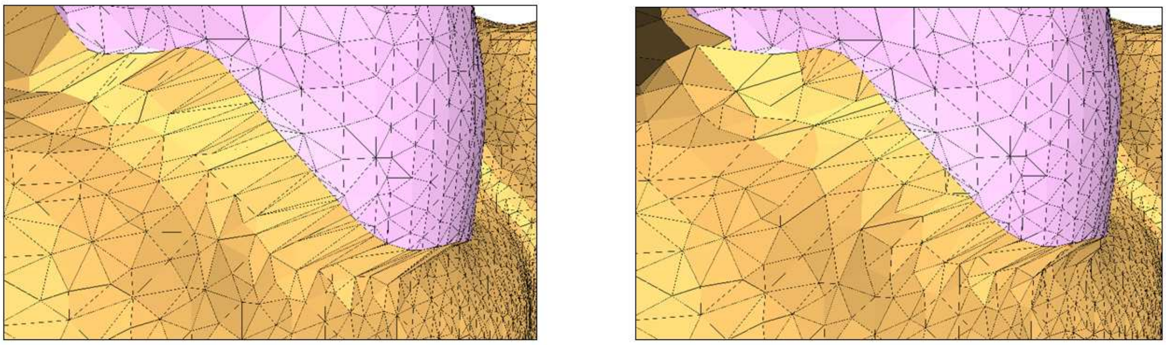

Fig.7: *Simulation at 100 iterations (left) and 500 iterations (right) of mandible.*

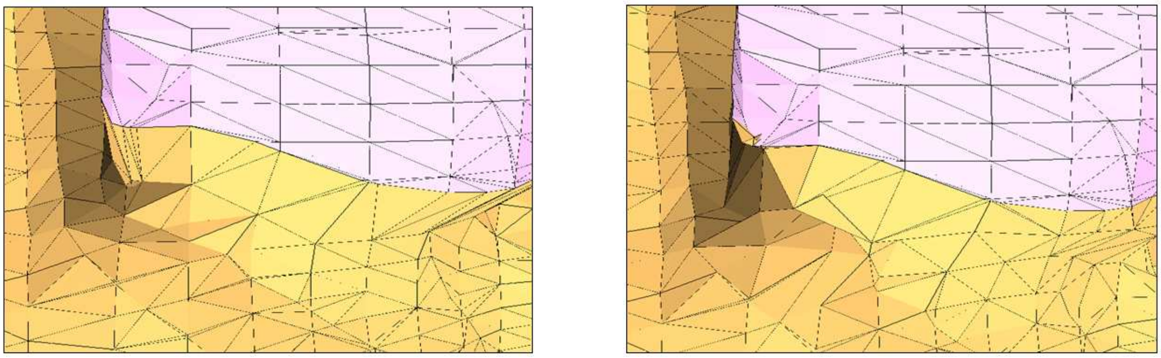

Fig.8: *Simulation at 100 iterations (left) and 500 iterations (right) of maxilla.*

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