

# Design of Static Var Compensator for Power Systems Subject to Voltage Fluctuation Satisfying Bounding Conditions

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## ABSTRACT

This paper presents the design of a static var compensator for power systems subject to voltage fluctuation by using a known framework comprising the principle of matching and the method of inequalities. In the design formulation, all possible voltage fluctuations are treated as persistent signals having uniform bounds on magnitude and slope. The principal design objective is to ensure that the rotor angle, the generator terminal voltage and the voltage of the nearby bus always deviate from their nominal values within the allowable ranges for all time in the presence of any possible voltage fluctuation. By virtue of the framework, it is clear that once a solution is found, the system is guaranteed to operate satisfactorily in regard to the design objective. The numerical results demonstrate that the framework adopted here is suitable and effective, thereby giving a realistic formulation of the design problem.

**Keywords:** Power Systems Control, Static Var Compensator, Voltage Fluctuation, Design Formulation, Control Systems Design, Principle of Matching, Method of Inequalities

## 1. INTRODUCTION

In power system operation, there are occasions in which large and rapid voltage fluctuations have a great influence on the system's dynamic stability and performance. As a well-known example, the operation of an Electric Arc Furnace (EAF) can affect the operation of other equipment or machines in the nearby factories and the nearby distribution systems. The instantaneous fluctuation with large amplitudes of EAF's voltage is a source of power quality disturbances in an electric power system [1–3]. Moreover,

the stability and the performance of nearby generators are deteriorated.

Compensators such as a power system stabilizer (PSS), a static var compensator (SVC) or a static synchronous (STATCOM) have been utilized to mitigate the undesirable effects caused by EAF. In addition, they can be used for improving the power system's stability (or damping) and dynamic performance. In this regard, the subject of tuning such compensators have been investigated by a number of authors, for example, [1, 4–8] and the references cited therein. In these studies, the authors used the control theories that are based on the system properties defined in connection with deterministic test inputs (for example, step functions or sinusoids), which never happen in practice. Accordingly, their design problems were not formulated in a realistic manner.

The objective of this paper is to present the design of an SVC for improving the dynamic performance of a power system operating under load voltage fluctuation, in which all *possible voltage fluctuations* (defined as voltage fluctuations that can happen or are likely to happen in practice) are modelled as persistent signals having uniform bounds on magnitude and slope (see (6) and (29) for more details). Accordingly, the design problem is formulated realistically in the sense that the uncertain characteristic of the possible voltage fluctuations is explicitly taken into account.

For the system to have sufficient damping as well as satisfactory performance, the SVC will be designed so as to ensure that the generator rotor angle, the generator terminal voltage and the voltage of the nearby bus always deviate from their nominal values within certain acceptable ranges for all the possible voltage fluctuations. The violation of the bound for the generator rotor angle may give rise to the system's instability and consequently may cause blackout in a widespread area.

The design methodology adopted in the paper is based on Zakian's framework [9, 10] (see also the references therein), which comprises the principle of matching and the method of inequalities. The framework has been successfully applied to various engineering problems (see, for example, [10–13]). See Section 2 for a recapitulation of the framework.

Recently, the framework has been used by [11] in tuning a PSS to improve the stability and the performance of a power systems operating with load voltage

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fluctuation. The work reported in the present paper is carried out in a more elaborate and refined way. The SVC device is used, the configuration of the power system is more complex, and the representation of the voltage fluctuation is simpler.

In this work, attention is restricted only to the tuning of SVC devices. However, it should be noted that the method used here can readily be applied to the tuning of other types of power system compensator as well.

The structure of the paper is as follows. Section 2 provides necessary background of Zakian's framework and also includes the fundamental theory used for formulating the design problem. Section 3 describes the model of the power system used in the paper. In Section 4, the characterization of the set of the possible inputs is explained. Section 5 explains how the design problem is formulated. In Section 6, the numerical results are presented. Finally, the conclusions are given.

## 2. BACKGROUND

Section 2.1 briefly reviews the general concept of Zakian's framework, and Section 2.2 provides the mathematical results which are used specifically for solving the design problem considered in the paper.

### 2.1 Zakian's framework

Two essential and complementary constituents of Zakian's framework [9, 10] are the method of inequalities (MoI) and the principle of matching (PoM). The MoI [9, 14, 15] suggests that a multiobjective design problem should be cast as a set of inequalities that can be solved in practice, whereas the PoM [9, 16] suggests what kind of design inequalities should be used so that the control objectives (2) are satisfied. A detailed account of the framework can be found in [9, 10] and also the references cited therein.

#### 2.1.1 The method of inequalities

The MoI [9, 14, 15] is a general design principle requiring that a design problem should be formulated as a set of inequalities

$$\phi_i(p) \leq C_i \quad (i = 1, 2, \dots, m) \quad (1)$$

where  $p \in \mathbb{R}^n$  is a vector of design parameters,  $\phi_i : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{\infty\}$  represents a quality or a property or an aspect of the behaviour of the system, and the numbers  $C_i$  is the largest value of  $\phi_i(p)$  that can be accepted. Any point  $p$  satisfying (1) characterizes an acceptable design solution.

The set of inequalities (1) includes two principal subsets, one subset representing constraints and the other representing required performance. Whereas constraints have traditionally been represented by inequalities, the representation of desired performance by a set of inequalities is a significant departure from

the tradition in which the performance is represented by a single objective function to be minimized. The MoI recognizes that desired performance is appropriately stated by means of several distinct criteria, with each criterion represented by one or more inequalities, thus allowing greater insight into the design process.

In many cases, some of the principal  $\phi_i$  are non-convex functions of the design parameter  $p$ . Consequently, a numerical algorithm is usually employed to determine a solution of the inequalities (1) by searching in the parameter space  $\mathbb{R}^n$ . Considerable experience over the last thirty years has shown that a wide range of practical problems can be formulated in the form (1) and can be solved by numerical methods. Because there are a large number of articles (more than 40 articles) on the successful applications of the MoI, readers are referred to the book [10] for a comprehensive list of references.

#### 2.1.2 The principle of matching

The PoM [9, 16] is a general concept which requires that the system and the environment in which the system is to operate should be designed so that both are matched in the sense defined below.

Assume that the environment subjects the system to an input  $f$  which causes responses (or outputs)  $y_i$  ( $i = 1, 2, \dots, m$ ) in the system. The input  $f$  generated by the environment is assumed to be known only to the extent that it belongs to a set  $\mathcal{P}$ , called *the possible set*. This set comprises all inputs that can happen or are likely to happen or are allowed to happen in practice. Then the system and the environment are said to be *matched* [16] if the following criteria are satisfied.

$$|y_i(t, f, p)| \leq \varepsilon_i \quad \forall t \forall f \in \mathcal{P} \quad (i = 1, 2, \dots, m) \quad (2)$$

where  $y_i(t, f, p)$  denotes the value of  $y_i$  at time  $t$  in response to  $f$  with the design parameter  $p \in \mathbb{R}^n$ , and  $\varepsilon_i$  is the largest value of  $y_i$  that can be accepted. The design problem is usually to determine any value of  $p \in \mathbb{R}^n$  that satisfies (2).

For economic reasons, it can be further required that the set  $\mathcal{P}$  should not contain too many *fictitious inputs*, that is, inputs that never happen in practice. However, it is not within the scope of the paper to deal with this requirement; see [20] for further details on this.

Indeed, the criteria (2) have been used in practice by engineers to monitor the performance of the control system. They are used to guarantee that the outputs  $y_i$  stay within the specified ranges  $\pm\varepsilon_i$  for all time and for all  $f \in \mathcal{P}$ . The criteria (2), though conceptually useful, are not computationally tractable.

It is easy to show that the criteria (2) are equivalent to

$$\hat{y}_i(p) \leq \varepsilon_i \quad (i = 1, 2, \dots, m) \quad (3)$$

where the performance measures  $\hat{y}_i$  are called the *peak*

outputs for the set  $\mathcal{P}$  and given by

$$\hat{y}_i(p) \triangleq \sup_{f \in \mathcal{P}} \sup_{t \geq 0} |y_i(t, f, p)| \quad (i = 1, 2, \dots, m). \quad (4)$$

The criteria (3) become useful design inequalities, provided that  $\hat{y}_i$  are computable. In connection with the PoM, a number of researches have been devoted to the development of readily computable inequalities that can be used to ensure that (2) are satisfied for various models of the possible set. See, for example, [15, 17–20] and also the references therein.

When there are a large number of possible inputs, the usefulness of the criteria (2), as well as (3), becomes very clear. Specifically, when the performance measures used in formulating the design problem are hardly related to (2), engineers will have to perform simulations with a number of input waveforms after obtaining a design so as to check whether or not the system operates satisfactorily. This process usually involves repeated redesign and simulation of performance, a process that can be very time-consuming. It is clear that only the framework adopted in this paper can deal with the design problem in connection with the requirements (2) in a systematic manner.

## 2.2 Fundamental design theory

Consider a linear, time-invariant and nonanticipative system whose input  $f$  and outputs  $y_i$  ( $i = 1, 2, \dots, m$ ) are related by the convolution integral

$$y_i(t, f, p) = \int_0^t h_i(t - \tau, p) f(\tau) d\tau \quad (5)$$

where  $h_i$  denotes the impulse response of the output  $y_i$  and depends on the design parameter  $p$ . For the rest of the paper, consider the possible set  $\mathcal{P}$  characterized by

$$\mathcal{P} \triangleq \{f : \|f\|_\infty \leq \mathcal{M}, \|\dot{f}\|_\infty \leq \mathcal{D}\} \quad (6)$$

where the bounds  $\mathcal{M}$  and  $\mathcal{D}$  are positive numbers that can be determined from the physical properties or the past records of the system.

In connection with (6), it should be noted that the set  $\mathcal{P}$  contains an important class of bounded signals that persistently vary for all time (see [9, 10] for details).

### 2.2.1 Computation of performance measures

For the possible set  $\mathcal{P}$  in (6), there are available methods for computing the peak outputs  $\hat{y}_i$ . See, for example, [19, 20] and the references therein. However, none of them are simple.

For clarity in explaining how to apply Zakian's framework to the design problem, this paper considers the use of the majorants  $\tilde{y}_i$  instead of the peak outputs  $\hat{y}_i$ . Note, in passing, that the majorants  $\tilde{y}_i$  have a property that  $\tilde{y}_i(p) \geq \hat{y}_i(p)$  for all  $p \in \mathbb{R}^n$ .

It is known [10, 15] that the majorants  $\tilde{y}_i$  are given by

$$\tilde{y}_i = |s_{i,ss}| \mathcal{M} + \|s_i - s_{i,ss}\|_1 \mathcal{D} \quad (7)$$

where  $s_i$  are the responses  $y_i$  when the input  $f$  is the unit step function, and  $s_{i,ss}$  denote the steady-state values of  $s_i$ . Expression (7) provides a useful and simple formula for computing the majorants  $\tilde{y}_i$ .

From the above, one can easily see that

$$\tilde{y}_i(p) \leq \varepsilon_i \quad (i = 1, 2, \dots, m) \quad (8)$$

are readily computable inequalities that can be used in practice to ensure that (3) are satisfied. Hence, in this paper, (8) are used as the design inequalities instead of (3).

In using (7) to compute  $\tilde{y}_i$ , once the step responses  $s_i$  are obtained, the rest of the computation can be carried out very easily by standard numerical algorithms. Since the calculation of  $\tilde{y}_i$  is very simple, one can focus one's attention mainly on understanding the design methodology without having to worrying too much about the detail of how to compute  $\tilde{y}_i$ .

### 2.2.2 Stabilization & finiteness of performance measures

Following previous works [14, 15, 21], it is readily appreciated that in solving the inequalities

$$\tilde{y}_i(p) \leq \varepsilon_i \quad (i = 1, 2, \dots, m) \quad (9)$$

by numerical methods, it is necessary to obtain a *stability point*, that is, a point  $p \in \mathbb{R}^n$  satisfying

$$\tilde{y}_i(p) < \infty \quad \forall i. \quad (10)$$

This is because numerical search algorithms, in general, are able to seek a solution of (10) only if they start from such a point (see [14, 21] and also [10] for details).

It is important to note [14, 15, 21] that condition (10) is not soluble by numerical methods and therefore needs to be replaced with conditions (usually in the form of inequalities) that can be satisfied by numerical methods.

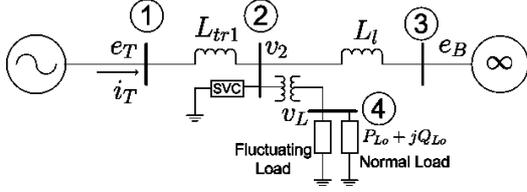
Assume that a state-space realization of the system is  $\{A, B, C, D\}$ . Then one can easily prove that  $\tilde{y}_i(p) < \infty$  for all  $i$  if all the eigenvalues of  $A$  have negative real parts; that is to say, if

$$\alpha(p) < 0 \quad (11)$$

where  $\alpha(p)$  denotes *the spectral abscissa* of  $A$ , given by

$$\alpha(p) \triangleq \max_i \{\operatorname{Re} \lambda_i(A)\}$$

and  $\lambda_i(A)$  denotes an eigenvalue of  $A$ . Notice that (11) is in fact equivalent to many well-known stability conditions for finite dimensional systems, which shows a close relationship between the finiteness of  $\tilde{y}_i(p)$  and the stability property of the system.



**Fig.1:** Power system configuration

In practice, especially when used in conjunction with the MoI, condition (11) is usually replaced with

$$\alpha(p) \leq -\varepsilon_0 \quad (12)$$

where a small positive bound  $\varepsilon_0$  is given (for example,  $\varepsilon_0 = 10^{-4}$  is used here).

Accordingly, a stability point can readily be obtained by solving the inequality (12). Once the stability point is obtained, the inequality (11) or (12) can also be used to prevent the algorithm from stepping out of the stability region during the search process. See [14, 15, 21] for more details.

### 3. MODEL OF POWER SYSTEM

The power system considered in this study is modelled as a single generator connecting to an infinite bus with an SVC installed at bus 2. The single line diagram of the system is shown in Figure 1 where  $i_T$  denotes the generator current,  $e_T$  the terminal voltage of the generator,  $e_B$  the voltage at the infinite bus,  $v_2$  the voltage at bus 2,  $v_L$  the load voltage at bus 4,  $L_{tr1}$  an inductance of the transformer connecting the generator to bus 2,  $L_{tr2}$  an inductance of the transformer connecting the bus 2 to bus 4 and  $L_l$  an inductance of the transmission line connecting bus 2 to the infinite bus.

By assuming that the power system operates under a three-phase balanced condition, it follows that

$$\begin{aligned} e_T(t) &= E_T(t) \sin(\omega t + \varphi_1) \\ v_2(t) &= V_2(t) \sin(\omega t + \varphi_2) \\ v_L(t) &= V_L(t) \sin(\omega t + \varphi_3) \\ i_T(t) &= I_T(t) \sin(\omega t + \varphi_4) \end{aligned} \quad (13)$$

where  $\varphi_i$  ( $i = 1, \dots, 4$ ) are the phase angles of  $e_T$ ,  $v_2$ ,  $v_L$  and  $i_T$  at the steady state.

Let  $(E_{T,d}, V_{2,d}, V_{L,d}, I_{T,d})$  and  $(E_{T,q}, V_{2,q}, V_{L,q}, I_{T,q})$  denote the  $d$ - and  $q$ -components of  $(E_T, V_2, V_L, I_T)$ , respectively. One can readily show that the variables in the  $abc$  reference frame is related to the  $dqo$  reference frame by

$$\begin{aligned} E_T(t) &= \sqrt{E_{T,d}^2(t) + E_{T,q}^2(t)} \\ V_2(t) &= \sqrt{V_{2,d}^2(t) + V_{2,q}^2(t)} \\ V_L(t) &= \sqrt{V_{L,d}^2(t) + V_{L,q}^2(t)} \\ I_T(t) &= \sqrt{I_{T,d}^2(t) + I_{T,q}^2(t)} \end{aligned} \quad (14)$$

The power system considered here consists of a generator, an excitation system, a governor control loop and an SVC. In the following subsections, the mathematical models of each component are described, grouped as an interconnected system, and linearized about a nominal operating condition.

#### 3.1 The synchronous generator model

The dynamics of the generator is described by the following nonlinear differential equations [22].

$$\begin{aligned} \frac{dE'_q}{dt} &= \frac{1}{T'_{d0}}(-E'_q + (X_d - X'_d)I_{T,d} + E_{fd}) \\ \frac{dE'_d}{dt} &= \frac{1}{T'_{q0}}(-E'_d + (X_q - X'_q)I_{T,q}) \\ \frac{d\delta}{dt} &= \omega_b \Delta\omega \\ \frac{d\Delta\omega}{dt} &= \frac{1}{2H}(T_m - T_e - K_D \Delta\omega) \\ T_e &= E'_d I_{T,d} + E'_q I_{T,q} + (X'_q - X'_d)I_{T,d} I_{T,q} \end{aligned} \quad (15)$$

where  $E'_d$  and  $E'_q$  are the transient electromotive forces,  $T'_{d0}$  and  $T'_{q0}$  are the open circuit field time constants,  $X_d$  and  $X_q$  are the reactances,  $X'_d$  and  $X'_q$  are the transient reactances,  $E_{fd}$  is the field voltage,  $\delta$  is the rotor angle of the generator,  $\Delta\omega$  is the deviation of the angular speed of the generator from the synchronous speed,  $\omega_b$  is the base angular speed,  $H$  is the inertia constant,  $T_m$  is the mechanical input torque,  $T_e$  is the electrical input torque, and  $K_D$  is the damping coefficient of the generator.

#### 3.2 The excitation system

The excitation system comprises a voltage transducer, an automatic voltage regulator (AVR) and a power system stabilizer (PSS) [22]. The block diagram of the excitation system is given in Figure 2 and its state-space representation is

$$\begin{aligned} \frac{dm_1}{dt} &= \frac{1}{T_R}(E_T - m_1) \\ \frac{dm_2}{dt} &= K_s \frac{d\Delta\omega}{dt} - \frac{1}{T_W} m_2 \\ \frac{dm_3}{dt} &= \frac{1}{T_2}(T_1 \frac{dm_2}{dt} + m_2 - m_3) \\ \frac{dm_4}{dt} &= \frac{1}{T_4}(T_3 \frac{dm_3}{dt} + m_3 - m_4) \\ E_{fd} &= K_A(V_{ref} - m_1 - v_s) \end{aligned} \quad (16)$$

where  $m_i$  and  $T_i$  ( $i = 1, \dots, 4$ ) denote the state variables and the lead-lag time constants of the excitation system,  $T_R$  is the transducer time constant,  $T_W$  is the washout time constant,  $K_s$  is the PSS gain, and  $v_s$  is the output signal of the PSS described by

$$v_s = \begin{cases} v_s^{\min}, & m_4 < v_s^{\min} \\ m_4, & v_s^{\min} \leq m_4 \leq v_s^{\max} \\ v_s^{\max}, & m_4 > v_s^{\max} \end{cases} \quad (17)$$

where  $v_s^{\min}$  and  $v_s^{\max}$  are some constants.

See Section 5 for further discussion on how to obtain the PSS parameters  $K_s$ ,  $T_W$  and  $T_i$  ( $i = 1, \dots, 4$ ) as shown in Table 1.

### 3.3 The governor control model

Figure 3 shows the block diagram of the governor control loop. The differential equations describing this subsystem are given by

$$\begin{aligned}\frac{dg_1}{dt} &= \frac{1}{T_G}(\frac{\Delta\omega}{R} - g_1) \\ \frac{dg_2}{dt} &= \frac{1}{T_P}(g_1 - g_2) \\ T_m &= T_{\text{ref}} - g_2\end{aligned}\quad (18)$$

where  $g_i$  ( $i = 1, 2$ ) are the state variables,  $K_I$  is the integrator gain,  $R$  is the droop constant,  $T_G$  is the governor time constant, and  $T_P$  is the prime mover time constant.

### 3.4 The SVC model

The SVC model used in this study is taken from [23] and shown in Figure 4. The dynamic behaviour of the SVC is described by

$$\begin{aligned}\frac{ds_1}{dt} &= \frac{1}{T_v}(K_v(V_{20} - V_2) - s_1) \\ \frac{ds_2}{dt} &= \frac{1}{T_{v2}}(T_{v1}\frac{ds_1}{dt} + s_1 - s_2) \\ \frac{dB}{dt} &= \frac{1}{T_b}(B_{\text{ref}} - B)\end{aligned}\quad (19)$$

where  $V_{20}$  is the voltage of bus 2 at the steady state,  $K_v$  is the SVC gain,  $T_v$  is the SVC time constant,  $s_1$  and  $s_2$  are state variables,  $T_{v1}$  and  $T_{v2}$  are the lead-lag time constants,  $T_b$  is the thyristor firing time constant,  $B$  is the susceptance of the SVC and  $B_{\text{ref}}$  is the output susceptance of the voltage regulator given by

$$B_{\text{ref}} = \begin{cases} B_{\text{min}}, & s_2 < B_{\text{min}} \\ s_2, & B_{\text{min}} \leq s_2 \leq B_{\text{max}} \\ B_{\text{max}}, & s_2 > B_{\text{max}} \end{cases} \quad (20)$$

It is worth noting that, in order to ensure that the PSS and SVC models always operate in the linear ranges, the control signals  $m_4$  and  $s_2$  need to satisfy

$$\left. \begin{aligned} v_s^{\text{min}} &\leq m_4(t) \leq v_s^{\text{max}} \\ B_{\text{min}} &\leq s_2(t) \leq B_{\text{max}} \end{aligned} \right\} \text{ for all } t \quad (21)$$

where  $v_s^{\text{min}}$ ,  $v_s^{\text{max}}$ ,  $B_{\text{min}}$  and  $B_{\text{max}}$  are specified constants.

### 3.5 The overall system equations

Owing to the very fast transient responses of the transmission network [22], it suffices to represent the

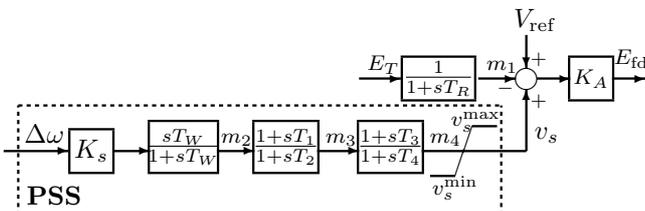


Fig.2: Block diagram of excitation system

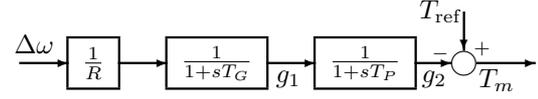


Fig.3: Block diagram of governor loop

transmission network with the algebraic equation

$$I(x, V) = Y_N V \quad (22)$$

where  $x$  is the state vector of the system,  $V$  is the bus voltage vector and  $I$  is the current injection vector.

Accordingly, the overall system equations, including the differential equations for all the devices and the algebraic equations for the transmission network (22), are expressed as

$$\frac{dx}{dt} = h(x, V) \quad (23)$$

where  $h$  is a vector of corresponding nonlinear function.

Define the output vector of interest  $y$ , the input vector  $u$  and the state vector  $x$  as follows:

$$\begin{aligned} y &\triangleq [\delta \quad E_T \quad V_2 \quad m_4 \quad s_2]^T \\ u &\triangleq V_L \angle \theta \\ x &\triangleq [\delta \quad \Delta\omega \quad E'_q \quad E'_d \quad m_1 \quad m_2 \\ &\quad m_3 \quad m_4 \quad g_1 \quad g_2 \quad s_1 \quad s_2 \quad s_3]^T \end{aligned}\quad (24)$$

where  $V_L \angle \theta$  is the phasor of the voltage of bus 2.

The generator parameters (in per unit with respect to 2220 MVA base) are shown in Table 1, and the power system is in the steady state with the following conditions:

$$\begin{aligned} P_0 &= 0.9 \text{ pu}, & Q_0 &= 0.436 \text{ pu}, \\ E_{T0} &= 1.0 \text{ pu}, & E_{B0} &= 0.9 \text{ pu}. \end{aligned}$$

By applying the steady-state analysis given in [22], one obtains a nominal operating point

$$x_0 = [\delta_0 \quad \Delta\omega_0 \quad E'_{q0} \quad E'_{d0} \quad m_{10} \quad m_{20} \\ m_{30} \quad m_{40} \quad g_{10} \quad g_{20} \quad s_{10} \quad s_{20} \quad s_{30}]^T \quad (25)$$

where  $\delta_0 = 46.26^\circ$ ,  $E'_{q0} = 0.3170$  pu,  $E'_{d0} = 1.0875$  pu, all others are zero and the nominal voltage of the load bus  $V_{L0} = 0.732$  pu.

Equation (23) is linearized about the nominal operating point given in (25) and the incremental linear model is given by

$$\begin{aligned} \frac{d\Delta x}{dt} &= A\Delta x + B\Delta u \\ \Delta y &= C\Delta x + D\Delta u \end{aligned}\quad (26)$$

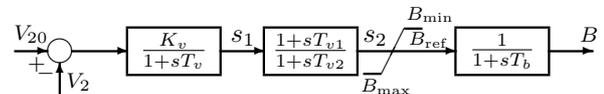


Fig.4: Block diagram of SVC

**Table 1:** Generator parameters in per unit base

$K_D = 10$	$H = 3.5$ sec
$K_A = 100$	$K_s = 19.03$
$L_{tr} = 0.15$ pu	$L_{l_1} = 0.5$ pu
$L_{l_2} = 0.93$ pu	$T'_{d0} = 8.0$ sec
$T'_{q0} = 1.0$ sec	$T_G = 0.2$ sec
$T_P = 0.3$ sec	$T_R = 0.015$ sec
$T_W = 0.844$ sec	$T_1 = 0.137$ sec
$T_2 = 0.058$ sec	$T_3 = 0.225$ sec
$T_4 = 0.158$ sec	$R = 0.05$
$X_d = 1.81$ pu	$X_q = 1.76$ pu
$R_s = 0.003$ pu	$X'_d = 0.30$ pu
$X'_q = 0.65$ pu	$\omega_b = 120\pi$ rad/sec

where  $A$ ,  $B$ ,  $C$  and  $D$  are constant matrices, the incremental output vector  $\Delta y$  is defined by

$$\Delta y = [\Delta\delta \ \Delta E_T \ \Delta V_2 \ \Delta m_4 \ \Delta s_2] \quad (27)$$

and  $\Delta u$ ,  $\Delta x$  are the incremental input and state vectors respectively.

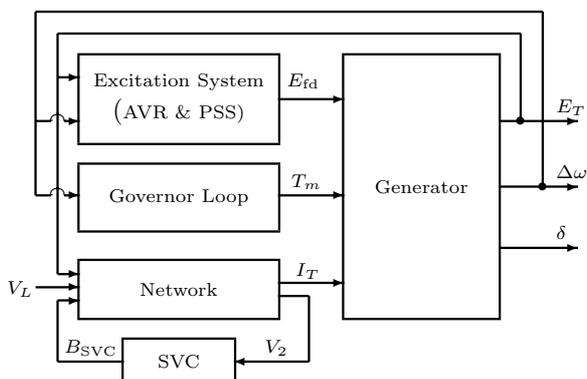
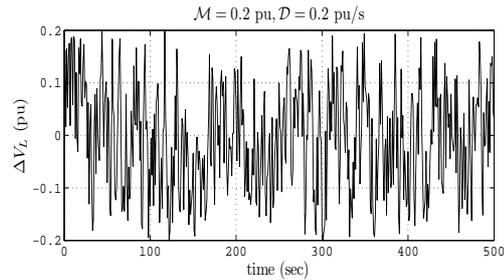
#### 4. CHARACTERIZATION OF POSSIBLE SET

This section describes how to characterize the possible set  $\mathcal{P}$ , which contains all the possible load voltage fluctuations in the power system. From the previous section, one can see that the input in the linearized model is the incremental load voltage  $\Delta V_L$ . Assume that the power system is in the steady state for  $t \leq 0$  and the load voltage changes for  $t > 0$ . That is,

$$v_L(t) = \begin{cases} V_{L0} \sin(\omega t + \theta), & t \leq 0 \\ (V_{L0} + \Delta V_L(t)) \sin(\omega t + \theta), & t > 0 \end{cases} \quad (28)$$

where  $V_{L0}$  is constant and depends upon the nominal operating condition of the system. Accordingly, the set  $\mathcal{P}$  is defined as

$$\mathcal{P} \triangleq \left\{ \Delta V_L : \|\Delta V_L\|_\infty \leq \mathcal{M}, \|\Delta \dot{V}_L\|_\infty \leq \mathcal{D} \right\} \quad (29)$$

**Fig.5:** Configuration of the power system**Fig.6:** Test input waveform

where  $\mathcal{M}$  and  $\mathcal{D}$  are positive numbers specified by designers.

At this point, the possible set  $\mathcal{P}$  is to be defined. Accordingly, the bound  $\mathcal{M}$  is chosen with respect to the change of real power of the load bus that used to happen or is likely to happen in practice. The bound  $\mathcal{D}$  is related to the rate of change of the power flow from the transmission network to the fluctuating load. In this study, the bounds used are chosen, for example, by

$$\mathcal{M} = 0.2 \text{ pu} \quad \text{and} \quad \mathcal{D} = 0.2 \text{ pu/s.} \quad (30)$$

To shed some light on the physical meaning of  $\mathcal{M} = 0.2$  pu, suppose that the system was in sinusoidal steady-state. By performing load-flow calculation, it is found that if the load voltage decreased (or increased) 0.2 pu from its nominal value, then the real power would be drawn from (or injected into) the load bus by 275.66 MW (or 210.90 MW).

#### 5. DESIGN FORMULATION

This section explains how the design problem is formulated according to the PoM and the MoI.

Assume that the power system is subjected to the incremental load voltage  $\Delta V_L(t)$  for  $t > 0$  where  $\Delta V_L$  is characterized by (28)–(30). The SVC will be used for improving the performance of the power system with the signal  $V_2$  being fed back to the SVC. The configuration of the feedback control system is shown in Figure 5.

To ensure good performance for the power system, the design problem considered here is to determine the design parameters so that the following requirements are fulfilled for any disturbance  $\Delta V_L \in \mathcal{P}$  and for all time  $t \geq 0$ .

**R1.** The incremental rotor angle ( $\Delta y_1$ ) should not be too large so as to ensure that the system has a sufficient amount of damping and that the generator is always in stable operation (that is, the rotor angle  $\delta$  does not reach the stability limit). From the linearization in Section 3,  $\delta_0 = 46.26^\circ$  and thus we set

$$\varepsilon_1 = 20^\circ. \quad (31)$$

**R2.** The incremental terminal voltage of the generator ( $\Delta y_2$ ) and the incremental voltage of the nearby

bus ( $\Delta y_3$ ) should remain strictly within  $\pm 3\%$  and  $\pm 5\%$ , respectively, so that both the generator and the nearby bus have good voltage regulation. Hence,

$$\varepsilon_2 = 0.03 \text{ pu} \quad \text{and} \quad \varepsilon_3 = 0.05 \text{ pu}. \quad (32)$$

**R3.** The signals  $\Delta m_4$  and  $\Delta s_2$  (defined as  $\Delta y_4$  and  $\Delta y_5$ , respectively) should remain within their linear ranges of operation so as to ensure that the linear model used is valid throughout the design process. Accordingly, the bounds  $\varepsilon_4$  and  $\varepsilon_5$  are chosen as

$$\varepsilon_4 = 0.2 \text{ pu} \quad \text{and} \quad \varepsilon_5 = 1 \text{ pu}. \quad (33)$$

From the above discussion, the principal design specifications to be considered in Section 6 are

$$\begin{aligned} \Delta \tilde{y}_1 &\leq \varepsilon_1 \\ \Delta \tilde{y}_2 &\leq \varepsilon_2 \\ \Delta \tilde{y}_3 &\leq \varepsilon_3 \\ \Delta \tilde{y}_4 &\leq \varepsilon_4 \\ \Delta \tilde{y}_5 &\leq \varepsilon_5 \end{aligned} \quad (34)$$

where  $\Delta \tilde{y}_i$  denotes the majorant of  $\Delta \hat{y}_i$  and is given by the formula (7),  $\Delta \hat{y}_i$  is the peak value of the output  $\Delta y_i$  for the possible set  $\mathcal{P}$  in (29)–(30), the bound  $\varepsilon_i$  is given in (31)–(33). Notice that (34) clearly shows that the design problem is multiobjective.

## 6. NUMERICAL RESULTS

This section presents the numerical results of the design problem formulated in Section 5.

To verify the design results, a test input  $\Delta V_L^*$  is generated randomly as a piecewise linear function such that  $\Delta V_L(0) = 0$  pu,  $\|\Delta V_L\|_\infty = 0.2$  pu and  $\|\Delta \dot{V}_L\|_\infty = 0.2$  pu/s. Its waveform is displayed in Figure 6. All the simulations in this section are performed by using the actual nonlinear systems.

In this work, the inequalities (34) are solved by a numerical search algorithm called the moving boundaries process (MBP). See [10, 14] for details of the algorithm. Note, in passing, that other algorithms for solving inequalities can be found, for example, in [24, 25].

### 6.1 Tuning the SVC by the PoM and the MoI

Before employing the SVC, we attempt to tune the PSS in the excitation system so that the following design criteria are satisfied.

$$\begin{aligned} \Delta \tilde{y}_1(\tilde{p}) &\leq \varepsilon_1 \\ \Delta \tilde{y}_2(\tilde{p}) &\leq \varepsilon_2 \\ \Delta \tilde{y}_3(\tilde{p}) &\leq \varepsilon_3 \\ \Delta \tilde{y}_4(\tilde{p}) &\leq \varepsilon_4 \end{aligned} \quad (35)$$

where  $\tilde{p} = [K_s, T_W, T_1, T_2, T_3, T_4]$  denotes the vector of the PSS parameters.

After a large number of trials, it appears that the MBP algorithm cannot a vector  $\tilde{p} \in \mathbb{R}^6$  satisfying

(35). This may indicate that the inequalities (35) are unlikely to have a solution. By performing a number of iterations, the best possible  $\tilde{p}$  that can be found is

$$\tilde{p} = [19.03, 0.844, 0.137, 0.058, 0.225, 0.158] \quad (36)$$

and the corresponding performance measures of the system are

$$\begin{aligned} \Delta \tilde{y}_1(\tilde{p}) &= 14.62^\circ \quad (\leq 20^\circ) \\ \Delta \tilde{y}_2(\tilde{p}) &= 0.023 \text{ pu} \quad (\leq 0.03 \text{ pu}) \\ \Delta \tilde{y}_3(\tilde{p}) &= 0.092 \text{ pu} \quad (> 0.05 \text{ pu}) \\ \Delta \tilde{y}_4(\tilde{p}) &= 0.015 \text{ pu} \quad (\leq 0.2 \text{ pu}) \end{aligned} \quad (37)$$

The PSS parameters in (36) are listed in Table 1 and used throughout the paper.

The time responses  $\Delta y_i$  due to the test input  $\Delta V_L^*$  for the system without the SVC are plotted in Figure 7. It can be seen from (37) and Figure 7 that only the requirement on the voltage regulation of the nearby bus cannot be achieved. Hence, the use of SVC is inevitable.

Now, the SVC (19) is installed at the nearby bus. Let  $p \in \mathbb{R}^4$  denote the design parameter vector of the SVC and be given by

$$p_1 \triangleq [K_v \quad T_v \quad T_{v1} \quad T_{v2}]^T. \quad (38)$$

The SVC parameters are determined by solving the inequalities (34). By starting from  $p_1 = [0, 0, 0, 0]^T$  and after a number of iterations, the MBP algorithm locates a design solution  $p_1$  where

$$p_1 = [28.0 \quad 0.150 \quad 0.169 \quad 0.033]^T \quad (39)$$

and the corresponding performance measures are

$$\begin{aligned} \Delta \tilde{y}_1(p_1) &= 5.16^\circ \quad (\leq 20^\circ) \\ \Delta \tilde{y}_2(p_1) &= 0.008 \text{ pu} \quad (\leq 0.03 \text{ pu}) \\ \Delta \tilde{y}_3(p_1) &= 0.034 \text{ pu} \quad (\leq 0.05 \text{ pu}) \\ \Delta \tilde{y}_4(p_1) &= 0.004 \text{ pu} \quad (\leq 0.2 \text{ pu}) \\ \Delta \tilde{y}_5(p_1) &= 0.94 \text{ pu} \quad (\leq 1 \text{ pu}) \end{aligned} \quad (40)$$

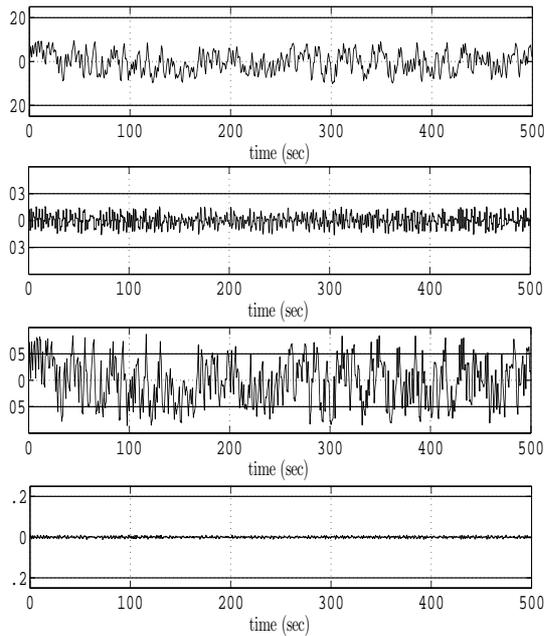
Figure 8 shows the time responses  $\Delta y_i$  due to the test input  $\Delta V_L^*$  for the system with the SVC (39). It is clear from (40) and Figure 8 that the obtained SVC not only significantly reduces the fluctuation in the voltage of the near by bus and the terminal voltage, but also provides a better damping to the system.

### 6.2 Comparison with the design obtained by the MoI

To illustrate what can go wrong when the PoM is not used, the design  $p_1$  in (39) will be compared with another design obtained by using the MoI in conjunction with the step-response criteria.

In the following, assume that the incremental load voltage  $\Delta V_L$  is a step disturbance given by

$$\Delta V_L(t) = 0.2 \text{ pu} \quad \text{for } t \geq 0. \quad (41)$$



**Fig.7:** Responses of the system without SVC due to the test input  $\Delta V_L^*$

Note that  $\Delta V_L$  does not belong to the possible set  $\mathcal{P}$ . The step responses of the power system with the SVC (39) are shown in Figure 9.

For the purpose of comparison, define  $\bar{\phi}_i$  ( $i = 1, 2, \dots, 5$ ) as the two norm of the transient deviation of the step response of  $\Delta y_i$ . That is to say,

$$\bar{\phi}_i \triangleq \sqrt{\int_0^{\infty} \{Y_i(t) - Y_{i,ss}\}^2 dt} \quad (42)$$

where  $Y_i$  denotes the step response of  $\Delta y_i$  due to the  $\Delta V_L$  given in (41) and  $Y_{i,ss}$  denotes the steady-state value of  $Y_i$ .

Then the MoI is used to determine another set of the SVC parameters, denoted by  $p_2$ , so that  $\bar{\phi}_i(p_2)$  is not worse than  $\bar{\phi}_i(p_1)$  for every  $i$  where  $p_1$  is given in (39). That is to say,  $p_2$  is obtained by solving the following criteria:

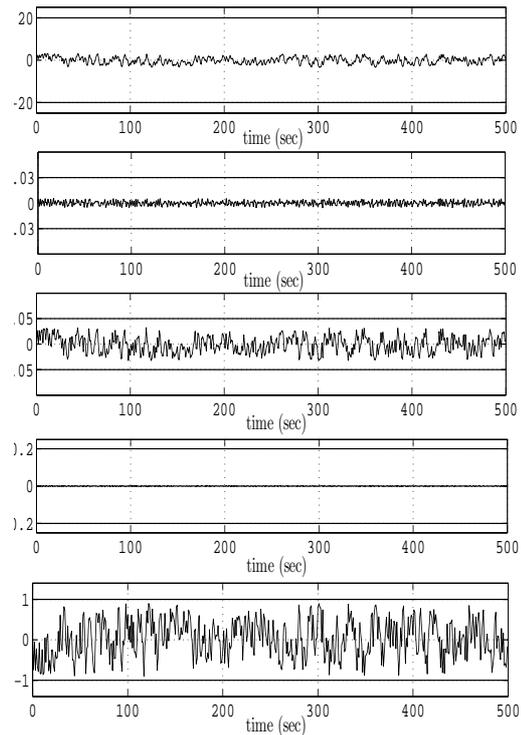
$$\begin{aligned} \bar{\phi}_1(p_2) &\leq \bar{\phi}_1(p_1) \triangleq 1.6535 \\ \bar{\phi}_2(p_2) &\leq \bar{\phi}_2(p_1) \triangleq 0.0029 \\ \bar{\phi}_3(p_2) &\leq \bar{\phi}_3(p_1) \triangleq 8.8515 \times 10^{-2} \\ \bar{\phi}_4(p_2) &\leq \bar{\phi}_4(p_1) \triangleq 7.4501 \times 10^{-3} \\ \bar{\phi}_5(p_2) &\leq \bar{\phi}_5(p_1) \triangleq 0.0123 \end{aligned} \quad (43)$$

By using the MBP algorithm, the following solution  $p_2$  is obtained where

$$p_2 = [ 31.7 \quad 0.21 \quad 0.142 \quad 0.016 ]^T \quad (44)$$

and the corresponding  $\bar{\phi}_i(p_2)$  are given by

$$\begin{aligned} \bar{\phi}_1(p_2) &= 1.6314 \\ \bar{\phi}_2(p_2) &= 0.0028 \\ \bar{\phi}_3(p_2) &= 8.1724 \times 10^{-2} \\ \bar{\phi}_4(p_2) &= 7.1697 \times 10^{-3} \\ \bar{\phi}_5(p_2) &= 0.0117 \end{aligned} \quad (45)$$



**Fig.8:** Responses of the system with SVC (39) due to the test input  $\Delta V_L^*$

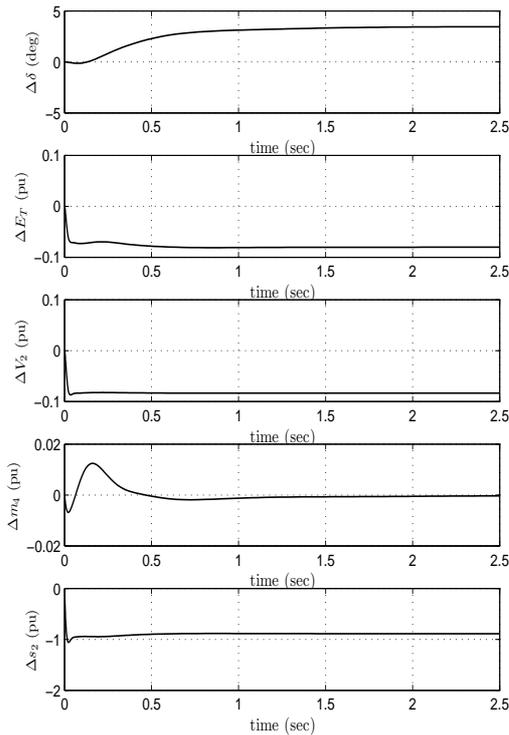
The step response of the system with the SVC (44) are displayed in Figure 10.

From (45), it is evident that the step-response of the system with  $p_2$  is slightly better than the response of the system with  $p_1$  in the sense of the performance measures  $\bar{\phi}_i$ . By comparing Figures 9 and 10, the step responses of both systems are not much different.

In connection with the actual specifications (34), we compare the performance of the SVC (39) with that of the SVC (44). To this end, the time responses  $\Delta y_i$  of the system with the SVC (44) due to the test input  $\Delta V_L^*$  are plotted in Figure 11. From this, it follows that when the SVC (44) is used, the incremental voltage of the nearby bus violates the requirement **R2**, and thereby the specifications (34) are not satisfied. From Figures 8 and 11, one can see that by using only the MoI with the step-response criteria, the specifications (34) are not explicitly taken into account in formulating the design problem. In this case, it is very difficult to determine the design parameters so that (34) are satisfied. By contrast, the problem can be solved effectively by using the PoM in conjunction with the MoI. Accordingly, the value of the design method used here is evident.

## 7. CONCLUSIONS

This paper describes a general procedure for designing a device such as an SVC for power systems subject to load voltage fluctuation by Zakian's framework, comprising the PoM and the MoI. The PoM provides a set of design inequalities that can be used

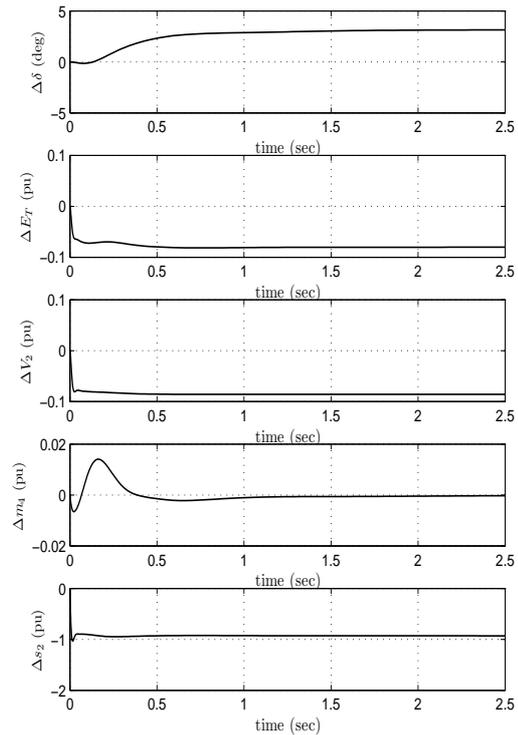


**Fig.9:** Step responses of the system with SVC (39)

in practice to ensure the satisfaction of the criteria (2). The resultant inequalities then give rise to the multi-objective design problem that are solved effectively by the MoI. The framework facilitates designers to arrive at a realistic formulation of the design problem.

The study in the paper shows that the framework can effectively handle the design problem in which the system possesses conditionally linear elements. Specifically, in spite of the saturations in the SVC and the excitation system, the power system with the obtained design solution is guaranteed to always operate within the linear ranges of the devices. Consequently, ubiquitous theories for linear systems can be utilized in the analysis and the design.

The numerical results clearly show that all the outputs  $\Delta y_i$  of interest (namely, the rotor angle, the terminal voltage and the voltage of the nearby bus) can be ensured to remain strictly within the prescribed bounds as long as the incremental load voltage  $\Delta V_L$  belongs to the possible set  $\mathcal{P}$ . Evidently, the criteria (29) or (2) are realistic in the senses that they are actually used by the plant engineers to monitor the performance of the system in practice, and that all the inputs that happen in reality are explicitly taken into account in the formulation. It is interesting to note, however, that in power systems operation, the rotor angle, the generator terminal voltage and the nearby bus voltage can be allowed to exceed the bounds for a short period of time. In this case, the use of the criteria (2) may cause some conservatism, which can usefully provide a safety margin for the design pro-



**Fig.10:** Step responses of the system with SVC (44)

vided that it is not too excessive.

The comparison in Section 6.2 shows what can go wrong if the design problem is formulated by using the step-response criteria, which do not represent the actual requirements (34).

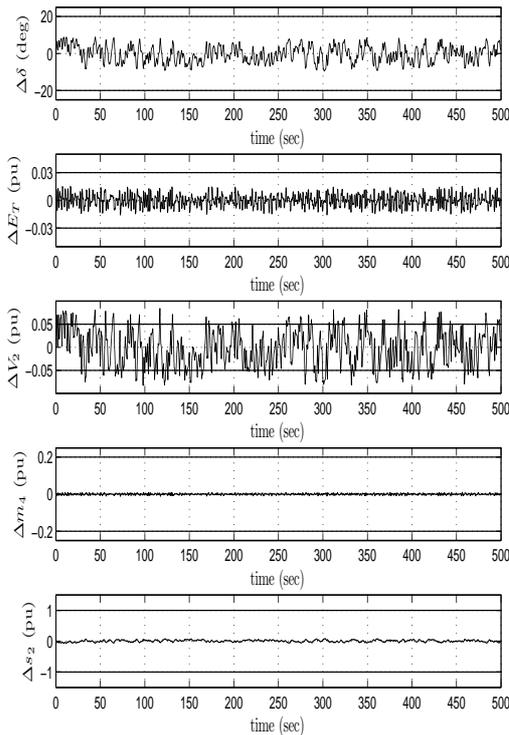
In this work, the possible inputs are considered in the form of the deviation of the voltage at the load bus from its nominal value. However, it is worth pointing out that in power systems operation, the information of the real power and the reactive power of the load is easily obtainable (for example, in the case of EAF's). Because of the potential of the framework, it may be fruitful to develop in the future a practical method for determining the bounds  $\mathcal{M}$  and  $\mathcal{D}$  of  $\Delta V_L$  due to the load fluctuation using such information so that the possible inputs are characterized more accurately and the conservatism can therefore be reduced.

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**Fig.11:** Responses of the system with SVC (44) due to the test input  $\Delta V_L^*$

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