Hardware Implementation of Flatness Based Nonlinear Sensorless Control Level for Three-Tank System

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ABSTRACT

This paper deals with the flatness-based approach for sensorless control of the three-tank system. Two main characteristics of the proposed control deserve to be mentioned. Firstly, the relative simplicity of implementation compared with the other advanced output feedback controllers. Secondly, the estimation of the non-measurable states using a continuous-discrete time observer which can provide, even for weakly sampled output, all the missing states. Simulation and experimental results show the applicability and efficiency of the proposed control scheme.

Keywords: Nonlinear State Observer, Continuous-Discrete Time Observer, Three-Tank System, Flatness Control, PID Control

1. INTRODUCTION

Liquid level control has been widely used in different industrial fields such as nuclear power, chemical and wastewater treatment plants. In these kinds of dangerous and complex processes, highly sensitive liquid level systems must be employed. In order to precisely adjust the liquid flow between tanks and to achieve a correct mixture ratio, well-designed controllers must be also used.

The three-tank system is a good prototype of many applications in industrial processes, such as water treatment, food industry, chemical and petrochemical plants, oil and gas systems. It is used in water conditioning systems, which provide the user with an abundant supply of luxuriously conditioned water, and in the craft brewing system. For this reason, it has proved to be one of the most effective benchmark systems in modeling, identification, control, fault detection, and diagnosis as well as for fault-tolerant control. It exhibits typical characteristics of nonlinearities with possibilities of disturbance solicitation which provides a rich ground that can serve as a test environment for advanced control and observation schemes.

A variety of control algorithms have been proposed by the researchers to efficiently handle this kind of system. Due to their simple structures and easy parameter adjustment, classical PID controllers are used to control the liquid level [1–3]. However, the downside of the classical PID controllers generally appears in the experimental applications where sudden changes can occur.

Due to the intrinsic coupling that exists in such plant variables, some nonlinear MIMO state feedback advanced controllers have been developed. Namely, the backstepping controller has been presented to control systems where its dynamics can be written in a lower triangular form [4–5]. It presents the ability to guarantee the global stabilization of the system and offers good performance, even for uncertain systems; however, the choice of an appropriate Lyapunov function at each step is still a difficult problem. The sliding mode control approach is also frequently adopted. It is characterized by its design simplicity and robustness [6–7]. But, despite the presence of some works to alleviate its effect, the chattering phenomenon remains the major inconvenience of such method [8].

Exploiting the flat property for certain systems presents the main feature of the flatness control law w.r.t the previously mentioned approaches. Indeed, the flatness approach enables us to linearize the nonlinear system through an appropriate choice of dynamic state feedback. In addition to its simplicity of implementation and input-output decoupling, this method asserts itself as a real solution to directly reconstruct each system's variable. This latter is presented as a function of the flat outputs and a finite number of their time derivatives [9–10].

All these nonlinear control techniques assume that the whole state variables of the considered system are measured. However, from a practical point of view, generally, a few state variables are available for online measurements because of technical and/or economic constraints. In order to perform sensorless control techniques, there is a great need for a reliable and accurate estimation of the unavailable state variables.

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For that purpose, a nonlinear state observer may be used. Several solutions are presented in the literature including linearization techniques [11], extended Kalman filter [12], high gain observers [13] and sliding mode observers [6, 14–15]. Most of the available contributions in this field concern the case with continuous-time measurements. However, this is not always the case, for example when the output measurements are shared through a digital communication network that is available only at discrete-time instances. Indeed, it is well-known that the stability and convergence properties of these continuous-time observers may be lost if the sampling process is relatively slow. A practical solution is to employ a state observer that performs an estimation of the state vector with discrete-time output measurements. Several observation schemes have been proposed for some classes of continuous-time systems with sampled output measurements [16–19], but very few results for an effective hardware implementation are achieved [20–21].

The main contribution of this paper concerns the design of a flatness controller based on a continuous-discrete time high gain observer CDHGO for a three-tank system. Thus, the designed controller does not need liquid level sensors in both tank 2 and tank 3. An observation procedure is used to accurately estimate these levels from the discrete measurements of liquid level in the tank 1 and provides them to the flatness controller. As a feature of the designed observer, it offers the possibility to use low-cost digital hardware that can work with a relatively low sample frequency.

The remainder of the paper is organized as follows: in the second section, we represent the three-tank process and its dynamical model. The continuous discrete-time observer and its application to the process are detailed in section three. Section four is devoted to the proposed controller. Finally, the simulation and experimental results are reported in section five before drawing some conclusions.

2. THREE TANK SYSTEM

2.1 Description of the Plant

The hydrographical process consists of 3 columns T_1, T_2, and T_3 with the same section S, coupled serially by transfer valves and which can be drained into a reservoir by leakage valves. These leakage valves denoted V_{l1}, V_{l2}, V_{l3} and V_e have identical effective sections S_n. Two rotary valves V_{t12} and V_{t23} can be used to change the channel section and therefore change the flow characteristics between the columns. Each leakage valve can be used as a manually adjustable disturbance. Tanks 1 and 3 are supplied with fluid via 2 pumps of maximum flow Q_{max}. The maximum flow rate of the pump (6.66×10^{-5} m^3/s) is reached when a voltage of 12 V is applied to the pump. Each column is equipped with a pressure sensor providing

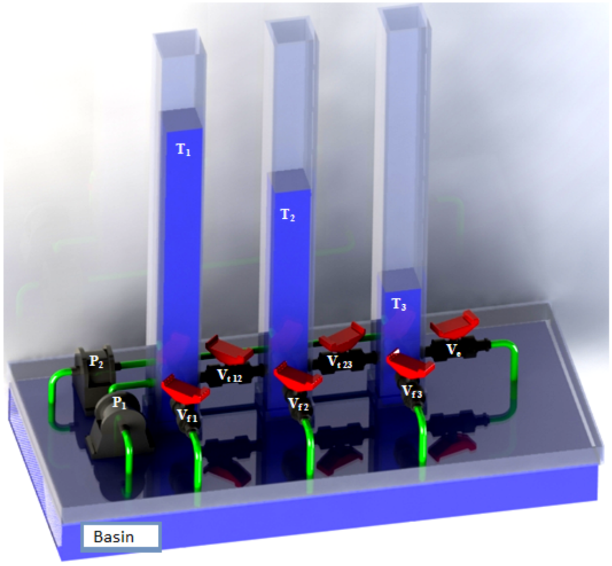

Fig.1: *Full structure of the computer design plant.*

the liquid level in the tank [16]. The sensors are calibrated to provide output signals ranging from 0 to 5 V corresponding to a change in water level from 0 to 100 cm for each column. The experimental system shown in Fig. 1 is equipped with sensors and actuators that communicate via an acquisition card and a computer.

2.2 The Mathematical Model

The variation of the volume of water in a tank i ($i = 1:3$) is equal to the difference between the sum of the inflows and the sum of the outflows. Using the mass equations (flow equilibrium), the system can be easily represented by the following equation:

$$\dot{V}_i = S_i \frac{dh_i}{dt} = \sum Q_{in,i} - \sum Q_{out,i} \qquad (1)$$

where $\sum Q_{in,i}$ and $\sum Q_{out,i}$ stand for the total water inflows and outflows in the tank, respectively and S_i is the cross-section of the tank i. Then the mathematical model is specified by the following equations:

$$\begin{aligned} \dot{h}_1(t) &= \frac{1}{S}\left(Q_1(t) - Q_{12}(t) - Q_{l1}(t)\right) \\ \dot{h}_2(t) &= \frac{1}{S}\left(Q_{12}(t) - Q_{23}(t) - Q_{l2}(t)\right) \\ \dot{h}_3(t) &= \frac{1}{S}\left(Q_2(t) + Q_{23}(t) - Q_e(t) - Q_{l3}(t)\right) \end{aligned} \qquad (2)$$

where t represents the time; h_1, h_2, and h_3 represent the liquid levels in each tank; S represents the cross-section of the tanks; Q_1 and Q_2 designate the flow rates of both pumps 1 and 2; Q_{ij} denotes the flow rates between tank T_i and system T_j; and Q_{fi} represent the output flow of the corresponding tank when the leak valve is open.

The flows Q_{ij} and Q_e in Eq. (2) are calculated using Torricelli's law as follow:

$$Q_{ij}(t) = a_{zi} S_n \operatorname{sgn}(h_i - h_j)\sqrt{2g\,|h_i - h_j|} \quad (3)$$

$$Q_e(t) = a_{z3} S_n \sqrt{2gh_3} \quad (4)$$

where a_{zi} is the outflow coefficients, sgn(.) is the sign of the argument, and g is the acceleration due to gravity. Note that the inflow rates into Tank 1 and Tank 3 are given by:

$$Q_i(t) = k_i u_i(t) \quad (5)$$

where k_i is the pump constant (cm^3/V) and $u_i(t)$ is the voltage applied to the pump.

Consequently, we obtain:

$$\begin{aligned}
\frac{dh_1}{dt} &= -a_1 \operatorname{sgn}(h_1 - h_2)\sqrt{|h_1 - h_2|} + \frac{k_1}{S} u_1 \\
\frac{dh_2}{dt} &= a_1 \operatorname{sgn}(h_1 - h_2)\sqrt{|h_1 - h_2|} \\
&\quad - a_2 \operatorname{sgn}(h_2 - h_3)\sqrt{|h_2 - h_3|} \\
\frac{dh_3}{dt} &= a_2 \operatorname{sgn}(h_2 - h_3)\sqrt{|h_2 - h_3|} - a_3\sqrt{h_3} + \frac{k_2}{S} u_2
\end{aligned} \quad (6)$$

where a_i are the system parameters given by:

$$a_i = \frac{1}{S} a_{zi} S_n \sqrt{2g}; \quad i = 1, \ldots, 3 \quad (7)$$

3. CONTINUOUS-DISCRETE TIME OBSERVER

3.1 Basic Concepts

Consider the class of MIMO systems that are diffeomorphic to the following triangular form:

$$\begin{cases} \dot{x}(t) &= Ax(t) + \varphi\left(u(t), x(t)\right) \\ y_k &= Cx(t_k) = x^1(t_k); \forall k \in \mathbf{N} \end{cases} \quad (8)$$

with:

$$x = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix}; \ \varphi(u,x) = \begin{pmatrix} \varphi^1(u, x^1) \\ \varphi^2(u, x^1, x^2) \\ \vdots \\ \varphi^{q-1}(u, x^1, ..., x^{q-1}) \\ \varphi^q(u, x) \end{pmatrix}$$

$$A = \begin{bmatrix} 0_p & I_p & 0_p & \cdot & 0_p \\ \cdot & \cdot & I_p & \cdot & \cdot \\ 0_p & \cdot & \cdot & \cdot & 0_p \\ 0_p & \cdot & \cdot & \cdot & I_p \\ 0_p & \cdot & \cdot & 0_p & 0_p \end{bmatrix}; \ C = \begin{bmatrix} I_p \\ 0_p \\ \vdots \\ 0_p \end{bmatrix}^T \quad (9)$$

where $x(t) \in \mathbf{R}^n$ is the state vector with $x^j \in \mathbf{R}^p$ for $j = [1, ..., q]$, the input $u(t) \in U$ a compact subset of $\mathbf{R}^s$. The output $y \in \mathbf{R}^p$ is available only at the sampling instant with respect to the following condition:

$$0 \le t_0 < \ldots < t_k < t_{k+1} < \ldots \text{ such as } \lim_{k \to \infty} t_k = \infty$$

The observer design requires the following assumptions:

A1. The function $\varphi^i(.)$ for $i \in [1, q]$ are globally Lipschitz with respect to x uniformly in u, i.e $\exists L > 0$ such that the following inequality holds for $i \in [1, q]$ and x and $\bar{x} \in \mathbf{R}^n$:

$$\|\varphi^i(u, x) - \varphi^i(u, \bar{x})\| \le L\|x - \bar{x}\|$$

A2. Assume that the time intervals τ_k are bounded away from zero and are upperly bounded by the upper bound of sampling partition diameter τ_M, i.e

$$0 < \tau_k = \tau_{k+1} - t_k \le \tau_M, \forall k \ge 0$$

A candidate continuous-discrete time observer for system Eq. (8) is given by [21]:

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + \varphi\left(u(t), \hat{x}(t)\right) \\ &\quad - \theta\Delta_\theta^{-1} K e^{-\theta k^1 (t - t_k)} \left(C\hat{x}(t_k) - y(t_k)\right) \end{aligned} \quad (10)$$

with $\hat{x} = \left(\hat{x}^{1T}, \ldots, \hat{x}^{qT}\right)^T$ is the vector of the estimated state,

$$\Delta_\theta = \operatorname{diag}\left(I_p, \frac{1}{\theta} I_p, \ldots, \frac{1}{\theta^{q-1}} I_p\right)$$

where θ is the observer design parameter.

$$K = \left[K^{1T}, \ldots, K^{qT}\right]^T$$

is a gain matrix that is designed such that the matrix $\bar{A} = A - KC$ is Hurwitz. In the particular case when we have chosen the eigenvalues located at (–1), we have:

$$K = \operatorname{diag}\left(C_1^q I_{n1}, C_2^q I_{n2}, \ldots, C_q^q I_{n1}\right)$$

for $i = [1, \ldots, q]$, with

$$C_i^j = \frac{j!}{i!(j-i)!}$$

for $1 \le i,\ j \le q$.

3.2 Application to the Three-Tank System

The main purpose of this section is to accurately estimate the two levels h_2 and h_3, when the output measurement h_1 is available only at discrete-time instants. The dynamics of the three-tank system given by Eq. (8) can be rewritten by the following form:

$$\dot{h} = f(h) + g(Q) \quad (11)$$

with:

$$h=\begin{bmatrix}h_1\\h_2\\h_3\end{bmatrix},\ Q=\begin{bmatrix}Q_1\\Q_2\end{bmatrix},\ g(Q)=\begin{bmatrix}\frac{Q_1}{S}\\0\\\frac{Q_2}{S}\end{bmatrix} \tag{12}$$

$$f(h)= \tag{13}$$
$$\begin{bmatrix}-a_1\,\mathrm{sgn}(h_1-h_2)\sqrt{|h_1-h_2|}\\ a_1\,\mathrm{sgn}(h_1-h_2)\sqrt{|h_1-h_2|}-a_2\,\mathrm{sgn}(h_2-h_3)\sqrt{|h_2-h_3|}\\ a_2\,\mathrm{sgn}(h_2-h_3)\sqrt{|h_2-h_3|}-a_3\sqrt{h_3}\end{bmatrix}$$

Table 1 summarizes the typical values of the three-tank system. These values are later used in observer and controller implementation.

Now, in order to transform the system Eq. (11) to the system Eq. (8), we use the following Lipshitzian diffeomorphism:
$\Phi:\mathbf{R}^n\rightarrow\mathbf{R}^n$

$$x(t)\mapsto z(t)=\Phi(x(t),u(t))=\begin{bmatrix}h(x(t))\\L_fh(x(t))\\\vdots\\L_f^{n-1}h(x(t))\end{bmatrix} \tag{14}$$

where $L_f^jh(x)$ is the j^{th} Lie derivative of the function h by f, such as $h(x)$ and $f(x)$ are differentiable functions of x up to the order n.

Considering the case of the studied system with the only measurement of h_1 ,Φ is given by the following expression [22].
$\Phi:\mathbf{R}^3\rightarrow\mathbf{R}^3$; $x(t)\mapsto z(t)=\Phi(x(t),u(t))$

$$\Phi(h)=\begin{bmatrix}h_1\\L_fh_1\\L_f^2h_1\end{bmatrix}=\begin{bmatrix}h_1\\-a_1r(h_1-h_2)\\\frac{a_1^2}{2}\mathrm{sgn}(h_1-h_2)-\frac{a_1a_2}{2}\frac{r(h_2-h_3)}{\sqrt{|h_1-h_2|}}\end{bmatrix} \tag{15}$$

where $r(x)=\mathrm{sgn}(x)\sqrt{|x|}$.

The three-tank system is described in the Z coordinate, as follows:

Table 1: *Physical parameters of the three-tank system.*

Tank cross-section areas	$S_1=S_2=S_3=S=0.01$ m^2
Pipe cross-section areas	$S_N=0.0000786$ m^2
Coefficients*	$a_1=0.1$; $a_2=0.086$; $a_3=0.099$; $k_1=6.4665$; $k_2=5.7596$
Maximum in-flow rate	$Q_{max}=6.66\times10^{-5}$ m^3/s

$$\begin{cases}\dot{x}(t) &= Az(t)+\varphi\left(u(t),z(t)\right)\\ y_k &= Cz(t_k)=z^1(t_k);\forall k\in\mathbf{N}\end{cases} \tag{16}$$

We can adapt a continuous-discrete time observer in the Z coordinate as given in Eq. (10). When using the inverse transformation, the continuous-discrete time observer can be given by

$$\dot{\hat{h}}=f(\hat{h})+g(Q)-\theta\Delta_\theta^{-1}\left(\frac{\partial\Phi(h)}{\partial h}\right)^{-1}Ke^{-\theta^1(t-t_k)}(C\hat{h}-y) \tag{17}$$

with:

$$f(\hat{h})=$$
$$\begin{bmatrix}-a_1\,\mathrm{sgn}(\hat{h}_1-\hat{h}_2)\sqrt{\left|\hat{h}_1-\hat{h}_2\right|}\\ a_1\,\mathrm{sgn}(\hat{h}_1-\hat{h}_2)\sqrt{\left|\hat{h}_1-\hat{h}_2\right|}-a_2\,\mathrm{sgn}(\hat{h}_2-\hat{h}_3)\sqrt{\left|\hat{h}_2-\hat{h}_3\right|}\\ a_2\,\mathrm{sgn}(\hat{h}_2-\hat{h}_3)\sqrt{\left|\hat{h}_2-\hat{h}_3\right|}-a_3\sqrt{\hat{h}_3}\end{bmatrix}$$

and

$$\left(\frac{\partial\Phi(h)}{\partial h}\right)^{-1}=\begin{pmatrix}1&0&0\\1&\frac{1}{a}&0\\1&\frac{1}{a}+\frac{b}{ca}&\frac{1}{c}\end{pmatrix}$$

where:

$$a=\frac{a_1}{2\sqrt{|h_1-h_2|}};$$
$$b=\frac{a_1a_2r(h_2-h_3)}{4r(h_1-h_2)\left|h_1-h_2\right|};$$
$$c=\frac{a_1a_2}{4\sqrt{|h_1-h_2|\cdot|h_2-h_3|}}$$

4. CONTROLLER DESIGN

4.1 Basic concept of flatness

The property of flatness of a system is closely related to the general ability to linearize a nonlinear system by an appropriate choice of dynamic state feedback. Roughly speaking, the flatness is a structural property of a nonlinear class of systems, for which all system variables can be written in terms of a set of specific variables (the widely known flat outputs) and a finite number of their time derivatives. Let us consider the following general nonlinear system:

$$\begin{cases}\dot{x}(t) &= f(x(t),u(t))\\ y(t) &= h(x(t))\end{cases} \tag{18}$$

where $x(t)$ is the state vector of the considered system, $u(t)$ and $y(t)$ are respectively the input control and the output. The functions $f(.)$ and $h(.)$ are assumed to be smooth with respect to their arguments. The nonlinear system Eq. (18) is said to be (differentially) flat if and only if there exists an output vector $\xi(t)$, called flat or linearizing output, such that:

- The flat output $\xi(t)$ and a finite number of its time derivatives $\dot{\xi}(t), \ddot{\xi}(t), \ldots, \xi^{(n)}(t)$ are independent.
- The number of independent components of the flat output is equal to the number of independent inputs.
- Every system state and inputs variables may be expressed as a function of the flat output and of a finite number of its time derivatives as:

$$\begin{aligned} x(t) &= \phi\left(\xi(t), \dot{\xi}(t), \ldots, \xi^{(n-1)}(t)\right) \\ u(t) &= \psi\left(\xi(t), \dot{\xi}(t), \ldots, \xi^{n}(t)\right) \end{aligned} \tag{19}$$

The functions $\phi(.)$ and $\psi(.)$ are assumed to be smooth.

The choice of the flat output is not a very restrictive condition in the case of real systems. It can be a physical variable as a position, a velocity, a current, a voltage, etc. In the case of flat outputs that are unavailable for on-line measurements, a state observer can be designed to estimate them. The main advantage of the differential flatness is that the differentiation of the chosen flat output, up to order n yields the necessary information to reconstruct the state and input trajectories of the considered system relying on the above-mentioned relations.

4.2 Flatness Control of the Three-Tank System

The three-tank model Eq. (6) has two flat outputs:

$$y_i = h_i; i = 1, 2 \tag{20}$$

Let us consider the reference trajectories as follows:

$$h_i^* = r_i; i = 1, 2 \tag{21}$$

So, a reference model can be given by:

$$\begin{aligned} \dot{r}_1 &= -a_1 \operatorname{sgn}(r_1 - r_2)\sqrt{|r_1 - r_2|} + \frac{k_1}{S} u_1^* \\ \dot{r}_2 &= a_1 \operatorname{sgn}(r_1 - r_2)\sqrt{|r_1 - r_2|} - a_2 \operatorname{sgn}(r_2 - x_3^*)\sqrt{|r_2 - h_3^*|} \\ \dot{h}_3^* &= a_2 \operatorname{sgn}(r_2 - x_3^*)\sqrt{|r_2 - h_3^*|} - a_3\sqrt{|h_3^*|} + \frac{k_2}{S} u_2^* \end{aligned} \tag{22}$$

Consequently, we can generate input control references as follows:

$$\begin{aligned} u_1^* &= \frac{S}{k_1}\left(\dot{r}_1 + a_1 \operatorname{sgn}(r_1 - r_2)\sqrt{|r_1 - r_2|}\right) \\ u_2^* &= \frac{S}{k_2}\left(\dot{h}_3^* - a_2 \operatorname{sgn}(r_2 - h_3^*)\sqrt{|r_2 - h_3^*|} + a_3\sqrt{|h_3^*|}\right) \end{aligned} \tag{23}$$

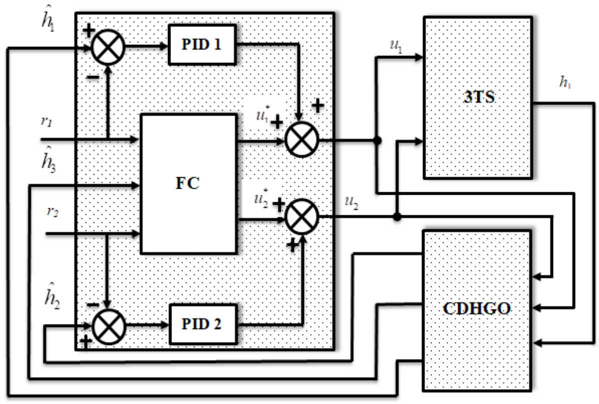

Fig.2: *Block diagram of the output feedback flatness controller.*

with:

$$h_3^* = r_2 - \left(\frac{a_1 \operatorname{sgn}(r_1 - r_2)\sqrt{|r_1 - r_2|} - \dot{r}_2}{a_2}\right)^2 \tag{24}$$

By adding the PID regulators effects, the control laws applied to the system are given by:

$$\begin{aligned} u_1 &= u_1^* + \frac{S}{k_1}\left(\delta e_1 - G_1 e_1 - G_2 \int e_1\right) \\ u_2 &= u_2^* + \frac{S}{k_2}\left(\delta e_2 - G_3 e_2 - G_4 \int e_2\right) \end{aligned} \tag{25}$$

where:

$$\begin{aligned} \delta e_1 &= a_1 \operatorname{sgn}(y_1 - y_2)\sqrt{|y_1 - y_2|} - a_1 \operatorname{sgn}(r_1 - r_2)\sqrt{|r_1 - r_2|} \\ \delta e_2 &= a_2 \operatorname{sgn}(y_2 - y_3)\sqrt{|y_2 - y_3|} + a_3\sqrt{|y_3|} \\ &\quad + a_2 \operatorname{sgn}(y_2 - h_3^*)\sqrt{|y_2 - h_3^*|} - a_3\sqrt{|h_3^*|} \end{aligned}$$

$e_i = y_i - r_i$ is the tracking error and $G_i (i = 1, \ldots, 4)$ are the design parameters of the PID regulator. The control laws u_1^* and u_2^* are generated from the flatness controller (FC).

The whole block diagram given in Fig. 2 shows the structure of the proposed output feedback flatness controller. Relying on the only discrete output measurement h_1, the continuous-discrete time high gain observer CDHGO performs an estimation of the state vector $\left[\hat{h}_1, \hat{h}_2, \hat{h}_3\right]^T$. This last is exploited by the controller in order to generate to control law $u = (u_1, u_2)^T$ to the three-tank system. Note that each component of the last vector is composed of the sum of two signals: the first is given from the corresponding PID and the other is provided by the Flatness reference generator.

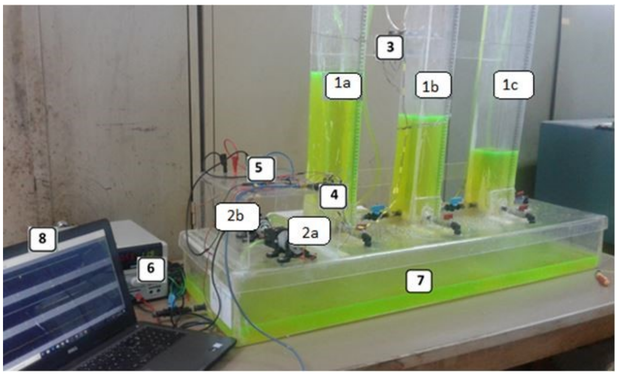

Fig.3: *Experimental set-up: (1a, 1b, 1c) 3 tanks; (2a, 2b) 2 DC pumps; (3) liquid level sensors; (4) Fio std STM32F10 board; (5) motor driver board; (6) power supply; (7) basin; (8) control desk.*

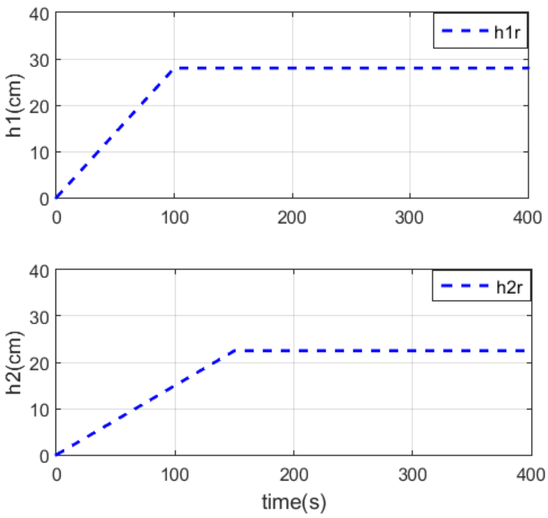

Fig.4: *Benchmark trajectory.*

5. SIMULATION AND EXPERIMENTAL RESULTS

5.1 Test Bench Description

The proposed controller performances should be now evaluated not only by the numerical simulation but also, by the hardware implementation on the three-tank process. The test bench used in Fig. 3, is located in the laboratory of Industrial Systems and Renewable Energies Studies "ESIER" at the National Engineering School of Monastir, Tunisia.

5.2 Simulation Results

The benchmark used in simulation as well as in experimental validation, presents two reference level trajectories of h_1 and h_2 given in Fig. 4.

The observer design parameter is chosen as $\theta = 1$, the initial value of the estimated vector is $\hat{h} = [0, 5, 10]^T$. Using the Zeigler-Nichol's tuning method, we have got the following PID control parameters:

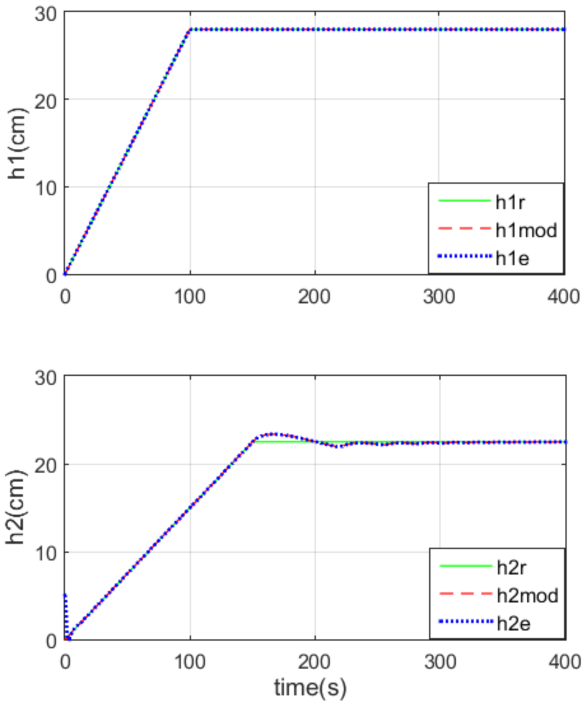

Fig.5: *Level reference, estimated and resulting flat outputs.*

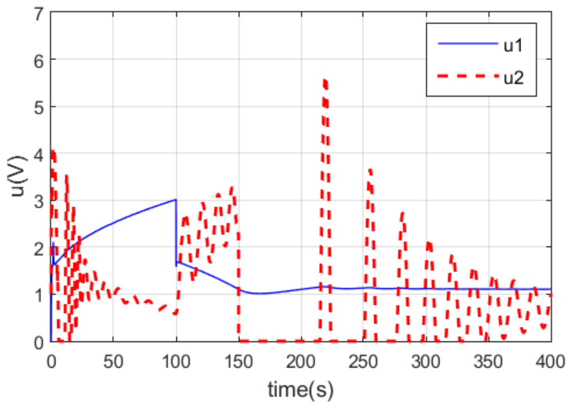

Fig.6: *Both the control signals.*

$$G_1 = 5; G_2 = 2 \times 10^{-2}; G_3 = 10; G_4 = 2 \times 10^{-4}$$

As shown in Fig. 5, the liquid level h_1 tracks well the desired trajectory; whereas the liquid level h_2 presents some overshoots (less than 5%) when the reference changes. In the same figure, we remark after the transient, that the estimation error between the estimated and the simulated states is null. Both signals of control which should be applied to the DC moto-pumps are illustrated in Fig. 6. In the steady-state, it is shown that the signal u_1 stabilizes at a constant value, while u_2 stays in damped oscillation because the liquid level h_2 is indirectly controlled by the moto-pump 2 which only fed the third tank.

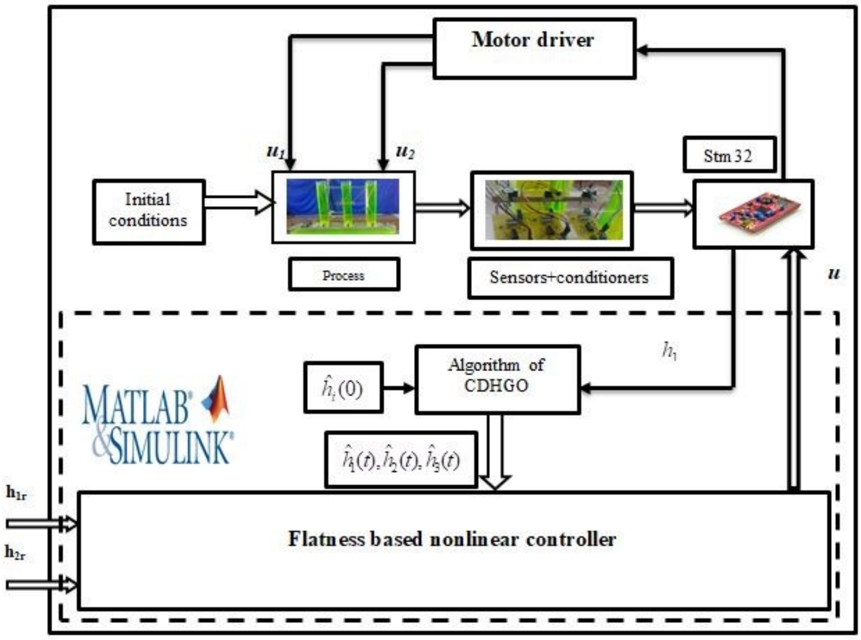

Fig.7: *Synoptic scheme of the setup.*

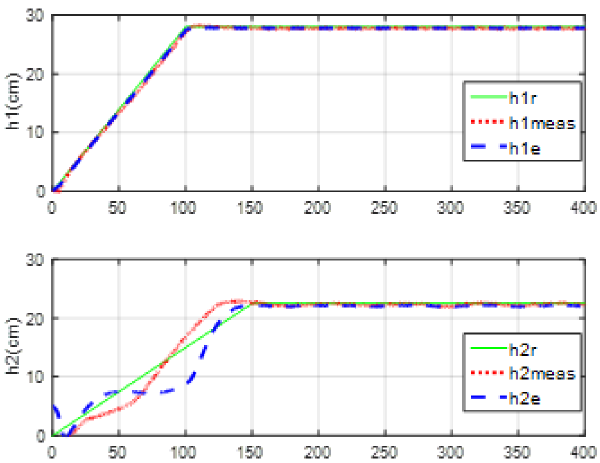

Fig.8: *Level reference, estimated and measured outputs.*

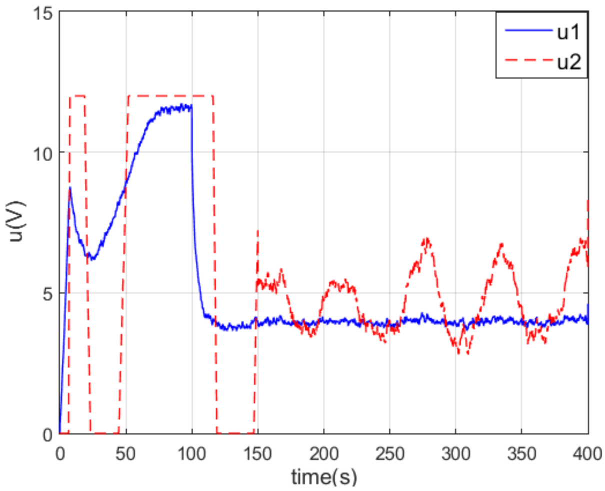

Fig.9: *Experimental control signals for both moto-pumps.*

5.3 Experimental results

The proposed output feedback control algorithm has been implemented in the MATLAB/Simulink environment incorporated with the real-time interface associated with the STM32Fio microcontroller data acquisition device. An interface card is used to establish the connection, via USB port, between the control desk using Matlab/Simulink environment and the three piezo-resistive differential pressure sensors as input and the moto-pump drive board as an output.

Each sensor and its conditioner card deliver a voltage varying between 0 and 5 V corresponding respectively to a height of 0 to 100 cm. Using the PWM interface in Simulink library, the interface card associated with the dual-full bridge LM298, generates to both pumps a variable voltage from 0 to +12 V, corresponding to a flow rate ranging from 0 to Q_{max}.

The synoptic scheme of the proposed controller used in the experimental setup is presented in Fig. 7.

In a practical case, the CDHGO reconstruct states h_2 and h_3 using the discrete measurement of h_1 available only at sampling times. The control law is implemented in real-time using a sampling period $T_e = 0.5$s. It's shown in Fig. 8 that after 10 s the liquid level h_1 tracks well its reference, but for the signal h_2, 150 s are required to obtain the convergence to the reference trajectory. This large transient time arises from the observation procedure which, when choosing the setting parameter θ to be small ($\theta = 0.1$), requires a not negligible time for achieving the convergence of the estimation error to zero. This choice of θ is performed in order to ensure the compromise between the output noise rejection aim and the convergence of the estimation error.

The control signals are illustrated in Fig. 9. It is shown that u_1 is almost constant in the steady-state regime because it has a direct effect on the flat output h_1. Whereas, similarly to the numerical simulation case, the signal u_2 presents an oscillating dynamic. Such behavior is due to the indirect effect of the moto-pump 2 on the flat output h_2.

6. CONCLUSION

In this paper, a flatness-based approach for nonlinear sensorless control of the three-tank system has been presented. A continuous-discrete time high gain observer has effectively contributed to the estimation of the unmeasurable liquid levels that are required for the proposed controller. Simulation and experimental results show that the output feedback flatness control strategy ensures a perfect tracking to level references.

In our future work, to ensure the tracking objective for more difficult predefined trajectories, we will be interested in the hybrid control of the studied system. Indeed, we should not only control the moto-pump flows, but also the water solenoid valve between the liquid tanks.

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