A Composite Nonlinear Controller for Power Systems with STATCOM under External Disturbances

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ABSTRACT

This paper concentrates on the design of a composite nonlinear stabilizing state feedback control for power systems with static synchronous compensator (STATCOM) with the help of a combination of backstepping strategy and a nonlinear disturbance approach. The disturbance observer is used to estimate unavoidable external disturbances. Thus, the obtained control law can be used to successfully stabilize the system and reject undesired external disturbances. In order to demonstrate the effectiveness of the developed process design, numerical simulation results are provided to indicate that the presented composite controller can improve dynamic performances, rapidly suppress system oscillations of the overall closed-loop dynamics, and despite having inevitable external disturbances, perform better than a conventional backstepping control technique.

Keywords: Backstepping Control, STATCOM, Generator Excitation, Disturbance Observer

1. INTRODUCTION

It is well-known that modern power systems have seen a rapid increase in size and complexity. When power system operation is confronted with unavoidable disturbances, maintaining power system stability is one of the most important problems. Therefore, this problem has attracted much attention from a number of researchers. Currently, there are three effective and promising methods that are used to improve system stability under unpredictable disturbances. The first method is a utilization of generator excitation control [1]–[7]. The second method is a combination of the excitation and energy storage system [23]. The third method is an coordination of the excitation and Flexible AC Transmission System (FACTS) devices [9, 10]. These schemes focus on improving power system stability and accomplishing the desired control objectives.

Since there are recent fast developments in power electronic devices, FACTS devices have been applied to not only provide an opportunity to effectively tackle the existing transmission facilities, but also deal with several constraints to build new transmission lines. In this paper, the Static Synchronous Compensator (STATCOM)[9, 10] of particular interest can be employed to increase the grid transfer capability through enhanced voltage stability, significantly provide smooth and rapid reactive power compensation for voltage support, and enhance both damping oscillation and transient stability. So far, the generator excitation controller [1] and STATCOM controller [10] have been separately designed. However, in order to further enhance the power system stability of power systems, the combination of generator excitation and STATCOM is a promising and effective method and has attracted much attention in literature for years.

To the best of our knowledge, based directly on the nonlinear control strategy, there is little prior work that has been devoted to a coordination of generator excitation and STATCOM. In [11, 12], an adaptive coordinated generator excitation and STATCOM control strategy was designed via generalized Hamiltonian control for stability enhancement of large-scale power systems. With the help of the zero dynamic design and pole-assignment scheme, a coordinated controller [13] for the single-machine infinite bus system was investigated. A nonlinear coordinated controller [14] has been developed through a combination of the passivity design and backstepping technique. Kanchanaharuthai et al. [15] has developed an interconnection and damping assignment-passivity based control (IDA-PBC) strategy for coordination of generator excitation and STATCOM/battery energy storage for transient stability and voltage regulation enhancement of multi-machine power systems. In [16], a coordinated immersion and invariance (I&I) control scheme has been developed for transient sta-
bility improvement and voltage regulation. Kan-
chanaharutthi [17] presented an adaptive I&I control and adaptive backstepping scheme to enhance
trajectory stability and voltage regulation for power sys-
tems with STATCOM in the presence of unknown
parameters. Recently, based on a Takagi-Sugeno (T-
S) fuzzy scheme, a nonlinear stabilizer design [18] for
power systems with random loads and STATCOM
was presented and tested on both single and multi-
machine power systems.

It has been found that in practice, most engineering
systems have frequent disturbances capable of inev-
itably degrading the desired control performance of
the closed-loop dynamics. The disturbances consid-
ered include external disturbances, parametric un-
certainties and other unknown nonlinear terms. There-
fore, the desired control design method needs to in-
clude the disturbance dynamics to reject the effects
of the abovementioned disturbances. Disturbance ob-
serving observer method is an approach for compensating
the result from external disturbances and mismatched
disturbances/uncertainties. This method has been
widely accepted in compensating the effects of dis-
turbances. The disturbance observer is utilized to es-
timate disturbances appearing in the system. There
is current development of disturbance observer design
combined with most popular nonlinear control meth-
ods such as backstepping method [19] and sliding
mode method [20], as presented in [20]–[26]. Based on
the abovementioned references, disturbance observer-
based control is a promising method capable of reject-
ing external disturbances and improving robustness
against uncertainties [20] simultaneously. It also pro-
vides an effective way to handle external disturbances
and system uncertainties. Additionally, disturbance
observer design method can be further extended to
several problems in control system societies, such as
adaptive control [27], finite-time control [28], track-
ing control [29], and so on. Further, this method
is successfully applied for numerous kinds of real
engineering systems such as flight control systems
[29], permanent magnet synchronous motors [29], air-
breathing hypersonic vehicle systems [21], power sys-
tems [22, 23], active suspension system [24], electro-
hydraulic actuator systems [29], and so on. Those
indicate important application potentials of the distur-
bance observer-based control method to deal with the
effect of unavoidable external disturbances. However,
even though the control design methods presented in
[11]–[16] have good control performances, external
disturbances and uncertainties have not been taken
into account before. These disturbances may lead to
poor performances, and eventually make the system
unstable.

This paper presents a systematic procedure to syn-
thetize a nonlinear feedback stabilizing control law
on the basis of a backstepping control [19] combined
with the disturbance observer design, it is developed
to cope with the adverse effects of external distur-
bances. Therefore, the merits of this work are as fol-
lows: (a) The use of a nonlinear disturbance observer-
based backstepping control strategy to stabilize the
system in the presence of external disturbances has
not been investigated before for power systems with
STATCOM; (b) The overall closed-loop system is
input-to state despite external disturbances; (c) In
comparison with a backstepping control, the devel-
oped control law offers better dynamic performances
and a satisfactory disturbance rejection ability.

The rest of this paper is organized as follows. Sim-
pified synchronous generator and STATCOM models
are briefly described, and the problem statement is
given in Section 2. Control design is given in Section
3. Simulation results are given in Section 4. Conclu-
sions are given in Section 5.

2. POWER SYSTEM MODEL DESCRIPTION

2.1 Power system models with STATCOM

The complete dynamical model [16, 17] of the syn-
chronous generator (SG) connected to an infinite bus
with STATCOM dynamics can be expressed as fol-
lows:

$$
\begin{align*}
\dot{\delta} & = \omega - \omega_s + d_1 \\
\dot{\omega} & = \frac{1}{M} \left( P_m - P_e - P_s - D(\omega - \omega_s) \right) + d_2 \\
\dot{P}_e & = (-a + \cot \delta(\omega - \omega_s))P_e + \frac{\varphi_{\omega e} \sin 2\delta}{2(X_1 + X_2)} + \frac{V_s \sin \delta}{(X_1 + X_2)} \cdot \frac{u_f}{T_p} + d_3, \\
\dot{P}_s & = N(\delta, P_e, P_s)(-a + \cot \delta(\omega - \omega_s))P_e + \frac{N(\delta, P_e, P_s)V_s \sin \delta}{2(X_1 + X_2)} \cdot \frac{u_f}{T_p} + d_3, \\
\dot{V}_d & = \frac{P_e - X_1 P_s}{2(X_1 + X_2)} \cdot \frac{1}{T_f} \left( \frac{P_e \Delta(\delta, P_e)}{X_1} - I_{q0} \right) + u_q + d_4,
\end{align*}
$$

with $\Delta(\delta, P_e), N(\delta, P_e), P_e, P_s, a$ and $b$ given in [16],
where $\delta$ is the power angle of the generator, $\omega_s$
 denotes the relative speed of the generator, $D \geq 0$ is a
damping constant, $P_m$ is the mechanical input power,
$E$ denotes the generator transient voltage source,
$P_E = \frac{E V_{\omega e} \sin \delta}{X_{q0}}$ is the electrical power, without STAT-
COM, delivered by the generator to the voltage at
the infinite bus $V_\infty$, $\omega_s$ is the synchronous machine
speed, $\omega_s = 2\pi f$, $H$ represents the per unit inertial
constant, $f$ is the system frequency and $M = 2H/\omega_s$.
$X_d'$ denotes the direct axis transient reactance of SG
and $X_d$ denotes the direct axis reactance of SG. $X_T$
is the reactance of the transformer, and $X_L$ denotes
the reactance of the transmission line. For simplicity,
$X_1$ is the reactance consisting of the direct axis transient
reactance of SG and the reactance of the transformer,
and $X_2$ is the reactance of the transmission line. $T_f$
is the direct axis transient short-circuit time constant.
$u_q$ is the field voltage control input to be designed.
$I_{q0}$ denotes the injected or absorbed STATCOM current
as a controllable current source, $I_{q0}$ is an equilibrium.
point of STATCOM currents, \( u_q \) is the STATCOM control input to be designed, and \( T \) is a time constant of the STATCOM model. \( d_j(t), (j = 1, 2, 3, 4) \) are external disturbances and system parameter variations.

For convenience, let us introduce new state variables as follows:

\[
x_1 = \delta - \delta_e, x_2 = \omega - \omega_s, x_3 = P_e, x_4 = P_s,
\]

(2)

Subsequently, after differentiating the state variables (2), we have the power system with STATCOM which can be written in the following form of an affine nonlinear system:

\[
\dot{x} = f(x) + g(x)u(x) + d(t),
\]

(3)

where \( f(x), g(x), u(x) \) and \( d(t) \) are given at the top of the next page. The region of operation is defined in the set \( \mathcal{D} = \{ x \in \mathbb{R}^4, 0 < x_1 < \frac{\pi}{2} \} \).

The open loop operating equilibrium is denoted by

\[
\dot{x} = x_2 + d_1,
\]

\[
\dot{x}_2 = \frac{1}{m}(P_m - Dx_2 - x_3 - x_4) + d_2,
\]

\[
\dot{x}_3 = f_3(x) + g_31(x) \frac{u_1}{T_1} + d_3,
\]

\[
\dot{x}_4 = f_4(x) + g_41(x) \frac{u_1}{T_1} + g_42(x) \frac{u_2}{T_2} + d_4
\]

(4)

For the sake of simplicity, the power system considered (3) can be expressed in the following form:

\[
\begin{align*}
\dot{x}_1 &= x_2 + d_1, \\
\dot{x}_2 &= \frac{1}{m}(P_m - Dx_2 - x_3 - x_4) + d_2, \\
\dot{x}_3 &= f_3(x) + g_31(x) \frac{u_1}{T_1} + d_3, \\
\dot{x}_4 &= f_4(x) + g_41(x) \frac{u_1}{T_1} + g_42(x) \frac{u_2}{T_2} + d_4
\end{align*}
\]

2.2 Preliminaries

In this subsection, some important lemmas are mentioned as follows for convenience of the reader. Consider the following system

\[
\dot{y} = f(t, y, u), \quad y \in \mathbb{R}^n, \quad u \in \mathbb{R}^m.
\]

Definition 1: [30] A continuous function \( \alpha : [0, a) \rightarrow [0, +\infty) \) belongs to class \( \mathcal{K} \) if it is strictly increasing and \( \alpha(0) = 0 \). It belongs to class \( \mathcal{K}_\infty \) if \( a = +\infty \) and \( a(r) \rightarrow +\infty \) as \( r \rightarrow +\infty \).

Lemma 1: [30] Let \( V : [0, \infty) \times \mathbb{R}^n \rightarrow \mathbb{R} \) be a continuously differentiable function such that

\[
\frac{\partial V}{\partial t} + \frac{\partial V}{\partial y} f(t, y, u) \leq -W_3(y), \forall y \geq \rho(y) > 0,
\]

for all \( (t, y, u) \in [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \), where \( \alpha_1 \) and \( \alpha_2 \) are class \( \mathcal{K} \) functions, \( \rho \) is a class \( \mathcal{K} \) function, and \( W_3(y) \) is a continuous positive definite function on \( \mathbb{R}^n \). Then, system (5) is input-to-state stable (ISS).

Lemma 2: [30] Consider the following system (5). If the following conditions are satisfied

- system \( \dot{y} = f(t, y, u) \) is input-to-state stable.
- \( \lim_{t \rightarrow +\infty} u = 0 \).

then the states of the system (4) will asymptotically converge to zero, that is, \( \lim_{t \rightarrow +\infty} y(t) = 0 \).

Before stating the problem statement and control design, the following assumption needs to be made as well.

Assumption 1: The external disturbances \( d_j(t), (j = 1, 2, 3, 4) \) are bounded. Also, the first derivatives of the disturbances above are also bounded.

Problem statement: The objectives of this paper are to stabilize the power system including STATCOM (4) with the external disturbance \( d \) and to accomplish the desired control performances, which can be formulated as follows: with the help of the nonlinear disturbance observer-based backstepping-like control technique [22], find out, if possible, a stabilizing (state) feedback controller \( u(y) \) and disturbance estimation \( \hat{d} \) as follows:

\[
\begin{cases}
\dot{u} = \phi(x, \hat{d}) \\
\dot{\hat{d}} = \varphi(x, u, d)
\end{cases}
\]

(6)

such that the overall closed-loop system (4) and (6) is input-to-state stable, where \( \hat{d} \) is the estimate of \( d \).

For the developed design procedure in the next section, a combination of the backstepping scheme and disturbance observer design will be developed to obtain a composite nonlinear controller (6). In comparison with the conventional backstepping and integral backstepping methods, the proposed approach will use the full information of the disturbance estimation into each step. Such information is also used for compensating the external disturbances at each step, and the estimation error dynamics are included for the closed-loop stability analysis. In addition, as the system is subjected to external disturbances, the proposed composite controller can offer the capability to maintain the power system stability, to reject undesired disturbances, and to improve transient control performances. In the following section, the developed controller is to achieve the desired performances.

3. CONTROLLER DESIGN AND STABILITY ANALYSIS

This section aims to determine the control laws for stabilizing the power system with STATCOM. The proposed design procedure is divided into three parts.

- The first part introduces a nonlinear disturbance observer technique to estimate the unknown, but bounded, disturbances and to compensate for the external disturbance.
- The second part proposes an approach consisting of the backstepping control method and the result of the disturbance estimator from the first part to find the desired controller in each design step.
- The last part shows that Lyapunov stability theorem is used to analyze the overall closed-loop system stability. Despite having disturbances in the system,
the results indicate that it is capable of achieving both system stability and the desired control performances by the obtained controller.

3.1 Nonlinear disturbance observer design

The aim of designing the disturbance observer is to estimate the external disturbance and other uncertainties so that the effect of disturbances is removed and the whole system performance can be enhanced. The disturbance observer proposed in [20]–[22] is used to estimate the disturbance and is applied with the control input.

Therefore, the nonlinear disturbance observer for the system (4) is designed as

\[
\begin{cases}
\dot{d}_1 = \lambda_i(x_i - p_i), i = 1, 2, 3, 4, \\
\dot{p}_1 = f_1(x) + d_1, \\
\dot{p}_2 = f_2(x) + d_2, \\
\dot{p}_3 = f_3(x) + g_{31}(x) \frac{\partial}{\partial x_1} + \hat{d}_3, \\
\dot{p}_4 = f_4(x) + g_{41}(x) \frac{\partial}{\partial x_1} + g_{42}(x) \frac{\partial}{\partial x_2} + \hat{d}_4,
\end{cases}
\]

where \( \lambda_j > 0 \) is a design parameter. Thus, based on (7) the disturbance estimation dynamics can be expressed in the following form:

\[
\dot{d}_j = \lambda_j(x_j - \hat{p}_j) = \lambda(d_j - \hat{d}_j), j = 1, 2, 3, 4.
\]

Let us define the disturbance estimation error as \( e_j = d_j - \hat{d}_j \), the estimation error dynamics can be expressed as follows.

\[
\dot{e}_j = -\lambda_j e_j + \hat{d}_j.
\]

3.2 Backstepping design

According to the concept reported in [22], the stabilization problem for the system (4) is solved by designing a backstepping control. The design procedure is developed step by step as follows.

Step 1: Considering, the first subsystem (4), a Lyapunov function is selected as

\[
V_1 = \frac{1}{2} e_1^2 + \frac{1}{2} e_j^2,
\]

where \( z_1 = x_1 \). Then the time derivative of \( V_1 \) along the system trajectories becomes

\[
\dot{V}_1 = -\lambda_1 e_1^2 + z_1 x_2 + z_1 (x_2 - x_2^*_{\theta}) + z_1 d_1 + e_1 \dot{d}_1.
\]

From (11), it is seen that \( x_2^*_{\theta} \) is regarded as the virtual control variable with the disturbance estimate \( d_1 \) as follows.

\[
x_2^* = -\left( k_1 + \frac{1}{4\epsilon_1} \right) x_1 - \dot{d}_1.
\]

where \( k_1 > 0 \) and \( \epsilon_1 > 0 \). After substituting (12) into (11), we have

\[
\dot{V}_1 = -\left( k_1 + \frac{1}{4\epsilon_1} \right) z_1^2 - \lambda_1 e_1^2 + \frac{1}{4\epsilon_1} z_2^2 + \epsilon_1 e_2^2 + z_1 z_2 + c_1 d_1.
\]

where \( z = x_2 - x_2^*_{\theta} \).

Step 2: Let us define the Lyapunov function of Step 1 as \( V_2 = V_1 + \frac{1}{2} z^2 + \frac{1}{2} e_2^2 \). Then the time derivative of \( V_2 \) along the system trajectories is as follows:

\[
\dot{V}_2 = -k_1 z_2^2 - (\lambda_1 - \epsilon_1) e_1^2 - \lambda_2 e_2^2 + z_1 z_2 - \frac{1}{M} z_2 x_3^* - \frac{1}{M} z_2 x_2^* - z_2 \frac{\partial x_2^*}{\partial x_1} x_2 + \frac{1}{M} \left( p_m - D x_2 \right)
\]

\[
\left( x_3 - x_3^* \right) - (x_4 - x_4^*) - \frac{\partial x_2^*}{\partial x_1} x_2 + d_1 \right)
\]

\[
\dot{e}_2 = -\frac{\partial x_2^*}{\partial d_1} \lambda_1 e_1 + e_1 \dot{d}_1 + e_2 \dot{d}_2.
\]

From (14), it can be observed that \( x_3^* \) and \( x_4^* \) are considered as the virtual control variables with the disturbance estimates \( d_1 \) and \( d_2 \) as follows.

\[
\begin{cases}
x_3^* = \frac{M}{2} \left[ -\left( k_2 + \frac{1}{4\epsilon_2} + \hat{c}_2 \right) z_2 - P_m - z_1 + \frac{D x_2}{M} - d_2 + \frac{\partial x_2^*}{\partial x_1} (x_2 + \dot{d}_1) \right],
\end{cases}
\]

\[
x_4^* = x_5^* + \frac{M P_m}{2}.
\]
where $k_2 > 0, \epsilon_2 > 0$ and $\hat{c}_2 = \frac{1}{4\epsilon_2} \left( \frac{\partial x_2^2}{\partial z_1} + \frac{\partial x_2^2}{\partial d_1} \lambda_1 \right)^2$. After substituting (15) into (14), we obtain

\[ \dot{V}_2 = -k_1 z_1^2 - \lambda_1 e_1^2 - \left( k_2 + \frac{1}{4\epsilon_2} + \hat{c}_2 \right) z_2^2 - \lambda_2 e_2^2 - \frac{1}{M} z_2 (x_3 - x_3^*) - \frac{1}{M} z_2 (x_4 - x_4^*) + z_2 e_2 \\
= \frac{dz_2}{2} \left( \frac{\partial x_2^2}{\partial z_1} + \frac{\partial x_2^2}{\partial d_1} \lambda_1 \right) e_1 + e_1 \dot{d}_1 + e_2 \dot{d}_2. \quad (16) \]

Based on Young inequality [31], the terms in (16) can be straightforwardly computed as

\[ e_2 z_2 \leq \frac{1}{4\epsilon_2} z_2^2 + \epsilon_2 e_2^2, \quad (17) \]
\[ -z_2 \left( \frac{\partial x_2^2}{\partial z_1} + \frac{\partial x_2^2}{\partial d_1} \lambda_1 \right) e_1 \leq \hat{c} \frac{z_2 e_2}{2} + \epsilon_1 e_1^2, \quad (18) \]

where $\hat{c} = \frac{1}{4\epsilon_2} \left( \frac{\partial x_2^2}{\partial z_1} + \frac{\partial x_2^2}{\partial d_1} \lambda_1 \right)^2 \epsilon_1^2$.

Substituting (17)-(18) into (16) and then defining $z_i = x_i - x_i^*, i = 3, 4$, we get

\[ \dot{V}_2 \leq -k_1 z_1^2 - k_2 z_2^2 - (\lambda_1 - 2\epsilon_1) e_1^2 - (\lambda_2 - 2\epsilon_2) e_2^2 - \frac{z_2}{M} (z_3 + z_4). \quad (19) \]

**Step 3:** we select a Lyapunov function as follows:

\[ V_3 = V_2 + \frac{1}{2} \sum_{i=3}^{4} (z_i^2 + e_i^2). \quad (20) \]

After taking derivatives of both sides of (20), one has

\[ \dot{V}_3 \leq -k_1 z_1^2 - k_2 z_2^2 - (\lambda_1 - 2\epsilon_1) e_1^2 - (\lambda_2 - 2\epsilon_2) e_2^2 - \frac{z_2}{M} (z_3 + z_4) \]
\[ - \sum_{i=3}^{4} \left[ \left( \xi_i - \frac{\partial x_i^2}{\partial z_1} \hat{z}_1 - \frac{\partial x_i^2}{\partial z_2} \hat{z}_2 - \frac{\partial x_i^2}{\partial d_1} \lambda_1 e_1 - \frac{\partial x_i^2}{\partial d_2} \lambda_2 e_2 \right) - \lambda_i e_i^2 + e_i \dot{d}_i \right]. \quad (21) \]

Substituting $\dot{x}_i, (i = 3, 4)$ from (4), $\dot{z}_1, \dot{z}_2$ and $x_2^*$ into (21) yields

\[ \dot{V}_3 \leq -k_1 z_1^2 - k_2 z_2^2 - (\lambda_1 - 2\epsilon_1) e_1^2 - (\lambda_2 - 2\epsilon_2) e_2^2 - \sum_{i=3}^{4} \left[ (\lambda_i e_i^2 - \epsilon_i d_i) + z_3 \left( \frac{z_2}{M} + f_3(x) \right) \right. \\
+ g_{31}(x) \frac{u_{sT}}{T_0} + d_3 - \frac{\partial x_3^2}{\partial z_1} (x_2 + \hat{d}_1) \\
+ \frac{\partial x_3^2}{\partial z_2} (f_2(x) + d_2) + \frac{\partial x_3^2}{\partial d_1} (x_2 + d_1) - \frac{\partial x_3^2}{\partial d_2} \lambda_1 e_1 \right] \\
\left. + d_4 - \frac{\partial x_4^2}{\partial z_1} (x_2 + \hat{d}_1) - \frac{\partial x_4^2}{\partial z_2} (f_2(x) + d_2) + \frac{\partial x_4^2}{\partial d_1} (x_2 + d_1) - \frac{\partial x_4^2}{\partial d_2} \lambda_1 e_1 \right] \\
\left. - \frac{z_4}{M} + f_4(x) + g_{41}(x) \frac{u_{sT}}{T_0} + g_{42}(x) \frac{u_{qT}}{T} \right) \\
+ d_4 - \frac{\partial x_4^2}{\partial z_1} (x_2 + \hat{d}_1) - \frac{\partial x_4^2}{\partial z_2} (f_2(x) + d_2) + \frac{\partial x_4^2}{\partial d_1} (x_2 + d_1) - \frac{\partial x_4^2}{\partial d_2} \lambda_1 e_1 \right]. \quad (22) \]

From (22), in order to achieve the desired control performance, we choose the control law as follows:

\[ \begin{align*}
\frac{u_{sT}}{T_0} &= \frac{1}{g_{31}(x)} \left[ \frac{z_2}{M} + f_3(x) - \hat{d}_3 + \frac{\partial x_3^2}{\partial z_1} (x_2 + \hat{d}_1) \\
+ \frac{\partial x_3^2}{\partial z_2} (f_2(x) + \hat{d}_2 - \frac{\partial x_3^2}{\partial d_1} (x_2 + \hat{d}_1) \right] \\
- \left( k_3 + \frac{1}{4\epsilon_3} + \hat{c}_31 + \hat{c}_32 + \hat{c}_33 + \hat{c}_34 \right) z_3 \right) \\
\frac{u_{qT}}{T} &= \frac{1}{g_{41}(x)} \left[ \frac{z_4}{M} + f_4(x) - g_{41}(x) \frac{u_{sT}}{T_0} + \frac{\partial x_4^2}{\partial z_1} (x_2 + \hat{d}_1) \\
+ \frac{\partial x_4^2}{\partial z_2} (f_2(x) + \hat{d}_2 - \frac{\partial x_3^2}{\partial d_1} (x_2 + \hat{d}_1) \right] - \hat{d}_4 \\
- \left( k_4 + \frac{1}{4\epsilon_4} + \hat{c}_41 + \hat{c}_42 + \hat{c}_43 + \hat{c}_44 \right) z_4 \right].
\end{align*} \quad (23) \]

where

\[ \begin{align*}
\hat{c}_1 &= \frac{1}{4\epsilon_1} \left( \frac{\partial x_1^2}{\partial z_1} + \frac{\partial x_1^2}{\partial d_1} \lambda_1 \right)^2, \\
\hat{c}_2 &= \frac{1}{4\epsilon_2} \left( \frac{\partial x_2^2}{\partial z_1} + \frac{\partial x_2^2}{\partial d_1} \lambda_2 \right)^2, \\
\hat{c}_3 &= \frac{1}{4\epsilon_3} \left( \frac{\partial x_3^2}{\partial z_1} + \frac{\partial x_3^2}{\partial d_1} \lambda_1 \right)^2, \\
\hat{c}_4 &= \frac{1}{4\epsilon_4} \left( \frac{\partial x_4^2}{\partial z_1} \right)^2, i = 3, 4.
\end{align*} \quad (24) \]
After computing the time derivative of the Lyapunov function candidate (27) and selecting $\lambda_1 = a_{01} + 6\epsilon_1, \lambda_2 = a_{02} + 2\epsilon_2, \lambda_i = a_{0i} + \epsilon_i, (i = 3, 4), a_{0j} > 0, (j = 1, 2, 3, 4)$, we obtain

$$V_3 \leq - 4 \sum_{j=1}^{4} k_j z_j^2 - \sum_{j=1}^{4} a_{0j} e_j^2 + \sum_{j=1}^{4} e_j d_j,$$

(28)

where $e = [e_1, e_2, e_3, e_4]^T, d = [d_1, d_2, d_3, d_4]^T, a_0 = \min\{a_{01}, a_{02}, \ldots, a_{04}\}$. The inequality (28) can be rewritten as

$$V_3 \leq - 4 \sum_{j=1}^{4} k_j z_j^2 - (1 - \theta) a_0 ||e||^2 - \theta a_0 ||e||^2 + ||e|| ||d||,$$

(29)

where $0 < \theta < 1$. If we select $||e|| \leq \frac{||d||}{\alpha_0 a_0}$, it is easy to obtain that $V_3 \leq - \sum_{j=1}^{4} k_j z_j^2 - (1 - \theta) a_0 ||e||^2 \leq 0$.

Thus, the conditions of Lemmas 1 and 2 are satisfied with $\alpha_1(r) = e_1 r^2, \alpha_2(r) = e_2 r^2$, and $\rho(r) = (1/\alpha_0 r) r$, and we can conclude that the overall closed-loop system is input-to-state stable. This completes the proof.

**Assumption 2:** The disturbances satisfy the condition of \( \lim_{t \to +\infty} d_j(t) = 0 \).

**Theorem 2:** Under Assumptions 1 and 2, the closed-loop dynamics under the control law (23) and the disturbance estimation (7) will asymptotically converge to zero.

**Proof:** It is seen from the closed-loop system that $d_j$ can be regarded as an input of the system. After combining Theorem 1 with Assumption 2, it follows from Lemma 2 that $V_3 \leq 0$. In accordance with Lyapunov control theory, it can be concluded that all trajectories of $z_j$ and $e_j$ of the closed-loop dynamics converge to zero. This means that $z_j \to 0$ and $e_j \to 0$ as $t \to +\infty$. This completes the proof.

5. SIMULATION RESULTS

In this section, in order to verify the effectiveness of the proposed nonlinear controller. The proposed controller is evaluated via simulations of a single-machine infinite bus (SMIB) power system consisting of dynamic model of synchronous generators and STATCOM as shown in Fig. 1. The performance of the proposed control scheme is evaluated in MATLAB environment under the presence of undesired external disturbances.

The physical parameters (pu.) and initial conditions ($\delta_e, \omega_s, P_{se}, P_{se}, d_{10}, \ldots, d_{40}$) for this power system model are are the same as those used in [16]. Additionally, the external disturbances ($d_j, j = 1, 2, 3, 4$) acting on the underlying system are assumed...
The controller parameters are set as $\epsilon_j = 10, k_j = 20, \lambda_j = 50, (j = 1, 2, \ldots, 4)$. The time domain simulations are carried out to investigate the system stability enhancement and the dynamic performance of the designed controller, as given in (23), in the system in the presence of external disturbances. The control performance of the proposed controller (nonlinear disturbance observer-based backstepping controller) is compared with that of a conventional backstepping controller (CBSC) [19] as shown below.

$$\begin{align*}
t^* &= \frac{1}{g(x)} \left[ -k_3 z_3 - \frac{3}{M} - f_3(x) 
+ \frac{1}{2} \left( D z_2 - M(k_2 z_2 + \dot{z}_1 + k_1 \dot{x}_2) \right) \right], \\
t^* &= \frac{1}{g(x)} \left[ -k_4 z_4 - \frac{3}{M} - f_4(x) 
- \frac{1}{2} \left( D z_2 - M(k_2 z_2 + \dot{z}_1 + k_1 \dot{x}_2) \right) \right],
\end{align*}$$

(34)

with $z_j = x_j - x_j^*, (j = 1, 2, 3, 4), x_1^* = 0, x_2^* = -k_1 z_1, x_3^* = -\frac{3}{2} (D x_2 - M(k_2 z_2 + z_1 + k_1 x_2)) + P_m, x_4^* = x_4^* - P_m, \dot{z}_1 = -c_1 z_1 + \dot{z}_2, \dot{z}_2 = f_2(x) + c_1 \dot{x}_2$.

The controller parameters of this scheme are set as $c_j = 50, (j = 1, 2, \ldots, 4)$.

The simulation results are presented and discussed as follows. Time histories of the power angle, frequency, transient internal voltage, and STATCOM current under two controllers are presented in Fig. 2. Also, the results of disturbance estimation and external disturbances together with disturbance estimation error is demonstrated in Figs. 3 and 4.

From these Figs., it can be seen that the developed method and the CBSC method is able to successfully stabilize the system despite having external disturbances given in (30)-(33). In addition, it can be observed that the presented control has not only better dynamic performances, but also satisfactory disturbance rejection ability such as a shorter settling time, a short rise time, and a faster convergence rate. Clearly, all time responses are significantly more damped with the proposed scheme than with the CBSC scheme. Compared with the presented method, the CBSC scheme has worse dynamic performances such as unsatisfactory overshoots and slowly suppressing system oscillations. This is because in the developed control framework the proposed nonlinear control combines the advanced feedback control law with the full use of disturbances information in each step to mitigate the effects of inevitable disturbances. In contrast, the BSC method does not include the effects of disturbances in the designed control law. Fig. 3 shows the disturbance estimators can rapidly track the unknown external disturbances with fast convergence rate and no oscillations. Also, the error between unknown disturbances and disturbance estimator is shown in Fig. 4.

From the simulation results mentioned above, it is evident that as the presented method combined with the disturbance observer design is applied to the SMIB power system with STATCOM under external disturbances, the advantages over the CBSC method are as follows.

- The proposed control law is effectively designed to stabilize the system in the presence of undesired disturbances.
- The developed control strategy can make the overall closed-loop dynamics converge more quickly to a desired equilibrium point. In particular, it obviously performs well and has considerably effective disturbance rejection ability. It offers obviously superior transient performances illustrated by the rapid suppression of system oscillations in all time trajectories in spite of having external disturbances.
- The process of designing the desired control law includes some auxiliary terms into the virtual control laws and the final controller. These terms can counteract the crossing terms arising from disturbances, compensation errors, and system states. In contrast, these terms are not included in the CBSC method, thereby leading to unsatisfactory control performances.

6. CONCLUSION

In this paper, a composite nonlinear control strategy has been developed for power systems with STATCOM under external disturbances. The presented composite control law has been designed based
Fig. 2: Controller performance–Power angles ($\delta$) (rad.), frequency ($\omega - \omega_s$) rad/s., transient voltage, and STATCOM current (Solid: The proposed control, Dashed: Backstepping control without disturbance observer)

Fig. 3: External disturbances and disturbance estimations
on a combination of backstepping control and a disturbance observer method. This combination can offer not only better dynamic performance, but also disturbance rejection ability as compared with the conventional backstepping control (CBSC). With the help of Lyapunov control theory, the stability analysis of closed-loop system has been provided in spite of having both non-vanishing and vanishing disturbances. The simulation results have confirmed that even though the CBSC method is an effective method to stabilize the overall closed-loop dynamics, the nonlinear composite control can clearly improve the transient performances and has better disturbance rejection property than the CBSC method.

**References**


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