Performance Analysis of ML-based Low Complex CFO Estimation for MIMO-OFDMA Uplink Systems

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ABSTRACT

Carrier frequency offset (CFO) estimation in multiuser multiple-input multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) system is investigated in this study. MIMO-OFDM is very sensitive to CFOs due to oscillator frequency mismatch and/or Doppler shift. Inaccurate CFO estimation results in intercarrier interference (ICI) through the loss of orthogonality among subcarriers. In this paper, the performance of the ML and APFE algorithm is analyzed for CFO estimation. ML becomes extremely complex due to the multidimensional exhaustive search issue, which is the basic concern in ML estimation. However, making use of the iterative low complex APFE method, here this multidimensional search is replaced with a sequence of mono-dimensional searches. This results in an estimation algorithm of reasonable complexity which is suitable for practical applications. In addition, ML accuracy is compared with the Cramer-Rao bound (CRB).

Keywords: Orthogonal Frequency-Division Multiplexing, OFDM, Carrier Frequency Offset, CFO, Multiple-Input Multiple-Output, MIMO, Maximum Likelihood, ML, Cramer-Rao Bound, CRB

1. INTRODUCTION

In digital communications, orthogonal frequency-division multiplexing (OFDM) has proven to be a viable strategy for addressing frequency-selective and multipath channels. Carrier frequency offset (CFO) exists between base stations (BS) and mobile stations (MS) in OFDM systems. In the OFDMA system, many users submit their distinct data simultaneously by modulating an exclusive set of orthogonal subcarriers. In the OFDMA uplink system, CFO estimation and channel estimation are two critical problems. Inaccurate CFO estimation produces intercarrier interference (ICI) due to the loss of orthogonality among subcarriers. Timing issues create inter-block interference (IBI) within successive OFDMA blocks, which must be avoided to prevent significant bit error reduction [1].

Since OFDM is more susceptible to CFOs, exact CFO estimation and compensation is very important [2]. Multiple-input multiple-output orthogonal frequency-division multiplexing (MIMO-OFDM) has drawn substantial research interest as a major element in broadband wireless communication systems. MIMO can exploit spatial resources and give high capacity, whereas OFDM is not affected by frequency-selective fading. However, these benefits are strongly reliant on accurate CFO estimation and compensation between the transceivers [3].

To fully utilize the capabilities of MIMO-OFDM systems, three aspects should be considered: channel estimation, time synchronization, and frequency synchronization. In similarity to other multicarrier-based techniques, the MIMO-OFDM system is very sensitive to CFO caused by oscillator mismatch and/or Doppler shift. For coherent data detection, knowledge of the channel impulse response (CIR) is indispensable at the receiver. Since OFDM systems have some tolerances to timing errors due to the cyclic prefix (CP) addition, time synchronization is less critical than frequency synchronization and channel estimation [4, 5].

The CFO for single-input, single-output (SISO) OFDM systems are estimated using periodic training sequences in [6] and [7]. It is demonstrated that a CFO estimator of this type achieves the Cramer-Rao bound (CRB) with minimal computational effort. Comparative thinking is employed to analyze the MIMO-OFDM scheme [8–10], in which a concentrated local oscillator (LO) drives all the transmitting and receiving connections, as well as the complete receiver equipment. MIMO systems utilize multiple antennas at the source and destination to significantly improve the capacity of such rich scattering radio fading channels in comparison to SISO systems [11, 12].

The challenge of simultaneously predicting CFOs, timing offsets, and channel responses of all active users in the uplink transmission of an OFDMA system is addressed in [1]. It is assumed that each user transmits a training block (carrying known symbols) at the beginning of the uplink frame. A quasi-synchronous system is considered in [1], where the CP of a training block is sufficiently

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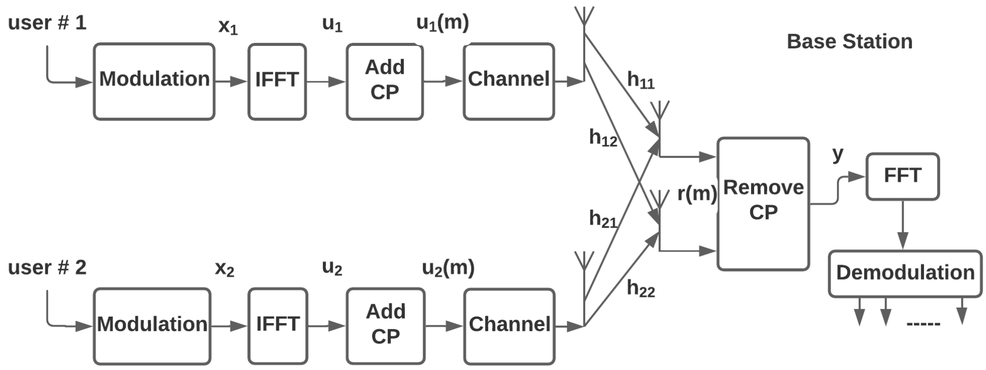

***Fig. 1**: Signal model for the MIMO-OFDMA system.*

long to accommodate both the channel delay spread and propagation delays experienced by user signals.

Like OFDM, the MIMO-OFDM system has recently gained significant attention and is widely considered as a viable choice for next generation wireless networks because of its ability to reduce inter symbol interference (ISI) and increase system capacity [13]. By simultaneously sending independent data streams through distinct antennas, the system throughput can be increased by spatial multiplexing. The frequency-selective fading channel is converted into a group of frequencies at fading channels with OFDM, making equalization considerably easier than is the case with single carrier systems. MIMO-OFDM systems offer substantially greater spectrum efficiency than other OFDM-based techniques, such as MIMO-OFDMA, due to the utilization of spatial multiplexing [14].

CFO is well-known in the OFDMA system and according to the literature, maximum likelihood (ML) estimation of CFO is the best technique and provides an optimum solution to this issue. Since an exhaustive multidimensional search is required in the multiuser scenario, estimation with ML is difficult, especially with a large number of active users [1, 15]. Non-ML approaches with a low level of complexity tend to perform below expectations [16, 17].

In recent years, there has been a focus on reducing the complexity of ML estimation for CFO. Alternating projection frequency estimation (APFE) reduces the number of total evaluations by converting a multidimensional exhaustive search into a one-dimensional search. APFE is a simplified approach that estimates CFO sequentially rather than jointly [1]. It uses an iterative approach for changing just one user's trial CFO value at a time while holding the values of the others constant. For accurate estimation, this approach requires a higher resolution of CFO trials and a reasonable initial guess. Furthermore, the suggested method for calculating an initial guess [18] contributes to the overall complexity of the CFO estimation process. In [19], a new iterative approach is developed based on dividing and updating the frequencies of distinct users, promising APFE accuracy with reduced complexity.

The authors in [1] built up a CFO estimation method for the OFDMA uplink. In this paper, the CFO estimation for the MIMO-OFDMA system is investigated and assessed for two users.

The content of this work is structured as follows. Section 2 introduces the signal model, followed by the ML and APFE estimation methods in Sections 3 and 4, respectively. Section 5 details the computational complexity of APFE while the performance of the ML and APFE estimation method is evaluated in section 6 through the simulation results. Section 7 presents the conclusion and future scope of the paper.

2. SIGNAL MODEL FOR MIMO-OFDMA SYSTEMS

Fig. 1 shows a multiuser MIMO-OFDM system in which many users, each with one or more antennas, can broadcast at the same time utilizing the same frequency band. A base station with many antennas is used as the receiver. The performance of ML is assessed in this paper for CFO estimation in a MIMO-OFDMA uplink system with two users, each with one transmitting antenna.

Here, N denotes the number of total subcarriers and $S_k(n)$ indicates the k^{th} user's n^{th} block of frequency-domain symbol, where $k \in \{1, 2, \dots, K\}$. The sequel to this investigation focuses on a single block with the temporal index n omitted for notational simplicity. The j^{th} entry of S_k, say $S_{k,j}$, is nonzero if, and only if, the j^{th} subcarrier is modulated by the k^{th} user, with $j \in \{1, 2, \dots, N-1\}$. This implies that S_k has only N_k nonzero components, where N_k denotes the number of subcarriers allotted for the k^{th} user. The equivalent time-domain vector is given by

$$X_k = F^H S_k \tag{1}$$

where F is the N-point discrete Fourier transform (DFT) matrix with entries $[F]_{n,k} = \left(1/\sqrt{N}\right) \exp\left(-j2\pi nk/N\right)$ for $0 \le n, k \le N-1$, and the Hermitian transpose is indicated by the superscript $(\cdot)^H$. A Cyclic Prefix length, which is denoted by N_g is appended in front of x_k to eliminate

IBI. The resulting vector u_k of length $N_B = N + N_g$ is then sent across the channel. Referring to $h_k(l)$ as the discrete-time composite channel impulse response of the k^{th} user (encompassing the transmit/receive filters and the communication channel), the corresponding channel response vector can be composed as $h_k = [h_k(0), h_k(1), \ldots, h_k(L_k - 1)]^T$, where $(\cdot)^H$ refers to the transpose vector and L_k the channel order. Since L_k is usually unknown, in practice, h_k is replaced with an L_h-dimensional vector

$$h'_k = \left[h_k^T \quad 0_{(L_k - L_k)}\right]^T \tag{2}$$

where $L_h \geq \max_k\{L_k\}$ is the design parameter that depends on the duration of the transmit/receive filters and on the "maximum expected" channel delay spread.

A MIMO-OFDM system with n_t users and a single transmit antenna for each user is considered. The base station contains n_r receiving antennas, with $n_r \geq n_t$. The major focus of this study is complexity reduction, hence two users with two receiving antennas on the base station are investigated. As the number of users increases, the complexity of ML and APFE grows exponentially and linearly, respectively. The OFDM system includes 128 subcarriers with a CP length of 8. The exact 128 subcarriers are separated similarly for every user. At the i^{th} receiving antenna, the received signal can be made up of the following components

$$r_i(m) = \sum_{k=1}^{n_i}\left\{e^{j\omega_k m}\sum_{l=0}^{L_h-1} h_{ik}(l)u_k(m-l)\right\} + v(m) \tag{3}$$

where the CFO value for the k^{th} user is ω_k, the timing index is m and the number of multipath segments in the channel is L. It can be seen from Eq. (3) that there are no separate CFO values to estimate. In an equivalent matrix form, the receiving signal in Eq. (3) can be described as

$$y = \sum_{k=1}^{n_t}\Gamma(\omega_k)A_k\xi_k + v \tag{4}$$

where $\omega_k = 2\pi\Delta f_k/N$ and Δf_k represents the k^{th} CFO normalized to the subcarrier spacing, v represents the noise contribution with variance $\sigma_v^2 = 2N_0$, with $N_0/2$, being the two-sided power spectral density of the thermal noise.

$$\Gamma(\omega_k) = \operatorname{diag}\left\{e^{j\omega_k N_g}, e^{j\omega_k(N_g+1)}, \ldots, e^{j\omega_k(N_g+N-1)}\right\}$$

$$[A_k]_{p,q} = [x_k]_{|p-q|_N}, \quad 1 \leq p \leq N, \ 1 \leq q \leq N_g$$

$$\xi_k = \left[0^T_{\mu_k\times 1} \quad h'^T_k \quad 0^T_{(N_g-\mu_k-L_h)\times 1}\right]^T$$

In the above conditions, $[x_k]_l$ signifies the l^{th} element of $[x_k]$ for $0 \leq l \leq N-1$, and the modulo-N operation $|i|_N$ indicates that i is decreased to the interval $[0, N-1]$. The signal model in Eq. (4) is used for CFO estimation in this study, which can be used for K users. The subsequent sections provide further details.

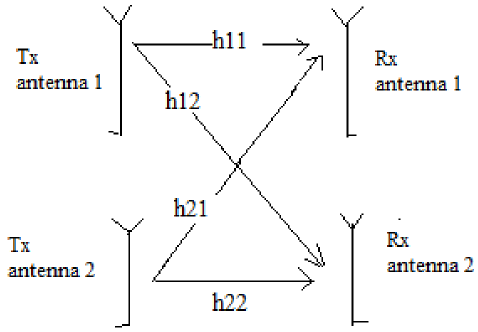

Fig. 2: *Channel model.*

3. ML ESTIMATION

The ML process for CFO estimation mentioned in [1] is briefly discussed in this section. A channel impulse response vector in between the k^{th} user and i^{th} receiving antenna is indicated as h_{ki} (Fig. 2). Gathering the message signals received by all antennas we have,

$$y = Q(\omega)\xi + v \tag{5}$$

The above equation is the equivalent form of Eq. (4), where $Q(\omega) = [\Gamma(\omega_1), \Gamma(\omega_2), \ldots, \Gamma(\omega_k)]$.

Recalling that the entries of v are independent Gaussian random variables having zero mean and variance σ_v^2, the log likelihood factor for the unknown parameters ω and ξ takes the form

$$\wedge(\widetilde{\omega}, \tilde{\xi}) = -N\ln\left(\pi\sigma_v^2\right) - \frac{1}{\sigma_v^2}\left\|y - Q(\widetilde{\omega})\tilde{\xi}\right\|^2 \tag{6}$$

where $\widetilde{\omega}$ and $\tilde{\xi}$ is the trial values of ω and ξ respectively, whereas $\|x\|$ is the Euclidean norm of the enclosed vector x. Searching for the maximum of $\wedge(\widetilde{\omega})$ yields the joint ML estimations of ω.

$$\widetilde{\omega} = \arg\max_{\widetilde{\omega}}\left\{\left\|P_Q(\widetilde{\omega})y\right\|^2\right\} \tag{7}$$

where $P_Q(\widetilde{\omega}) = Q(\widetilde{\omega})\left[Q^H(\widetilde{\omega})Q(\widetilde{\omega})\right]^{-1}Q^H(\widetilde{\omega})$.

For the joint ML estimation of ω and ξ, $N_k \geq N_g$ and $K < N/N_g$ are required conditions.

$$y = [y_1 \quad y_2]_{n_t\times n_r} \tag{8}$$

where n_t denotes the number of total transmitting antenna and n_r denotes the receiving antenna

$$\xi = [\xi_1 \quad \xi_{n_t}] \tag{9}$$

$$\xi_k = [\xi_{1,k} \quad \xi_{n_r,k}] \tag{10}$$

where ξ represents the channel matrix and ξ_k is the k^{th} user's channel matrix.

$$\widetilde{\omega} = \arg\max_{\widetilde{\omega}} \{tr\left(y^H P_Q(\widetilde{\omega})y\right)\} \tag{11}$$

Moreover, $tr(\cdot)$ denotes a matrix's trace.

To acquire the ML estimation of the CFO vector, a search with all potential CFO values from all users is required. The complexity of this search rises rapidly as the user percentage grows, making it impractical. The maximization in Eq. (7) requires a grid-search across the multidimensional domain spanned by $(\widetilde{\omega})$, which may be too cumbersome in practice. APFE replaces the multidimensional maximization issue in Eq. (7) with a sequence of simpler 1-D searches.

The asymptotic CRB ($asCRB$) for ω is characterized in [4] as

$$asCRB(\omega) = \frac{6\sigma_v^2}{N^3}\left[\left(\Re\left\{H^H R H\right\}\right)^{-1}\right] \tag{12}$$

where $H = \left[H_1^T \ H_2^T \ \cdots \ H_{M_r}^T\right]$, $H_j = \text{diag}\left([h_{1j} \ h_{2j} \ \cdots \ h_{kj}\right)$, and R is a positive-definite matrix, as shown in [4]

$$d_k(n) = \exp\left(j\xi_n\right)\exp\left(j\phi_k\right)\exp\left(-\frac{j2\pi(k-1)n}{k}\right) \tag{13}$$

where $k = 1, 2, \ldots, K$ and $n = 0, 1, \ldots, N-1$.

4. ALTERNATING PROJECTION FREQUENCY ESTIMATOR (APFE) METHOD FOR MIMO-OFDMA

The alternating projection algorithm for a OFDMA uplink system is derived in [1]. Here, the same method is extended for multiuser MIMO-OFDM uplink systems with multiple antennas. The iterative APFE method is used to solve the multidimensional optimization issue. This technique is now exploited to reduce the K-dimensional maximization in Eq. (7) into a sequence of 1-D maximization issues, as previously described. The technique that emerges is made up of cycles and steps. Each of the K stages in a cycle updates the CFO of a single user while retaining the other CFOs at their most updated values. In order to maintain generality, the user's CFO is updated in the natural ordering $k = 1, 2, \ldots, K$. In addition, $\widehat{\omega}_k^{(i)}$ is referred to as the estimate of ω_k at the i^{th} cycle, and define the $(K-1)$-dimensional vector $\widehat{\omega}_k^{(i)}$ as

$$\widehat{\omega}_k^{(i)} = \left[\widehat{\omega}_1^{(i+1)}, \ldots, \widehat{\omega}_{(k-1)}^{(i+1)}, \ \widehat{\omega}_{(k+1)}^{(i)}, \ldots, \widehat{\omega}_k^{(i)}\right]^T \tag{14}$$

At the k^{th} step of the $(i+1)^{\text{th}}$ cycle, the alternating projection approach updates the estimate of ω_k by solving the following 1-D maximising issue:

$$\widehat{\omega}_k^{(i+1)} = \arg\max_{\widetilde{\omega}_k}\left\{\left\|P_Q\left(\widetilde{\omega}_k, \widehat{\omega}_k^{(i)}\right)y\right\|^2\right\} \tag{15}$$

where $P_Q\left(\widetilde{\omega}_k, \widehat{\omega}_k^{(i)}\right)$ denotes the relationship between $\widehat{\omega}_k^{(i)} = \left[\widehat{\omega}_1^{(i+1)}, \ldots, \widehat{\omega}_{(k-1)}^{(i+1)}, \ \widehat{\omega}_{(k+1)}^{(i)}, \ldots, \widehat{\omega}_k^{(i)}\right]^T$ and P_Q, continuing in this manner until the computation of $\widehat{\omega}_k^{(i+1)}$, which concludes the $(i+1)^{\text{th}}$ cycle. The results gathered so far are further refined in the $(i+2)^{\text{th}}$ cycle, where $\widehat{\omega}_k^{(i+1)}$ is used to compute $\widehat{\omega}_k^{(i+2)}$ for $k = 1, 2, \ldots, K$. Multiple cycles are executed until the CFO estimations reach a steady conclusion. Even though it is possible for the solution to converge to a local maximum of $\left\|P_Q(\widetilde{\omega})y\right\|^2$ depending on the particular initialization, the approach converged to the true CFO in several cycles during all the experiments. In Eq. (15), the estimator is referred to as iterative estimation via alternating projection.

5. COMPUTATIONAL COMPLEXITY

The APFE estimation scheme is less complex than ML estimation. Starting with the ML metric $\left\|P_Q(\widetilde{\omega})y\right\|^2$ on the right-hand side of Eq. (7), which must be evaluated for each and every trial value, demanding a greater number of operations than APFE. ML has a complexity of $N_C \times (N_\omega)^k \times N_0$, where, N_ω represents the trial values of $\widetilde{\omega}$ and N_0 represents the total number of operations requiring estimation. Since a new metric must be generated for all users at each cycle, the complexity of APFE is $N_C \times N_\omega \times K \times N_0$. As a result, in ML estimation, the complexity of APFE increases linearly rather than exponentially. APFE achieves convergence in two cycles, but $N_C > 2$ yields no significant benefits. As a result, $N_C = 2$ is used in all subsequent simulations, helping to minimize the computational complexity. At any given time, only the CFO of one user can be modified as a trial value, while the CFOs of all other users remain unchanged. CFO is an iterative process, converging for all users.

6. ANALYSIS RESULT

In a MIMO-OFDMA scenario, the performance of ML and APFE is investigated using a computational model as shown in Fig. 3. The number of subcarriers per user in an OFDMA system with N subcarriers distributed among K users is $N_k = N/K$. Every user is assigned subcarriers at random and the fractional CFO of each is accepted in all simulations. The length of the cyclic prefix is specified as 8. To evaluate the performance of CFO estimation, the mean squared error (MSE) in the estimation of normalized CFO is characterized as MSE = $\|\text{estimated CFO} - \text{actual CFO}\|^2$.

The outcomes of the APFE algorithm [1] are also reproduced as a comparison measure. The parameters used for simulation are presented in Table 1.

The accuracy of the ML within sight of two users with equivalent power is examined in this study. The MSE as a measure of SNR is shown in Fig. 4. The CRB is also used as a benchmark. As can be observed from Fig. 4, the ML error cannot cross the CRB because the latter is

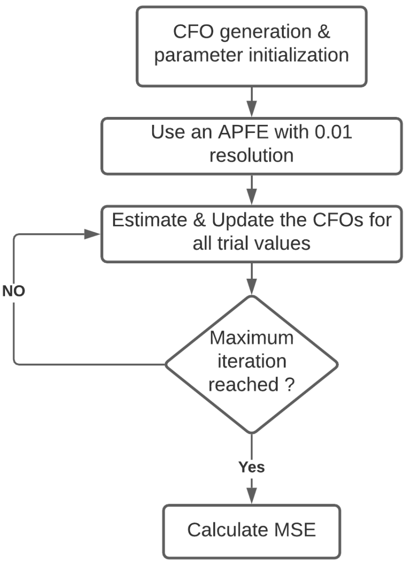

Fig. 3: *Flowchart of the APFE scheme.*

Table 1: *MIMO-OFDMA system parameters.*

Parameter	Values
Total users (K)	2
Transmitting antennas	2
Receiving antennas	2
DFT size (N)	128
Cyclic Prefix length (N_g)	8
Modulation method	QPSK
CFO	Fractional
Channel	Rayleigh fading

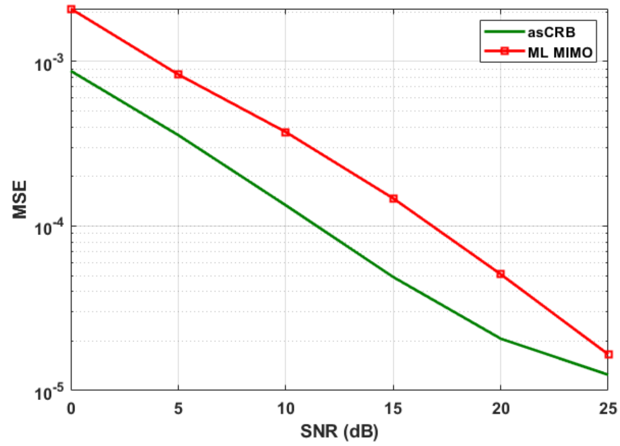

Fig. 4: *Comparison between asCRB and ML.*

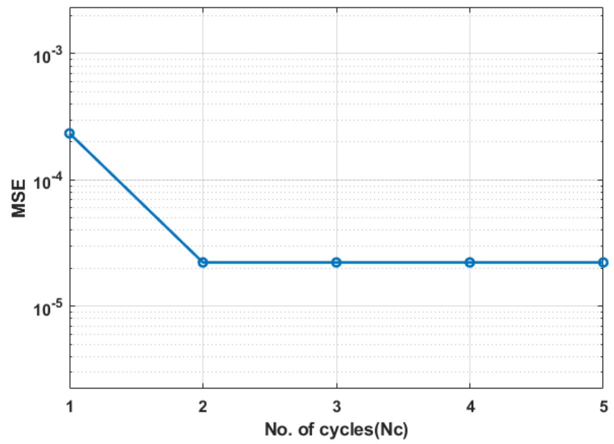

Fig. 5: *Convergence of APFE.*

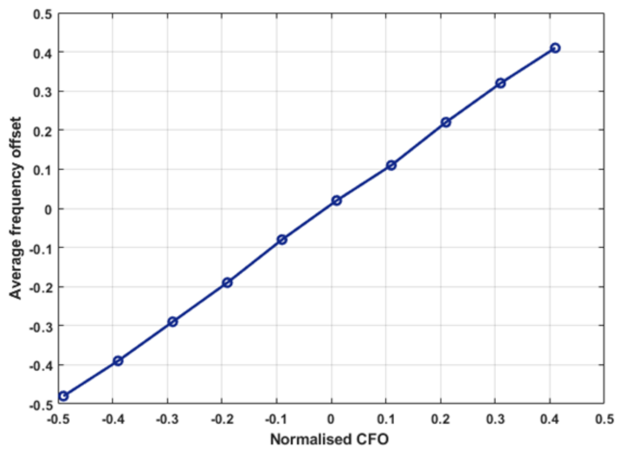

Fig. 6: *Average Frequency estimates versus $\Delta f1$.*

in the lower reaches of the mean square error. The ML estimation strategy is an asymptotic way to deal with CRB, yet at the same time, there is a hole as far as MSE is concerned. In the event any estimation technique can address the CRB issue, at that point, it will become an optimal method.

The number of cycles required to attain convergence is a key design parameter in CFO estimators. The MSE of the normalized frequency estimates provided by APFE as a function of $N_C = 2$ for $K = 2$ is shown in Fig. 5. All users have the same amount of power with 20 dB of SNR. It can be observed that APFE reaches convergence in two cycles and there are no significant benefits with $N_C > 2$. As a result, $N_C = 2$ is taken into account in all following simulations. The average frequency estimates versus $\Delta f1$ are shown in Fig. 6, assuming two users are present in the system. For comparison, the ideal line $E\left\{\widehat{\Delta f1}\right\} = \Delta f1$ is also drawn. It can be observed that APFE provides accurate estimation over the range $|\Delta f1| \leq 0.5$.

ML-MIMO performs better than the ML scheme, as observed in Fig. 7. The MSE of 1.2×10^{-4} achieves at 15 dB of SNR in ML-MIMO and the MSE of 2×10^{-4} achieves at 15 dB of SNR in ML.

The APFE-MIMO scheme performs better than the APFE system, as shown in Fig. 8. The MSE of 1.8×10^{-4}

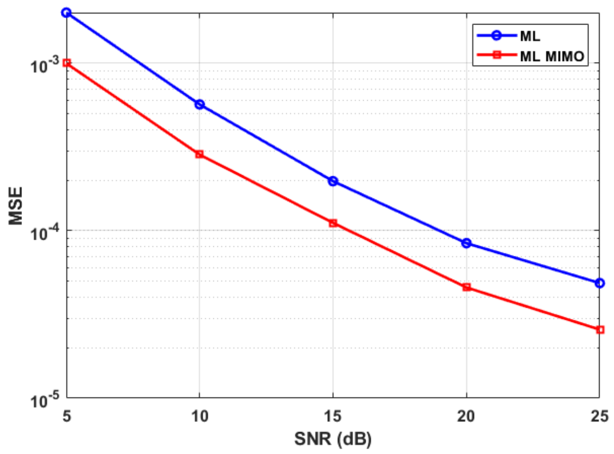

***Fig. 7**: Comparison between ML and ML-MIMO.*

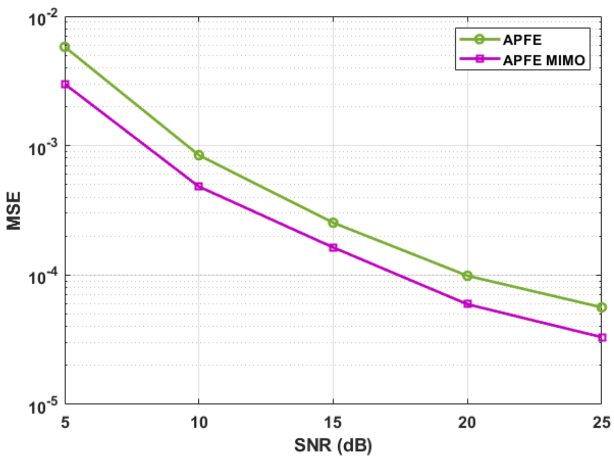

***Fig. 8**: Comparison between APFE and APFE-MIMO.*

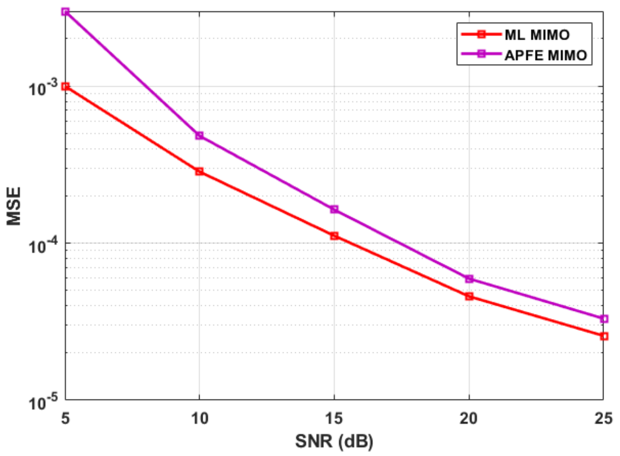

***Fig. 9**: Comparison between ML-MIMO and APFE-MIMO.*

achieves at 15 dB of SNR in APFE-MIMO whereas the MSE of 2.5×10^{-4} achieves at 15 dB of SNR in APFE.

In terms of accuracy, ML-MIMO performs better than APFE-MIMO, as shown in Fig. 9. The MSE of 1.2×10^{-4} reaches at 15 dB of SNR in ML-MIMO whereas the MSE of 1.8×10^{-4} provides at 15 dB of SNR in APFE-MIMO.

Section 5 and Table 2 explore and summarize the

***Table 2**: Evaluations required in ML and APFE.*

Estimation Technique	Complexity
ML	$N_C \times (N_\omega)^K \times N_0$
APFE	$N_C \times N_\omega \times K \times N_0$

***Table 3**: Complexity comparison of ML and APFE for the two user cases.*

Estimation Technique	Complexity
ML	$20000 \times N_0$
APFE	$400 \times N_0$

***Table 4**: MSE Comparison of ML, APFE, ML-MIMO and APFE-MIMO with the two user cases.*

Sr. No.	SNR	ML	ML-MIMO	APFE	APFE-MIMO
1	5	2×10^{-3}	1×10^{-3}	5.8×10^{-3}	3×10^{-3}
2	10	5.8×10^{-4}	2.8×10^{-4}	8.5×10^{-4}	4.9×10^{-4}
3	15	2×10^{-4}	1.2×10^{-4}	2.5×10^{-4}	1.8×10^{-4}
4	20	8.5×10^{-5}	4.6×10^{-5}	1×10^{-4}	6×10^{-5}
5	25	5×10^{-5}	2.6×10^{-5}	5.6×10^{-5}	3.3×10^{-5}

computational complexity of the ML and APFE, respectively. The attributes presented in Table 1 are used to estimate the fractional CFO of two users. There are only a few options for varying the complexity of APFE. The results of the APFE method with a resolution of 0.01 are considered. In Table 2, N_C denotes the number of cycles, N_ω represents the trial values of $\tilde{\omega}$, which are the same as the number of resolutions and N_0 represents the total number of operations needed for the CFO estimation. In Table 3, N_0 can be varied based on the requirement since the N_0 is directly proportional to both accuracy and complexity. The complexity of ML is relatively high in comparison to APFE for two user cases as shown in Tables 2 and 3. ML complexity grows exponentially along with the number of users, but APFE minimizes it to linear growth.

Table 4 shows the comparison between ML, APFE, ML-MIMO, and APFE-MIMO systems with two user cases. Here it is observed that ML-MIMO performs better in terms of accuracy. ML reduces the complexity in comparison to ML-MIMO in terms of antenna configuration, but its performance degrades in terms of accuracy for the same number of users. On the other hand, APFE-MIMO performs better in comparison to APFE while APFE decreases the complexity of antenna designs when compared to APFE-MIMO. It can also be observed that the accuracy of ML is superior when compared to APFE. ML-MIMO and APFE-MIMO show a similar result.

7. CONCLUSION AND FUTURE SCOPE

The CFO estimation issue in the uplink of an OFDMA and MIMO-OFDMA for two users is examined in this study. ML performs better for the multiuser MIMO-OFDM uplink in terms of accuracy compared to the OFDMA uplink. CRB is within the lower limit of mean squared error. The ML estimation technique is an asymptotical way of dealing with CRB, yet at the same time, there is a hole as far as MSE is concerned. If any estimation technique can address the CRB issue, at that point, it is an optimal method. The accuracy of ML is better than APFE, but more complex due to the multidimensional search. APFE achieves convergence in two cycles, which helps to minimize its computational complexity. The complexity of APFE increases linearly rather than exponentially in ML estimation.

In this study, a simulated system with two users is proposed, each with their own transmitting antenna, and multiple receiving antennas on the base station. In the future, this study can be further extended by increasing the number of users. Since it is proven that, by increasing the number of users, complexity also increases, enabling different algorithms to be applied.

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