Efficient User Grouping for MU-MIMO Visible Light Communications Based on Block Diagonalization

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ABSTRACT

This paper investigates efficient user grouping methods for multi-user multi-input multi-output (MU-MIMO) visible light communication (VLC) systems. Block diagonalization (BD) precoding is considered for interference avoidance. In addition, time division multiplexing (TDM) is applied to perform user grouping when the number of users exceeds the limit of BD precoding based on the number of light emitting diode (LED) transmitters and the total number of users' photodiodes (PDs). User grouping methods are proposed based on pairwise interference considerations among users in the same group. The proposed methods can be implemented through integer linear programming (ILP), which requires less computation than exhaustive search. The numerical results on the average minimum user throughputs over random scenarios indicate that the proposed hybrid method can significantly outperform random user grouping and performs reasonably well compared to exhaustive search. Finally, this study demonstrates that, when BD precoding greatly attenuates the desired user signals, user grouping can help improve minimum user throughputs even though BD precoding can support all users as a single group.

Keywords: Visible Light Communications, Multi-Input Multi-Output Transmissions, Multi-User Transmissions, Block Diagonalization, Time Division Multiplexing, Optimization

1. INTRODUCTION

In recent years, visible light communication (VLC) is considered a promising candidate to complement indoor radio frequency (RF) communications for high data rate wireless access. The growing interest in VLC from researchers as well as industrial communities has resulted in a rapid increase in the usage of light emitting diodes (LEDs) for both illumination and communication purposes. The benefits of VLC include much higher unregulated bandwidth compared to RF communications, immunity to RF interferences, and insusceptibility to eavesdropping. Based on these properties, VLC is expected to make significant inroads in future indoor networking [1, 2].

Several applications of VLC have been reported in [3–5]. Notable applications include localization for indoor navigation, location-based services, asset tracking, and inventory management. As a communication infrastructure, VLC appears in intelligent transportation systems, home automation, underwater communication systems, and indoor high-speed data transmissions known as light fidelity (Li-Fi). Recently, VLC has been used to provide connections among a large number of Internet-of-Things (IoT) devices.

Multiple LEDs are typically used in indoor environments to provide sufficient illumination. In such scenarios, a multi-input multi-output (MIMO) scheme, which utilizes multiple LEDs and multiple photodiodes (PDs), can naturally be implemented to support high data rates in VLC systems. Several MIMO techniques for VLC have been investigated [1, 2]. Most of the literature on MIMO VLC has been inspired by MIMO schemes for RF communications, either to increase data rates through spatial multiplexing or to improve reliability through spatial diversity [6]. In addition, through spatial multiplexing, MIMO VLC systems can support the simultaneous transmission of multiple data streams from multiple LEDs to multiple users at different locations. Such systems are referred to as multi-user MIMO (MU-MIMO) VLC systems.

To date, there have been several studies on MU-MIMO VLC systems. A common challenge is how to mitigate or prevent multi-user interference (MUI), which has adverse effects on the transmission performance at each user's receiver. A number of transmission precoding schemes have been proposed to overcome MUI, including dirty paper coding (DPC), zero-forcing (ZF) precoding, and block diagonalization (BD) precoding [7, 8]. Among them, DPC is considered computationally insensitive since it involves nonlinear signal processing. The applications of linear processing, ZF precoding, and BD precoding are more practical in terms of computational complexity.

ZF precoding can prevent interference among all data streams transmitted from multiple LEDs, even among the data streams going to the same user. However, ZF precoding suffers from a noise amplification problem.

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BD is therefore considered as a compromise. Unlike ZF precoding, BD precoding prevents interference among individual users. Furthermore, signal processing, e.g., precoding based on singular value decomposition (SVD), is required to prevent interference among the individual data streams of each user. As a result, BD precoding does not suffer as much from noise amplification as ZF precoding.

Several existing works investigate BD precoding for MU-MIMO VLC [9–16]. In all these works, it is assumed that the total number of PDs at the receivers of all users does not exceed the number of LEDs. For example, in [9], there are four LED transmitters and two users each with two PDs, for a total of four PDs. This assumption is made since BD precoding is not applicable when the number of PDs exceeds the number of LED transmitters.

The above assumption is relaxed in this research since it may be possible to have many user devices in the same room, yielding more PDs than LEDs. To apply BD precoding, user grouping is performed while time division multiplexing (TDM) is applied with BD precoding for each user group. For example, with four LED transmitters and four users each with two PDs, the users can be divided into two groups of two, and TDM is then used with BD precoding applied in each timeslot.

This paper investigates how to group users in an MU-MIMO VLC system with BD precoding so that the transmission performance is maximized. In particular, for fairness, the goal is to maximize the minimum user throughput. First, it is demonstrated that efficient user grouping can lead to significant throughput improvements over the worst-case grouping. Then, a user grouping heuristic is proposed and shown to perform reasonably well compared to the optimal grouping found through exhaustive search. In addition, based on the proposed heuristic, an integer linear programming (ILP) formulation is provided to perform user grouping. Finally, user grouping is shown to help improve minimum user throughputs in scenarios where BD precoding can accommodate all users but greatly attenuates the desired user signals to prevent MUI.

Section 2 describes the system model used in this investigation. Section 3 provides an example scenario to demonstrate the importance of efficient user grouping for MU-MIMO VLC systems. Section 4 presents the proposed user grouping methods. The numerical performance results of user grouping in terms of the minimum user throughputs are provided in Section 5. Finally, Section 6 summarizes the work and provides suggestions for further research.

2. SYSTEM MODEL

Consider downlink data transmission in an indoor MU-MIMO VLC system consisting of N_{tx} LED transmitters and K users (i.e., receivers) each with N'_{rx} PDs. The total number of PDs from all users is $N_{\text{rx}} = KN'_{\text{rx}}$. Assume that $N_{\text{rx}} > N_{\text{tx}}$. For simplicity, let us also assume that $N_{\text{rx}}/N_{\text{tx}}$ and $N_{\text{tx}}/N'_{\text{rx}}$ are positive integers. Accordingly, there will be $G = N_{\text{rx}}/N_{\text{tx}}$ groups each containing $K' = N_{\text{tx}}/N'_{\text{rx}}$ users. When this integer assumption does not hold, user group sizes may vary. For example, when $N_{\text{tx}} = 6$, $N_{\text{rx}} = 10$, and $N'_{\text{rx}} = 2$, two groups of three and two users can be formed, respectively. It should be noted that all the user grouping methods being investigated can be adjusted to accommodate unequal group sizes.

2.1 Transmitted and Received Signals

Consider a TDM timeslot used by an arbitrary group of K' users. Index the K' users by $1, \dots, K'$. The transmitted signals from LED transmitters can be expressed as

$$\mathbf{s} = P\left(\mathbf{1} + \mu \sum_{k=1}^{K'} \mathbf{F}_k \mathbf{x}_k\right), \tag{1}$$

where $\mathbf{s}$ is a $N_{\text{tx}} \times 1$ vector (i.e., a matrix with N_{tx} rows and 1 column) containing the transmitted signal values, P is a scaling factor, $\mathbf{1}$ is the $N_{\text{tx}} \times 1$ all-one vector, μ is the modulation index, $\mathbf{F}_k$ is a $N_{\text{tx}} \times N'_{\text{rx}}$ precoding matrix for user k, and $\mathbf{x}_k$ is a $N'_{\text{rx}} \times 1$ vector of data symbols for user k.

Assuming intensity modulation (IM) and direct detection (DD), μ is selected to make transmitted signal values non-negative, i.e., $\mathbf{s} \geq \mathbf{0}$. In addition, since typical data symbols (e.g., from standard constellations used in pulse amplitude modulation) have mean 0, the average of $\mathbf{x}_k$ is a zero vector, yielding the average signal value at each LED transmitter equal to P. Since the transmitted optical power depends on the average signal value, P is the optical power per LED transmitter.

The received signals at the N'_{rx} PDs of user k can be written as

$$\mathbf{r}_k = \alpha_{\text{A/W}} \mathbf{H}_k \mathbf{s} + \mathbf{n}_k, \tag{2}$$

where $\mathbf{r}_k$ is a $N'_{\text{rx}} \times 1$ vector containing received signal values at the N'_{rx} PDs for user k, $\alpha_{\text{A/W}}$ is the receiver responsivity, $\mathbf{H}_k$ is a $N'_{\text{rx}} \times N_{\text{tx}}$ MIMO channel matrix for user k, and $\mathbf{n}_k$ is a $N'_{\text{rx}} \times 1$ noise vector for user k that contains independent and identically distributed (IID) Gaussian random variables with mean 0 and variance σ^2_{AWGN}.

The MIMO channel matrix

$$\mathbf{H}_k = \begin{bmatrix} h_{k,11} & \cdots & h_{k,1N_{\text{tx}}} \\ \vdots & \ddots & \vdots \\ h_{k,N'_{\text{rx}}1} & \cdots & h_{k,N'_{\text{rx}}N_{\text{tx}}} \end{bmatrix} \tag{3}$$

of user k contains the values of path losses, where $h_{k,ij}$ denotes the path loss from LED transmitter j to PD i for user k. For VLC systems, $h_{k,ij}$ can be computed using the line-of-sight (LoS) channel model expressed as [17–20]

$$h_{k,ij} = \begin{cases} \frac{(m+1)A}{2\pi d_{k,ij}^2} \cos^m \phi_{k,ij} \cos \psi_{k,ij}, & |\psi_{k,ij}| \leq \Psi_{\text{FoV}} \\ 0, & |\psi_{k,ij}| > \Psi_{\text{FoV}} \end{cases} \tag{4}$$

where $d_{k,ij}$ is the distance from LED transmitter j to PD i for user k, $\phi_{k,ij}$ is the emission angle from LED transmitter j to PD i for user k, $\psi_{k,ij}$ is the incidence angle from LED transmitter j to PD i for user k, A is the PD detection surface area, Ψ_{FoV} is the field-of-view (FoV) of each PD, and m is the order of Lambertian radiation related to the LED semiangle at half the maximum power $\mathbf{\Phi}_{1/2}$ by $m = -\log(2)/\log(\cos \mathbf{\Phi}_{1/2})$.

While non-LoS components of light propagation also exist, e.g., due to reflections, LoS components dominate for most indoor usage scenarios [18, 20]. Therefore, only LoS components are considered. In addition, shadowing can occur due to random movements of people and objects, potentially causing data transmission outages [3, 21, 22]. This work considers the optimization of user grouping based on long-term channel conditions. Therefore, intermittent outages of data transmission due to shadowing are not considered.

2.2 BD Precoding and Combining

BD precoding is applied within each group of K' users. Consider an arbitrary group of users indexed by $1, \ldots, K'$. For user $k \in \{1, \ldots, K'\}$, the precoding matrix $\mathbf{F}_k$ is computed using the following steps [9].

Step 1: Form the $(K'-1)N'_{\mathrm{rx}} \times N_{\mathrm{tx}}$ matrix

$$\widetilde{\mathbf{H}}_k = \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_{k-1} \\ \mathbf{H}_{k+1} \\ \vdots \\ \mathbf{H}_{K'} \end{bmatrix}. \tag{5}$$

Step 2: Use singular value decomposition (SVD) to write

$$\widetilde{\mathbf{H}}_k = \widetilde{\mathbf{U}}_k \widetilde{\mathbf{\Sigma}}_k \widetilde{\mathbf{V}}_k^{\mathrm{T}}. \tag{6}$$

Step 3: Write

$$\widetilde{\mathbf{V}}_k = \begin{bmatrix} \widetilde{\mathbf{V}}_k^1 & \widetilde{\mathbf{V}}_k^0 \end{bmatrix}, \tag{7}$$

where $\widetilde{\mathbf{V}}_k^0$ contains the last N'_{rx} columns of $\widetilde{\mathbf{V}}_k$ and is the basis matrix for the null space of $\widetilde{\mathbf{H}}_k$. It should be noted that $\widetilde{\mathbf{V}}_k^0$ is $N_{\mathrm{tx}} \times N'_{\mathrm{rx}}$.

Step 4: Form the $N'_{\mathrm{rx}} \times N'_{\mathrm{rx}}$ matrix

$$\overline{\mathbf{H}}_k = \mathbf{H}_k \widetilde{\mathbf{V}}_k^0. \tag{8}$$

Step 5: Use SVD to write

$$\overline{\mathbf{H}}_k = \overline{\mathbf{U}}_k \overline{\mathbf{\Sigma}}_k \overline{\mathbf{V}}_k^{\mathrm{T}}. \tag{9}$$

Step 6: Form the $N_{\mathrm{tx}} \times N'_{\mathrm{rx}}$ precoding matrix

$$\mathbf{F}_k = \widetilde{\mathbf{V}}_k^0 \overline{\mathbf{V}}_k. \tag{10}$$

From Eq. (10) and the fact that $\widetilde{\mathbf{V}}_k^0$ is the basis for the null space of $\widetilde{\mathbf{H}}_k$, which contains the MIMO channel matrices of all the users except for user k, it follows that the signals transmitted to user k will not be received by any other user. Therefore, BD precoding can prevent MUI.

On the receiving side, user k performs signal combining using the $N'_{\mathrm{rx}} \times N'_{\mathrm{rx}}$ matrix $\overline{\mathbf{U}}_k^{\mathrm{T}}$. Overall, BD precoding and combing yield N'_{rx} spatial channels for user k with the effective path losses $\sigma_{k,1}, \ldots, \sigma_{k,N'_{\mathrm{rx}}}$ equal to the diagonal entries of the $N'_{\mathrm{rx}} \times N'_{\mathrm{rx}}$ diagonal matrix $\overline{\mathbf{\Sigma}}_k$.

2.3 User Throughputs

After signal combining using $\overline{\mathbf{U}}_k^{\mathrm{T}}$ and removing the signal averages which do not contain any information, the combined signals can be expressed as

$$\mathbf{v}_k = \overline{\mathbf{U}}_k^{\mathrm{T}} \mathbf{r}_k = \alpha_{\mathrm{A/W}} P \mu \overline{\mathbf{\Sigma}}_k \mathbf{x}_k + \mathbf{n}'_k, \tag{11}$$

where the noise vector $\mathbf{n}'_k = \overline{\mathbf{U}}_k^{\mathrm{T}} \mathbf{n}_k$ has the same statistics as the noise vector $\mathbf{n}_k$ before signal combining, i.e., containing IID Gaussian random variables with mean 0 and variance σ^2_{AWGN}.

Let E_{s} denote the mean square of data symbols, i.e., average symbol energy. Using Shannon's capacity formula [23], the average total throughput of user k (in bit/s/Hz) over N'_{rx} spatial channels with G TDM timeslots (for G user groups) is

$$\begin{aligned} C_k &= \frac{1}{G} \sum_{n=1}^{N'_{\mathrm{rx}}} \log_2 \left(1 + \frac{\alpha^2_{\mathrm{A/W}} P^2 \mu^2 \sigma^2_{k,n} E_{\mathrm{s}}}{\sigma^2_{\mathrm{AWGN}}} \right) \\ &= \frac{1}{G} \sum_{n=1}^{N'_{\mathrm{rx}}} \log_2 \left(1 + K_{\mathrm{SNR}} \sigma^2_{k,n} \right), \end{aligned} \tag{12}$$

where

$$K_{\mathrm{SNR}} = \frac{\alpha^2_{\mathrm{A/W}} P^2 \mu^2 E_{\mathrm{s}}}{\sigma^2_{\mathrm{AWGN}}}. \tag{13}$$

In a multi-user scenario, user throughputs can differ depending on their path losses. For fairness among users, the minimum throughput among users is considered as the performance measure.

3. EXAMPLE USER GROUPING SCENARIOS

Consider downstream data transmissions in a 5 m × 5 m × 2.5 m (width × length × height) room as illustrated in Fig. 1. There are four LED transmitters and four users each with two PDs, i.e., $N_{\mathrm{tx}} = 4$, $K = 4$, and $N'_{\mathrm{rx}} = 2$. Accordingly, to apply BD precoding, users need to be divided into two groups each with two users, i.e., $K' = 2$.

The four LED transmitters are located at coordinates (1.25, 1.25, 2.5), (1.25, 3.75, 2.5), (3.75, 1.25, 2.5), and (3.75, 3.75, 2.5). For each user, the PDs are separated equally on the radius-ε circle centered at the user location, where $\varepsilon = 1$ cm. Assume that $A = 1\,\mathrm{cm}^2$ and $\Phi_{\mathrm{FoV}} = 70°$ for each PD.

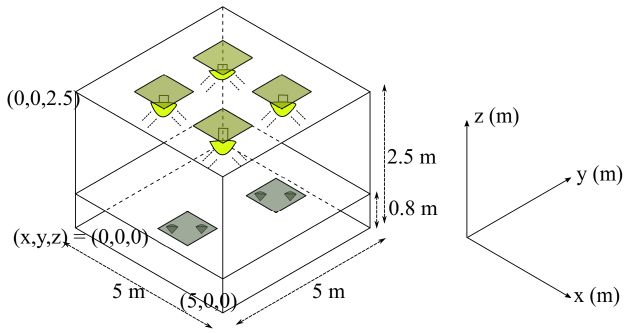

***Fig. 1**: Room dimensions and coordinates (in m) with example LED locations on the ceiling (four LEDs) and example PD locations at the table height of 0.8 m (two users each with two PDs).*

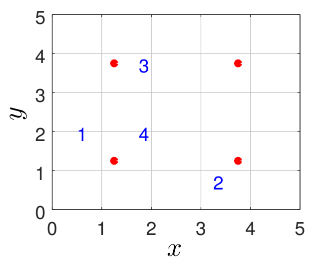

***Fig. 2**: Bird's eye view of user locations shown as blue digits and transmitter locations shown as red dots for scenario 1.*

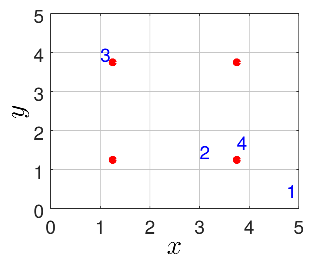

***Fig. 3**: Bird's eye view of user locations shown as blue digits and transmitter locations shown as red dots for scenario 2.*

For throughput evaluation using Eq. (12), assume that K_{SNR} in Eq. (13) is such that the receiver signal-to-noise ratio (SNR), which is $K_{\mathrm{SNR}}\sigma_{k,n}^2$, is equal to 15 dB for typical path loss $\sigma_{k,n} = 10^{-6}$. In other words, $K_{\mathrm{SNR}} \approx 3.1623 \times 10^{13}$. It should be noted that other values of K_{SNR} can also be used, as demonstrated in the subsequent sections.

Scenario 1: Assume user and PD locations as shown in Table 1, where the angle 0° points towards the positive x-axis. Fig. 2 illustrates the user as well as the LED transmitter locations.

Table 2 presents all three possible user groupings as well as the corresponding minimum user throughputs based on computation using Eq. (12). From the throughputs, the best-case grouping yields the minimum throughput (3.63 bit/s/Hz) that is 1.31 times the value for the worst-case grouping (2.78 bit/s/Hz).

***Table 1**: User and PD locations for scenario 1.*

User	Location	PD locations on radius-ε circle
1	$(0.5, 2, 0.8)$	0°, 180°
2	$(3.25, 0.75, 0.8)$	45°, 225°
3	$(1.75, 3.75, 0.8)$	90°, 270°
4	$(1.75, 2, 0.8)$	135°, 315°

***Table 2**: Minimum user throughputs for scenario 1.*

User grouping	Minimum user throughput (bit/s/Hz)
$\{1,2\}$ and $\{3,4\}$	2.84
$\{1,3\}$ and $\{2,4\}$	3.63
$\{1,4\}$ and $\{2,3\}$	2.78

***Table 3**: User and PD locations for scenario 2.*

User	Location	PD locations on radius-ε circle
1	$(4.75, 0.5, 0.8)$	60°, 240°
2	$(3, 1.5, 0.8)$	120°, 300°
3	$(1, 4, 0.8)$	100°, 280°
4	$(3.75, 1.75, 0.8)$	105°, 285°

***Table 4**: Minimum user throughputs for scenario 2.*

User grouping	Minimum user throughput (bit/s/Hz)
$\{1,2\}$ and $\{3,4\}$	1.58
$\{1,3\}$ and $\{2,4\}$	3.54
$\{1,4\}$ and $\{2,3\}$	0.63

Scenario 2: Assume user and PD locations as shown in Table 3. Fig. 3 illustrates the user as well as the LED transmitter locations.

Table 4 presents all three possible user groupings as well as the corresponding minimum throughputs. From the throughputs, the best-case grouping yields the minimum throughput (3.54 bit/s/Hz) that is as high as 5.62 times the value for the worst-case grouping (0.63 bit/s/Hz).

The two example scenarios demonstrate that efficient user grouping can help increase the minimum user throughputs significantly compared to the worst-case values. The next section discusses the proposed methods for user grouping.

4. USER GROUPING METHODS

User throughputs in Eq. (12) depend the user grouping in a complicated fashion. In particular, the SNRs

($K_{\text{SNR}}\sigma^2_{k,n}$) in Eq. (12) depend on path losses ($\sigma^2_{k,n}$) that are obtained after the computation of BD precoding described in Section 2.2. As a result, while the optimization problem in user grouping can be formulated, it is a general nonlinear problem and numerically difficult to solve. Therefore, the following heuristics for user grouping are proposed, with their performances evaluated in subsequent sections.

4.1 Heuristics for User Grouping

Intuitively, user grouping should be performed to avoid "highly correlated" channel matrices among users in the same group, where different correlation measures are applied in the following different heuristics. This is because BD precoding relies on projecting the user's MIMO channel matrix onto the null space of the other users' MIMO channel matrices in the same group. Three different heuristics to prevent high pairwise channel correlations among users in the same group are proposed.

Method 1: Group the users to maximize the average distance between users in the same group, where the distance between users k and l is defined as

$$D_{kl} = \|\mathbf{u}_k - \mathbf{u}_l\|, \tag{14}$$

where $\mathbf{u}_k$ and $\mathbf{u}_l$ denote the location vectors of users k and l, respectively. Roughly speaking, since users located close to each other tend to have similar MIMO channel matrices, maximizing the average distance within each group should prevent highly correlated channel matrices.

Method 2: Group the users to minimize the average correlation coefficient as defined in [24] between users in the same group, where the correlation coefficient is given by[1]

$$R_{kl} = \frac{|\text{tr}(\mathbf{H}_k\mathbf{H}_l^{\mathsf{T}})|}{\|\mathbf{H}_k\|_{\mathsf{F}}\|\mathbf{H}_l\|_{\mathsf{F}}}. \tag{15}$$

The definition of R_{kl} tries to capture the similarity between channel matrices of users k and l. Its value is between 0 and 1.

Method 3: Group the users to maximize the average normalized matrix norm of its projection on the other user's null space, where the pairwise normalized matrix norm is defined as

$$N_{kl} = \min\left(\frac{\|\mathbf{H}_k\mathbf{V}_l^0\|_{\mathsf{F}}}{\|\mathbf{H}_k\|_{\mathsf{F}}}, \frac{\|\mathbf{H}_l\mathbf{V}_k^0\|_{\mathsf{F}}}{\|\mathbf{H}_l\|_{\mathsf{F}}}\right), \tag{16}$$

where $\mathbf{V}_k^0$ denotes a $N_{\text{tx}} \times (N_{\text{tx}} - N'_{\text{rx}})$ basis matrix for the null space of $\mathbf{H}_k$. Note that $\mathbf{V}_k^0$ contains the last $(N_{\text{tx}} - N'_{\text{rx}})$ columns of $\mathbf{V}_k$ obtained from the SVD $\mathbf{H}_k = \mathbf{U}_k\mathbf{\Sigma}_k\mathbf{V}_k^{\mathsf{T}}$.

Method 3 is proposed on the basis of the BD precoding operation. As previously mentioned, the user's precoding matrix is chosen to be in the null space of all the other users' channel matrices. Each fraction in the definition of N_{kl} captures how much energy survives in the transmitted signals if projected onto the null space of the other user's MIMO channel matrix; its value is between 0 and 1.

Finally, a hybrid method is considered, where all three methods are applied and the best user grouping among them is selected as the final answer.

4.2 ILP Formulation for User Grouping

The three methods of user grouping as proposed in the previous section can be formulated as integer linear programming (ILP) problems that can be practically solved using ILP software, e.g., GNU Linear Programming Kit (GLPK). To avoid repetition, each method can be formulated to minimize the maximum average pairwise parameter γ_{kl} between all pairs of users in the same group, where the pairwise parameter is defined below.

$$\gamma_{kl} = \begin{cases} -D_{kl}, & \text{method 1} \\ R_{kl}, & \text{method 2} \\ -N_{kl}, & \text{method 3} \end{cases} \tag{17}$$

It should be noted that the values of γ_{kl} for all user pairs can be precomputed using Eqs. (14), (15), and (16). They can then be used as input parameters for solving the optimization problem.

Define two sets of decision variables for optimization. Let $x_k^g \in \{0,1\}$, where $k \in \{1,\ldots,K\}$ and $g \in \{1,\ldots,G\}$, be equal to 1 if user k is assigned to group g, and 0 otherwise. Let $y_{kl}^g \in \{0,1\}$, where $k,l \in \{1,\ldots,K\}$ and $g \in \{1,\ldots,G\}$, be equal to 1 if, and only if, users k and l are both assigned to group g.

The following is an optimization problem for each user grouping method. For notational convenience, let $\mathcal{K} = \{1,\ldots,K\}$ and $\mathcal{G} = \{1,\ldots,G\}$.

$$\min \max_{g\in\mathcal{G}} \frac{1}{K'(K'-1)} \sum_{k,l\in\mathcal{K},k\neq l} \gamma_{kl} y_{kl}^g \tag{18}$$

$$\text{s.t. } \forall k \in \mathcal{K}, \quad \sum_{g=1}^{G} x_k^g = 1 \tag{19}$$

$$\forall g \in \mathcal{G}, \quad \sum_{k=1}^{K} x_k^g = K' \tag{20}$$

$$\forall k,l \in \mathcal{K}, k \neq l, \forall g \in \mathcal{G}, \quad y_{kl}^g \geq x_k^g + x_l^g - 1 \text{ and } y_{kl}^g \leq x_k^g \text{ and } y_{kl}^g \leq x_l^g \tag{21}$$

$$\forall k \in \mathcal{K}, \forall g \in \mathcal{G}, \quad x_k^g \in \{0,1\} \tag{22}$$

$$\forall k,l \in \mathcal{K}, k \neq l, \forall g \in \mathcal{G}, \quad y_{kl}^g \in \{0,1\} \tag{23}$$

[1] tr($\mathbf{A}$) denotes the trace of matrix $\mathbf{A}$, which is the sum of diagonal entries of $\mathbf{A}$. $\|\mathbf{B}\|_{\mathsf{F}}$ denotes the Frobenius norm of matrix $\mathbf{B}$, which is the square root of the sum of all entries squared. For example, if $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$, then

$$\text{tr}(\mathbf{A}) = a_{11} + a_{22}$$

$$\|\mathbf{B}\|_{\mathsf{F}} = \sqrt{b_{11}^2 + b_{12}^2 + b_{13}^2 + b_{21}^2 + b_{22}^2 + b_{23}^2}$$

The objective in Eq. (18) is to maximize the average pairwise parameter γ_{lk} in each group. The set of constraints in Eq. (19) ensures that each user is assigned to exactly one group. The set of constrains in Eq. (20) ensures that exactly K' users are assigned to each group. The set of constraints in Eq. (21) ensures that $y_{kl}^g = 1$ when users k and l are assigned to group g, and $y_{kl}^g = 0$ otherwise.

By introducing one additional decision variable z to take the value of $\max_{g\in\mathcal{G}} \frac{1}{K'(K'-1)} \sum_{k,l\in\mathcal{K},k\neq l} \gamma_{kl} y_{kl}^g$, the above optimization problem can be written as an ILP problem as follows. It should be noted that the first set of constraints is added to ensure that z takes the maximum value over $g \in \mathcal{G}$.

$$
\begin{aligned}
\min \;& z \\
\text{s.t. } & \forall g \in \mathcal{G},\; z \geq \frac{1}{K'(K'-1)} \sum_{k,l\in\mathcal{K},k\neq l} \gamma_{kl} y_{kl}^g \\
& \forall k \in \mathcal{K},\; \sum_{g=1}^{G} x_k^g = 1 \\
& \forall g \in \mathcal{G},\; \sum_{k=1}^{K} x_k^g = K' \\
& \forall k,l \in \mathcal{K}, k\neq l, \forall g \in \mathcal{G},\; y_{kl}^g \geq x_k^g + x_l^g - 1 \\
& \quad \text{and } y_{kl}^g \leq x_k^g \text{ and } y_{kl}^g \leq x_l^g \\
& \forall k \in \mathcal{K}, \forall g \in \mathcal{G},\; x_k^g \in \{0,1\} \\
& \forall k,l \in \mathcal{K}, k\neq l, \forall g \in \mathcal{G},\; y_{kl}^g \in \{0,1\}
\end{aligned}
$$

The above problem formulation assumes that each user group contains $K' = N_{\text{tx}}/N'_{\text{rx}}$ users. For unequal group sizes, denote the size of group g by K_g. Then, the constant K' in the ILP problem can be replaced by K_g to accommodate unequal group sizes.

5. NUMERICAL RESULTS FOR PERFORMANCE EVALUATION

For the numerical results, consider downstream transmissions in a 5 m × 5 m × 2.5 m (width × length × height) room as illustrated in Fig. 1. Consider scenarios with N_{x} LED transmitters and K users each with two PDs, where $N_{\text{tx}} \in \{4,6,8\}$ and $K = N_{\text{tx}}$. In these scenarios, two groups of $K/2$ users are formed to apply BD precoding. The transmitter locations are shown in Fig. 4.

User locations are randomly and uniformly generated to be anywhere in the room. Their PDs are located at a fixed height of 0.8 m. The PDs of each user are equally separated on the radius-ε circle centered at the user location, where $\varepsilon = 1$ cm and the first PD is equally likely to be anywhere on the circle. For the PD of each user, assume that $A = 1\text{ cm}^2$ and $\Phi_{\text{FoV}} = 70°$.

For throughput evaluations using Eq. (12), K_{SNR} in Eq. (12) is set such that the SNR ($K_{\text{SNR}}\sigma_{k,n}^2$) is 10, 15, 20 dB for $\sigma_{k,n} = 10^{-6}$. The three values of K_{SNR} are used to represent low, medium, and high SNR scenarios, respectively.

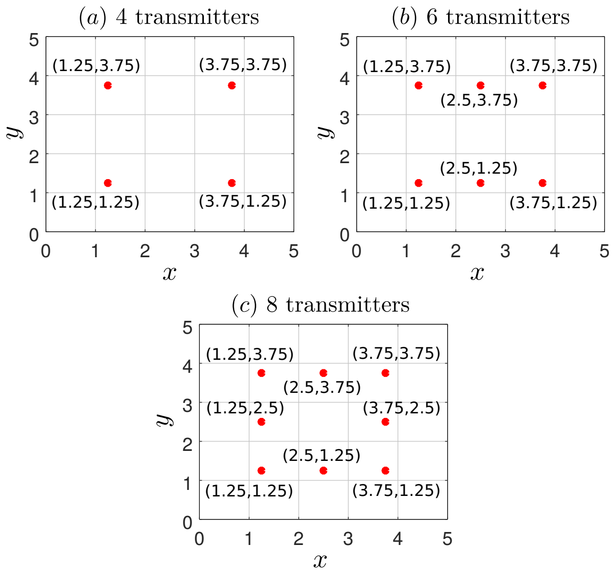

Fig. 4: *Bird's eye view of LED transmitter locations for (a) scenarios with four transmitters, (b) scenarios with six transmitters, and (c) scenarios with eight transmitters.*

All numerical results are obtained using the GNU Octave software [25], which is license-free. For each value of N_{tx} (number of LED transmitters) and K_{SNR} (SNR parameter), 1000 random scenarios of user locations are generated. The average minimum user throughputs are evaluated from the three methods of user grouping, the hybrid method (choosing the best answer from the three methods), and exhaustive search. It should be noted that exhaustive search always provides optimal user grouping, but the associated computational complexity is too high for large numbers of LED transmitters and users.

5.1 Average Minimum User Throughputs

Fig. 5 shows the average minimum user throughputs for different methods of user grouping for scenarios with $N_{\text{tx}} = 4$, $K = 4$, and $N'_{\text{rx}} = 2$. It can be observed that random grouping is significantly less efficient than other methods. Method 3 performs the best among the three proposed. In addition, method 3 performs close to hybrid and exhausive search methods. The relative performances are similar for low, medium, and high SNR values.

Fig. 6 shows the average minimum user throughputs for different methods of user grouping for scenarios with $N_{\text{tx}} = 6$, $K = 6$, and $N'_{\text{rx}} = 2$. Similar to Fig. 5, random grouping is significantly less efficient than other methods. While method 3 still outperforms methods 1 and 2, there is now a noticable gap between method 3 and the hybrid method, and also between the hybrid method and exhaustive search.

The fact that method 3 and the hybrid approach cannot perform as well as exhaustive search in Fig. 6 is because the proposed user grouping methods consider pairwise interferences among users. When the user group size is

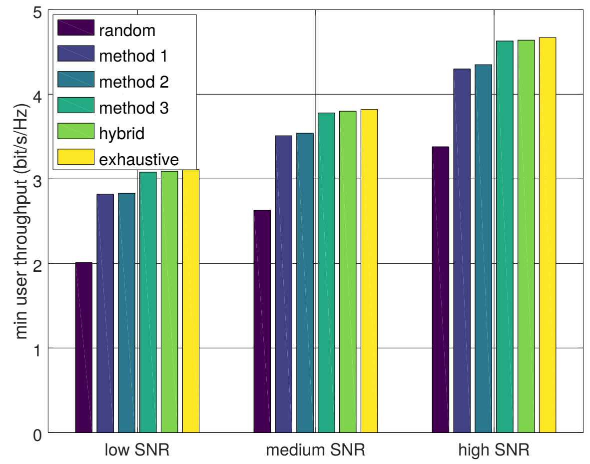

***Fig. 5**: Average minimum user throughputs for scenarios with $N_{tx} = 4$, $K = 4$, and $N'_{rx} = 2$.*

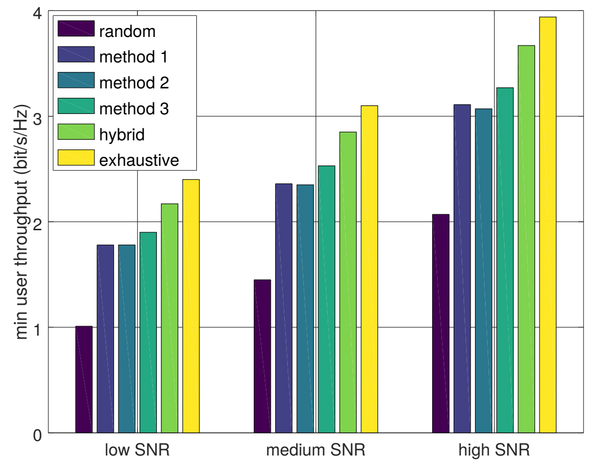

***Fig. 6**: Average minimum user throughputs for scenarios with $N_{tx} = 6$, $K = 6$, and $N'_{rx} = 2$.*

$K' = N_{\mathrm{tx}}/N'_{\mathrm{rx}} = 6/2 = 3$, optimal grouping will need to consider mutual interferences among three users instead of pairwise interferences only. Nevertheless, the hybrid method performs much better than random grouping, and is reasonably efficient compared to optimal grouping from exhaustive search. In particular, the throughputs of the hybrid method are 90.4%, 91.9%, and 93.1% of the values from exhaustive search for low, medium, and high SNR scenarios, respectively.

Fig. 7 shows the average minimum user throughputs for different methods of user grouping for scenarios with $N_{\mathrm{tx}} = 8$, $K = 8$, and $N'_{\mathrm{rx}} = 2$. Similar to Figs. 5 and 6, random grouping is significantly less efficient than other methods. Similar to Fig. 6, method 3 outperforms methods 1 and 2 but is less efficient than the hybrid method, which in turn is less efficient than exhaustive search. Nevertheless, the hybrid method performs reasonably well, with the throughputs equal to 81.1%, 83.3%, and 85.9% of the values from exhaustive search for low, medium, and high SNR scenarios, respectively.

As previously mentioned, for each SNR scenario in

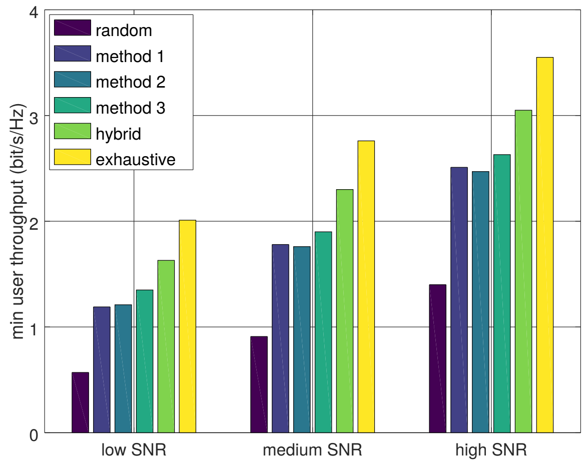

***Fig. 7**: Average minimum user throughputs for scenarios with $N_{tx} = 8$, $K = 8$, and $N'_{rx} = 2$.*

***Table 5**: Percentage of random scenarios in which each method is chosen by the hybrid method.*

$(N_{\mathrm{tx}}, K, N'_{\mathrm{rx}})$	SNR	Method		
		1	2	3
(4, 4, 2)	low	65.2%	12.9%	21.9%
	medium	67.2%	11.6%	21.2%
	high	62.8%	12.7%	24.5%
(6, 6, 2)	low	52.4%	23.0%	24.6%
	medium	53.0%	23.7%	23.3%
	high	49.7%	26.0%	24.3%
(8, 8, 2)	low	43.7%	27.1%	29.2%
	medium	44.4%	28.4%	27.2%
	high	44.4%	30.0%	25.6%

Figs. 5–7, 1000 random scenarios of user locations are simulated. Based on these simulation results, Table 5 shows the percentage of random scenarios in which methods 1, 2, and 3 are chosen by the hybrid method. As can be observed, all methods are used at least 10% of the time for each system configuration.

5.2 Comparison of Computational Complexity

To compare the computational complexity between the hybrid and exhaustive search methods, for each value of $(N_{\mathrm{tx}}, K, N'_{\mathrm{rx}})$, 20 random scenarios of user and PD locations are simulated and the average run time taken by each method computed. Table 6 compares the average run times using GNU Octave on a laptop computer with a Core i7 Intel processor and 8 GB of RAM. In particular, the command "`glpk`" in Octave is used to solve ILP problems in the hybrid method. It can be observed that, as the problem size increases (i.e., increasing K), the average run time of the hybrid method becomes significantly lower than that of the exhaustive method.

***Table 6**: Average run times for the hybrid method and the exhaustive method.*

$(N_{\text{tx}}, K, N'_{\text{rx}})$	Method	
	Hybrid	Exhaustive
$(6, 6, 2)$	0.0217 s	0.0102 s
$(6, 8, 2)$	0.0908 s	0.0462 s
$(6, 10, 2)$	0.358 s	0.444 s
$(6, 12, 2)$	1.72 s	9.89 s

5.3 User Grouping for Throughput Improvement when $N_{\mathbf{rx}} \leq N_{\mathbf{tx}}$

This section considers user grouping when $N_{\text{rx}} \leq N_{\text{tx}}$, corresponding to scenarios in which all users can be supported as a single group using BD precoding. However, numerical results indicate that user grouping, while unnecessary for BD precoding, can help improve the average minimum user throughputs.

In particular, consider scenarios with $N_{\text{tx}} = 8$, $K = 4$, and $N'_{\text{rx}} = 2$. Four grouping schemes are compared as follows. The first scheme puts all four users into a single group and applies BD precoding. The other three schemes separate users into two groups of two users, with user grouping based on the random method, the proposed hybrid method, and exhaustive search, respectively. For fair comparison, when computing the user throughput using Eq. (12), a factor of 1/2 is multiplied to each user throughput in the last three schemes since TDM with two timeslots is applied to support two user groups.

Fig. 8 shows the average minimum user throughputs obtained from the four schemes in low, medium, and high SNR scenarios. It can be observed that, with four users in a single group, the minimum user throughputs are lower than the values for two groups of two users, even with random user grouping. This is because BD precoding completely prevents interference among users by projecting each user's signals onto the null space of all the other users' signals, yielding possibly high attenuation of the desired user signals in the process. Compared to random user grouping, the proposed hybrid method can further improve the throughputs. In addition, the throughputs from the proposed hybrid method are close to the values from exhaustive search.

The results of this study indicate that efficient user grouping can possibly benefit even scenarios with $N_{\text{rx}} \leq N_{\text{tx}}$, which in principle do not require user grouping to operate BD precoding. Hence, when the number of users becomes high, efficient user grouping should be considered while operating MU-MIMO VLC systems with BD precoding.

6. CONCLUSION

User grouping for MU-MIMO VLC systems with BD precoding is investigated in this study to enable BD precoding to be applied to scenarios with a large number of users through the use of TDM timeslots. Three different user grouping methods as well as the hybrid method are proposed and compared with random user grouping and exhaustive search. The numerical results on the average minimum user throughputs over random user location scenarios indicate that the proposed hybrid method can significantly outperform random user grouping and performs reasonably well compared to exhaustive search. In particular, in a 5 m × 5 m × 2.5 m room with 4, 6, and 8 LED transmitters with four, six, and eight users each with two PDs, the proposed hybrid user grouping method can provide at least 80% of the throughputs provided by optimal grouping through exhaustive search in all scenarios. Finally, for scenarios with eight LED transmitters and four users each with two PDs, it is demonstrated that user grouping can help improve minimum user throughputs even though BD precoding can simultaneously support all users. Therefore, the proposed user grouping method can be useful for operating MU-MIMO VLC systems with BD precoding when the number of users in the same room becomes large. For future works, it is recommended that further investigations be conducted to ascertain when users should be divided into smaller groups for throughput improvement. In addition, user grouping can also be investigated under other precoding schemes for MU-MIMO VLC.

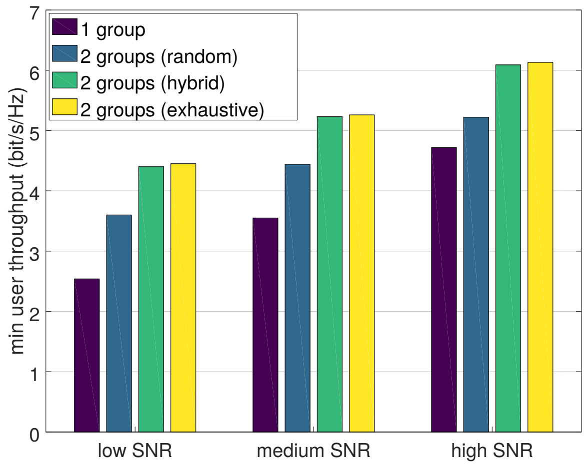

***Fig. 8**: Average minimum user throughputs for scenarios with $N_{tx} = 8$, $K = 4$, and $N'_{rx} = 2$.*

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