

A Novel Adaptive Neuro-Fuzzy Control Scheme For Eliminating Rule Explosion In Dynamic Systems

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ABSTRACT

This article presents a neuro-fuzzy adaptive control scheme for avoiding the problem of rule explosion inherently associated with fuzzy control systems. The study proposes a novel neuro-fuzzy model designed using only two membership functions and four if-then fuzzy rules for control of highly nonlinear two-stage cart and pendulum system. The neuro-fuzzy controller has been designed with the minimum possible number of rules thus solving the problem of rule explosion completely. The study further compares proposed ANFIS controller with conventional proportional-integral-derivative (PID) and artificial neural network controllers in terms of settling time, overshoot ranges and steady state error. The results show better performance of ANFIS controller compared to PID and neural controllers.

Keywords: Artificial Neural Networks, Nonlinearity, PID, Rule Explosion

1. INTRODUCTION

Inverted pendulum on cart is a very popular nonlinear system which requires highly robust controllers for their stabilization [1]. These system comprises of an inverted pendulum or pole mounted over a movable cart. The objective is to balance the pendulum at vertical upright position while the cart is moving in linear direction [2]. These systems are widely used for realizing experimental models, validating effectiveness of new control approaches and verifying their implementation [3]. Due to their inherent nonlinear dynamics these systems finds numerous practical applications including launching of rocket [4], missile guidance [5], designing of robotic limbs [6], Segway balancing [7], etc. Control of these systems becomes more challenging when the number of links attached to the cart is increased. The two-stage cart and pendulum is an excellent example of such system which constitutes a series of two inverted pendulums mounted on a movable cart [8]. The objective

is to balance both the pendulums in the vertical upright position by moving the cart in linear direction.

Several control approaches have been successfully adopted for optimal control of two-stage cart and pendulum systems. For an instance, a linear motor driven double inverted pendulum system to eliminate the difficulties associated with rotary transmission has been proposed in [9]. The authors designed a rapid controller prototyping platform named Cspace which resulted in reducing the time of programming and debugging. The platform has been extensively tested with linear quadratic regulator (LQR) and sliding mode controllers for validation. Furthermore, an input-output inversion approach for open-loop control of an overhead crane modelled as a double inverted pendulum has been highlighted in [10]. The study mathematically analyzed parametric trajectory of the system which resulted in reduction of residual oscillations. The results further highlights that proposed technique can also be incorporated with industrial drives after neglecting post-actuators. A fractional order controller based on Bode's ideal transfer function for control of double inverted rotary pendulum has been discussed in [11]. The study further compared the proposed controller with an integer order controller based on state space approach. The experimental results indicate better performance of fractional order controller demonstrating a smooth voltage control with better robustness against external disturbances.

In [12], the dynamics of a base excited double inverted pendulum for generating electricity has been examined. The authors considered a magnet attached to the tip of lower pendulum surrounded by a series of equally spaced coils. Small fluctuations in magnetic field resulted in generation of electricity. The authors further conducted parametric studies to optimize the proposed system and harvest energy on a greater scale. Further, in [13] a fuzzy based LQR and linear quadratic gaussian (LQG) controllers for control of double link rotary inverted pendulum has been demonstrated. The study compared the proposed controllers with conventional LQR and LQG controllers. The results indicate better performance of fuzzy based LQR and LQG controllers in terms of settling time, peak overshoot, steady state error, total root mean square error and robustness. Recently [14] proposed a LQR based sliding mode controller (SMC) for stabilization of rotary double inverted pendulum. The authors adopted Euler-Lagrange formulation for developing a dynamic model of proposed system and considered Lyapunov

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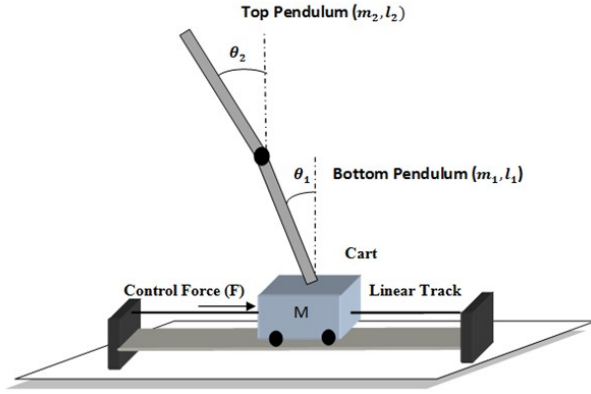


Fig. 1: Two-stage cart and pendulum system.

stability theory approach for determining its stability. The results of LQR-SMC controller were further compared with conventional SMC controller which clearly indicates better performance of LQR-SMC controller in terms of integral absolute error, integral time absolute error and integrated square error.

The fuzzy control systems are widely used for solution of numerous complex engineering problems. These systems intrinsically suffer from the problem of rule explosion in which the number of rules needed to model the system increases exponentially with the increase in number of inputs. As a result of which interpretability and computation complexity of the fuzzy system degrades. Therefore, this study proposes a novel neuro-fuzzy adaptive control scheme based on ANFIS technique for avoiding problem of rule explosion commonly associated with fuzzy controllers. The ANFIS controller has been designed using only two membership functions of triangular shape thereby resulting in only four if-then fuzzy rules thus solving the problem of fuzzy rule explosion completely. A mathematical model of the proposed system has been developed and simulated. The results of PID controllers have been considered for training of ANFIS and neural controller. The objective is to stabilize the complete system within 4.0 sec with optimal overshoot and steady state response. The article has been briefly divided into following sections.

2. METHOD AND EXPERIMENTAL PROCEDURE

The design of two-stage cart and pendulum system comprises of two rigid pendulums of mass ($m_1 = m_2 = 0.5\text{kg}$) and length ($l_1 = l_2 = 0.1\text{meter}$) inclined at an angle θ_1 and θ_2 respectively with the vertical as shown in Fig.1. The bottom pendulum is pivoted to a cart of mass ($M=1\text{kg}$). The cart can travel in linear direction under action of control force (F) and gravity ($g=9.81\text{m/s}^2$). The moment of inertia (I) and friction (b) considered for analysis were 0.006 Kg m^2 and 0.1 N s/m^2 respectively.

The nonlinear differential equations for the two-stage cart and pendulum system has been derived using

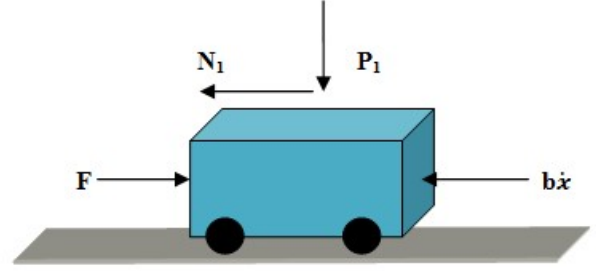


Fig. 2: Free-body-diagram of cart sub-system.

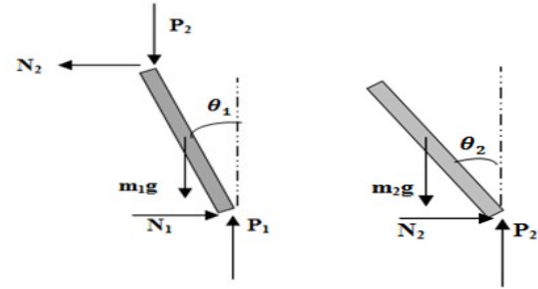


Fig. 3: Free-body-diagram of bottom and top Pendulum.

Newton's second law [15]. These equations were used for determining linear acceleration of cart (\ddot{x}) and angular accelerations of pendulums ($\ddot{\theta}$) respectively. A free-body-diagram of cart sub-system is shown in Fig.2. In the Figure N_1 and P_1 indicates the interaction forces between bottom pendulum and cart in horizontal and vertical directions respectively. The force of friction acting on the cart is indicated by $b\dot{x}$.

Summing forces in horizontal direction gives the following expression for linear acceleration of the cart as indicated by Equation (1).

$$\ddot{x} = \frac{1}{M}(F - N_1 - b\dot{x}) \quad (1)$$

In above equation, \dot{x} and \ddot{x} represents the linear velocity and linear acceleration of the cart respectively. The free-body-diagram of bottom and top pendulum is shown in Fig.3. The interaction forces acting between the pendulums in horizontal and vertical directions are given by N_2 and P_2 respectively.

Further, m_1g and m_2g represents the gravitational forces acting on the bottom and top pendulum respectively. The final expression for angular acceleration ($\ddot{\theta}_1$) of the bottom pendulum is given by Equation (2).

$$\ddot{\theta}_1 = \frac{1}{I_1}(N_1 l_1 \cos \theta_1 + P_1 l_1 \sin \theta_1 - b_1 \dot{\theta}_1 + N_2 l_1 \cos \theta_1 + P_2 l_1 \sin \theta_1) \quad (2)$$

In the above equation, $\dot{\theta}_1$ and I_1 are angular velocity and moment of inertia for bottom pendulum.

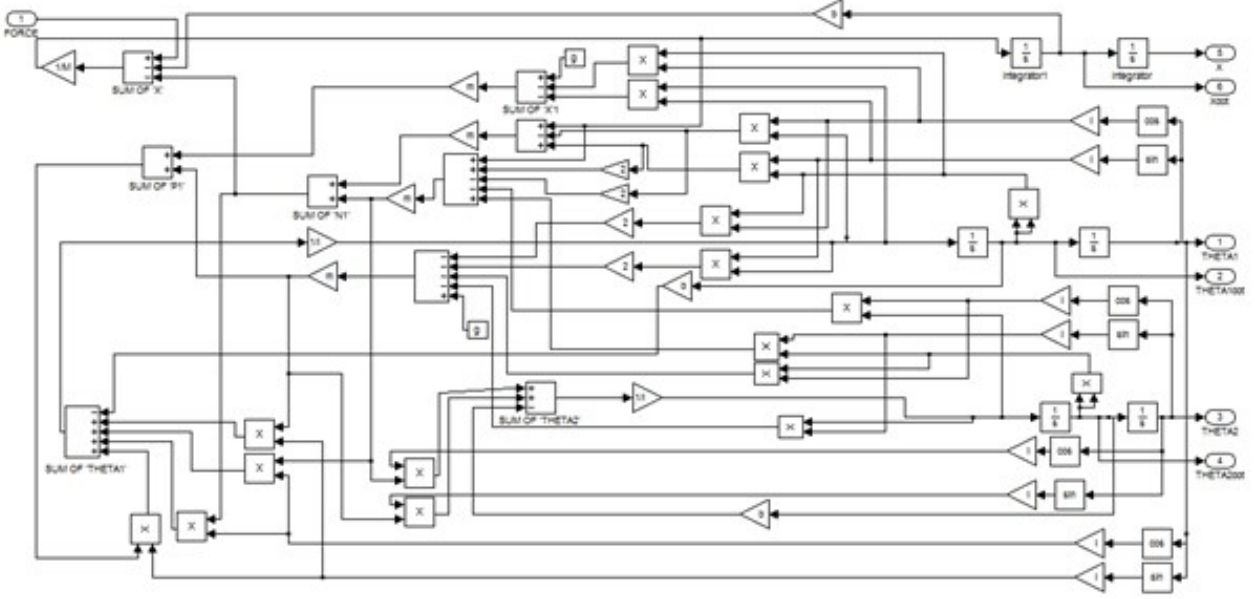


Fig. 4: Simulink model of two-stage cart and pendulum system.

The final expression for angular acceleration ($\ddot{\theta}_2$) of the top pendulum is given by Equation (3).

$$\ddot{\theta}_2 = \frac{N_2 I_2 \cos \theta_2 + P_2 I_2 \sin \theta_2 - b_2 \dot{\theta}_2}{I_2} \quad (3)$$

In the above equation, $\dot{\theta}_2$ and I_2 are angular velocity and moment of inertia of top pendulum. The above equations were considered for developing a simulink model of two-stage cart and pendulum system as shown in Fig. 4.

3. PID BASED CONTROL OF TWO-STAGE CART AND PENDULUM SYSTEM

PID are popular closed loop feedback control mechanism extensively employed in various industrial control applications [16-18]. PID controller computes an error value as the difference between output and target value and further applies a correction based on proportional (K_p), integral (K_i) and derivative (K_d) gains to minimize the error. In this study the PID gains for two-stage cart and pendulum system has been optimized using auto-tuned function in Matlab [19]. PID gains obtained after tuning for different sub-systems of two-stage cart and pendulum system are given in Table 1.

4. NEURAL NETWORK BASED CONTROL OF TWO-STAGE CART AND PENDULUM SYSTEM

Artificial neural networks are computational paradigms designed based on biological neurons. Their architecture comprises of collection of several nodes or junctions attached in different layers which aid in transmitting signal just like synapses in a biological brain [20]. During learning of networks, the weights associated with the nodes get adjusted according to training data [21]. In

Table 1: PID gains obtained after tuning.

| Sub-system | Integral gain (K_i) | Proportional gain (K_p) | Differential gain (K_d) |
|-----------------|-------------------------|-----------------------------|-----------------------------|
| Top pendulum | 10 | 1 | -1 |
| Bottom pendulum | 10 | 1 | -1 |
| Cart | 10 | 1 | -1 |

Table 2: Samples collected for training.

| Type of sample | Percentage share | Sample size |
|----------------|------------------|-------------|
| Training | 70% | 111 |
| Validation | 15% | 24 |
| Testing | 15% | 24 |

this study the neural controller for two-stage cart and pendulum system has been trained considering data samples from simulations of PID controller. A total of 158 data samples were collected and divided into training, validation and testing samples as shown in Table 2. Three neural controllers were designed for each sub-system i.e. cart, top pendulum and bottom pendulum.

The neural network architecture has been constructed using 20 neurons in hidden layer and 1 neuron in output layer. Further two inputs and one output were considered for designing of neural controller as shown in Fig.5.

In this study, the neural controller has been trained using Levenberg-Marquardt Back-Propagation learning algorithm [22]. The learning accuracy of neural con-

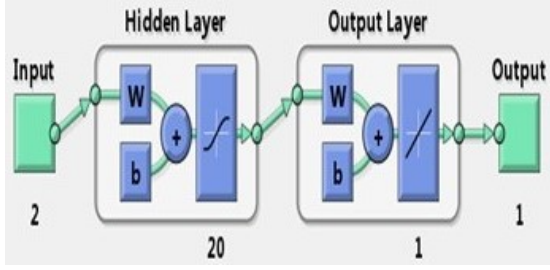


Fig. 5: Neural network architecture.

Table 3: MSE and Regression results.

| Sample | MSE | Regression |
|----------------------------|----------|------------|
| Top pendulum controller | | |
| Training | 1.08e-9 | 1 |
| Validation | 1.44e-8 | 1 |
| Testing | 2.43e-8 | 1 |
| Bottom pendulum controller | | |
| Training | 9.53e-10 | 1 |
| Validation | 1.03e-6 | 1 |
| Testing | 2.58e-6 | 1 |
| Cart controller | | |
| Training | 6.76e-9 | 1 |
| Validation | 4.57e-9 | 1 |
| Testing | 8.98e-8 | 1 |

troller has been demonstrated with the help of regression plots obtained between the output and target values. The regression plots obtained for training, validation and testing samples for top pendulum controller are shown in Fig.6. The results clearly indicate a regression value of unity i.e. $R=1$ for all the samples which indicate an excellent correlation between outputs and targets. Further, the mean squared error (MSE) was used for measuring the performance of these controllers. MSE measures the average squared difference between outputs and targets [23]. Excellent results for MSE (nearly equal to zero) were also obtained for all the controllers as indicated in Table 3.

5. ANFIS BASED CONTROL OF TWO-STAGE CART AND PENDULUM SYSTEM

ANFIS belongs to a category of artificial neural networks that integrates features of both neural networks and fuzzy logic controllers for its operation. ANFIS architecture is based on takagi-sugeno fuzzy inference system and is considered as a universal estimator [24]. Similar to a fuzzy controller, ANFIS utilizes a set of if-then fuzzy rules and also has learning and predicting capability of artificial neural networks. Therefore, it is widely used for approximating nonlinear functions [25]. ANFIS controller has been trained using data samples collected from simulation of PID controller [26]. Three different ANFIS controllers were designed for control

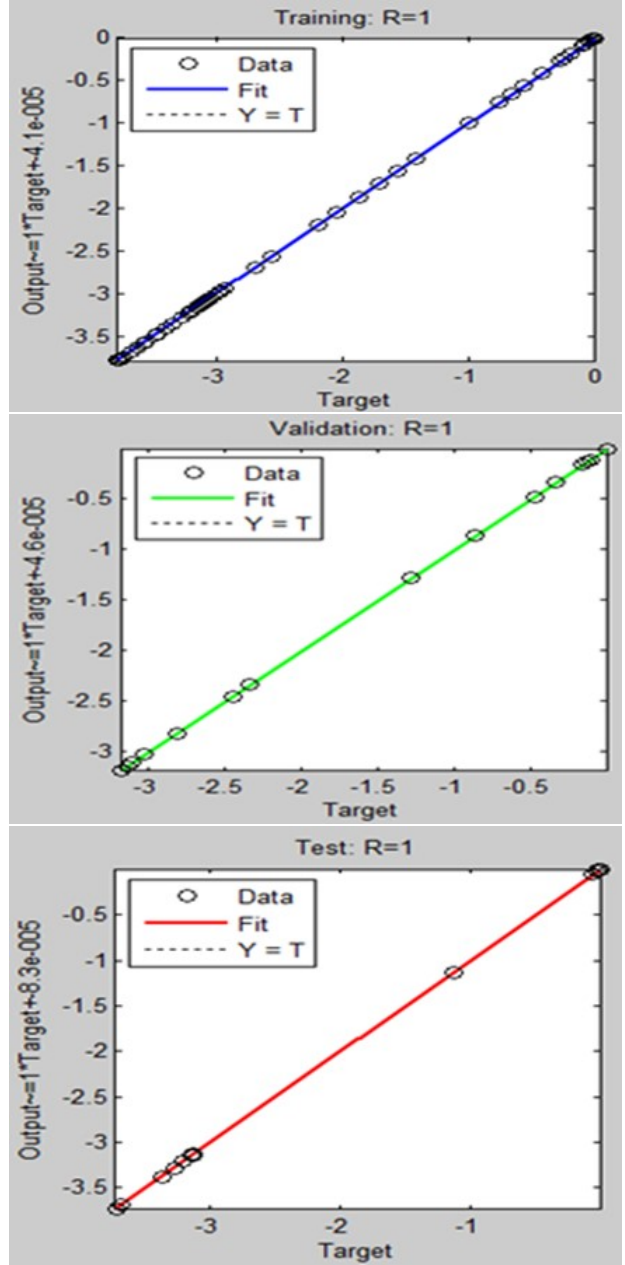


Fig. 6: Regression plots.

of cart, top pendulum and bottom pendulum. These controllers were tuned using hybrid learning algorithm incorporating a takagi-sugeno fuzzy inference system [27]. The inputs considered for pendulum controllers were pendulum angle and pendulum angular velocity, whereas for cart controller were cart position and cart velocity. The output is same for all the three controllers i.e. control force (F). The loading of data samples in ANFIS for training of top pendulum, bottom pendulum and cart controller is shown in Fig.7.

This study also determines the effect of shape and number of membership functions on training of ANFIS controllers [28]. Four different shapes of membership functions i.e. gaussian, gbell, trapezoidal and trian-

Table 4: Training errors obtained using different shape and number of membership functions.

| No. of memberships | Gaussian | Gbell | Trapezoidal | Triangular |
|--|----------|----------|-------------|------------|
| Training errors for top pendulum controller | | | | |
| 2 | 2.9e-006 | 3.1e-006 | 4.6e-006 | 5.0e-008 |
| 3 | 3.8e-005 | 3.1e-005 | 2.5e-005 | 4.0e-005 |
| 4 | 4.5e-005 | 3.5e-005 | 4.1e-005 | 2.3e-005 |
| 5 | 6.7e-005 | 6.8e-005 | 6.1e-005 | 1.6e-005 |
| 6 | 5.3e-005 | 6.1e-005 | 6.7e-005 | 1.8e-005 |
| 7 | 4.5e-005 | 4.2e-005 | 4.6e-005 | 2.3e-005 |
| 8 | 4.4e-005 | 3.6e-005 | 6.9e-005 | 9.1e-006 |
| 9 | 3.0e-005 | 2.9e-005 | 2.1e-005 | 1.5e-005 |
| 10 | 5.2e-005 | 4.9e-005 | 4.3e-005 | 3.0e-005 |
| Training errors for bottom pendulum controller | | | | |
| 2 | 1.8e-006 | 1.3e-006 | 8.5e-007 | 1.4e-007 |
| 3 | 3.3e-005 | 2.8e-005 | 1.4e-005 | 1.7e-005 |
| 4 | 6.8e-005 | 6.2e-005 | 1.2e-005 | 3.0e-005 |
| 5 | 8.3e-005 | 7.2e-005 | 4.5e-005 | 1.9e-005 |
| 6 | 3.5e-005 | 6.8e-005 | 6.0e-005 | 3.2e-005 |
| 7 | 5.8e-005 | 6.6e-005 | 5.1e-005 | 1.8e-005 |
| 8 | 5.9e-005 | 8.2e-005 | 7.8e-005 | 1.7e-005 |
| 9 | 3.7e-005 | 8.2e-005 | 6.1e-005 | 1.2e-005 |
| 10 | 4.2e-005 | 8.6e-005 | 4.7e-005 | 1.1e-005 |
| Training errors for cart controller | | | | |
| 2 | 1.8e-006 | 3.3e-005 | 2.9e-005 | 8.1e-009 |
| 3 | 6.0e-005 | 5.8e-005 | 5.0e-005 | 2.6e-007 |
| 4 | 3.0e-005 | 7.1e-005 | 8.5e-005 | 2.0e-006 |
| 5 | 2.4e-005 | 5.6e-005 | 9.6e-005 | 1.3e-006 |
| 6 | 2.0e-005 | 4.6e-005 | 8.4e-005 | 2.5e-006 |
| 7 | 1.1e-005 | 4.8e-005 | 7.4e-005 | 1.8e-006 |
| 8 | 1.5e-005 | 2.8e-005 | 6.7e-005 | 1.4e-006 |
| 9 | 1.7e-005 | 4.3e-005 | 3.8e-005 | 2.4e-006 |
| 10 | 1.2e-005 | 4.3e-005 | 7.0e-005 | 2.7e-006 |

gular were considered for analysis. The numbers of membership functions were varied from 2 to 10. The training errors obtained for different shape of membership functions are given in Table 4. The results indicate that the minimum training error was obtained using two membership functions of triangular shape. Therefore, ANFIS controller has been designed using only two triangular shape membership functions resulting in only four if-then fuzzy control rules [29]. The proposed ANFIS controller aided in solving the problem of fuzzy rule explosion completely. The minimum training error obtained for top pendulum, bottom pendulum and cart controller were 5.0e-008, 1.4e-007 and 8.1e-009 respectively.

During training in ANFIS, the shapes of membership functions and if-then fuzzy rules gets tuned according to

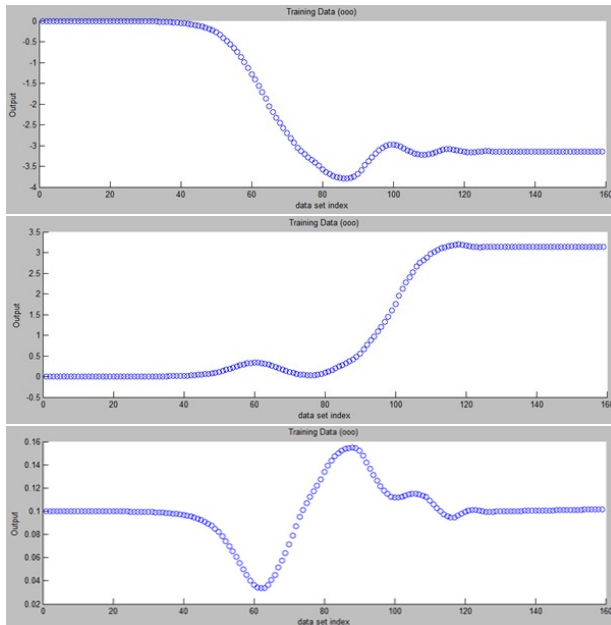
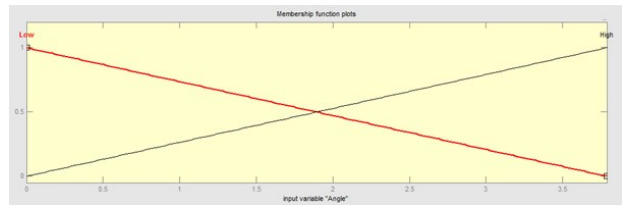
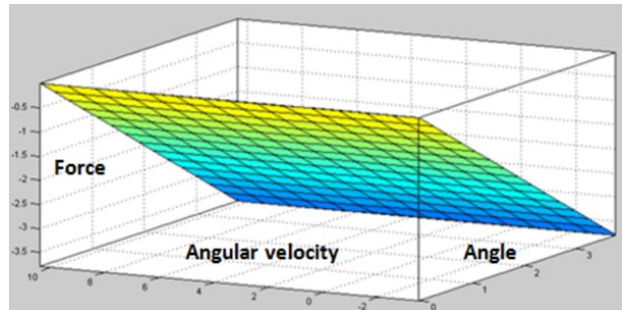
training data samples. The tuned triangular shape fuzzy membership functions are shown in Fig.8. The tuned fuzzy rules can be represented in three-dimensional space with the help of a surface viewer which demonstrates relationship between inputs and output. The surface viewer obtained after training for input 'top pendulum angle' is shown in Fig.9. The inputs i.e. pendulum angle and angular velocity are depicted along x and y axis respectively whereas outputs i.e. control force is depicted along z-axis.

6. RESULTS AND DISCUSSIONS

The PID, Neural and ANFIS controllers designed in the previous sections of the study has been simulated through a masked simulink model incorporating all the three controllers together. The masked simulink model of

Table 5: Simulation results.

| Controller | Settling time(seconds) | Overshoot Ranges (degree) | Steady state error |
|----------------------------------|------------------------|---------------------------|--------------------|
| Top pendulum angle | | | |
| ANFIS | 2.6 sec | 3.75° | 0 |
| PID | 2.8 sec | 3.75° | 0 |
| Neural | 3.0 sec | 3.75° | 0 |
| Top pendulum angular velocity | | | |
| ANFIS | 3.0 sec | 10.1° to -3.5° | 0 |
| PID | 3.2 sec | 10.3° to -3.5° | 0 |
| Neural | 3.5 sec | 10.3° to -3.5° | 0 |
| Bottom pendulum angle | | | |
| ANFIS | 2.7 sec | 0.05° to -3.3° | 0 |
| PID | 3.0 sec | -3.2° | 0 |
| Neural | 3.2 sec | -3.2° | 0 |
| Bottom pendulum angular velocity | | | |
| ANFIS | 3.3 sec | 2.5° to -4.5° | 0 |
| PID | 3.6 sec | 2.4° to -4.7° | 0 |
| Neural | 3.7 sec | 2.3° to -4.8° | 0 |
| Cart position | | | |
| ANFIS | 3.3 sec | 0.20° to -0.68° | 0 |
| PID | 3.0 sec | 0.07° to -0.06° | 0 |
| Neural | 3.1 sec | 0.07° to -0.06° | 0 |
| Cart velocity | | | |
| ANFIS | 2.0 sec | 0.07° to -0.05° | 0 |
| PID | 3.5 sec | 0.20° to -0.68° | 0 |
| Neural | 3.6 sec | 0.20° to -0.69° | 0 |

**Fig. 7:** Loading of samples in ANFIS.**Fig. 8:** Tuned triangular shape membership.**Fig. 9:** Three-dimensional rule surface.

two-stage cart and pendulum system is shown in Fig.10.

Further, the simulation responses obtained for two-

stage cart and pendulum system using above controllers are shown with the help of Fig.11 to Fig.16. The time considered for simulation was 5 seconds. A comparison

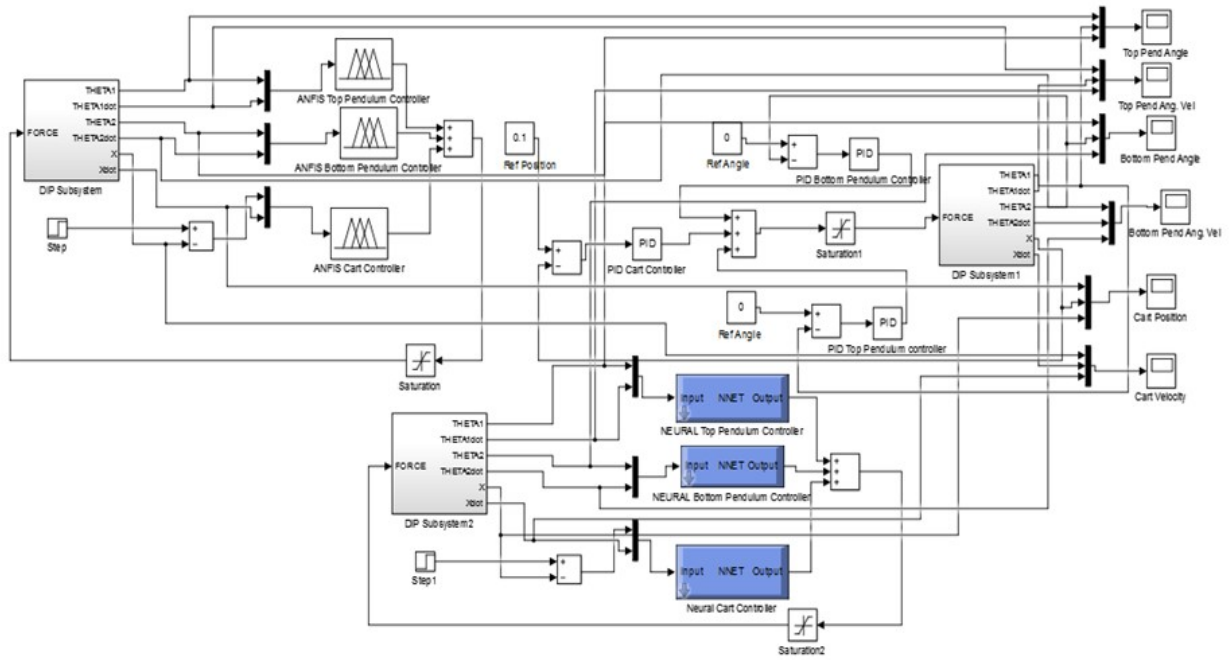


Fig. 10: Masked simulink model of proposed system.

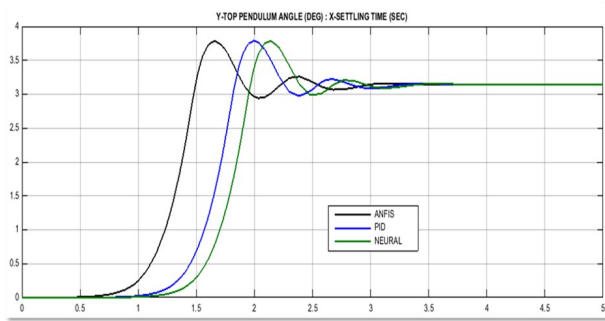


Fig. 11: Comparing simulation responses for 'Top pendulum angle'.

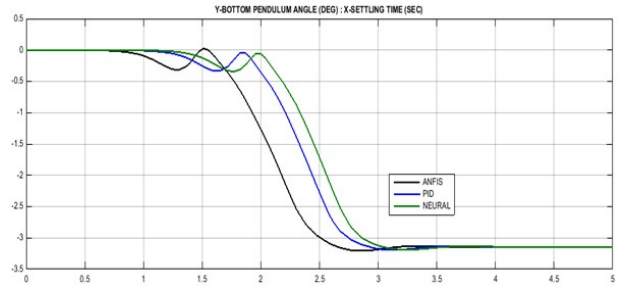


Fig. 13: Comparing simulation responses for 'Bottom pendulum angle'.

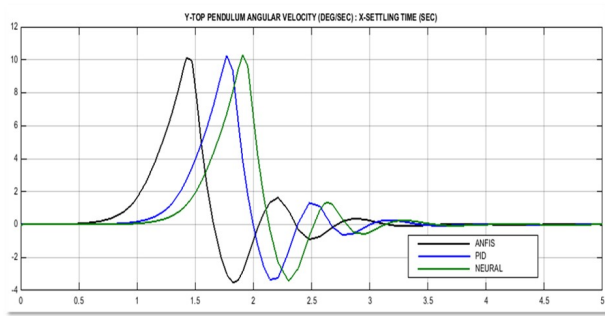


Fig. 12: Comparing simulation responses for 'Top pendulum angular velocity'.

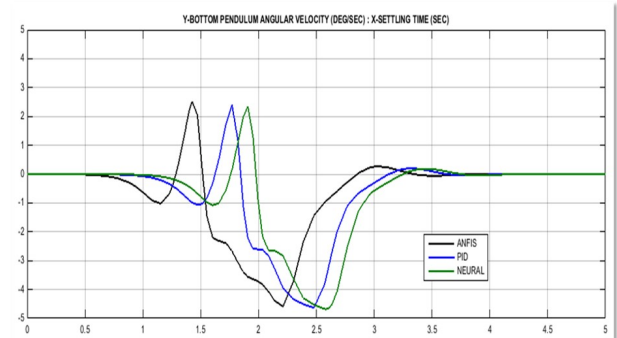


Fig. 14: Comparing simulation responses for 'Bottom pendulum angular velocity'.

of simulation results obtained for different controllers is shown in Table 5. The parameters considered for comparison includes settling time, overshoot ranges and

steady state error.

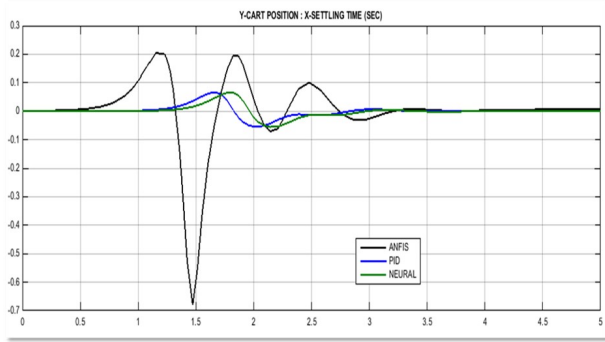


Fig. 15: Comparing simulation responses for 'Cart position'.

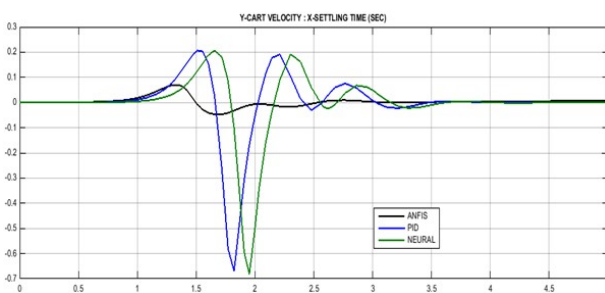


Fig. 16: Comparing simulation responses for 'Cart velocity'.

7. CONCLUSION

This study has successfully proposed a novel ANFIS controller designed using only two membership functions of triangular shape thereby resulting in only four if-then fuzzy rules for control of highly nonlinear two-stage cart and pendulum system. The proposed ANFIS controller has successfully solved the problem of rule explosion associated with fuzzy controllers. The study considered three different control techniques namely PID, Artificial neural networks and ANFIS for control of two-stage cart and pendulum system. A mathematical model of proposed system has been developed using Newton's second law. The results of PID controller have been successfully used for training of Neural and ANFIS controller. Excellent results for regression and MSE values were obtained for neural controllers. The study also highlights the effect of number and shape of membership functions on training error of ANFIS controller. It has been observed that minimum training error has been obtained using two membership functions of triangular shape. The results clearly indicate better performance of ANFIS controller in terms of settling time and overshoot ranges. The settling time duration and overshoot ranges response has been considerably improved using ANFIS controller. It can be observed that ANFIS, PID and Neural controllers were able to stabilize the complete system within 3.3s, 3.6s and 3.7s respectively. Lastly, it has been observed that all the three controllers provided excellent

steady state response. As an extension for future work, efforts can be made to further reduce the settling time of proposed system within 3.0 sec.

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