The Parameterization of All Disturbance Observers for Periodic Input Disturbances

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ABSTRACT

In this paper, we examine the parameterizations of all disturbance observers and all linear functional disturbance observers for periodic input disturbances. The plant disturbance observers have been used to estimate the disturbance in the plant. Several papers on design methods for disturbance observers have been published. Recently, the parameterization of all disturbance observers and all linear functional disturbance observers for plants with any input disturbance was clarified. However, no paper examines the parameterization of all disturbance observers or all linear functional disturbances for periodic input disturbances. In this paper, we propose parameterizations of all disturbance observers and all linear functional disturbance observers for periodic input disturbances.

Keywords: Estimation, Disturbances, Disturbance observer, Periodic input disturbance, Parameterization

1. INTRODUCTION

In this paper, we examine the parameterizations of all disturbance observers and all linear functional disturbance observers for periodic input disturbances. A disturbance observer is used to estimate the disturbances in the factory plant [1–6]. Several papers on design methods for disturbance observers have been published [4–8]. Currently, the applications of disturbance observers have been used in many control systems, such as motion-control fields [3, 5, 9]. The disturbance observer is used in motion control to cancel the disturbance or to make the closed-loop system robustly stable [1, 2, 7–13]. Typically, disturbance observers include a disturbance signal generator and an observer. Disturbances that are normally considered step disturbances are estimated by the observer. Since the disturbance observer's structure is simple to understand, it is used in many cases [1, 2, 8–13].

Mita et al. point out that disturbance observers are not the only alternative design for complete controllers [7]. That is, a control system with a disturbance observer does not guarantee robust stability. Extended H_∞ control in [7] has therefore been proposed as an effective motion control method that cancels disturbances. This implies that using the method in [7], the control system with the disturbance observer could be designed to guarantee robust stability. From another point of view, Kobayashi et al. considered an observer design method for obtaining phase compensation based on disturbance observers [8]. Compared to using phase compensation, the control system in [8] is simple and easy to design. In this way, a robustness analysis of the control system that has observed disturbances has been considered.

Another important control problem is the parameterization problem, which is the problem of finding all stabilizer controllers for the plant [14–20]. If the parameterization of all disturbance observers for any disturbance could be obtained, we could express the results of previous studies of disturbance observers in a uniform manner. In addition, disturbance observers for any disturbances could be designed systematically. From this point of view, Yamada et al. examine the parameterizations of all disturbance observers and all linear functional disturbance observers for plants with any input-output disturbances [21–26]. However, the methods in [21–26] could not apply to periodic disturbances. Phukapak et al. overcome this problem and examine the parameterizations of all disturbance observers for periodic output disturbances [27]. However, no paper examines the parameterization of all disturbance observers for periodic input disturbances. Methods in [21–26] could estimate disturbances with a finite number of frequency components but could not estimate disturbances with an infinite number of frequency components. In addition, when we control practical systems, many disturbances appear as periodic disturbances, such as robot arms, heat-flow experiments, multi-axis manipulators, positioning systems, noises, and vibrations [3, 25, 28–30]. Therefore, it is important for the control system to attenuate periodic disturbances. In addition, the parameterization and the observations of disturbance observers are taken into account for periodic input disturbances in this paper. Based on a control system to attenuate periodic disturbances, a strategy is proposed to improve the performance of the parameterization of all disturbance observers for periodic

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input disturbances. For these reasons, the purpose of this study is to propose the parameterization of all disturbance observers for periodic input disturbances.

In this paper, we clarify that the periodic input disturbances could estimate by using disturbance observers and propose the parameterization of all disturbance observers for periodic input disturbances. First, the necessary structure and characteristics of disturbance observers for periodic input disturbances are defined. In addition, the problem consideration of this paper is explained. Condition to estimate the periodic input disturbances are clarified. The parameterization of all disturbance observers for periodic input disturbances and that of all linear functional disturbance observers for periodic input disturbances are clarified. In addition, a design method for the linear functional disturbance observer and a procedure for linear functional disturbance observers for periodic input disturbances are clarified. Finally, we offered a numerical example to illustrate the features of the proposed design method. Fig. 1 shows the flowchart for the research process. This paper is organized comprising: In Section 2, the necessary structure and characteristics of disturbance observers for periodic input disturbances are defined. In addition, the problem considered in this paper is explained. In Section 3, conditions to estimate the periodic input disturbances are clarified. In Section 4, parameterization of all disturbance observers for periodic disturbances is clarified. In Section 5, parameterization of all linear functional disturbance observers for periodic input disturbance is clarified. In Section 6, using obtained parameterizations, a design method for the linear functional disturbance observer is presented. In Section 7, a design procedure for linear functional disturbance observers for periodic input disturbances is shown. In Section 8, we offer a numerical example to illustrate the features of the proposed design method. In Section 9 gives some concluding remarks.

Notations

R	the set of real number.
$R(s)$	the set of real rational function with s.
RH_∞	the set of stable proper real rational functions.
U	the unimodular procession in RH_∞. That is, $P(s) \in U$ means that $P(s) \in RH_\infty$ and $P^{-1}(s) \in RH_\infty$.
$\bar{\sigma}(\{\cdot\})$	largest singular value of $\{cdot\}$.
$diag(a_1, ..., a_n)$	an nxn diagonal matrix with a_i as its $i-th$ diagonal element.
$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	represents the state space description $C(sI - A)^{-1}B + D$.
$L\{\cdot\}$	the Laplace transformation of $\{\cdot\}$.

2. PROBLEM FORMULATION

Consider the plant described by

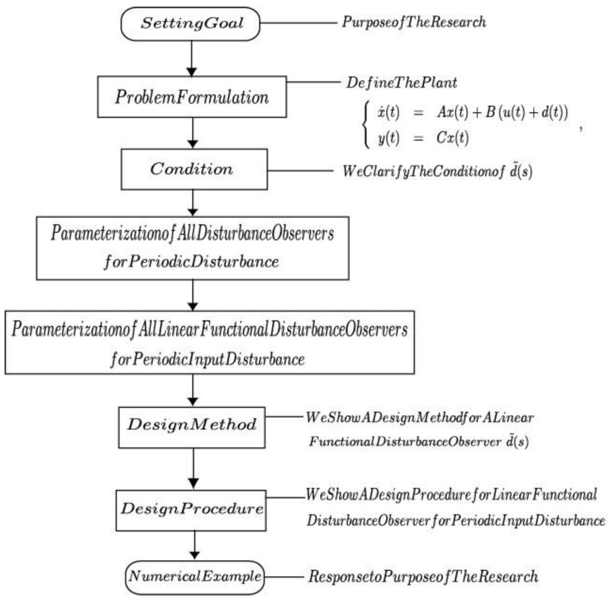

***Fig. 1**: Flowchart for The Research Process.*

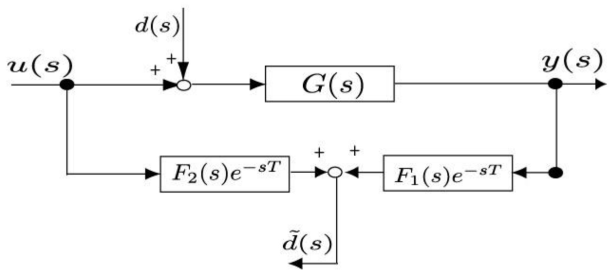

***Fig. 2**: Structure of A Disturbance Observer.*

$$\begin{cases} \dot{x}(t) = Ax(t) + B(u(t) + d(t)), \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $A \in R^{n \times n}$, $B \in R^{n \times p}$ and $C \in R^{m \times n}$, $x \in R^n$ is the state variable, $u \in R^p$ is the control input, $y \in R^m$ is the output, $d(t) \in R^m$ is periodic disturbances with period $T \geq 0$ satisfying

$$d(t + T) = d(t) \quad (\forall t \geq 0) \tag{2}$$

It is assumed that (A, B) is stabilizable, (C, A) is detectable, A has no eigenvalue on the imaginary axis and $u(t)$ and $y(t)$ are available, but $d(t)$ is unavailable. The transfer function from $u(s)$ to $y(s)$ in Eq. (1) is denoted by

$$y(s) = G(s)u(s) + G(s)d(s), \tag{3}$$

where

$$G(s) = C(sI - A)^{-1}B \in R^{mxp}(s). \tag{4}$$

When the disturbances $d(t)$ is unavailable, a disturbance estimator called the disturbance observer is frequently used. The disturbance observer estimates the disturbance of the periodic input disturbance $d(t)$ by using available measurements. Since available measurements of the plant in Eq. (1) are $u(t)$ and $y(t)$, that is, the input disturbance $d(t)$ satisfies Eq. (2), the periodic input disturbance $d(s)$ could be estimated by the form in

$$\tilde{d}(s) = F_1(s)e^{-sT}y(s) + F_2(s)e^{-sT}u(s), \tag{5}$$

where $F_1(s) \in RH_\infty^{m\times m}$, $F_2(s) \in RH_\infty^{m\times p}$ and $\tilde{d}(t) \in R^m$. The structure of disturbance observer $\tilde{d}(s)$ in Eq. (5) is shown in Fig. 2. In the following, we call the system $\tilde{d}(s)$ in Eq. (5) the disturbance observer for periodic input disturbances, if $e(t)$ written by

$$e(t) = d(t) + \tilde{d}(t) \tag{6}$$

satisfies

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} \left(d(t) - \tilde{d}(t)\right) = 0 \tag{7}$$

for any initial state $x(0)$, control input $u(t)$ and periodic disturbances $d(t)$. We denote $e(s)$ the Laplace transformation of $e(t)$.

If the parameterization of all disturbance observers for periodic input disturbances is obtained, there is a possibility to attenuate periodic input disturbances without using the repetitive control system. In addition, we can systematically design the disturbance observers for periodic input disturbances here is a possibility to attenuate periodic input disturbances without using the repetitive control system. In addition, we can systematically design the disturbance observers for periodic input disturbances. However, no paper examines the parameterization of all disturbance observers for periodic input disturbances.

The problem considered in this paper is to propose the parameterization of all disturbance observers for periodic input disturbances. Thus, we obtain the parameterization of all disturbance observers for $\tilde{d}(s)$ in Eq. (5) for periodic input disturbances.

3. CONDITION TO ESTIMATE THE PERIODIC INPUT DISTURBANCES

In this section, we clarify the condition of $\tilde{d}(s)$ in Eq. (5) to satisfy Eq. (7).

The condition of $\tilde{d}(s)$ in Eq. (5) satisfying Eq. (7) is summarized in the following theorem.

Theorem 1: $\tilde{d}(s)$ in Eq. (5) works as a disturbance observer for periodic input disturbances if and only if

$$F_1(s)N(s) + F_2(s)D(s) = 0 \tag{8}$$

$$(1-\bar{s}_i)^{l_i}(I - F_1(\bar{s}_i)e^{-\bar{s}_iT})G(\bar{s}_i) = 0 \quad (i = 1, ..., q), \tag{9}$$

$$(1 - e^{-s_iT})e(s_i) = 0 \ \ \forall s_i(i = 0, 1, ...) \tag{10}$$

are satisfied, where $\bar{s}_i(i = 1, \dots, q)$ is unstable pole of $G(s)$, $l_i(i = 1, \dots, q)$ is its multiplicity, q is the number of unstable poles,

$$s_i = j\omega_i \tag{11}$$

$$\omega_i = \frac{2\pi i}{T} \quad (i = 0, 1, ...) \tag{12}$$

and j is the imaginary unit.

Proof: First the necessity is shown, that is, if $\tilde{d}(s)$ in Eq. (5) satisfies Eq. (7), then Eq. (8) and Eq. (10) are satisfied. $\tilde{d}(s)$ in Eq. (5) is rewritten by

$$\begin{aligned}\tilde{d}(s) = &\left(F_1(s)e^{-sT}N(s) + F_2(s)e^{-sT}D(s)\right)\xi(s)\\ &+ F_1(s)G(s)e^{-sT}d(s),\end{aligned} \tag{13}$$

where $\xi(s)$ is the pseudo-state variable satisfying

$$u(s) = D(s)\xi(s), \tag{14}$$

$N(s) \in RH_\infty{}^{m\times p}$ and $D(s) \in RH_\infty{}^{m\times m}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = N(s)D^{-1}(s). \tag{15}$$

$\xi(s)$ in Eq. (13) is factorized as

$$\begin{aligned}\xi(s) &= \tilde{\xi}(s) + \bar{\xi}(s)\\ &= \frac{1}{1-e^{-sT}}\tilde{\xi}(s) + \bar{\xi}(s),\end{aligned} \tag{16}$$

where $\tilde{\xi}(s)$ is denoted by

$$\tilde{\xi}(s) = \int_0^T e^{-sT}\xi(\tau)d\tau, \tag{17}$$

$\tilde{\xi}(s)/\left(1 - e^{-sT}\right)$ means the periodic signal with period T and $\bar{\xi}(s)$ includes all other signals. From Eq. (2), $d(s)$ in Eq. (13) is written by

$$d(s) = \frac{1}{1-e^{-sT}}\hat{d}(s) \tag{18}$$

where

$$\hat{d}(s) = \int_0^T e^{-sT}d(\tau)d\tau. \tag{19}$$

From Eq. (13) and Eq. (18), $e(s)$ is given by

$$\begin{aligned}e(s) = &\left(I - F_1(s)e^{-sT}\right)G(s)\frac{1}{1-e^{-sT}}\hat{d}(s)\\ &- \left(F_1(s)N(s) + F_2(s)D(s)\right)\frac{e^{-sT}}{1-e^{-sT}}\tilde{\xi}(s)\\ &- \left(F_1(s)N(s) + F_2(s)D(s)\right)e^{-sT}\bar{\xi}s\end{aligned} \tag{20}$$

From the assumption that $e(t)$ satisfies Eq. (7) for any $\bar{\xi}(s)$,

$$\left(F_1(s)N(s) + F_2(s)D(S)\right)e^{-sT}\bar{\xi}s = 0 \tag{21}$$

is satisfied for any $\bar{\xi}s$. That is, we have Eq. (8). Substitution of Eqs. (8)- (20) gives

$$\begin{aligned}e(s) = &\left(I - F_1(s)e^{-sT}\right)G(s)\frac{1}{1-e^{-sT}}\hat{d}(s)\\ &- \left(F_1(s)N(s) + F_2(s)D(s)\right)\frac{e^{-sT}}{1-e^{-sT}}\tilde{\xi}(s).\end{aligned} \tag{22}$$

From the assumption that $e(t)$ satisfies Eq. (7), $\left(I - F_1(s)e^{-sT}\right)G(s)$ has no unstable pole. Therefore Eq. (9) holds true. From the internal model principle

[33-35], Eq. (10) is satisfied. We have thus proved the necessity.

Next the sufficiency is shown. That is, if Eq. (8) and Eq. (10) are satisfied, then $e(s)$ in Eq. (6) satisfies Eq. (7). From Eq. (8), $e(s)$ in Eq. (6) is written by

$$e(s) = \left(I - F_1(s)e^{-sT}\right)G(s)\frac{1}{1-e^{-sT}}\hat{d}(s) - \left(F_1(s)N(s) + F_2(s)D(s)\right)\frac{e^{-sT}}{1-e^{-sT}}\tilde{\xi}(s). \quad (23)$$

From the assumption that Eq. (7) is for $F_1(s) \in RH_\infty^{m\times m}$ satisfying Eq. (9). Substituting Eq. (8) and Eq. (9) for Eq. (23), $e(s)$ is rewritten by

$$e(\bar{s}_i) = \left(I - F_1(\bar{s}_i)e^{-\bar{s}_iT}\right)G(\bar{s}_i)\frac{1}{1-e^{-\bar{s}_iT}}\hat{d}(\bar{s}_i) - \left(F_1(\bar{s}_i)N(\bar{s}_i) + F_2(\bar{s}_i)D(\bar{s}_i)\right)\frac{e^{-\bar{s}_iT}}{1-e^{-\bar{s}_iT}}\tilde{\xi}(\bar{s}_i) = 0 \quad (24)$$

From Eq. (10), $e(t)$ in Eq. (6) satisfies Eq. (7). Thus the sufficiency is shown.

We have thus proved Theorem 1. Note that from Theorem 1, Eq. (8) is a condition of $\widetilde{d}(s)$ disturbance observers for any state variable. In addition, Eqs. (9)-(10) are conditions for estimating any periodic signals. Therefore, this is the most important condition to estimate the periodic input disturbances, and the mentioned condition could solve the problem.

In this section, we obtained the conditions to estimate the periodic input disturbances. In the next section, using the result of Theorem 1, we clarify the parameterization of all disturbance observers for periodic input disturbances.

4. PARAMETERIZATION OF ALL DISTURBANCE OBSERVERS FOR PERIODIC DISTURBANCE

In section, we propose the parameterization of all disturbance observers $\widetilde{d}(s)$ in Eq.(5) for periodic input disturbance.

The parameterization is summarized in the following theorem.

Theorem 2: The system $\widetilde{d}(s)$ in Eq. (5) is the disturbance observer for periodic input disturbance if and only if $F_1(s)$ and $F_2(s)$ are written by

$$F_1(s) = \widetilde{D}(s) + Q(s)\widetilde{D}(s), \quad (25)$$

$$F_2(s) = -\widetilde{N}(s) - Q(s)\widetilde{N}(s) \in RH_\infty^{m\times p}, \quad (26)$$

and

$$(1-\bar{s}_i)^{li}(I - F_1(\bar{s}_i)e^{-\bar{s}_iT}G(\bar{s}_i) = 0 \quad (i = 1, \ldots, q), \quad (27)$$

where $\widetilde{N}(s) \in RH_\infty^{m\times p}$ and $\widetilde{D}(s) \in RH_\infty^{m\times m}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \widetilde{D}^{-1}(s)\widetilde{N}(s), \quad (28)$$

and $Q(s) \in RH_\infty$ is any function satisfying

$$\widetilde{D}(s_i) + Q(s_i)\widetilde{D}(s_i) = I \quad \forall s_i\,(i = 0, \ldots), \quad (29)$$

satisfying Eqs. (10) and (11). Proof of Theorem 2 requires following lemma.

Lemma 1: [17] Assume that A $(s) \in RH_\infty^{m\times n}$, $B(s) \in RH_\infty^{q\times p}$, $C(s) \in RH_\infty^{m\times p}$ and

$$rank\begin{bmatrix} A^T(s) & B^T(s)\end{bmatrix} = \gamma. \quad (30)$$

are satisfied. There exist $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ satisfying

$$X(s)A(s) + Y(s)B(s) = C(s) \quad (31)$$

if and only if there exists $U(s) \in \mathcal{U}$ satisfying

$$\begin{bmatrix} A(s) \\ B(s) \\ C(s)\end{bmatrix} = U(s)\begin{bmatrix} A(s) \\ B(s) \\ 0\end{bmatrix}. \quad (32)$$

When $X_0(s) \in RH_\infty$ and $Y_0(s) \in RH_\infty$ are solution to Eq. (31), then all solutions to Eq. (31) are given by

$$\begin{bmatrix} X(s) & Y(s)\end{bmatrix} = \begin{bmatrix} X_0(s) & Y_0(s)\end{bmatrix} + Q(s)\begin{bmatrix} W_1(s) & W_2(s)\end{bmatrix} \quad (33)$$

where $W_1(s)$ and $W_2(s)$ satisfy

$$W_1(s)A(s) + W_2(s)B(s) = 0 \quad (34)$$

and

$$rank\begin{bmatrix} W_1(s) & W_2(s)\end{bmatrix} = n + q + \gamma \quad (35)$$

and $Q(s) \in RH_\infty^{p\times(N+q-\gamma)}$ is any function.

Using Theorem 1 and Lemma 1, Theorem 2 is proved.

Proof: From Theorem 1, $\widetilde{d}(s)$ works a disturbance observer for periodic disturbance if and only if $F_1(s) \in RH_\infty^{m\times m}$ and $F_2(s) \in RH_\infty^{m\times p}$ satisfy Eq. (8). From Lemma 1, all solution of $F_1(s)$ and $F_2(s)$ to satisfy Eq. (8) are given by Eqs. (25) and (26), respectively, since

$$\widetilde{D}(s)N(s) - \widetilde{N}(s)D(s) = 0, \quad (36)$$

and Lemma 1, where $\widetilde{D}(s) \in RH_\infty^{p\times p}$ and $\widetilde{N}(s) \in RH_\infty^{p\times m}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \widetilde{D}^{-1}(s)\widetilde{N}(s). \quad (37)$$

The rest is to prove $\widetilde{d}(s)$ in Eq. (5) works as a periodic input disturbance observer if and only if $Q(s)$ in Eqs. (25) and (26) satisfy Eq. (29). From Theorem 1, $\widetilde{d}(s)$ in Eq. (5) works as a periodic input disturbance observer if and only if $e(s)$ satisfy Eq. (10). The necessity is shown. That is if $\widetilde{d}(s)$ in Eq. (5) works as a periodic input disturbance observer, then $Q(s)$ in Eqs. (25) and (26) satisfy Eq. (29). From Eqs. (25) and (26), $e(s)$ is written by

$$e(s) = \left\{I - F_1(s)e^{-sT}\right\}G(s)\frac{1}{1-e^{-sT}}\hat{d}(s). \quad (38)$$

From the assumption that Eq. (7) is for $F_1(s) \in RH_\infty^{m\times m}$ satisfying Eq. (27). Substituting Eq. (27) for Eq. (38), $e(s)$ is rewritten by

$$(1-e^{-\bar{s}_iT}e(\bar{s}_i) = \{I - F_1(\bar{s}_i)e^{-\bar{s}_iT}\}G(\bar{s}_i)\widetilde{d}(\bar{s}_i) \quad (39)$$

This equation yields

$$(1-e^{-\bar{s}_iT}e(\bar{s}_i) = \{I - (\widetilde{d}(\bar{s}_i) + Q(\bar{s}_i)D(\bar{s}_i)G(\bar{s}_i)\widetilde{d}(\bar{s}_i) = 0. \quad (40)$$

We have Eq. (29). Thus we have proved the necessity.

Next, sufficiency is shown. That is, we show that if $Q(s)$ in Eqs. (25) and (26) satisfy Eq. (29), then Eq. (10) is satisfied. $e(s)$ is written by Eq. (38). Substituting Eq. (29) for Eq. (38), it is obvious that Eq. (10) is satisfied. In this way, sufficiency has been proven. From the above discussion, we have thus proved Theorem 2. Note that from Theorem 2, when $G(s)$ is stable, if $Q(s)$ is settled by

$$Q(s) = \widetilde{D}^{-1}(s) - I, \quad (41)$$

then $Q(s)$ in Eq. (41) satisfies Eq. (29). However, when $G(s)$ is unstable, it is difficult to set $Q(s)$ satisfying Eq. (29). For the unstable plant $G(s)$, a disturbance observer for periodic input disturbances is often used to attenuate disturbances effectively [36], even if the system $\widetilde{d}(s)$ in Eq. (5) satisfying Eq. (10) could not be designed. This means that in order to attenuate periodic disturbances, it is enough to estimate $(I - F(s))G(s)\widetilde{d}(s)$, where $F(s) \in RH_\infty$ is any function. From this point of view, in the next section, when $G(s)$ is unstable, we define a linear functional disturbance observer for periodic input disturbance observers and clarify the parameterization of all linear functional disturbance observers for periodic input observers.

5. PARAMETERIZATION OF ALL LINEAR FUNCTIONAL DISTURBANCE OBSERVERS FOR PERIODIC INPUT DISTURBANCE

In this section, we define a linear functional disturbance observer and present the parameterization of all linear functional disturbance observers for periodic input disturbance.

We call $\widetilde{d}(s)$ in Eq. (5) the linear functional disturbance observer for periodic input disturbances if $\widetilde{d}(s)$ written by

$$\left(1-e^{-s_iT}\right)e\left(s_i\right) = F\left(s_i\right)G\left(s_i\right)\hat{d}\left(s_i\right) \quad (42)$$

is satisfied, where $F(s) \in RH_\infty$ is any function satisfying

$$\bar{\sigma}\{F(s_i)\} \simeq 0 \quad \forall s_i(i=1,\ldots,n_{max}) \quad (43)$$

and n_{max} is the maximum frequency satisfying Eq. (43). Since the available measurements of the plant $G(s)$ in Eq. (1) are $u(t)$ and $y(t)$ and the input disturbance $d(t)$ satisfies Eq. (2), the periodic disturbance $d(t)$ is estimated by the form in Eq. (5), where $F_1(s) \in RH_\infty^{m\times m}$ and $F_2(s) \in RH_\infty^{m\times p}$.

The parameterization of the linear functional disturbance observer for periodic input disturbance is summarized as follows:

Theorem 3: The system $\widetilde{d}(s)$ in Eq. (5) is the linear functional disturbance observer for periodic input disturbance if and only if $F_1(s)$, $F_2(s)$ and $F(s)$ are described by

$$F_1(s) = \widetilde{D}(s) + Q(s)\widetilde{D}(s), \quad (44)$$

$$F_2(s) = -\widetilde{N}(s) - Q(s)\widetilde{N}(s), \quad (45)$$

and

$$\begin{aligned} F(s) &= I - F_1(s) \\ &= I - \left(\widetilde{D}(s) + Q(s)\widetilde{D}(s)\right), \end{aligned} \quad (46)$$

respectively, where $Q(s)$ is any function satisfying

$$\begin{aligned} (1-\bar{s}_i)^{l_i}(I - F_1(\bar{s}_i)e^{-\bar{s}_iT})G(\bar{s}_i) &= (1-\bar{s}_i)^{l_i}\{I - (\widetilde{D}(\bar{s}_i) \\ &\quad + Q(\bar{s}_i)\widetilde{D}(\bar{s}_i)e^{-\bar{s}_iT})\}G(\bar{s}_i) \\ &= 0 \quad (i=1,\ldots,q), \end{aligned} \quad (47)$$

and

$$\begin{aligned} \bar{\sigma}(I - F_1(s_i)) &= \bar{\sigma}\{I - (\widetilde{D}(s_i) + Q(s_i)\widetilde{D}(s_i))\} \\ &\simeq 0 \quad \forall s_i(i=1,...,n_{max}) \end{aligned} \quad (48)$$

Proof: First, the necessity is shown. That is, we show that if the system $\widetilde{d}(s)$ in Eq. (5) is a linear functional disturbance observer for periodic input disturbances, then Eqs. (44), (45), (46), (47), and (48) are satisfied. From Eqs. (5), (13), (14), (15), (16), and (17), for the system $\widetilde{d}(s)$ in Eq. (5), $e(s)$ is written as Eq. (20). From the assumption that $e(s)$ satisfies Eq. (7) for any $\bar{\xi}(s)$, Eq. (22) holds for any $\bar{\xi}(s)$. That is, we have Eq. (10). From Eq. (36) and Lemma 1, all solutions of $F_1(s)$ and $F_2(s)$ to satisfy Eq. (10) are given by Eqs. (44) and (45), respectively. Substitution of Eqs. (10) to (22) gives Eq. (23). From Eq. (23) the assumption that $e(s)$ satisfies Eq. (42), we have Eqs. (46), (47) and (48). In this way, the necessity has been proven.

Next, sufficiency is shown. That is, we show that if Eqs. (44), (45), (46), (47), and (48) are satisfied, then the $\widetilde{d}(s)$ is a linear functional disturbance observer. Since $e(s)$ is written by Eq. (20), substituting Eqs. (44), (45), (46), (47), and (48) to (20), it is obvious that Eq. (42) is satisfied. In this way, sufficiency has been proven.

From the above, we have thus proved Theorem 3.

Note that from Theorem 3, $\widetilde{d}(s)$ satisfying Eqs. (42) and (43), then Eqs. (44), (45), (46), (47) and (48) are satisfied to be solved by Theorem 3.

6. DESIGN METHOD FOR LINEAR FUNCTIONAL DISTURBANCE OBSERVERS

In this section, we show a design method for a linear functional disturbance observer $\widetilde{d}(s)$. In order to design the linear functional disturbance observer $\widetilde{d}(s)$ for periodic input disturbances, $Q(s)$ in Eqs. (44) and (45), one needs to satisfy Eq. (48). When $G(s)$ is unstable, $Q(s)$ is set as

$$Q(s) = \hat{Q}(s)\left(I - \widetilde{D}(s)\right)\widetilde{D}_o^{-1}(s), \tag{49}$$

and

$$\begin{aligned}(1-\bar{s}_i)^{li}(I - (\widetilde{D}(\bar{s}_i) + Q(\bar{s}_i)\widetilde{D}(\bar{s}_i))e^{-\bar{s}_i T})G(\bar{s}_i) \\ = 0 \quad (i = 1, \dots, q),\end{aligned} \tag{50}$$

where $\widetilde{D}_o(s) \in RH_\infty^{m\times m}$ is an outer function of $\widetilde{D}(s)$ satisfying

$$\widetilde{D}(s) = \widetilde{D}_o(s)\widetilde{D}_i(s), \tag{51}$$

$\widetilde{D}_i(s) \in RH_\infty^{m\times m}$ is a co-inner function of $\widetilde{D}(s)$ satisfying $\widetilde{D}_i(0) = I$ and $\widetilde{D}_i(s)\widetilde{D}_i(-s)^T = I, \hat{Q}(s) \in RH_\infty^{m\times m}$ is any function satisfying

$$\begin{aligned}(1-\bar{s}_i)^{li}\{I - \hat{Q}(\bar{s}_i)\widetilde{D}_i(\bar{s}_i)e^{-\bar{s}_i T}\}G(\bar{s}_i) \\ = 0 \quad (i = 1, \dots, q),\end{aligned} \tag{52}$$

and

$$\bar{\sigma}\{I - \hat{Q}(\bar{s}_i)\widetilde{D}(s_i)\} \simeq 0 \quad \forall s_i (i = 1, ..., n_{max}) \tag{53}$$

From the above, we showed a design of the linear functional disturbance observer $\widetilde{d}(s)$ for periodic input disturbances, $Q(s)$ in Eqs. (49) and (50), satisfying Eqs. (51), (52) and (53) based on Theorem 3. When $Q(s)$ in Eqs. (49) and (50), they are designed using the method described in Section 7.

7. DESIGN PROCEDURE FOR LINEAR FUNCTIONAL DISTURBANCE OBSERVERS FOR PERIODIC INPUT DISTURBANCE

In this section, we show a design procedure for a linear functional disturbance observer for periodic input disturbance satisfying Theorem 3. A design procedure is summarized as follows:

Procedure

1. Obtain coprime factors $\widetilde{N}(s) \in RH_\infty^{m\times p}$ and $\widetilde{D}(s) \in RH_\infty^{m\times m}$ of $G(s) \in R(s)^{m\times p}$ satisfying Eq. (28). The parameterization of all linear functional disturbance observers is given by Eq. (5), where $F_1(s)$, $F_2(s)$ and $F(s)$ are written by Eqs. (44), (45) and (46), respectively.
2. The maximum frequency range n_{max} in Eq. (48) to estimate the periodic disturbance $d(s)$ is settled.
3. Factorize $\widetilde{D}(s)$ as Eq. (51) satisfying $\widetilde{D}_i(0) = I$.
4. Settle $Q(s) \in RH_\infty^{m\times m}$ satisfying Eq. (48). In order to satisfy Eq. (48), $Q(s) \in RH_\infty^{m\times m}$ is set according to Eq. (49). Where $\hat{Q}(s)$ is a low-pass filter satisfying $\hat{Q}(0) = I$, as

$$\hat{Q}(s) = diag\left\{\frac{k_1}{\left(1+s\tau_1\right)^{\alpha_1}}, \dots, \frac{k_m}{\left(1+s\tau_m\right)^{\alpha_m}}\right\}, \tag{54}$$

α_i $(i = 1, 2, \dots, m)$ is an arbitrary positive integer and k_i $(i = 1, 2, \dots, m)$ satisfying τ_i $(i = 1, \dots, m)$

$$\begin{aligned}(1-\bar{s}_i)^{li}\{I - \hat{Q}(\bar{s}_i)\widetilde{D}_i(\bar{s}_i)e^{-\bar{s}_i T}\}G(\bar{s}_i) \\ = 0 \quad (i = 1, \dots, m),\end{aligned} \tag{55}$$

and

$$\bar{\sigma}\{I - \hat{Q}(\bar{s}_i)\widetilde{D}(s_i)\} \simeq 0 \quad \forall s_i (i = 1, ..., m) \tag{56}$$

are real numbers.

5. Substituting $Q(s)$ for Eqs. (44), (45) and (46), $F_1(s)$, $F_2(s)$ and $F(s)$ are obtained. Then we could design disturbance observer $\widetilde{d}(s)$ for periodic input disturbances as Eq. (5).

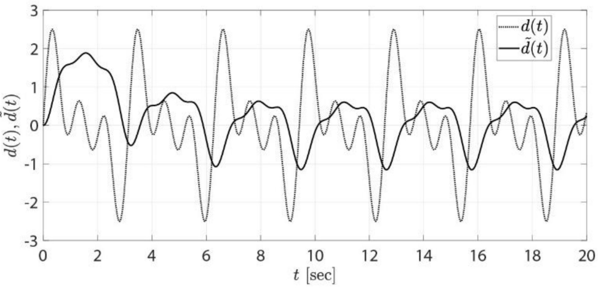

Fig. 3: *Response Curves of The Disturbance Estimation by Using A Design Method of [4].*

8. NUMERICAL EXAMPLE

In this section, we show numerical examples to illustrate the effectiveness of the proposed parameterizations.

Firstly, we show that the proposed design method of the disturbance observer for the stable plant in this paper could estimate the periodic disturbance more effectively than the other design methods of disturbance observers. To compare the effectiveness of the proposed design method in this paper, we show a result that the disturbance observer designed by using the design method described in this paper estimates the periodic disturbance for a single-input, single-output stable plant. Next, we show that the linear functional disturbance observer for the periodic disturbances designed by using the proposed design method in this paper could estimate the periodic disturbances for a single-input, single-output unstable plant.

8.1 Numerical example 1. A numerical example of disturbance observers for step disturbance for the stable plant

Consider the problem of estimating the periodic disturbance by designing a disturbance observer using a design method in [4] for a stable plant $G(s)$ given as

$$G(s) = \frac{s+1}{s^2+4s+5}. \tag{57}$$

The period T of the periodic disturbance $d(t)$ is

$$T = \pi \tag{58}$$

The disturbance observer is denoted as

$$\tilde{d}(s) = Q(s) G(s)^{-1} y(s) + Q(s) u(s), \tag{59}$$

where $Q(s)$ in Eq. (59) is the filter satisfying $\lim_{s \to 0} Q(s) = 1$. $Q(s)$ in Eq. (59) is settled by

$$Q(s) = \frac{1}{(s+1)^2}. \tag{60}$$

When the control input $u(t)$ and the periodic input disturbance $d(t)$ are given by

$$u(t) = 0 \tag{61}$$

and

$$d(t) = \sum_{i=1}^{3} sin(it), \tag{62}$$

respectively, the response curves of disturbance estimations are got by using the disturbance observers for the step disturbance. The response curves of disturbance estimations are shown in Figure 3. Here, the dotted line shows the periodic input disturbances of $d(t)$ and the solid line shows the disturbance observer of $\tilde{d}(t)$. Figure 3 shows that the disturbance observer $\tilde{d}(s)$ in Eq. (59) for step disturbance could not estimate $\tilde{d}(t)$ effectively.

8.2 Numerical example 2. A numerical example of disturbance observers for step disturbance for the stable plant

Consider the problem of obtaining the parameterization of all disturbance observers for stable plants $G(s)$ written by

$$G(s) = \frac{s+1}{s^2+4s+5} \tag{63}$$

The period T of the periodic disturbance $d(t)$ is

$$T = \pi. \tag{64}$$

Coprime factorization of $G(s)$ in Eq. (63) satisfying Eq. (28) is given by

$$\tilde{N}(s) = G(s) = \frac{s+1}{s^2+4s+5} \tag{65}$$

and

$$\tilde{D}(s) = \frac{s^2+4s+5}{s^2+13s+42} \tag{66}$$

From Theorem 2, the parameterization of all disturbance observers $\tilde{d}(s)$ for stable plant $G(s)$ in Eq. (63) is given by Eq. (5), where

$$F_1(s) = \frac{s^2+4s+5}{s^2+13s+42} + Q(s)\frac{s^2+4s+5}{s^2+13s+42}, \tag{67}$$

$$F_2(s) = -\frac{s+1}{s^2+4s+5} - Q(s)\frac{s+1}{s^2+4s+5} \tag{68}$$

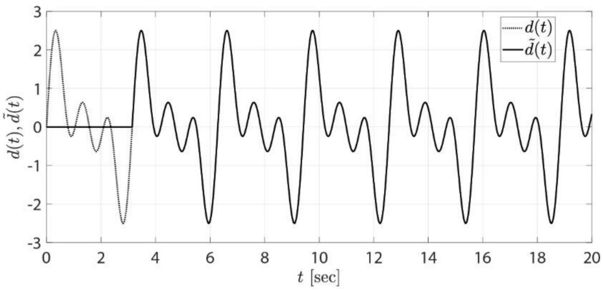

Fig. 4*: Response Curves of The Disturbance Estimation.*

and $Q(s) \in RH_\infty$ is any function.

Next using obtained parameterization, we design a disturbance observer $\tilde{d}(s)$ for the periodic input disturbances, that is, $Q(s)$ is settled satisfying Eq. (29). In order to satisfy Eq. (29), $Q(s)$ is settled by Eq. (41).

When the control input $u(t)$ and the periodic input disturbance $d(t)$ are given by

$$u(t) = 0 \tag{69}$$

and

$$d(t) = \sum_{i=1}^{3} \sin(it), \tag{70}$$

respectively, the response curves of disturbance estimations are got by using a proposed method. The response curves of the disturbance estimations are shown in Figure 4. Here, the dotted line shows the periodic input disturbances of $d(t)$ and the solid line shows the disturbance observer of $\tilde{d}(t)$. Figure 4 shows that the disturbance observer $\tilde{d}(s)$ in Eq. (5) could estimate $\tilde{d}(t)$ effectively. In this way, it is shown that using the obtained parameterization of all disturbance observers for periodic input disturbances, we could easily design a disturbance observer for periodic disturbances.

8.3 Numerical example 3. A numerical example of disturbance observers for periodic input disturbances

Consider the problem of obtaining the parameterization of all disturbance observers for stable plants $G(s)$ written by

$$G(s) = \frac{s+1}{s^2+2s+3} \tag{71}$$

The period T of the periodic disturbance $d(t)$ is

$$T = \pi. \tag{72}$$

Coprime factorization of $G(s)$ in Eq. (71) satisfying Eq. (28) is given by

$$\tilde{N}(s) = G(s) = \frac{s+1}{s^2+2s+3} \tag{73}$$

and

$$\tilde{D}(s) = \frac{s^2+2s+3}{s^2+13s+42}. \tag{74}$$

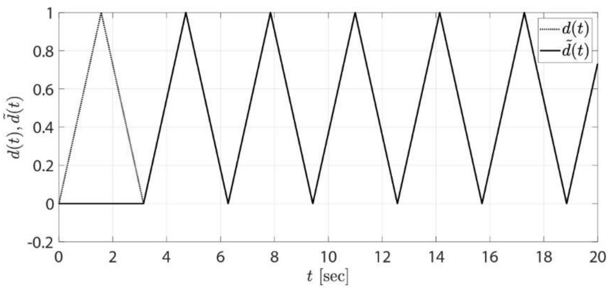

***Fig. 5**: Response Curves of The Disturbance Estimation.*

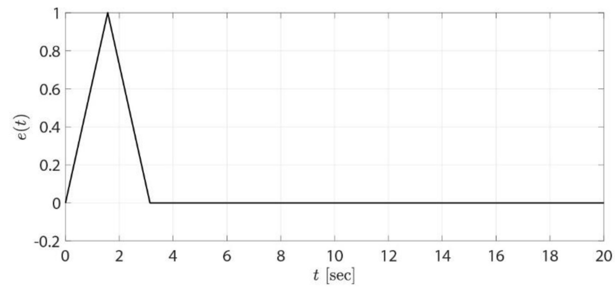

***Fig. 6**: The Response of The Error $e(t)$ in Eq. (6).*

From Theorem 2, the parameterization of all disturbance observers $\widetilde{d}(s)$ for stable plant $G(s)$ in Eq. (71) is given by Eq. (5), where

$$F_1(s) = \frac{s^2+2s+3}{s^2+13s+42} + Q(s)\frac{s^2+2s+3}{s^2+13s+42}, \tag{75}$$

$$F_2(s) = -\frac{s+1}{s^2+2s+3} - Q(s)\frac{s+1}{s^2+2s+3}, \tag{76}$$

and $Q(s) \in RH_\infty$ is any function.

Next using obtained parameterization, we design a disturbance observer $\widetilde{d}(s)$ for the periodic input disturbances, that is, $Q(s)$ is settled satisfying Eq. (29). In order to satisfy Eq. (29), $Q(s)$ is settled by Eq. (41).

When the control input $u(t)$ and the periodic input disturbance $d(t)$ are given by

$$u(t) = 0 \tag{77}$$

and

$$d(t) = \begin{cases} 1 + \frac{1}{\pi}t, & 2\pi i \le \pi + 2\pi i \\ \frac{1}{\pi}t - 1, & \pi + 2\pi i \le t < 2\pi + 2\pi i \end{cases}$$
$$(\forall i = 0, 1, ...), \tag{78}$$

respectively, the response curves of disturbance estimations by using the disturbance observers for the step disturbance. The response curves of the disturbance estimations are shown in Figure 5. Here, the dashed line shows the periodic input disturbances of $d(t)$ and the solid line shows the disturbance observer of $\widetilde{d}(t)$. Figure 5 shows that the disturbance observer $\widetilde{d}(s)$ in Eq. (5) for step disturbance could estimate $\widetilde{d}(t)$ effectively. The response to the error $e(t)$ in Eq. (6) is shown in Figure 6. Here, the solid line shows the response of $e(t)$. Figure 6 shows that the disturbance observer $\widetilde{d}(s)$ in Eq. (5) for periodic input disturbances could estimate $d(t) - \widetilde{d}(t)$ effectively.

In this way, it is shown that using the obtained parameterization of all disturbance observers for periodic input disturbances, we could easily design the disturbance observer for periodic input disturbances.

8.4 Numerical example 4. A numerical example for linear functional disturbance observer

Consider the problem of obtaining the parameterization of all linear functional disturbance observers for periodic input disturbances for unstable plants $G(s)$ described by

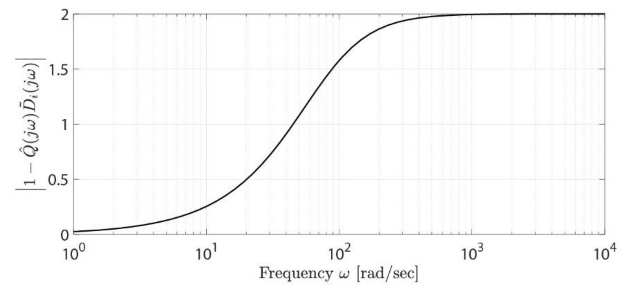

***Fig. 7**: The Gain Plot of $1 - \hat{Q}(s)\widetilde{D}_i(s)$.*

$$G(s) = \frac{s+1}{s^2-70s-624}. \tag{79}$$

The period T of the periodic disturbances is

$$T = \pi. \tag{80}$$

A pair of coprime factors $\widetilde{N}(s) \in RH_\infty$ and $\widetilde{D}(s) \in RH_\infty$ of $G(s)$ in Eq. (79) satisfying Eq. (28) is given by

$$\widetilde{N}(s) = \frac{-2s-2}{s^2+1008s+8000} \tag{81}$$

and

$$\widetilde{D}(s) = \frac{-2s+156}{s+1000}. \tag{82}$$

From Theorem 3, the parameterization of all linear functional disturbance observers $\widetilde{d}(s)$ is given by Eq. (5), where

$$F_1(s) = \frac{-2s+156}{s+1000} + Q(s)\frac{-2s+156}{s+1000}, \tag{83}$$

$$F_2(s) = \frac{2s+2}{s^2+1008s+8000} + Q(s)\frac{2s+2}{s^2+1008s+8000}, \tag{84}$$

$$F(s) = 1 - \frac{-2s+156}{s+1000} - Q(s)\frac{-2s+156}{s+1000} \tag{85}$$

and $Q(s) \in RH_\infty$ is any function.

Next, using the obtained parameterization, we design a linear functional disturbance observer $\widetilde{d}(s)$ for the periodic input disturbances by using the procedure described in Section 7, that is, $Q(s)$ is a settled satisfying observer Eq. (29). The maximum frequency range n_{max}

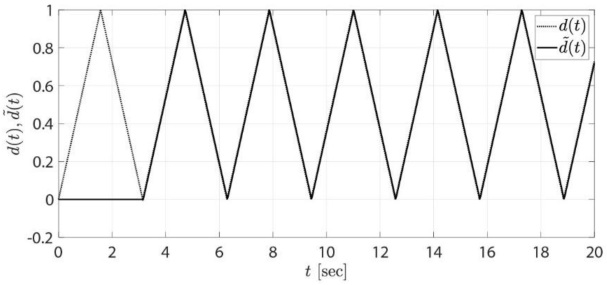

***Fig. 8**: Response Curves of The Disturbance Estimation.*

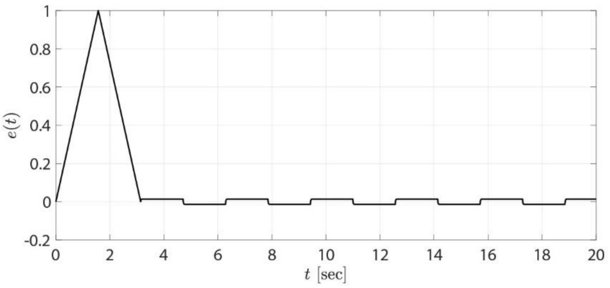

***Fig. 9**: The Response of The Error e(t) in Eq. (6).*

in Eq. (48) to estimate the periodic disturbance $d(s)$, is settled by

$$n_{max} = 3. \tag{86}$$

$\widetilde{D}(s)$ in Eq. (82) is factorized as Eq. (51), where

$$\widetilde{D}_o(s) = \frac{2s + 156}{s + 1000}, \tag{87}$$

and

$$\widetilde{D}_i(s) = \frac{-s + 78}{s + 78}. \tag{88}$$

In order to satisfy Eq. (48), $\hat{Q}(s)$ is settled by

$$\hat{Q}(s) = 1. \tag{89}$$

In order to confirm that $\hat{Q}(s)$ in Eq. (89) satisfy Eq. (48), we show the gain plot of $1 - \hat{Q}(s)\widetilde{D}_i(s)$ in Fig. 7. Figure 7 shows $\hat{Q}(s)$ in Eq. (89) satisfies Eq. (48). $Q(s)$ is set by Eq. (49) and written by

$$Q(s) = \frac{1.5s + 422}{s + 78}. \tag{90}$$

From Eqs. (44), (45) and (46), we have $F_1(s)$, $F_2(s)$ and $F(s)$ are designed as

$$F_1(s) = \frac{-5s^2 - 610s + 78000}{s^2 + 1078s + 78000}, \tag{91}$$

$$F_2(s) = \frac{-5s^2 - 1005s + 1000}{s^3 + 1068s^2 + 86620s + 624000}, \tag{92}$$

and

$$F(s) = \frac{6s^2 + 1688s}{s^2 + 1078s + 78000}. \tag{93}$$

When the control input $u(t)$ and the periodic input disturbance $d(t)$ are given by

$$u(t) = 0 \tag{94}$$

and

$$d(t) = \begin{cases} 1 + \frac{1}{\pi}t, \; 2\pi i \leq \pi + 2\pi i \\ \frac{1}{\pi}t - 1, \; \pi + 2\pi i \leq t < 2\pi + 2\pi i \end{cases}$$
$$(\forall i = 0, 1, \ldots), \tag{95}$$

respectively, the response curves of disturbance estimations by using the disturbance observers for the step disturbance. The response curves of the disturbance estimations are shown in Figure 8. Here, the dashed line shows the periodic input disturbances of $d(t)$ and the solid line shows the disturbance observer of $\widetilde{d}(t)$. Figure 8 shows that the disturbance observer $\widetilde{d}(s)$ in Eq. (5) for step disturbance could estimate $\widetilde{d}(t)$ effectively. The response to the error $e(t)$ in Eq. (6) is shown in Figure 9. Here, the solid line shows the response of $e(t)$. Figure 9 shows that the linear functional disturbance observer $\widetilde{d}(s)$ in Eq. (5) for periodic input disturbances could estimate $d(t) - \widetilde{d}(t)$ effectively. Although the error is maintained since we cannot completely estimate a disturbance when the plant has an unstable pole. We showed that using the parameterization of all linear functional disturbance observers for periodic input disturbances, we could easily design a linear functional disturbance observer for periodic input disturbances.

9. CONCLUSION

In this paper, we propose parameterizations of all disturbance observers and of all linear functional disturbance observers for periodic input disturbances. We have shown that the proposed method could attenuate periodic disturbances effectively without using repetitive controllers. A design method and a design procedure for linear functional disturbance observers are presented. Finally, we have shown features of the proposed design method through numerical examples. Using the obtained parameterizations, a design method for control systems will be discussed in another article.

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