

# Multi-level Optical On-Off Line Code for Flat-like Power Spectral Density: Outdoor Visible Light Communications

Samuel Nlend and Theo G. Swart<sup>†</sup>, Non-members

## ABSTRACT

This paper suggests a multi-level optical line code using an On-Off technique that enhances the performance of optical communication systems with flat-like power spectral density (PSD) such as outdoor visible light communications. The proposed technique based on a controlled multilevel coding and a multistage decoding of  $M = 2^l$  modulation signals, consists of designing the code using a balance distance rule that maximizes the minimum Euclidean distance and minimizes the number of nearest neighbours, with the aim to achieve a better system capacity and error protection. Through a controlled correlative level technique by an optical orthogonal code (OOC) time sequence, each obtained symbol is assigned an elementary signal within the same symbol duration using the conventional Miller coding. The constraints of an OOC sequence add a coding complexity that is dealt with using a synchronized error technique, where the error is recovered through an adapted time diversity decoding technique during the multi-stage decoding (MSD). The resulting  $M$  asymmetric signals are differentiated by  $l$  time transition levels and  $M = 2^l$  1-dimension space distances to produce  $M$ -ary On-Off Keying (OOK) signals. These signals are transmitted in accordance with the correlation properties of the OOC sequence, hence OOK OOC. An analytical investigation conducted on an 8-OOK OOC, compared with an amplitude shift keying (8-ASK) of the classical Ungerboeck (UG) and the unequal error protection (UEP) methods, has shown an improved level of signal-to-noise ratio (SNR), and consequently a better possible data rate.

**Keywords:** Duo-binary Coding, Miller Coding, Multi-level Coding, Optical Orthogonal code (OOC), On-off Keying (OOK), Partitioning, Synchronized Error Coding, Time Diversity

---

Manuscript received on February 3, 2023; revised on March 3, 2023; accepted on May 31, 2023. This paper was recommended by Associate Editor Piya Kovintavewat.

The authors are with the Centre For Telecommunications, University of Johannesburg, South Africa.

<sup>†</sup>Corresponding author: tgswart@uj.ac.za

©2023 Author(s). This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivs 4.0 License. To view a copy of this license visit: <https://creativecommons.org/licenses/by-nc-nd/4.0/>.

Digital Object Identifier: 10.37936/ecti-ec.2023213.251466

## 1. INTRODUCTION

Among the signals known for flat-like power spectral density (PSD), one can cite the outdoor visible light communications (VLC) signal. VLC plays a crucial role as a wireless communication system for intelligent transport systems (ITS) applications such as infrastructure to vehicle (I2V), vehicle to vehicle (V2V), and robot-to-robot (R2R) communications. It can serve as a replacement or as an alternative (redundancy) to the ubiquitous wired and wireless public networks, particularly, in the area not covered by these. However, it experiences transmission issues which limit the related VLC technologies to a low data rate, due to low signal-to-noise ratios (SNRs). This low SNR results from strong radiation of ambient day light noise, an issue this paper attempts to address through a coded modulation technique. Several studies are ongoing to overcome or to deal with these ambient effects. With a view to reduce the effects from light, [1] and [2] efficiently blocked or reduced the noise light for outdoor VLC data transmission. Another technique was proposed in ITS by [3], [4], that reduces the effects of ambient-light noise in data transmission using light sensing made of cameras or photodetectors at the receiver, an approach that showed good performance with a significant increase of the field-of-view for the camera. However, even if a photo-diode detection reduces the costs, high speed is still required, and this type of solution will require an improve field-of view for the camera, every time an improvement or a system upgrade is needed.

In this paper, we propose a coded modulation that increases the SNR, and consequently the transmission rate for these low SNR VLC systems. The VLC modulation technique makes use of light emitting diodes (LEDs) to transmit data using intensity modulation (IM) techniques, which intensity is detected at the receiver using direct detection (DD) [1]. Such simple modulation/demodulation techniques are favourable for outdoor VLC-ITS applications as shown Fig. 1 for this ITS system experiment conducted in [1] using the pulse position modulation (PPM) technique.

The VLC standard [5] specifies for outdoor application the usage of *On-Off Keying* (OOK) amplitude modulation and *Variable Pulse Position Modulation* (VPPM), but VPPM is efficient for dimming application, which is not always the case. In [6], it is noted that OOK, also known as a unipolar code, is a quite efficient transmission solution for these systems, hence our interest in the scheme.

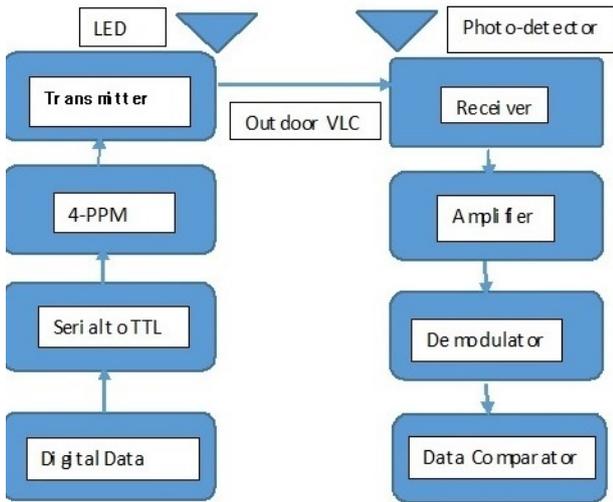


Fig. 1: Experimental ITS system.

We choose thus to associate this scheme with a less favourable line code spectrum wise, namely NRZ.

The contribution of this paper is to propose a controlled multi-level On-Off line coding, which improves the SNR level and consequently the data rate. This control is made possible due to a constraint code that manipulates the occurrence of binary digits, namely the *optical orthogonal code* (OOC). OOC has been used to sacrifice cardinality for the benefit of a better performance, and this performance is proven to be better than the well-known congruence counterparts [7]. The correlation properties responsible for that performance are herein exploited to produce an  $l$ -binary sequence, a correlative level technique suggested herein as a replacement for the modified duo-binary coding. The modified duo-binary technique, as its name implies, was introduced to avoid the possible recurrent error from the duo-binary coding, during detection. Therefore, in this paper, we introduce an  $l$ -binary coding technique while we control the introduced correlation using an OOC sequence. Furthermore, the OOC error detection and correction capabilities are of utmost importance for the synchronized error and time diversity coding techniques we are using in building the encoder. In synchronized error coding, any corruption of the transmitted symbols is possible while loss or gain of symbols are not considered. We can cite in this category, insertion/deletion and substitution correction codes. This paper focuses on substitution correction coding to transmit, and the time diversity technique is used for recovery. These techniques can be found respectively in [8] and [9].

Signalling wise, the VLC application generally adopts Manchester and Miller signalling codes to achieve the dc-balance, a condition required for signal reliability. Miller coding as used in *multilevel coding* (MLC) was proposed in [10], [11] as novel optical line codes with the introduction of a factor  $\alpha < 1$  that differentiates levels. This differentiation has resulted into 4 levels

labelled as order 1 Markov chain. However, in this Markov chain channel modelling, the inputs are not dependent of the channel state [11] and having  $0 < \alpha < 1$  that reduces the signal amplitude in outdoor VLC, where intensity detection matters, is more than a concern particularly for a multi-path fading channel. To keep all the levels without the above constraint on  $\alpha$  that reduces the amplitude as well as to keep using the conventional channel modelling, this paper suggests this conventional Miller coding for  $\alpha = 1$ , preceded by an  $l$ -binary coding pre-processing technique. The latter ensures that successive transitions always take place between  $l$  controlled bits with respect to the OOC sequence, to yield the required  $M$ -ary MLC.

Partitioning wise, MLC is an operation over  $D$ -dimensional signalling combined with a labelling of signal levels (herein  $D = 1$ ). Conventionally, *Ungerboeck partitioning* (UG) known as *Trellis Coded Modulation* (TCM), where individual bits are protected at each level, was proposed in [12]. During its encoding process, codes are chosen such that the minimum distance of the Euclidean space code is maximized. Exploiting the balanced distance rule design, and applying the capacity chain rule design, provides us with the required capacity. Another classical method of a block partitioning using *unequal error protection* (UEP) was introduced to improve the error performance by a power of  $2^w$  [13]. It is well known that such a technique is useful in allowing a better antenna symbol selection and transmission [14], as it results in reducing the number of *nearest neighbours* (NN). This is an important property, when more than one symbol is transmitted at each channel use, the channel efficiency increases, but the problem is that these symbols could be transmitted from the same antenna. This would require transmitting the modulation set from which these symbols are drawn, that means, the cardinality becomes the sum of these symbol modulation set sizes, and their size product as well. As a result, the Euclidean space becomes densely occupied, reducing the Euclidean distance. Therefore, one experiences the degradation of the *bit error rate* (BER) performance, because the detection depends on the minimum Euclidean distance among transmitted symbols [14]. An asymmetric coding based on UG's technique was suggested in [15] and [16]. The scheme considers the inter- and intra-subset minimum Euclidean distance with an intensive trellis code search over all possible reference sequences. Instead, in this paper, we achieve channel efficiency through a better capacity and BER performance by combining both methods without the cumbersome computer's code search and provide a direct clear correlation between Euclidean distance and antenna selection. Note that the Euclidean distance optimizes the antenna selection where the optimal antenna subset maximizes the minimum Euclidean distance, and the capacity optimizes antenna selection where the optimal antenna set maximizes the system capacity [17-24]. Note also in this paper that antenna refers to the VLC

system transmission. At the receiver side, each code is decoded individually starting from the lowest level and taking into account decisions of prior decoding stages. This procedure is called *multistage decoding* (MSD).

It is a known result that MLC and MSD suffice to approach capacity if the rates at each level are appropriately chosen. Also, the MLC with maximum likelihood decoding and the balance distance decoding rule [12] yields the same result. In this paper, we make use of the balance distance rule that protects the most significant bit for the code to achieve the required capacity and a better SNR at each level of transmission. The chain rule is used [12] to determine the capacity based on the capacity rule, and the balanced distance rule to reduce noise effects in such a way that the MLC and MSD optimize the overall channel performance in terms of SNR and transmission rate. This is crucial for outdoor VLC-ITS systems.

This paper is organized as follows: Section 2 gives the background of different techniques used in this paper. Section 3 suggests the system model which performance is analysed in Section 4, while Section 5 concludes the paper.

## 2. BACKGROUND

The reliability of our coded modulation is achieved by a constraint code. Specifically, OOC as it allows us to manipulate the sequence pattern to fit a certain requirement, the more so as we use correlative coding to yield our sequence. This code is fed to a Miller encoder. The following describes the theory behind our approach.

### 2.1 OOC and Correlative Coding Sequence

An OOC denoted  $(n, w, \lambda_a, \lambda_c)$ -OOC,  $C$ , is a family of  $(0, 1)$  sequences representing each a code word of length  $n$ , Hamming weight  $w$ , with the auto-correlation  $\lambda_a$  and cross-correlation  $\lambda_c$  satisfying for this case the following correlation properties [25]:

- Auto-correlation:  $\sum_{i=1}^n x_i x_{i+\tau} \leq \lambda_a$ ,  $\tau \neq 0$ ,
- Cross-correlation:  $\sum_{i=1}^n x_i y_{i+\tau} \leq \lambda_c$

with the subscript  $\tau$  a positive integer, taken modulo  $n$ , for any  $X = (x_1 x_2 \dots x_n)$ ,  $Y = (y_1 y_2 \dots y_n)$ , code vectors of the code. Throughout this paper, we will consider  $\lambda_a = \lambda_c = 1$  as imposed by the properties of light.

This definition can be extended to a difference family and formulated taking into account the difference correlation properties as follows [26]:

- Auto-correlation: For each  $x \in X$  a code vector of the cyclic difference packing, any integer  $d_{ij} = |x_i - x_j| \neq 0$ ,  $j \neq i = 1, 2, \dots, n$ , can be represented as the difference in at most  $\lambda = 1$  block.
- Cross-correlation: Similarly, for every pair  $x \in X$ ,  $y \in Y$ , code vectors of the cyclic difference packing, any integer  $d_{ij} = |x_i - y_j| \neq 0$ ,  $j \neq i = 1, 2, \dots, n$  can be represented as the difference in at most  $\lambda = 1$  block.

The latter definition is the one that inspires this paper as it allows not having some difference patterns more

than once. In fact, this characteristic, which complies with the properties of light, provides us with a good control of the sequence pattern. Particularly, with the use of a correlative coding extended to the  $l$ -binary coding introduced in this paper. This correlative coding makes use of the substitution code [8] with provision to correct this error with the time diversity coding [9], to achieve the required  $l$ -binary sequence. To better introduce the  $l$ -binary coding, it is necessary to present its duo-binary counterpart.

### 2.2 Duo-binary Encoding

The idea behind the duo-binary encoding is to achieve a signaling rate equal to the Nyquist rate of  $2W$  symbols per second in a channel of  $W$  Hz bandwidth [27]. Therefore, the duo-binary coding has a spectrum bounded within the Nyquist band, which makes it a minimum bandwidth coding-scheme. It is a partial response signal that controls *inter-symbol interference* (ISI) and allows the noise from ISI to remain small even when amplitude distortion is severe [28]. Mathematically, it changes input data sequence  $a_k$  of low spectrum, into a sequence  $\sigma_k$  that matches the spectral characteristics of the channel as follows.

Assume a sequence of binary symbol  $\{0, 1\}$  of  $T_b$  duration each which is applied to a pulse generator to produce a two-level sequence.

$$a_k = \begin{cases} +1, & \text{if symbol is 1} \\ -1, & \text{if symbol is 0.} \end{cases} \quad (1)$$

When applied to a duo-binary encoder, it produces  $\sigma_k = a_k + a_{k-1}$ , which yields three levels of two-unit pulses separated  $T_b$  seconds apart at the filter output [7].

However, with the high probability of spreading any error that may occur during detection in the entire sequence, this has led to the modified duo-binary, which requires a filter for signal processing. In this paper, instead of having a modified duo-binary system and a requirement of a filter, we choose  $l$ -binary coding and an  $l$ -level Miller encoder to replace the filter.

### 2.3 Miller Encoding

Miller's code is a process of encoding binary sequence with three-level rectangular pulses, also called the conventional Miller code. It is known as delay modulation or modified frequency modulation. It shows a good performance compared to the Manchester code due to its better resilience to noise [6], [11]. Miller coding provides us with an effective exploitation of information due to its appropriateness for outdoor VLC usage in ITS applications [6]. Originally introduced by Lender [29], it can be easily constructed using the Manchester code whereby a "1" is encoded as a transition on the mid-bit position, a "0" following a "1" is encoded as no transition on the entire bit period, whereas a "0" following a "0" is encoded as a transition on the beginning of the second bit period. The Miller code has very good timing content,

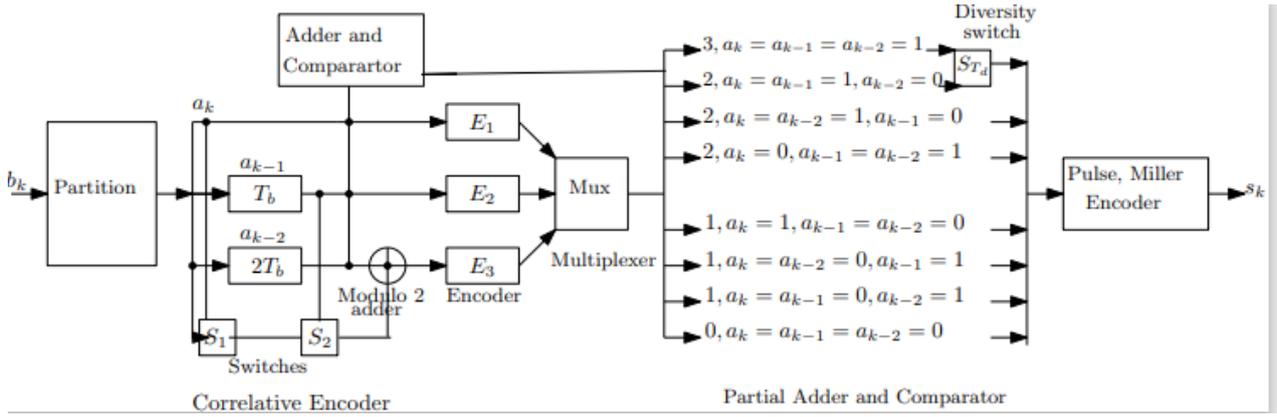


Fig. 2: System model.

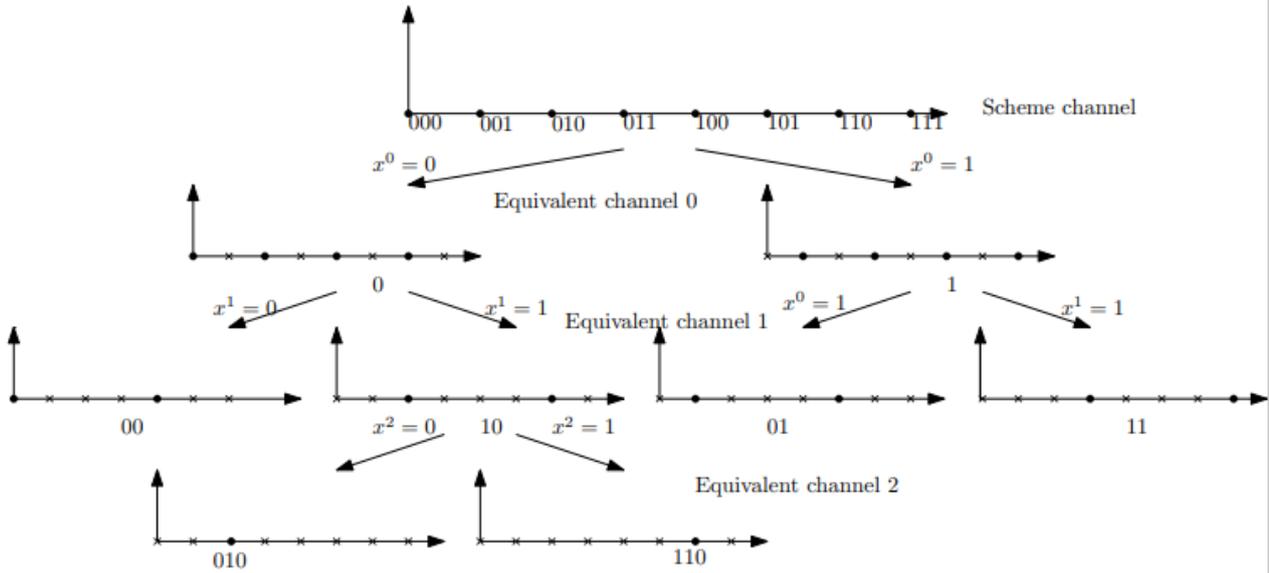


Fig. 3: Sequence partitioning.

and carrier tracking is easier compared to Manchester coding [6]. These capabilities are of upmost importance for multi-path systems hence our choice to use the Miller code in outdoor VLCs.

The OOC that controls and correct errors, the  $l$ -binary technique borrowed from the modified duobinary technique, the substitution correction coding, and Miller coding are then put together in our system model as next.

### 3. SYSTEM MODEL

#### 3.1 System Modeling and Diagram

The system is modelled as a multilevel encoder made of  $l$  parallel encoders receiving bits at  $T_b$  seconds apart respectively, from the set partitioning, with  $T_b$  being the bit duration. A set partitioning is obtained by protecting each level's most significant incoming bits. A correlative  $l$ -binary operation is applied to the  $E_l$  encoder through  $(l-1)$  logic switches,  $l = 1, 2, \dots, l-1$ , for synchronized

error coding, in particular substitution coding [8]. The respective  $l$ -bits are multiplexed to yield the  $l$ -binary OOC codewords from  $l$  binary parallel encoders, which are transmitted via a pulse modulator, consisting of a Miller encoder. Prior to the Miller encoder, the system contains a bit adder combined with a bit comparator, which, through a partial order addition of the current and the preceding bits, differentiates the input sequences such that the proper signal label is sent among the yielded  $M = 2^l$  signals at the  $l$ -th level. This modified  $l$ -binary partial addition is given by:

$$\sigma_k = a_k + a_{k-1} + a_{k-2}, a_k = \{0, 1\}. \quad (2)$$

A further time diversity switch  $T_d$  is added for time diversity coding [9], in accordance with the synchronized error introduced, for recovery purposes.

### 3.2 Set Partitioning

The codes are designed with the aim to maximize the minimum Euclidean distance that protects the most significant bit between signal points through set partitioning. To proceed, we consider in this model a general  $M$ -ary signal space consisting of signal points  $S = \{s_i, i = 0, 1, \dots, M - 1\}$  which represents an alphabet of  $M$  symbols  $\Sigma = \{\sigma_0, \sigma_1, \dots, \sigma_{M-1}\}$ . To each signal label  $s_i$  there is a bijective mapping of an  $l$ -bit vector that yields the following codes  $\{x_{i-1}^l \dots x_1^l x_0^l\}$ , where  $l = \log_2 M$  and  $x_j^i$  represents the  $j$ -th bit of the sequence at level  $i$ . These code sequences are obtained through partitioning of the binary input data as proposed in [30] with an extra condition of [13], with levels are labelled as  $x^{l-1}, \dots, x^2, x^1, x^0$ , respectively as shown in Fig. 2. Note that  $2^{l-1}$ -OOK OOC is directly seen within  $2^l$ -OOK OOC in the partitioning strategy shown in Fig. 3.

In this partitioning strategy, a signal point for instance  $s(x^3 x^2 x^1)$  can be  $s(0x^2 x^1)$  or  $s(1x^2 x^1)$ , which in turn originated in the case of  $s(0x^2 x^1)$ , from  $s(00x^1)$  or  $s(01x^1)$  (see Fig. 3). In this strategy, the most significant address bit  $x^i$  of the signal point is protected such that the distance between  $s(x^3 x^2 0)$  and  $(x^3 x^2 1)$  for instance is maximized. Consequently, we achieve a reduction of the number of NN code vectors, as compared to UG's protection of each address bit of an individual binary code at each level  $i$ . However, this does not change the parallel coding strategy with an encoder for level  $i$  denoted  $E_i$  as well as the condition that the data rate  $R_i$  is equal to the channel capacity  $C_i$ ,  $C_i = R_i$ . Furthermore, unlike UG where the transmission of these code vectors is separated into different parallel individual encoders that yield channels known as equivalent channels [31], the protection of the most significant bit allows through multiplexing and Miller coding, to directly achieve the system channel capacity meant for all included levels. That is: the channel capacity up to level  $i$  is  $C = C_i$ , while in UG, it is given by  $C = \sum_i C_i$ .

For some OOC with the code length given by  $n = w(w - 1) + 1$ , one may proceed with a substitution coding, with the decoder retrieving the sequence that undergoes substitution through time diversity, allowing therefore to recover sequences where such a synchronized error coding has taken place as shown in Fig. 2.

In Fig. 2, to achieve the transmission signal, the partitioned sequences are sent to a combined correlative-multi-level-OOC encoder, which, through a multiplexer and a combined adder-comparator, sends the code vectors (most significant bit first) to the Miller encoder for signal mapping. Whenever the sequence does not fit an OOC pattern, provisions are made for substitution and time diversity coding to take place in the correlative encoder through logic switches ( $S_1, S_2$ ) by using a modulo 2 addition, and signal time diversity introduced via switch ( $S_{T_d}$ ). To make it clearer, this synchronized error coding, when it happened through  $S_1, S_2$  modulo 2 addition, is corrected through level switch  $S_{T_d}$  that

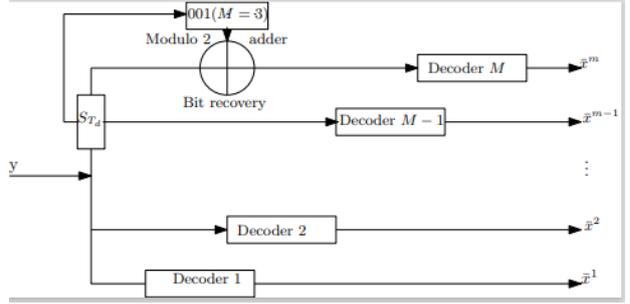


Fig. 4: Multi-stage decoding.

differentiates a same sequence belonging to two different signal points by using time diversity  $T_d$  at the decoder. As an example, sequence 111 is substituted by 110 (or any other valid sequence), with the comparator allowing to diversify through time  $T_d$ , between the original 110 and the substituted 110 sequence. Instead of having a Nyquist filter, the bit adder-comparator outputs are inputted into the Miller encoder that produces accordingly the different coded modulation signals.

At the receiver, a matched filter is used followed by an MSD using the reverse mapping process as shown in Fig. 4. Where the substitution coding has been applied, a modulo 2 addition is used between a pre-selected sequence and the received sequence to produce the original sequence. In this case for example, 001 is pre-selected since 111 was changed into 110.

### 3.3 An M-OOK-OOC Code Line Signals Generation for On-Off Signalling

We exploit Miller's code [26] for the definition of the required signals based on the properties of light and the OOC's correlation properties. These properties impose that any difference between two "1" bits can only be present at most once in an OOC, henceforth, some sequences defining a symbol can only be allowed once. Furthermore, each code at level  $i$  is represented by a signal point  $s_i$ , defining therefore the mapping of the signal to the code vector  $X = x^{l-1} \dots x^1 x^0$ . The signals outputted by Miller encoder are generated as follows.

An incoming binary sequence  $\{b_k\}$  of length  $K$ , is partitioned into  $l$  sequences  $\{a_k\}$  of length  $k_i$  each, such that  $K = \sum_i k_i$ , with the aim to produce at each level  $i$  an  $(n_i, w, 1)$ -OOC sequence of length  $n_i$ , weight  $w$  and correlation index 1 from  $l_i$  parallel encoders. Note that the minimum code length of such a sequence is given by  $n_i = w(w - 1) + 1$ . Each OOC sequence with a code rate  $R_i = \frac{n_i}{K}$ , is fed into the Miller encoder, where the mapping that differentiates signals in time takes place. At the Miller encoder output, the obtained signal point  $s_i$  represents the code  $x^{l-1} \dots x^1 x^0$  meant for channel  $i$  with capacity  $C_i = C$  and the code rate  $R_i = \frac{n_i}{K} = C$  at that particular time  $t$ . Taking into account the Miller code with the light properties, the  $M = 8$  pulses signals

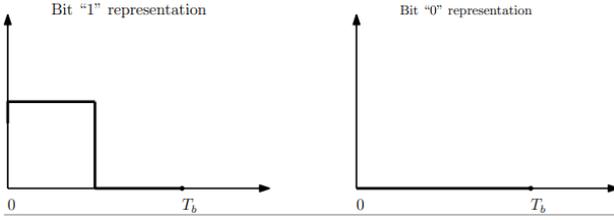


Fig. 5: Signal graphical representation.

Table 1: Sequence, symbols and their signal representations.

Sequence	Symbol	$g(\sigma_k)$	Signal point
111/0	$\sigma_7$	$A$	$A(p(t - T_d) + p(t - p(T_b + T_d)))$
110	$\sigma_6$	$A$	$A(p(t) + p(t - T_b))$
101	$\sigma_5$	$A$	$A(p(t) + p(t - 2T_b))$
100	$\sigma_4$	$A$	$A(p(t))$
011	$\sigma_3$	$A$	$A(p(t - T_b) + p(t - 2T_b))$
010	$\sigma_2$	$A$	$A(p(t - 2T_b))$
001	$\sigma_1$	$A$	$A(p(t - 2T_b))$
000	$\sigma_0$	$A$	0

are derived from Eq. (3):

$$p(t) = \text{rect}\left(\frac{t}{T_b}\right) = \begin{cases} 1, & 0 < t < \frac{T_b}{2} \\ 0, & \text{elsewhere} \end{cases}, \quad (3)$$

which corresponds graphically to the representations in Fig. 5. Based on Fig. 3, one can define the symbols  $\sigma_k$ ,  $k = 0, 1, 2, \dots$  of the codeword  $x_k = x_k^{l-1} \dots x_k^1 x_k^0$ , associated with signal  $s_k(t) = \sum_k g(\sigma_k) p(t - kT_b)$ ,  $g(\sigma_k)$ , and  $T_b$  the bit duration, as given in Table 1. Note that  $g(\sigma_k)$  defines the amplitude which might be variable if intensity detection is used. In our case, we keep it constant,  $g(\sigma_k) = A$ .

The mapping of the signals in Table 1 is done in accordance to the input sequence. As such, a signal 111 is replaced by 110 (hence 111|0 notation), which is differentiated with the original signal assigned to 110 by the time diversity  $T_d$ . This time diversity is introduced during the mapping operation through a switch  $S_{T_d}$ , which triggers the same operation on its counterpart at the decoder side. This operation is an addition of an incoming sequence with a selected sequence for recovery purposes. An example of the sequence is given in Fig. 5 for both 4-OOK and 8-OOK.

### 3.4 Channel Model

The pulses are then transmitted through a channel in compliance with a OOC code that exhibits Z-channel properties. A Z-channel is a channel where the transition  $0 \rightarrow 0$  is always with probability 1, while  $0 \rightarrow 1$  happens with probability 0, see Fig. 7.

We consider that the transmission of the symbol  $x_i$  is entirely dependent on the lower-level symbol  $x_j$ ,  $j < i$ , hence a standard characterization of the channel. In such a case, we consider the channel noise as Gaussian for

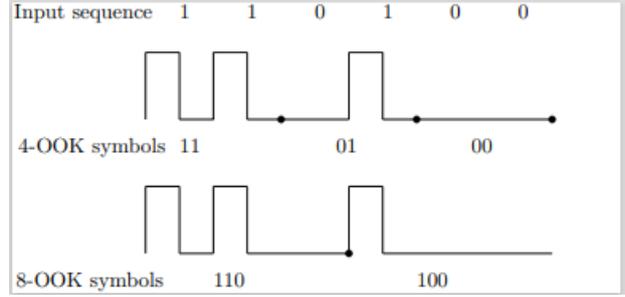


Fig. 6: Binary sequence representations ( $A = 1$ ).

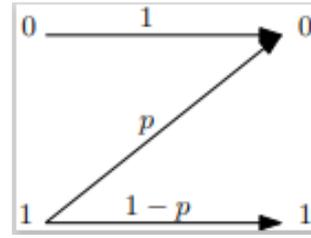


Fig. 7: Z-channel.

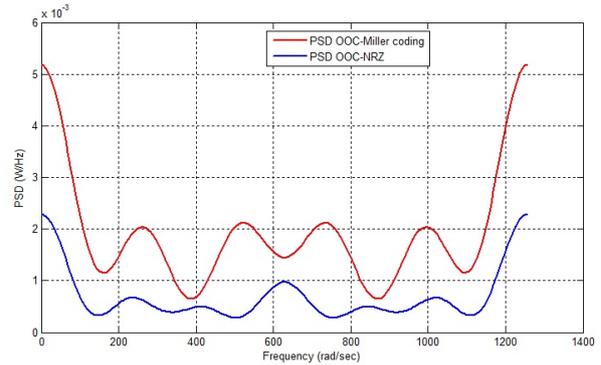


Fig. 8: PSD 8-OOK-Miller vs. 8-OOK-NRZ.

a channel that is a fading multi-path channel, because the Rayleigh distribution can be approximated for a large number as geometric distribution, which in turn can be approximated as a Gaussian distribution, characterizing therefore the transition probabilities. Furthermore, and for simplicity, we only consider in this analysis non-colored signals, the noise can be taken as an additive white Gaussian noise. That is:

$$P(y_i | x_j) = p = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-y_i)^2}{2\sigma^2}} \quad (4)$$

Furthermore, with the characterization of the Z-channel given in Fig. 7, and the channel model as prescribed by Eq. (4), at every unit of time, the channel, accepts a symbol from an alphabet  $\Sigma$ , and outputs a symbol from an alphabet  $\bar{\Sigma}$  with probability  $p$ . Therefore, the channel is described by the following probability

transitions matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ p & 1-p & 0 & 0 & 0 & 0 & 0 \\ p & 0 & 1-p & 0 & 0 & 0 & 0 \\ p^2 & p(1-p) & p(1-p) & (1-p)^2 & 0 & 0 & 0 \\ p & 0 & 0 & 0 & 1-p & 0 & 0 \\ p^2 & p(1-p) & 0 & 0 & p(1-p) & (1-p)^2 & 0 \\ p^2 & 0 & p(1-p) & 0 & p(1-p) & 0 & (1-p)^2 \end{bmatrix} \quad (5)$$

which corresponds to the codes or sequences  $\{000, 001, 010, 011, 100, 101, 110\}$  assigned to the symbols  $\Sigma = \{\sigma_0, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6(t)\}$  mapped into signal points  $s_0, s_1, s_2, s_3, s_4, s_5, s_6$  or  $s_7$ .

However, we do not control the input data bits to avoid 111 as inputs, since with the OOC, a difference between any two weighted positions can only be present at most once as the correlation properties dictate, the sequence 111 is not valid. To bypass this situation, we choose a partial code substitution solution. In that way, the most significant "1" bit from the input 111 sequence (which is inverted at the encoder output) is substituted by a bit "0" through a modulo-2 addition in our encoder, while at the decoder, a modulo 2 addition with 001 selected sequence is applied to retrieve the initial code vector once time diversity is detected. Typically,

$$\sigma_6(t) = \begin{cases} s_6, & t = 0. \\ s_7, & t = T_d \end{cases} \quad (6)$$

Furthermore, in an OOK light signaling, a "0" bit is equivalent to "no signal" transmission, which means, 110 OOK transmission is equivalent to 11 transmission, channel capacity wise. It is therefore convenient to consider the  $111 \rightarrow 110$  as a partial bit erasure channel, meaning that the channel capacity and the bit error rate can be considered as the ones produced by 110 on which a substitution error probability is added. Therefore, the signal points  $s_6 = s_7$  are differentiated only through time diversity ( $T_b + T_d$ ) and the switch  $s_{T_d}$ .

## 4. PERFORMANCE EVALUATION

### 4.1 Coding Complexity Evaluation

In this complexity evaluation, we consider two techniques, the conventional UG's coded modulation and our approach, both making use of an OOC without redundant bits.

With the UG's approach, one expects  $(l-1)$  differences and  $(l-1)$  comparisons (each difference compared with the previous ones) from the  $l$ -bits partitioned, followed by the  $l$  modulations, yielding therefore  $(l-1)^2 + 2l$  operations.

In our approach instead, the  $l$  partitioning are exclusively followed with  $(l-1)$  additions with another  $l$  addition for the substitution, and then  $l$  multiplexing with  $l$  comparisons, ending with  $l$  modulations, yielding therefore  $6l - 1$  operations.

Therefore, to produce the coded signals, our approach requires  $O(l)$  operations comparatively to the conventional UG that necessitates  $O(l^2)$  operations.

Note that during the decoding stage, the decorrelation uses a diagonal matrix which normally yields  $O(l)$  decoding operations. However, because of the time diversity recovery, our approach will require  $l + 1$  more operations prior to the decorrelation.

### 4.2 Channel Capacity

Since the mapping  $X \rightarrow Y$  is bijective, the mutual information  $I(Y, X)$  where  $Y$  is the received set, is obtained using the chain rule. Applying then this chain rule, the mutual information at level  $i$  is given as follows [12]:

$$I(Y, X) = I(Y; X^i | X^0, X^1, \dots, X^{i-1}) = I(Y; X^0) + I(Y; X^1 | X^0) + \dots + I(Y; X^{i-1} | X^0, X^1, \dots, X^{i-2}) \quad (7)$$

and the subsequent system capacity is given by:

$$C = \max_{(p, x_i)} I(Y, X)$$

with all the  $(Y; X^i | X^0, \dots, Y | X^0 X^1 \dots X^{i-1})$  being the equivalent channels.

It means for instance that  $I(01, 01) = I(01, 0) + I(01, 1 | 0)$ , considering that 01 is received when 01 was sent.

The capacity design rule suggests that the channel capacity is the sum of the equivalent channel capacities. That is, for an equivalent channel  $i$  with data rate  $R_i = C_i$ , the total coded modulation channel capacity is given by:

$$C = \sum_i C_i$$

This capacity is chosen and distributed accordingly to the different equivalent channel capacities, with the notation given as:  $C_1/C_2/\dots/C_i$  [6].

This channel capacity is computed using capacity rule in a system designed according to the balance distance rule. According to [30], [32], [33], the balance distance rule requirement is such that for any constellation Euclidean distance  $d$  and the related minimum Hamming distance  $d_{min}$  the particular equivalent channel  $i$ , the quantity  $(d^2 d_{min})$  such that:

$$(d^2 d_{min})_i \leq (d^2 d_{min})_j \leq \dots \leq (d^2 d_{min})_k, \quad i > j > k.$$

Since the coded modulation design is done under this condition, we only consider the capacity rule as defined above, to determine the channel capacity knowing that there is no change in the channel capacity, due to the substitution of "1" with "0", considered as erasure channel.

According to the capacity rule of Section 3.2, the transmission rate represents the sum of the individual channels known as equivalent channels, each with its own rate. In our model, the channel capacity is directly obtained at the level at which the most significant bit protection is achieved. However, we will use the term

**Table 2:** Capacity bits/s symbol ASK-OOK comparison.

Channel	Level	8-OOK	8-ASK
Equivalent 0	1	0.5.	0.5
Equivalent 1	2	1.729	1
Equivalent 2	3	2.573	1

equivalent channel for comparison purposes between the OOK and ASK channel schemes. The relevant rates obtained by exploiting the capacity design rule are given in Table 2.

According to the rate and capacity rules, the capacity of an 8-OOK is distributed as follows:  $C_0/C_1/C_2 = 0.5/1.729/2.573$ , compared to  $0.5/1/1$  for 8-ASK. Since with UG the total capacity is the sum of all capacities of the equivalent channels, while in our model the protection of the most significant bit makes it the actual capacity, it turns out that our model yields a 2.573 bits/symbol compared to 2.5 bits/symbol for the UG scheme for  $M = 8$ . In the case of 4-OOK, an 4-ASK UG allows 1.5 bits/symbol, while our scheme yields 1.729 bits/symbol.

### 4.3 Power Spectral Density

The power spectrum of these signals applied to an (7,3,1)-OOC for an 4-ary sequence are given by:

$$\begin{aligned}
 PSD_{NRZ} = & \frac{1}{T} \left[ \frac{3}{2} + \frac{5}{7} \cos(\omega T) + \frac{5}{6} \cos(2\omega T) \right. \\
 & + \frac{2}{5} \cos(3\omega T) + \frac{3}{4} \cos(4\omega T) + \frac{1}{3} \cos(5\omega T) \\
 & \left. + \frac{1}{2} \cos(6\omega T) \right] (T \sin c(\omega T/2))^2, \quad (8)
 \end{aligned}$$

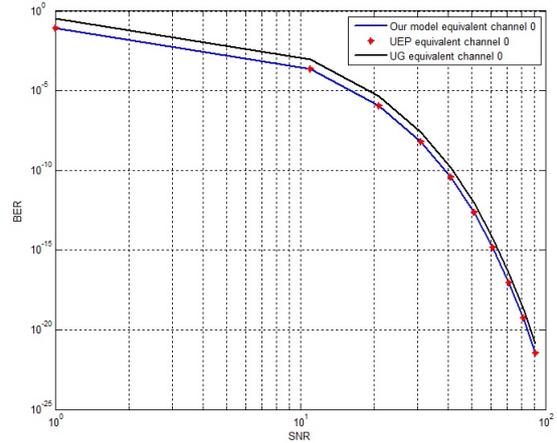
and for Miller sequence, one gets:

$$\begin{aligned}
 PSD_{Miller} = & \frac{2}{T} \left[ \frac{9}{7} + \frac{1}{2} \cos(\omega T) + \frac{3}{5} \cos(2\omega T) \right. \\
 & + \frac{1}{4} \cos(3\omega T) + \frac{1}{3} \cos(4\omega T) + \frac{1}{2} \cos(5\omega T) \left. \right] \\
 & \times \left( \frac{T}{2} (\sin c(\omega T/2))^2 \right). \quad (9)
 \end{aligned}$$

The power spectrum analysis of OOC sequence for both NRZ (8) and Miller (9) exhibiting the power distribution over the frequency is given in Fig. 8. The signal shows already that a non flat-like PSD can be achieved whether we make use of a worse scenario, which is NRZ, as longer the coded modulation, from the coding phase, up to the modulation phase, are controlled.

### 4.4 Bit Error Rate

In analysing the error performance of the  $i$ -th level, we assume that an all-zero sequence is transmitted at each level  $i$ , which means that all signals are equally likely to be transmitted with a probability of  $\frac{1}{2^i}$ . As for all-one sequence, when  $i > 2$ , a substitution code

**Fig. 9:** Equivalent channel 0 BER OOK-OOC vs ASK UG vs ASK UEP.

applies. Therefore, for an 8-OOK, an adjustment is required through the correlative coding in reducing the sequence 111 into a 110 sequence. It is important to note that one could improve the performance in terms of spectral efficiency by substituting 111 with 101 instead. However, we would like to focus on the worst-case scenario.

According to the  $Z$ -channel's characteristic, an error can occur whenever a non "all-zero" codeword is decoded, and as transition matrix (5) shows, only the "1" bits are of concern.

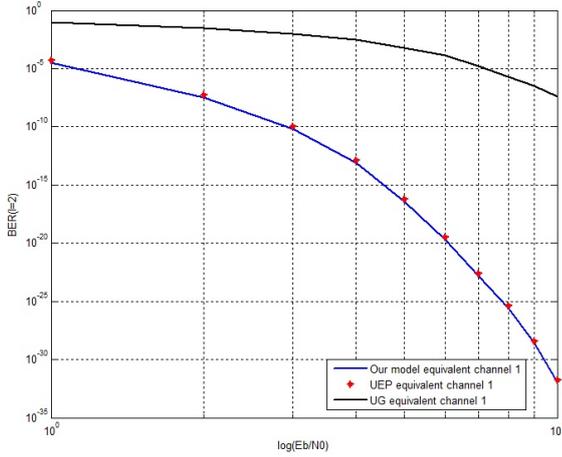
Assume  $d_i$  the Euclidean distance between two codewords  $X_1$  and  $X_2$  in the partitioning at level  $i$ , let  $w$  be the Hamming weight of the resulting codeword of length  $n$ , the decision region between both codewords is the plane characterized by  $\frac{X_1 + X_2}{2} = 0$ . A decoding error occurs whenever  $\frac{X_1 + X_2}{2} \neq 0$ , with probability at level  $i$  in accordance with [34] is given by:

$$Pr \left[ \frac{X_1 + X_2}{2} \neq 0 \right] = \binom{w}{i} \left( \frac{1}{2^i} \right)^w Q \left( \sqrt{\frac{2RE_b}{N_0}} d_i \right) \quad (10)$$

where  $Q(x) = \frac{1}{2\pi} \int_x^\infty e^{-\frac{u^2}{2}} du$ ,  $\frac{E_0}{N_0}$  is the energy-per-bit-to-noise ratio. Please note that at each level  $i$  where the substitution code takes place in a codeword, to the error probability above is added the probability of the resulting  $(i - 1)$  level codeword which is multiplied by the probability of occurrence of the "1" that have been substituted ( $Pr[101] \neq Pr[110]$  for instance).

Assume furthermore that there are  $|C_w|$  codewords of length  $n$  and Hamming weight  $w$  and knowing that a non-zero chip appears with probability  $\frac{w}{n}$ , then for the whole level known in [21] as block partitioning, the level  $i$  error probability is bounded by:

$$P_i = \sum_{w=d_i}^n |C_w| \frac{w}{n} Pr \left[ \frac{X_1 + X_2}{2} \neq 0 \right]$$



**Fig. 10:** Equivalent channel 1 BER OOK-OOC vs ASK UG vs ASK UEP.

$$\leq \sum_{w=d_i}^n |C_w| \frac{w}{n} \sum_{i=0}^w \binom{w}{i} \left(\frac{1}{2}\right)^w Q\left(\sqrt{\frac{2RE_b}{N_0} d_i^2}\right) \quad (11)$$

The probability of level 3 is provided in the following figures, in comparison to ASK from UEP and UG. Note that UEP's error probability of level  $i$  is bounded by [21]:

$$P_{UEP} = \sum_{w=d_i}^n |C_w| \frac{w}{n} \sum_{i=0}^w \binom{w}{i} \left(\frac{1}{2}\right)^w Q\left(\sqrt{\frac{2RE_b}{N_0} d_i^2}\right) \quad (12)$$

while UG provide us with the following error probability [21]. Note that we will only present level 3 for 8-ASK demonstrated by [21] in accordance with [20] and compare it with our 8-OOK.

$$P_{UG} = \sum_{w=d}^n |C_w| \frac{w}{n} \sum_{i=0}^w \binom{w}{i} (2)^w Q\left(\sqrt{\frac{2RE_b}{N_0} d_i^2}\right) \quad (13)$$

Figs. 9, 10, and 11 give the results of the bit error rate performance in comparison with two other methods that inspire this paper, namely UG and UEP.

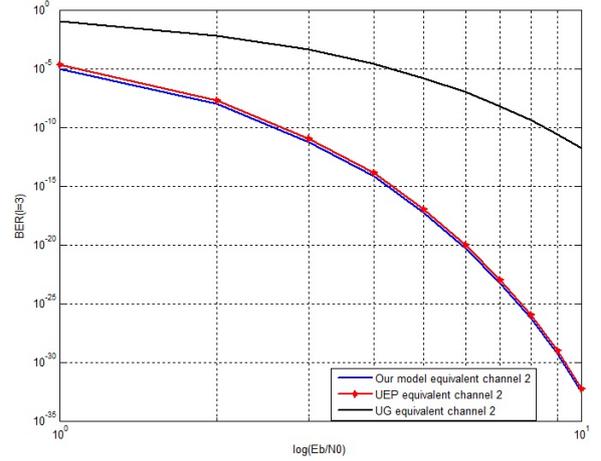
#### 4.5 Application: Outdoor VLC Channel

The outdoor VLC system of our interest keeps its signal spectrum unchanged over the entire signal bandwidth, which is a condition of *Line of sight* (LoS) communication, for simplicity. In that condition, the received power  $P_r$  is given by [35]:

$$P_r = P_t H(0),$$

where  $P_t$  is the transmitted power,  $H(0)$  is the DC channel gain, which is consequently flat due to the condition of pure LoS above.

Therefore,  $H(0)$  characterizes the entire medium, which characteristics can be enumerated as follows [36], [37].



**Fig. 11:** Equivalent channel 2 BER OOK-OOC vs ASK UG vs ASK UEP.

-  $H(0)$  is inversely proportional to the square of the distance  $d$  between the transmitter and the receiver.

-  $H(0)$  takes into consideration the distribution of the power per unit solid angle, defined through  $\theta$ . Note that  $\theta$  is the angle defined from positive to the negative axis perpendicular to the propagation plane, which with the azimuth in that plane  $\phi$  yield a function  $R(\theta, \phi)$  we will elucidate later.

-  $H(0)$  is proportional to the optical gain of the photodiode, which depends on the field-of-view angle  $\Phi_c$ ,  $g(\Phi_c)$ .

-  $H(0)$  is proportional to the surface of the photodiode  $A_p$ . Therefore, in a pure LoS VLC communication system we have:

$$H(0) = \frac{R(\theta, \phi) g(\Phi_c) A_p}{d^2}$$

Please note, it can be an inclination  $\Phi_i$  between the transmitter and the receiver, in that case, multiply this expression by  $\cos\Phi_i$ , that is:  $H(0)\cos\Phi_i$ . In this case we choose  $\Phi_i = 0$ .

To evaluate  $R(\theta, \phi)$ , we lean on Gfeller's assumption that the luminescence of a single LED can be considered the same in all directions, a condition evoking Lambertian radiators [38]. This yields consequently an uni-axial symmetry, leading to  $R(\theta, \phi) = R(\theta) = \frac{m+1}{2\pi} \cos^m \theta_n$ , with  $\theta_n \in [0, \pi]$ , which we consider in this paper as  $\theta = 60^\circ$ . According to Lambertian radiator,  $m = \frac{\ln 2}{\ln(\cos \theta_n)}$  where  $\theta_n$ , known as half power beamwidth, is the angle between the maximum power axis and the points at half power. In the VLC channel investigation, we consider that the received signal is the equal to the sensitivity of the photodiode detector, which can be obtained using the relation given by [38]:

$$S_i = K_T T_a B F \frac{E_b}{N_0}$$

where  $K_T = 1.38 \times 10^{-23} JK^{-1}$  is the Boltzmann constant,  $B = 463\text{MHz}$  is the bandwidth taken from

**Table 3:** Minimum effective distance.

SNR	1.5	1.6	1.7	1.8	1.9	2
d (m) 25°C	2168	1862	1584	1340	1185	1019
d (m) 30°C	2158	1852	1574	1330	1175	1009

the LED emitter's frequency modulation,  $T_a$  is the photodiode operating temperature  $T_a = [-40, 85]^\circ\text{C}$ ,  $F$  is the noise factor,  $F = 2$ . The result of this investigation provides with the distance as a function of SNR, which is chosen in the range of  $10^{-6}$  to  $10^{-8}$  of the BER value, for 25 and 30°C photodiode temperature, are given in Table 3.

#### 4.6 Results Analysis

From the obtained results, Fig. 8, it is shown that it is possible to improve the flat-like optical spectrum for outdoor visible light communications to a better level whether using OOK or ASK. It is also possible to improve the data rate by improving the transmission capacity as shown in the results in Table 2. With the protection of the most significant bit, the transmission at level  $i$  yields the rate of the entire system if limited at that level, such that the transmission rate at level 3 is 2.573 bits/symbol, getting closer to Shannon rate, than the UG technique at 2.5 bits/symbol. Not only about rate improvement, one can also observe, BER wise that protecting the most significant bit controlled better the noise, and consequently increases the SNR, which in turns reduces the BER as shown in Figs. 8, 9, and 10 respectively. This is the required characteristic for outdoors VLC-ITS transmission, as it requires only 1, 1 of SNR in a log-scale to achieve about  $10^{-7}$  bit error rate. As a result, our model is providing us with a gap of 0.29 in terms of efficiency compared to the other UEP method which inspired this paper, and less than  $\sim 10^6$  for UG scheme. Note that the gap is a function of the permissible probability of error and the encoding system of interest. The difference comes from the fact that the OOC used herein provides us with a sequence management tool that allows to adapt the sequence to the requirements of the channel.

Also, we recall that MLD associated with MSD can tend towards the Shannon rate if the rate is chosen appropriately. Though we did not explore that solution, it is again obvious analytically that in doing so, we will still achieve a better performance, SNR and consequently BER wise than the previous techniques.

When investigating the minimum distance of photodiode sensitivity, one can realize that a minimum distance of 2 km can be achieved at an SNR of 1.5 in ratio. Beyond that distance, power increase is needed, which means, the overheating noise increases and the diode is likely to reach its breakdown, which is materializes by the decrease of distance as the power increases.

## 5. CONCLUSION

In this paper, we showed that it is possible to transmit information at a better data rate, or at a better transmission rate obtained through a better SNR. We also showed that it is possible to improve a flat like PSD level of a wireless optical communication such as outdoors VLC, using M-ary coded modulation. The improvement is achieved by choosing a code that provides us with a sequence pattern that fits the constraints of the channel and controls the modulation. With only 1.1 dB of SNR and a rate getting closer to 3 bits/transmission, one can transmit and receive the so-called low rate from low SNR communication systems like ITS, VLC, and I2V at a minimum distance of 2000 m in ambient light. However, investigating the system in a non-LoS scenario or in the presence of smoke/fog is left for further studies.

## ACKNOWLEDGMENT

The first author would like to express his gratitude of thanks to the Centre of Telecommunications, as well as the Department of Electrical and Electronic Science in the Faculty of Engineering and the Built Environment at the University of Johannesburg.

## REFERENCES

- [1] Y. H. Kim and Y. H. Chung, "Experimental Outdoor Visible Light Data Communication Systems with Optical Filter," *Journal of the Korea Institute of Information and Communication Engineering*, vol. 18, no. 8, pp. 1840-1846, 2014.
- [2] Y. H. Chung and S. Oh, "Efficient optical filtering for outdoor visible light communications in the presence of sunlight or artificial light," *2013 International Symposium on Intelligent Signal Processing and Communication Systems*, Nov. 2013.
- [3] A. -M. Cailean, B. Cagneau, L. Chassagne, S. Topsu, Y. Alayli and M. Dimian, "Design and implementation of a visible light communications system for vehicle applications," *2013 21st Telecommunications Forum Telfor (TELFOR)*, Belgrade, Serbia, 2013, pp. 349-352.
- [4] T. Nagura, T. Yamazato, M. Katayama, T. Yendo, T. Fujii and H. Okada, "Improved Decoding Methods of Visible Light Communication System for ITS Using LED Array and High-Speed Camera," *2010 IEEE 71st Vehicular Technology Conference*, Taipei, Taiwan, 2010, pp. 1-5.
- [5] "IEEE Standard for Local and Metropolitan Area Networks-Part 15.7: Short-Range Wireless Optical Communication Using Visible Light," in *IEEE Std 802.15.7-2011*, pp.1-309, 6 Sept. 2011.
- [6] A. M Cailean, B. Cagneau, L. Chassagne, M. Dimian and V. Popa, "Miller code usage in VLC under the PHY I layer of the IEEE 802.15.7 standard," *2014 10th International Conference on Communications (COMM)*, May 2014.
- [7] G. Ge and J. Yin, "Construction of optimal (v,4,1)

- OOC," *IEEE Trans. Inform. Theory*, vol. 14, pp. 256-266, 2001.
- [8] M. C. Davey and D. J. C. MacKay, "Reliable communication over channels with insertions, deletions, and substitutions," *IEEE Trans. Inf. Theory*, vol. 47, no. 2, pp. 687-698, Feb. 2001.
- [9] A. Y. Hassan, "Code-Time Diversity for Direct Sequence Spread Spectrum Systems," *The Scientific World Journal, Special Issue: Recent Advances in Communications and Networking*: <https://doi.org/10.1155/2014/146186>, 2014.
- [10] F. R. K. Chung, J. A. Salehi and V. K. Wei, "Optical orthogonal codes: Design, analysis and applications," *IEEE Trans. Inform. Theory*, vol. 35, pp 595-604, May 1989.
- [11] A. J. Goldsmith and P. P. Varaiya, "Capacity of mutual information and coding for finite state Markov chain channel," *IEEE Trans. Inform. Theory*, vol. 42, no. 2, May 1996.
- [12] U. Wachsmann, R. F. H. Fischer, and J. B. Huber, "Multilevel Codes: Theoretical Concepts and Practical Design Rules", *IEEE Trans. Info. Theory*, vol. 45, pp. 1361-1391, no. 5, July 1999.
- [13] R. H. Morelos-Zaragoza, M. P. C. Fossorier, S. Lin, and H. Imai, "Multilevel Coded Modulation for Unequal Error Protection and Multistage Decoding—Part I: Symmetric Constellations", *IEEE Trans. Comm.*, vol. 48, no 2, pp. 204-213, Feb. 2000.
- [14] M. Mohaisen, "Increasing the minimum Euclidean distance of the complex quadrature spatial modulation," *IET*, <https://doi.org/10.1049/iet-com.2017.1198>, March 2018.
- [15] L. Gordon Stuber, and E. katz, "Systematic Trellis-coded Modulation with Asymmetric Constellations," in *Proceedings of GLOBECOM 1995 Mini*, 12-17 November 1995.
- [16] D. Divsalar, M. Simon, and J. Yuen, "Trellis coding with asymmetric modulations," *IEEE Trans. Commun.*, Vol. COM-35, pp. 130-141, February 1987.
- [17] J. Zang, "Fast Receive Antenna Subset Selection for Pre-Coding Aided Spatial Modulation," *IEEE Wirel. Comm. Letters*, vol 4, no 3, pp. 317-320, 2015.
- [18] S. Lee, M. Mohaisen, "Gram-aschmidt Orthogonalization-Based Antenna Selection for Pre-Coding Aided Spatial Modulation," *J. Telecom., Electron., Comput. Eng.*, vol. 8, no. 9, pp. 83-88, 2016.
- [19] P. Yang, Y. Xiao, L. Li, et al, "Link adaptation for Spatial Modulation with Limited Feedback," *IEEE Trans. Veh. Techno.*, vol. 61, no 8, pp. 3808-3813, 2012.
- [20] R. Rajaskekar, K. Hari, L. Hanzo, "Antenna Selection in Spatial Modulation Systems," *IEEE Comm. Letters*, vol. 17, no. 3, pp. 521-524, 2013.
- [21] N. Pillay, H. J. Xu, "Comments on Antenna Selection in Spatial Modulation Systems," *IEEE Comm. Letters*, vol. 17, no. 9, pp. 1681-1683, 2013.
- [22] Ntonlink, M. Di Renzo, A. Perez-Neira, et al, "A Low Complexity Method for Antenna Selection in Spatial Modulation Systems," *IEEE Comm. Letters*, vol. 17, no. 12, pp. 2312-2315, 2013.
- [23] N. Wang, W. Liu, H. Mar, et al, "Further Complexity Reduction Using Rotational Symmetry for EDAS in Spatial Modulation," *IEEE Comm. Letters*, vol. 18, no. 10, pp. 1835-1838, 2014.
- [24] R. Rajaskekar, K. Hari, L. Hanzo, "Quantifying the Transmit Diversity Order of Euclidean Distance Based Antenna Selection Spatial Modulation," *IEEE Sign. Proces. Letters*, vol. 22, no. 9, pp. 1434-1437, 2015.
- [25] R. Omnari and V. Kuman, "Code for optical CDMA," *SETA LNCS4086*, pp. 3446, 2006.
- [26] E. Forestieri and G. Prati, "Novel optical line codes tolerant to fiber chromatic dispersion," *J. Lightwave Tech.*, vol. 19, no. 11, Nov. 2001.
- [27] S. Haykin, *Baseband Pulse Transmission in Communication Systems*, 4th Edition, John Wiley and Sons, 2001.
- [28] R. Gharat and S. S. Thorat, "Miller encoder for outdoor MIMO VLC application," *Int. J. of Sci. techno. And Eng. (IJSTE)*, vol. 3, no. 11, May 2017.
- [29] A. Lender, *Correlative (partial response) techniques and applications to digital radio systems in Digital Microwave Application*, ed. C. Feller Englewood Cliffs: Prentice Hall, Sec. 7, pp. 144-182, 1981.
- [30] G. J. Pottie and D. P. Taylor, "Multilevel codes based on partitioning," *IEEE Trans. Inform. Theory*, vol. 35, pp. 87-98, Jan. 1989.
- [31] H. Imai and S. Hirakawa, "A new multilevel coding method using error correcting codes," *IEEE Trans. Inform. Theory*, vol. 23, pp. 371-377, May 1977.
- [32] V. V. Ginzburg, "Multidimensional Signals for a Continuous Channel" *Problems Inform. Transmission*, vol. 20, no. 1, pp. 20-34, 1984.
- [33] S. I. Sayegh, "A class of optimum block codes in signal space," *IEEE Trans. Commun.*, vol. 34, pp. 1043-1045, Oct. 1986.
- [34] M. P. C. Fossorier, S. Lin, and D. Rhee, "Bit error probability for maximum likelihood decoding of linear block codes and related soft-decision decoding methods," *IEEE Trans. Inform. Theory*, vol. 44, pp. 3083-3090, Nov. 1998.
- [35] V. Georlette, S. Bette, S. Brohez, R. Pérez-Jiménez, N. Point, and V. Moeyaert, "Outdoor Visible Light Communication Channel Modeling under Smoke Conditions and Analogy with Fog Conditions," *Optics*, vol. 1, no. 3, pp. 259-281, Nov. 2020.
- [36] Z. Ghassemlooy, L. N. Alves, S. Zvanovec, and M. A. Khalighi, *Visible light communications: theory and applications*, CRC press, 2017.
- [37] Z. Ghassemlooy, W. Popoola, and S. Rajbhandari, *Optical wireless communications: system and channel modelling with Matlab®*, CRC press, 2019.
- [38] L. Dennis, *Receiver Sensitivity and Equivalent Noise Bandwidth*, Available: <http://> High Frequency Electronics, 2014.



**Samuel Nlend** received the B.Sc. (Physics) degree in 1994, the M.Phil. (Electrical and Electronic) degree in 2013 and PhD (DPhil) in 2021. He is currently assistant researcher in the Department of Electrical and Electronic Engineering Science at the University of Johannesburg, South Africa. He worked from 1996 to 2010 respectively as the Regional Head of Department of Statistics and QoS and of Technical Department at Cameroon Telecommunications, Cameroon, and as a

Technical Engineer at Phonelines International, South Africa. Dr. Nlend is a member of the Engineering Council of South Africa since 2017 and a student member of IEEE since 2019.



**Theo G. Swart** received the B.Eng. and M.Eng. degrees (both cum laude) in electrical and electronic engineering from the Rand Afrikaans University, South Africa, in 1999 and 2001, respectively, and the D. Eng. degree from the University of Johannesburg, South Africa, in 2006. He is an associate professor in the Department of Electrical and Electronic Engineering Science and the director of the UJ Centre for Telecommunications. His research interests include digital communi-

cations, error-correction coding, constrained coding and power-line communications. Dr. Swart is a senior member of the IEEE, and was previously the chair of the IEEE South Africa Chapter on Information Theory. He is a former specialist editor for the SAIEE Africa Research Journal.