Multiple Access Interference Bit Error Rate Evaluation Technique for Direct Detection FFH-OCDMA Bragg Gratings Based Channels

Samuel Nlend and **Theo G. Swart**[†], Non-members

ABSTRACT

This paper proposes a more realistic optical code division multiple access (OCDMA) sequence bit error rate (BER) evaluation technique for multiple access interference (MAI) during transmission over optical fiber. The technique provides a filter for all possible non-zero correlations events that may occur during transmission, leaning on all positive time-frequency intervals for a 2-D OCDMA, or all positive time intervals for 1-D OCDMA. As a wavelength hopping/time sequence (WH/TS) suggests, this MAI evaluation on a 2-D OCDMA consists of one-coincidence frequency hopping code optical orthogonal code (OCFHC-OOC) Bragg gratings encoded signals. For a better assessment of the time-intervals, we also investigate the 1-D OCDMA OOC since the OOC is the time spreading component of the OCFHC-OOC sequence. In both cases, the signals of interest are transmitted using simple On-Off keying with non-return-to-zero signalling and direct detection at the receiver. The received signal contains multiple access interference from other users' coincidence correlations and the in-band random correlations from the photodiode due to square detection. The MAI mean and variance are analysed over all possible non-zero wavelength-time interval pairs of sequences and the standard Gaussian approximation is used to evaluate the bit error rate. Further, the resulting bit error rate is then compared with that of the user coincidence-based evaluation technique.

Keywords: Asymmetric Error Correcting Code, Bragg Gratings, Fast Frequency Hopping, Optical Code Division Multiple Access, Optical Orthogonal Code

1. INTRODUCTION

In several cases, bit error rate (BER) evaluation of optical code division multiple access (OCDMA) systems is performed using user coincidence-based correlations for multiple access interference (MAI) from other users [1], [2]. The number of coincidences is the number of hits experienced by a targeted bit in the superposition scenario of a particular sequence [1]. This approach is applied when evaluating the MAI BER in 1-D OCDMA optical orthogonal codes (OOC) [2] and in 2-D OCDMA one coincidence frequency hopping code-OOC (OCFHC-OOC) [3] sequences. However, applying this analytical approach where the square detection is performed on the signals from the photodiode detector and Bragg gratings encoder/decoder is used, raises two issues: the first is the number of coincidences, which leads to the second, the actual system MAI. The user coincidence-based approach either ignores, or assumes negligible, in-band frequencies from the photodiode due to the square detection which cause random correlations. In fact, during the opto-electrical conversion of the user-1 central frequency signal $s_1(t)$, the photodiode generates a signal $s_p(t)$ of the same frequency band, which is added and squared. The output of the squared signal from the low pass filter contains the quantity $2\eta s_1(t)s_p(t-\tau)$ called beat noise [4], [5], [6], with η being the transmittance constant. This implies that there is no delay at all at the time the photodiode generates $s_p(t)$ an approach that needs to be revisited for a better statistical analysis of the BER.

The approach requires therefore a technique that takes into account these in-band correlations by assuming that while the photodiode generates $s_p(t)$, the signal may experience a random delay τ, perhaps small but not neligible, which is uniformly distributed over the signalling time interval. As a consequence, the photodiode-generated noise is expressed as $2\eta s_1(t)s_p(t-\tau)$, which exhibits the correlation properties, has to be added to the user's coincidence-based correlations. Note that when $\tau = 0$, one reverts to the noise scenario in [4], which is called beat noise, since in [7], it is proven that the beat noise is proportional to the coherent time. However, this paper re-evaluates the MAI BER and consequently its signal-to-noise ratio (SNR) when $\tau \neq 0$, since the generation of $s_p(t)$ happens after detection of $s_1(t)$ by the photodiode. The difference might be small but taking it into account is crucial for small signals system analysis, particularly for a system such as OCDMA that never went beyond laboratories.

Further, the OCFHC-OOC user's coincidence-based approach in [3] is developed without the encoder requirements. The OCFHC-OOC is a wavelength hopping-time sequence (WH/TS), and the studies conducted by [18] proposed the Bragg gratings as encoder for such WH/TS

Manuscript received on February 3, 2023; revised on March 3, 2023; accepted on May 31, 2023. This paper was recommended by Associate Editor Piya Kovintavewat.

The authors are with the Centre For Telecommunications, University of Johannesburg, South Africa.

[†]Corresponding author: tgswart@uj.ac.za

Digital Object Identifier: 10.37936/ecti-eec.2023213.251467

sequences. Then using this encoder for the OCFHC as wavelength hopping sequence raises a concern. When a frequency is used in a sequence, it is no longer available. As a consequence, the number of coincidences given in [3] requires a recount.

Furthermore, considering the characteristics of light and the OCFHC-OOC sequence, the coincidences are no longer the coincidences in frequency alone as in [3], but also the coincidences in time. We consider therefore during analysis, the time at which a frequency is considered as coincidence frequency. This brings to the fore the time-intervals at which one observes zero-correlations.

The contribution of this paper is to provide a more realistic statistical tool in evaluating the MAI BER for the struggling OCDMA systems made of Bragg gratings encoder/decoder and direct detector. This is challenging for such OCDMA systems that never went far from the laboratories, but which are becoming compelling with the current high data traffic volume and number of users demands. Therefore, we evaluate the OCDMA system performance using: 1-D OCDMA OOC because of its better performance compared to other good correlation codes, and 2-D OCDMA OCFHC-OOC which shows a better performance than the first [11]. For that purpose, it requires a MAI BER evaluation technique that takes into account the coincidence-based correlations, the photodiode in-band frequencies random correlations, and the zero-correlation time-intervals as imposed by the characteristics of light and the encoder frequency requirements.

To capture the user's coincidence-based correlations, the in-band correlations from the photodiode and the time-intervals with zero-correlation, this paper proposes a technique that analyses all the non-zero signalling time intervals of the aggregated signal. All delay MAI expressions for direct sequence spread spectrum multiple access were studied in [8], [9]. We extend the same MAI BER evaluation that considers all possible non-zero signalling time-intervals for a fast frequency hopping-OCDMA (FFH-OCDMA) channel, using an OCFHC as optical FFH sequence as suggested in [10], [18], which is constructed according to Bin [19], an OOC constructed using the difference of position, and the BER evaluated using Gaussian approximation.

An OCFHC is a one-coincidence frequency hopping code of length L and Q frequencies available denoted (L, Q, H_{max}). This is a $(0, 1)$ sequence, with 0 if no frequency is present, and 1 otherwise, satisfying the Hamming correlation condition $H_{max} = 1$ and optimal cardinality $\left|\frac{Q(Q-1)}{L}\right|$ [11].

An $(n, w, 1)$-OOC is an optical orthogonal code of length n, Hamming weight w which is characterized by its correlation index 1 and optimal cardinality $\frac{n-1}{w(w-1)}$ [12].

When combined into 2-D OCFHC-OOC, the system cardinality is given by:

$$|C| = (L+1)^2 L \frac{n-1}{w(w-1)}.$$

Several techniques of Gaussian approximation have been used to determine the BER [13], [14], [15], and [16]. We determine the mean and variance of the random variable MAI and use standard or direct Gaussian approximations to obtain its BER. 1-D OCDMA OOC is used to investigate the generalization of the technique, leaning on its close properties with the OCFHC [19], while 2-D OCDMA OCFHC-OOC is the system reference. The improvement is evaluated fisrtly: by comparing through MATLAB investigations the theoretical BER results between our proposal and the coincidence-based correlations for 2-D OCDMA OCFHC-OOC and for 1-D OCDMA OOC BERs as evaluated in [1], [2] and [3]; secondly by comparing through MATLAB simulations, these theoretical BER results with the correlations outputted by the system detection Fig. 4.

This paper is organized as follows: we start in Section 2 with a brief description of the MAI in OCDMA sequences. In Section 3, we evaluate the MAI over all time-intervals using the system mode shown in Fig. 3. In Section 4, we present the performance of the approach, and a comparison is made with reported literature of OCDMA MAI.

2. BACKGROUND

In OCDMA networks, all users can send and receive signals asynchronously. We consider an OCFHC-OOC sequence consisting of Q frequencies and N time intervals from the i-th user expressed as

$$s_i(t) = \sum_{q=0}^{Q-1} \sum_{n=0}^{N-1} C_i(w_q, n) p(t - nT_c) \tag{1}$$

where $C_i(w_q, n)$ is the coded sequence and $p(t - nT_c)$ is the pulse shape of duration $T_c = \frac{T_d}{N}$ with T_d the data duration.

The received signal from K users is then

$$r(t) = \sum_{k=1}^{K} b_k(t - \tau_k) s_k(t - \tau_k), \tag{2}$$

with $b_k(t)$ being the user k transmitted information symbols and τ_k the delay experienced. The adaptive detector's filter picks up the i-th user signal and the photodiode generated in-band frequencies delayed by τ_k as well as the signals of the other $K-1$ users to output

$$y_i(t) = \int_0^{T_c} s_i(t) r(t) dt$$

$$= \int_0^{T_c} \sum_{k=1}^{K} b_k(t - \tau_k) s_i(t) s_k(t - \tau_k) dt.$$

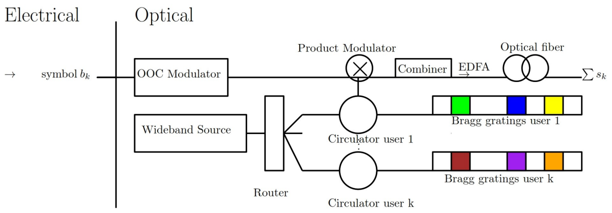


Fig. 1: *Electrical-optical conversion.*

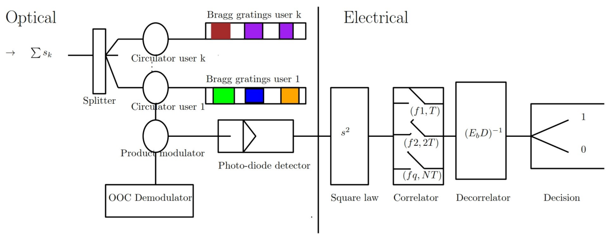


Fig. 2: *Optical-electrical conversion.*

It is important to mention that the different channels can induce different attenuation, resulting in different powers being received. However, throughout this paper, an assumption is made that the received power is the same for all. The square detector adds in-band signals with the same frequency which when squared yields:

$$y_i'(t) = \int_0^{T_c} b_i(t)\, s_i^2(t)\, dt$$

$$+2\int_0^{T_c} \sum_{\tau_i=1}^{\infty} b_i(t) s_i(t) s_i(t-\tau_i) dt$$

$$+\int_0^{T_c} \sum_{\tau_i=1}^{\infty} b_k(t) s_i(t) s_k(t-\tau_k) dt \tag{3}$$

The first term of Eq. (3) is the actual information, while the last two terms represent the photodiode in-band correlations and the users' coincidence-based correlations, respectively.

3. SYSTEM MODEL AND NUMERICAL ANALYSIS

3.1 System Model

The FFH-OCDMA is made of two parts, an electrical part and an optical part. We assume that the incoming bits are already electrically processed, only the electro-optical and opto-electrical conversions are our focus as shown in Fig. 1 and Fig. 2. The encoding system under analysis is presented in Fig. 3, as motivated by [8], [9], [16]. It is composed of K user sequences characterized each one by its frequency and time information, with each user transmitting asynchronously from its own or same network. Each network is characterized by an access code constituted of $A_i(w_q, n)$ frequency sequence, pulse modulated by $a_i(t)$ time sequence, to produce the user sequence $A_i(w_q, n)a_i(t)$ in compliance with the outputs of Fig. 1.

These Bragg gratings encoded sequences are transmitted in the format of a WH-TS, made of OCFHC-OOC as described in Fig. 3, over the optical fibre with a non-coherent receiver at the destination. The transmission is considered linearly polarized over an effective fibre length. These transmitted sequences are aggregated in a destination of a targeted network receiver assumed to be user-1 of network 1. These optical signals are received by a system detection via an opto-electric conversion in compliance with Fig. 2, though it is only important to focus on the second part of the Fig. 2, the electrical part. We assumed that the system detector is made of a correlator and the decorrelator after the square detector, before the decision device. According to the difference family called divergence of position as construction solution of the OOC given in [23], the weighted elements are in a particular position pi such that the correlator matrix P is reduced to a vector $D = [p_1 p_2 \cdots p_w]$ representing the sequence of user 1. The signal y will be correlated by the user 1 correlator in a fixed weighted OOC vector.

$$\boldsymbol{r} = Dy = [y_{11}p_1\ y_{12}p_2 \dots\ y_{1w}p_w]$$

According to the system detector (Fig. 4) where the correlation properties are exploited to determine which bits were sent, the correlator outputs $r = |D|E_b + N_0$ where N_0 represents the MAI density level while the decorrelator multiplies r by $(E_b|\mathrm{D}|)^{-1}$, subsequently yields $r^- = 1 + \frac{N_0}{E_b|D|}$, with the decision device producing d = 1 when the auto-correlation $E_b|D| = wE_b$ and the decorrelation produces $(Eb|D|)^{-1} = \frac{1}{wE_b}$ or d = 0 otherwise (see Fig. 4). Normally, $|D| = w$ which is the sum of non-zero bits or Hamming weight of the sequence, however, with a fixed weighted OOC representation, one can decide to reduce it to $|D| = w - 1$.

3.2 Multiple Access Interference Evaluation

In this analysis, we assume that the number of frequencies available for OCFHC is Q and N is the number of OOC time spreading pulses propagating in the optical fiber with power P. The k-th modulating signal of the user data is given:

$$s_k(t) = \sqrt{2P} A(w_q, n) b_k(t-\tau_k) a_k(t-\tau_k), \tag{4}$$

where $C_i\left(w_q, n\right) = A(w_q, n)b_k(t-\tau_k)a_k(t-\tau_k)$ is the OCFHC-OOC spreading sequence defined as:

$$A(w_q, n)a_k(t-\tau_k) = \begin{cases} a_k(t-\tau_k), & \text{if } \mathrm{w_q} \text{ is present at } \tau_\mathrm{k} \\ 0, & \text{otherwise,} \end{cases}$$

with

$$a_k(t) = \sum_{l=0}^{N-1} a_k^l p\left(t - lT_c\right) \tag{5}$$

$$= \begin{cases} \sum_l a_k^l, \ t = lT_c, \ l = n = q = 0,\ 1,\ \dots,\ w-1, & \\ 0 & \text{otherwise,} \end{cases} \tag{6}$$

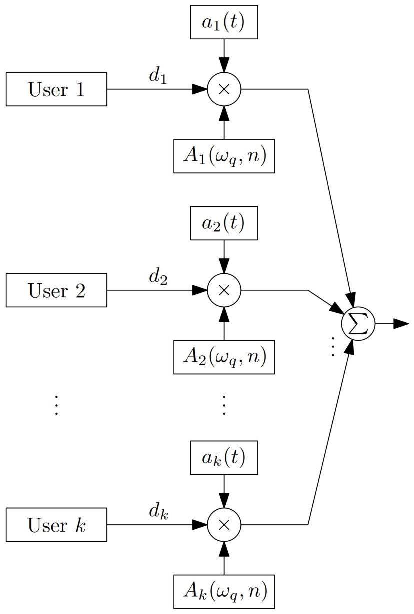


Fig. 3*: 2-D OCDMA modulator system diagram.*

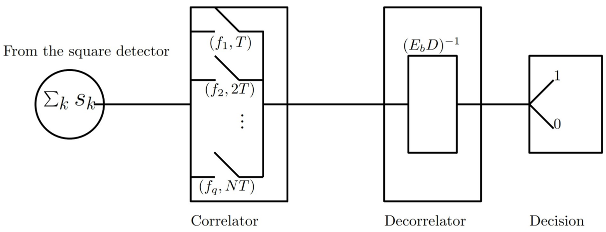


Fig. 4*: System detection.*

being the OOC time spreading sequence mapped into a rectangular pulse $p(t)$ of amplitude a_k^l | and width T_c, and a_k^l being the l-th chip from user k. The data sequence $d_k = b_k(t)$ is also mapped into a rectangular pulse as:

$$b_k(t) = \sum_{j=l=0}^{N-1} b_k^j a_k^j p\left(t - jT_c\right),$$

where b_k^j being the j-th data symbol of the k-th user taken from $b_k = \left\{b_k^0, b_k^1, \ldots\right\}$. However, in the following development, we consider that bk is binary, amplitude $|a_k^l| = 1$ in line with (6) such that $\sum_l a_k^l = w$, and that $(n, w, 1)$-OOC constructed using the difference of position [17], dictates the time mapping sequence, hence $n = N$. In that sequence, the number of "0" bits is bigger than the number of "1" bits which corresponds also to the spreading sequences. In an On-Off Keying (OOK) signalling, the presence of bit "0" is characterized by no signal and the presence of bit "1" means presence of the signal.

Therefore, the random variables a_k^l are the Bernoulli random variables with the probabilities within one time sequence, and b_k^j is deterministic in compliance with the OOK signalling and the characteristics of a Z-channel. The errors that occur ("1" becoming "0") are corrected using the OOC error correcting code capability, therefore we define the following filter:

$$\begin{cases} \text{if } b_k^j = 1, \text{ and } r(t) \neq 0, \text{then no error} \\ \quad \text{or } r(t) = 0, \text{ an error occured} \\ \text{if } b_k^j = 0, \ r(t) = 0, \text{ then no error} \end{cases}$$

Henceforth the following probabilities:

$$\Pr\left[a_k^l = 1\right] = \frac{w}{N}, \text{ and } \Pr\left[a_k^l = 0\right] = \frac{N-w}{N}$$

These probabilities lead to the following non-coherent receiver analysis. At the receiver, the K user signals are such that:

$$r(t) = \sum_{k=1}^{K} \sum_{n=0}^{N-1} \sum_{q=0}^{Q-1} \sqrt{2P} A(w_q, n) b_k^j a_k^l (t - \tau_k)$$

The non-coherent receiver listening to user 1 outputs:

$$y_1(t) = \int_0^{lT_c} s_1(t) r_1(t)\, dt = I_1$$

$$+2P \int_0^{lT_c} \sum_{k=1}^{K} \sum_{n=0}^{N-1} \sum_{q=0}^{Q-1} \left(A\left(w_q, n\right) b_k^j a_1^l p(t) a_k^l p\left(t - \tau_k\right)\right) dt$$

The quantity I_1 is the decision value for user 1 while. the second term represents the MAI.

$$I_{\text{mai}} = 2P \int_0^{lT_c} \sum_{k=1}^{K} \sum_{n=0}^{N-1} \sum_{q=0}^{Q-1} A\left(w_q, n\right)$$

$$\times (b_k^j a_1^l p(t) a_k^l p\left(t - \tau_k\right)) dt \tag{7}$$

I_1 is made of signals sampled at $t = \tau k = lT_c$, $n = j = l = N$ for which the correlation properties require that $A_1(w_q, n) A_k(w_q, n) b_k^j a_k^l = 1$. In that case, user-1 experiences a total interference given by:

$$I_{\text{mai}} = 2PN \int_0^{lT_c} \sum_{k=2}^{K} \sum_{q=1}^{Q} (b_k^j a_1^l p(t) a_k^l p\left(t - \tau_k\right)) dt \tag{8}$$

For what follows, we focus only on the correlation integral Eq. (8), $\int_0^{lT_c} a_1(t) a_k(t - \tau) dt$ of the spreading sequence as in [8] and [9], and we consider two consecutive data symbols of the k-th signal b_k^0, and b_k^1.

The continuous-time partial cross-correlation of user 1 and k-th user's spectral-spreading signal is defined as $R_{k,1}(\tau) = \int_0^{\tau} a_k(t - \tau) a_1(t) dt$ and its complement $\overline{R}_{k,1}(\tau) = \int_{\tau}^{lT_c} a_k(t - \tau) a_1(t) dt$. To this partial cross-correlation spreading signal, is associated the discrete aperiodic cross correlation of the same user' sequences C, which is a function of λ interdependent chips and written as in [15]

$$C_{k,1}(\lambda) = \begin{cases} \sum_{j=0}^{N-1-\lambda} a_k^j a_1^{j+\lambda}, & 0 \leq \lambda \leq N-1, \\ \sum_{j=0}^{N-1+\lambda} a_k^{j-\lambda} a_1^j, & 1 - N \leq \lambda < 0, \\ 0, & |\lambda| \geq N. \end{cases} \tag{9}$$

Noting that the signal is continuous while data is discrete, the total MAI for two consecutive data symbols after substituting $R_{k,1}$, $\overline{R}_{k,1}$ is therefore:

$$I_{\text{mai}} = P_b N \left[b_k^0 R_{k,1}(\tau) + b_k^1 \overline{R}_{k,1}(\tau) \right]$$

where $P_b = 2P$ is the power per bit.

The continuous partial cross-correlations $R_{k,1}$ depends only on user-1 and the k-th user signature sequences through $C_{k,1}$, and on the partial auto-correlation $R_p(s) = \int_0^s a(t)a(t+T_c-s)dt$ of the chip signal. As so, is $R_{k,1}$ for $\overline{R}_p(s) = \int_s^{T_c} a(t)a(t-s)dt$, such that $R_p(s) = \overline{R}_p(s) = 0$ for $s < 0$ or $s > T_c$. The overall cross-correlation becomes as in [22].

$$R_{k,1}(\tau) = C_{k,1}(\gamma - N)\overline{R}_p(\tau - \gamma T_c)$$

$+C_{k,1}(\gamma + 1 - N)R_p(\tau - \gamma T_c)$, and

$$R_{k,1}(\tau) = C_{k,1}(\gamma)\overline{R}_p(\tau - \gamma T_c)$$

$$+C_{k,1}(\gamma+1)R_p(\tau - \gamma T_c).$$

The random I_{mai} is obtained at $t = \tau_k - \gamma_k T_c$. Considering the delay $S_k = \tau_k - \gamma_k T_c$, which is a random variable uniformly distributed over $[0, T_c]$, γk a random shift uniformly distributed over $\{0, 1, \ldots, l-1\}$, while τ_k is uniformly distributed over $[0, lT_c]$, the MAI can be written as follows:

$$I_{\text{mai}} = P_b N \left[b_k^0 C_{k,1}\left(\gamma_k + 1 - N\right) + b_k^1 C_{k,1}\left(\gamma_k\right) \right] \overline{R}_p(S_k)$$

$$+ \left[b_k^0 C_{k,1}\left(\gamma_k + 1 - N\right) + b_k^1 C_{k,1}\left(\gamma_k + 1\right) \right] R_p(S_k) \quad (10)$$

Eq. (10) can be simplified by transforming the Bernoulli random variables a_k^l and b_k^j into other Bernoulli independent random variables P_k, Q_k and the sum of independent Bernouilli random variable namely X_k, Y_k as in [8], [9]. These random variables are obtained from Eq. (10) and letting $a_1^i = y_i$, $a_k^i = x_i$ leads to the following notations:

$$Z_j = \begin{cases} b_k^0 x_{j-\gamma+N} y_j, & j = 0, \ldots, \gamma - 1, \\ b_k^1 x_{j-\gamma} y_j, & j = \gamma, \ldots, N-2, \\ b_k^1 x_{N-\gamma-1} y_{N-1}, & j = N-1. \\ b_k^0 x_{N-\gamma} y_0, & j = N, \end{cases}$$

Rearranging Eq. (10) accordingly yields:

$$I_{\text{mai}} = N P_b [[\sum_{j=0}^{N-2} Z_j \left(R_p(S_k) + y_j y_{j+1} \bar{R}_p(S_k) \right)$$

$$+ Z_N R_p(S_k) + Z_{N-1} \bar{R}_p(S_k)]]$$

Noting that:
$P_k = Z_{N-1}, Q_k = Z_N$,
$X_k = \sum_{j \epsilon A} Z_j, Y_k = \sum_{j \epsilon B} Z_j$, where
$A = \left\{ x_j \,:\, a_1^j a_1^{j+1} = 1 \right\}$ and $B = \left\{ x_j \,:\, a_1^j a_1^{j+1} = 0 \right\}$.
X_k, Y_k, P_k, and Q_k can be interpreted as follows: X_k and Y_k represent the cross-correlation of the spreading sequence coefficients $\{a_k^j\}$; P_k and Q_k are the correlation coefficients at the end and at the beginning (after one cycle) of the sequence positions. Without loss of generality, $S = S_k$ is the random time correlation of the spreading rectangular pulse probability function, considered as uniformly distributed over $[0, T_c)$ and so is $(1-S)$. We can also, without loss of generality consider that the power bit $P_b = 1$. The MAI expression (10) of an OCFHC-OOC evaluated with OOC sequences for which $n \cong 1 \bmod (w(w-1))$ (the worst case), which is constructed using difference of position [17]. It is very important to understand that this sequence is made of at most 1 consecutive two pulses (X_k), and at least 1 parsed single pulse (Y_k), leading to $y_j y_{j+1} = 1$ or $y_j y_{j+1} = 0$ (in an OOC sequence, each dierence between any 2 positive bits can only be present at most once). The MAI can be reduced to:

$$I_{\text{mai}} = N[[P_k R_p(S) + Q_k R_p(S) + X_k(R_p(S)$$

$$+ \bar{R}_p(S_k)) + Y_k \bar{R}_p(S_k)]],$$

Assuming rectangular chip pulses $R_p(S) = S$ and $\bar{R}_p = 1 - S$, the MAI becomes:

$$I_{\text{mai}} = N \left[X_k + Y_k (1-S) + P_k (1-S) + Q_k S \right] \quad (11)$$

This MAI is a random variable I_{mai} consisting of the Bernoulli random variables X_k, Y_k, S, P_k, Q_k, which according to a_k(t) in (6) occur w times each. Since there is only interference when there is light in accordance with the filter definition, only the transitions $1 \rightarrow 0$ and $1 \rightarrow 1$ (two consecutive pulses can correlate if and only if there is presence of light on the first pulse) are valid, hence:

$|A| + |B| = w$.

This implies $Pr[Z_j = x_A] = \frac{1}{w}$ and $Pr[Z_j = y_B] = \frac{w-1}{w}$. Also, $Pr[Z_j = 1] = \frac{w}{n}$ and $Pr\left[Z_j = 0\right] = 1 - \frac{w}{n}$. Therefore, $Pr[Z_j = P_k] = Pr[Z_j = Q_k] = \frac{1}{2}$.

3.3 Gaussian Approximation

Since all variables are statistically independent Bernoulli random variables, the MAI is characterized by the mean (N is omitted here, however, we will revert to it):

$$E\left[I_{\text{mai}}\right] = E\left[X_k\right] + E\left[Y_k\right] E[(1-S)]$$

$$+ E\left[P_k\right] E[[(1-S)] + E\left[Q_k\right] E[S], \quad (12)$$

and the variance

$$\text{Var}[I_{\text{mai}}] = \text{Var}[X_k] + \text{Var}[Y_k]\text{Var}[1-S]$$

$$+ E^2[Y_k]\text{Var}[1-S] + \text{Var}[Y_k]E^2[1-S]$$

$$+ \text{Var}[P_k]\text{Var}[1-S] + E^2[P_k]\text{Var}[1-S]$$

$$+ \text{Var}[P]E^2[1-S] + \text{Var}[Q_k]\text{Var}[S]$$

$$+ \text{Var}[Q_k]E^2[S] + E^2[Y_k]\text{Var}[S]$$

The mean and the variance is evaluated using the following random variable statistics

$$E\left[X_k\right] = \frac{A}{w}, E\left[Y_k\right] = \frac{w-A}{w}, E\left[P_k\right] = E\left[Q_k\right] = \frac{1}{2},$$

$$E[S] = \frac{1}{T_c}\int_0^{T_c} s\,ds = \frac{T_c}{2}, E\left[S^2\right] = \frac{1}{T_c}\int_0^{T_c} s^2 ds = \frac{T_c^2}{2},$$

$$E[1-S] = \frac{1}{T_c}\int_0^{T_c}(1-s)\,ds = 1 - \frac{T_c}{2},$$

$$E\left[(1-S)^2\right] = \frac{1}{T_c}\int_0^{T_c}(1-s)\,ds = 1 - T_c - \frac{T_c{}^2}{3},$$

$$\mathrm{Var}[X_k] = \frac{A(w-1)}{w^2}, \mathrm{Var}[Y_k] = \frac{(w-A)(w-1)}{w^2},$$

$$\mathrm{Var}[P_k] = \mathrm{Var}[Q_k] = \tfrac{1}{4}, \mathrm{Var}[S] = \tfrac{T_c^2}{12}, \mathrm{Var}[1-S] = \tfrac{T_c^2}{12}$$

Since the MAI noise process I_{mai} is the Bernoulli distributed with mean $E[I_{\mathrm{mai}}]$ and variance $\mathrm{Var}[I_{\mathrm{mai}}]$, it is approximated as a Gaussian process using the central limit theorem with the probability density function of random variable i_{mai} described as:

$$f\left(i_{\mathrm{mai}}\right) = \frac{1}{\sqrt{2\pi}\mathrm{Var}[I_{\mathrm{mai}}]}\exp(-\frac{i_{\mathrm{mai}}{}^2}{2}) \qquad (13)$$

where $i_{\mathrm{mai}} = \frac{I_{\mathrm{mai}} - E[I_{\mathrm{mai}}]}{\sqrt{\mathrm{Var}[I_{\mathrm{mai}}]}}$

4. PERFORMANCE EVALUATION

4.1 Analytical Results

We have evaluated the MAI BER based on the time-interval correlations in a 2-D OCDMA OCFHCOOC sequence (BER1). However, in a Bragg gratings application, once a frequency is used in a sequence, it is no longer available, thus the previous given cardinality given in the preceding is reduced to $|C| = (L+1)L\frac{n-1}{w(w-1)}$. Therefore, there are $|C|\,w$ chips exposed to error. Since the probabilities are associated with the Bernoulli independent random variables, the probability of error from the standard Gaussian approximation of Eq. (13) is obtained as follows (used in [21]):

$$BER_1 = \int \frac{1}{\sqrt{2\pi}\mathrm{Var}[I_{\mathrm{mai}}]}\exp(-\frac{i_{\mathrm{mai}}{}^2}{2})di_{\mathrm{mai}} \qquad (14)$$

This Gaussian approximation for K users is averaged to:

$$BER_1 = \frac{(K-1)\,N}{|C|\,w-1}\mathrm{erfc}\left(\frac{wE[I_{\mathrm{mai}}]}{\sqrt{\mathrm{Var}[I_{\mathrm{mai}}]}}\right) \qquad (15)$$

Since the OOC is the time sequence of the OCFHC-OOC dictating the "1" bit positions, the time-interval analysis based on frequency-time $(\lambda,\ T)$ pairs is the same as the analysis conducted based on T. Therefore, BER1 is also valid for 1-D OCDMA OOC with the cardinality being the one for $(n,\ w,\ 1)$-OOC. We evaluate its MAI BER for the class $n \cong 1 \bmod (w(w-1))$ and investigate the case when n is lengthened by introducing "0" bits. We compare the time-interval correlations BER1 with the previous OCDMA results on MAI which are based on the user's coincidence correlations using the same Gaussian approximation as in [2] for OOC and in [3] for OCFHC-OOC. According to [2], [3], the number of coincidences a chip of a sequence experiences is w with a probability of $\frac{w}{n}$. The probability of appearance of a chip during the specific time interval is 0.5. Hence, the probability that a chip experiences coincidence is $p = \frac{w^2}{2n}$. According to [7], the bit error rate is then given by:

$$BER = \frac{1}{2}\sum_{i=w}^{K-1}\binom{K-1}{i}p^i(1-p)^{(k-1-i)}. \qquad (16)$$

One can realize that the above probability can be approximated using the central limit theorem as the number of users becomes large.

Accordingly, for a given K users in a 2-D OCDMA OCFHC-OOC, the BER [3] (BER2) can be reduced to

$$BER_2 = \frac{1}{2}\frac{(K-1-w)}{|C|-1}\left[\mathrm{erfc}\left(\frac{K-1-w}{\sqrt{(K-1)pq}}\right)\right.$$
$$\left.-\mathrm{erfc}\left(\sqrt{\frac{(K-1)q}{p}}\right)\right] \qquad (17)$$

Also, the approximation of the BER equation [7] for 1-D OCDMA OOC sequence (BER_3) can be given by:

$$BER_3 = \frac{1}{\sqrt{2\pi Kpq}}\mathrm{erfc}\left(\sqrt{\frac{Kq}{p}}\right) \qquad (18)$$

4.2 Simulations

We use MATLAB to investigate the analytical results presented by Eqs. (15), (17), and (18) considering the following numerical values: $n = N = 7, w = 3, L = 7, Q = 8$, which gives a cardinality of 392 in an OCFHC-OOC scheme for a Bragg gratings application. K is ranging from 50 to 400 for a 2-D OCDMA OCFHC-OOC sequence just to also investigate the case whereby the "0" bits are introduced to extend the code length.

We do the same for an OOC in a 1-D OCDMA OOC sequence with cardinality 7, and K ranging from 5 to 50 users for OOC assuming the "0" bits introduction into (7, 3, 1)-OOC, increasing its length to (49, 3, 1)-OOC and consequently the number of users. These theoretical results are compared through simulations, with the correlations from the direct detector output, using a MATLAB built-in script of Fig. 4. The quantity $\frac{N0}{E_b}$ represents the SNR. For an on-off keying with NRZ signalling with direct detection applied at the output of the Z-channel, the optimum threshold level for spatial (constellation) signal detection using Leibniz's approximation is given by $\frac{w}{n}\sqrt{\frac{|D|E_b}{T_b}}$. The average error probability of symbol 0 being detected when symbol 1 in a binary scheme was sent is therefore given by:

$$P_e = \frac{1}{2}\mathrm{erfc}\left(\frac{n-w}{n}\sqrt{\frac{|D|E_b}{N_0}}\right)$$

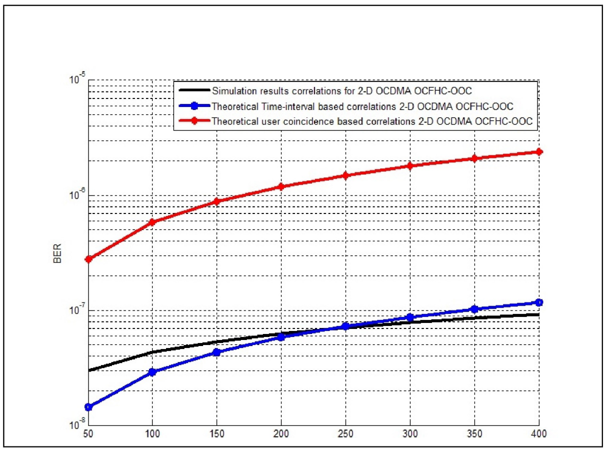


Fig. 5*: BER for time-interval-based vs. user coincidence-based correlations for OCFHC-OOC.*

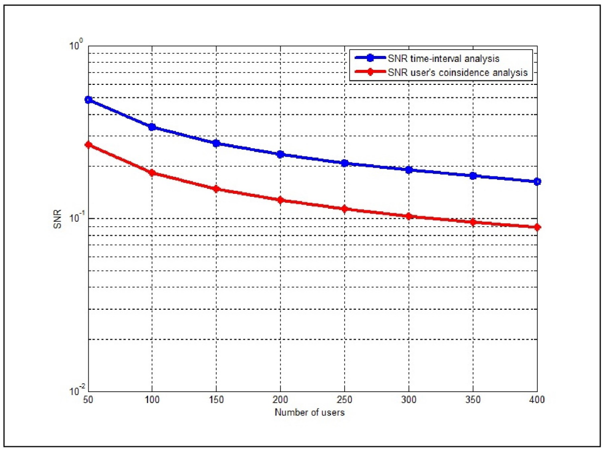


Fig. 7*: SNR time-interval based vs. user coincidence-based approaches.*

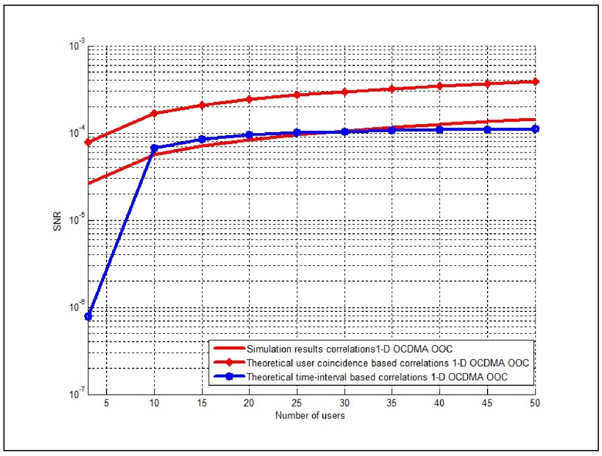


Fig. 6*: BER for time-interval-based vs. user coincidence-based correlations for OOC.*

The detection of the transmitted signal impaired by MAI, when considering $\boldsymbol{C_i(\omega_q, n)}$ in Eq. (4), has a probability of bit error

$$P_e = \frac{1}{2}\text{erfc}\left(\frac{n-w}{n}\sqrt{\frac{(w-1)E_b}{N_0}}\right)$$

The simulated results for 2-D OCDMA OCFHC-OOC and 1-D OCDMA OOC are both in Figs. 5 and 6 while the system SNR for OCDMA using OCFHC-OOC is given in Fig. 7.

4.3 Results

The results show an expression of the MAI random variable in Eq. (11) close to the one in [9, 15], which considered as a Gaussian random variable, allowed us to obtain the BER1 in Eq. (15), resulting from all positive time intervals. This BER1 is compared with BER2 in Eq. (17) for 2-D OCDMA OCFHC-OOC sequence, and BER3 in Eq. (18) for when reduced to 1-D OCDMA OOC from [3], [2] user coincidence-based techniques. We compare for each, 1-D and 2-D OCDMA, our approach with the user coincidence-based technique, and in both cases, we investigate their respective behaviours against the system detection output made of a correlator, a decorrelator and decision device (Fig. 4). The results of these investigations are shown in Figs. 5 and 6.

4.4 Discussion

The zero-correlation time-intervals improve the system performance. This is in line with the theory which implies that an increase in code length in 1-D OCDMA OOC increases the system performance up to a certain limit. In fact, the introduction of “0” bits increase the code length which is proportional to the cardinality, and which is related to the number of users. Therefore, the increase in number of users exploits the increase of zero-correlation timeintervals, explaining why the system shows a better performance level when the number of users increases. The user’s coincidence correlation-based are not the only correlations events that impact the system’s performance. One should count the users and the inband correlations to get a closer SNR required for the system performance. Thus, we can say that our approach of taking into account all the correlation events in evaluating MAI BER as developed in this paper, presents a better BER performance and subsequently SNR.

5. CONCLUSION

In this paper, we presented an improved evaluation of the MAI BER on OCDMA channel based OCFHC-OOC sequences, which is achieved using time-interval analysis. This analysis yielded an MAI random varitable from which the BER (BER1) is obtained using a Gaussian approximation. Since we suggested the OOC to be the time sequence for FFH-OCDMA, the technique is also valid for this scheme. In both cases, we confirmed the previous OCDMA results showing that as the number of users increases for the code, the performance of the system worsens but at a lesser rate for our technique, with the 2-D OCDMA being better than its 1-D counterpart. We showed that the coincidence-based correlations present not a good performance since the method omits other correlations occurring during transmission. Hence, the time-interval based correlations suggested herein, due to the fact that the zero-correlation time-intervals and the in-band frequencies are taken into account, seems to be a better tool for MAI BER evaluation, as it allows us to overcome the limit on the number of users supported by the user coincidence-based evaluation, improving accordingly the findings of [11]. Finally, we showed that the users-based correlations are not the only impairment responsible for the MAI in an OCDMA using photodiode detector in a direct detection scenario. One should count the in-band frequencies as well from the photodiode. The analysis to evaluate the bounds of code length for a required transmission power are left for further studies.

ACKNOWLEDGMENTS

The authors would like to thank B. Twala (previously with the Institute for Intelligent Systems, University of Johannesburg), C. Mbhowa (previously with the Faculty of Engineering and Built Environment, University of Johannesburg) and S. Motala (with the Postgraduate School, University of Johannesburg) for their financial support.

REFERENCES

[1] J. A. Salehi, "Code division multiple-access techniques in optical fibre networks. Part I: Fundamental principles," *IEEE Trans. Commun.*, vol. 37, no. 8, pp. 824-833, Aug. 1988.

[2] C. Malik and S. Tripathi, "Performance evaluation and comparison of optical CDMA networks," *Intl. J. Electron. Commun. Technol.*, vol. 2, no. 1, pp. 55-59, Mar. 2011.

[3] S. Shuron, H. Yin, Z. Wang and A. Xu, "A new family of 2-D optical orthogonal codes and analysis of its performance in optical CDMA access networks," *J. Lightwave Technol.*, vol. 24, no. 4, Apr. 2006.

[4] T. Demeechai and A. B. Sharma, "Beat noise in a non-coherent optical CDMA system," *Intl. Conf. Commun. Syst.*, pp. 899-902, Nov. 2002.

[5] E. D. J. Smith, P. T. Gough and D. P. Taylor, "Noise limits of optical spectral-encoding CDMA systems," *Electronics Letters,* vol. 31, no. 17, pp. 14691470, Aug. 1995.

[6] L.-P. Boulianne, "Système de Communication Optique à Accès multiple par Repartition de Code à Saut rapide de Fréquence (in French)," Master's dissertation, Université Laval, Québec, Canada, Nov. 2001.

[7] R. Yadav and G. Kaur, "Optical CDMA: Technique, parameters and applications," Proc. 4th Int. Conf. Emerging Trends Eng. Technol.", *Ku-rukshetra, India*, , pp. 90-98, Oct., 2013.

[8] J. S. Lehnert and M. B. Pursley, "Error probabilities for binary direct sequence spread-spectrum communications with random signature sequences", *IEEE Trans. Commun.*, vol. 35, no. 1, pp. 87-98, Jan. 1987.

[9] R. K. Morrov and J. S. Lehnert, "Bit-to-bit error dependence in slotted DS/SSMA packet systems with random signature sequences," *IEEE Trans. Commun.*, vol. 37, no. 10, pp. 1052-1061, Oct. 1989.

[10] W. R. Peng, W. P. Liu, "Improved fibre gratings array OFFH-CDMA system using a novel frequency lapping multigroup method," *IEEE/OSA J. Lightwave Technol.*, vol. 24, no. 3, pp. 1072-1081, Mar. 2006.

[11] H. Yin and D. J. Richardson, *Optical Code Division Multiple Access Communication Networks: Theory and Applications*, Tsinghua University Press/Springer-Verlag, 2007.

[12] S. Johnson, "A new upper bound for error correcting codes," *IEEE Trans. Inf. Theory*, vol. 8, no. 3, pp. 203-207, Apr. 1962.

[13] M. B. Pursley, D. V. Sarwate and W. E. Stark, "Error probability for direct-sequence spread-spectrum multiple-access communications: Part I: Upper and lower bounds," *IEEE Trans. Commun.*, vol. 30, no. 5, pp. 975-984, May 1982.

[14] R. M. Buehrer and B. D. Woerner, "Analysis of adaptive multistage interference cancellation for CDMA using an improved Gaussian approximation," *IEEE Trans. Comm.*, vol. 44, no. 10, pp. 1308-1321, Oct. 1996.

[15] M. B. Pursley, "Performance evaluation for phase-coded spread-spectrum multiple-access communication. Part I: System analysis," *IEEE Trans. Commun.,* vol. 25, no. 8, pp. 795-799, Aug. 1977.

[16] E. A. Geraniotis and M. B. Pursley, "Error probability for direct-sequence spread-spectrum multiple-access communications: Part II: Approximations," *IEEE Trans. Commun.*, vol. 30, no. 5, pp. 985-995, May 1982.

[17] F. R. K. Chung, J. A. Salehi and V. K. Wei, "Optical orthogonal codes: design, analysis and application," IEEE *Trans. Inf. Theory,* vol. 35, no. 3, pp. 595-604, May 1989.

[18] H. Fathallah, L. A. Rusch and S. LaRochelle, "Optical frequency-hop multiple access communications systems," *in Proc. IEEE ICC*, vol. 36, no. 2, Atlanta, Jun. 1998.

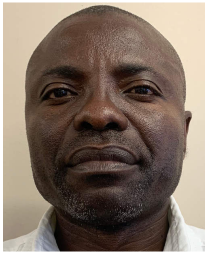
[19] L. Bin, "One-coincidence sequences with specified distance between adjacent symbols of frequency-hopping multiple access", *IEEE Trans. Commun.*, vol. 45, no. 4, pp. 408-410, Apr. 1997.

[20] A. Borude and K. Shobha, "Simulation of optical code division multiple access using OOC," *International Journal of Scientific Research Publications*, vol. 2, no. 5, May 2012.

[21] S. Nlend and T. G. Swart, "Enhanced 3-D OCDMA Code Family using Asymmetric Run Length Constraints," *2019 IEEE AFRICON*, Ghana, Sep. 2019.

[22] M. B. Pursley, F. D. Garber, and J. S. Lehnert, "Analysis of Generalized Quadriphase Spread-Spectrum Communications," *IEEE Int. Conf. Commun.*, June 1980.

[23] S. Nlend and T. G. Swart, "Construction and Characterization of Optical Orthogonal Codes (n,w,1) for Fast Frequency Hopping-Optical Code Division Multiple Access", *ECTI-EEC*, vol. 20, no. 1, pp. 62–73, Feb. 2022.



Samuel Nlend received the B.Sc. (Physics) degree in 1994, the M.Phil. (Electrical and Electronic) degree in 2013 and PhD (DPhil) in 2021. He is currently assistant researcher in the Department of Electrical and Electronic Engineering Science at the University of Johannesburg, South Africa. He worked from 1996 to 2010 respectively as the Regional Head of Department of Statistics and QoS and of Technical Department at Cameroon Telecommunications, Cameroon, and as a Technical Engineer at Phonelines International, South Africa. Dr. Nlend is a member of the Engineering Council of South Africa since 2017 and a student member of IEEE since 2019.

Theo G. Swart received the B.Eng. and M.Eng. degrees (both cum laude) in electrical and electronic engineering from the Rand Afrikaans University, South Africa, in 1999 and 2001, respectively, and the D.Eng. degree from the University of Johannesburg, South Africa, in 2006. He is an associate professor in the Department of Electrical and Electronic Engineering Science and the director of the UJ Centre for Telecommunications. His research interests include digital communications, error-correction coding, constrained coding and power-line communications. Dr. Swart is a senior member of the IEEE, and was previously the chair of the IEEE South Africa Chapter on Information Theory. He is a former specialist editor for the SAIEE Africa Research Journal.