

Transformer Parameters and Transmission Line Temperatures Estimation in Three-phase Power Networks

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ABSTRACT

This paper presents a method for temperature-dependent three-phase state estimation where not only bus voltage phasors but also tap ratio, leakage admittance parameters of transformers, line conductor temperature, and weather environment variables are considered as the state variables. The three-phase tap-changing transformer model and the weather-based transmission line model are integrated into the proposed state estimation, which is then formulated as a nonlinear optimization problem based on the weighted least squares objective function. The proposed method is demonstrated on the IEEE 30-bus and 118-bus systems modified as three-phase test systems. Simulation results indicate that the proposed formulation outperforms the conventional state estimations, which do not consider transformer parameters and line temperature, in terms of estimation accuracy and bad data detection performance.

Keywords: Three-phase State Estimation, Transformer Parameter, Transmission Line Temperature

1. INTRODUCTION

State estimation (SE) is an important function in energy management systems. It is based on a mathematical relationship between the actual measurements and the network topology of a power system. Operating state variables of power systems, such as voltage magnitudes and phase angles of all buses, can be approximated by employing SE [1]. The remote terminal units (RTUs) usually record the voltage and current magnitudes and the power measurements. These collected measurements are transmitted to a supervisory control and data acquisition (SCADA) system and used to determine the best

estimation of power system states. In modern power systems, phasor measurement units (PMUs), which measure voltage and current phasors at the synchronizing time via the global positioning system, are also utilized because the synchronized measurements obtained from PMUs are very accurate and can improve SE accuracy [2]–[5]. In traditional SE, the parameters of all transmission lines are usually obtained from off-line calculations at a certain temperature of conductors. However, in practice, the parameters of the transmission line vary depending on operating and environmental conditions. The transformer's tap ratio can be obtained by using tap position measurements. Nevertheless, tap position measurements are usually placed only on some transformers. Furthermore, the variation of the transformer's admittance parameter due to the tap position changing cannot be measured directly. The SE methods that employ inaccurate parameter values for the transformers and transmission lines of power systems can lead to erroneous estimated states.

To address the problem of SE in the existence of incorrect network parameters, an online tracking SE method for estimating transmission line parameters using SCADA data has been presented in [6]. A method using a dedicated parameter estimator based on a measurement system utilizing both RTUs and PMUs to determine and correct erroneous transmission line parameters has been presented in [7]. In [8], incorrect parameters of lines can be identified and estimated using PMU devices. The method for estimating the parameters of lines under non-Gaussian noise in PMUs has been reported [9]. Estimating the transformer's tap positions in power systems under certain parameters of both the transmission line and the transformer's admittance has been studied in [10]–[12]. A robust equality-constrained SE method that is able to prevent ill-conditioned problems of estimating transformer tap positions and bus voltages under erroneous zero-power injections has been presented in [13]. Parameter estimation of transmission line and transformer has been presented in [14]. RTUs are placed at both terminals of the line and the transformer to achieve the measured values of bus voltages and power flows through the line and the transformer. The linear SE model using multiple time instants has been proposed and solved with a non-iterative process. In [15], a method for estimating line parameters, tap ratios, and

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systematic errors based on PMU-based measurements has been proposed. The tap-changing transformer has been represented by a general model [16], and the compared results with a traditional tap-changing transformer model are discussed. Nevertheless, all these SE methods are developed based on single-phase representation and cannot be directly applied to unbalanced three-phase power systems under realistic conditions.

In the context of three-phase SE, the mutual coupling parameters between phases of a three-phase line cannot be neglected [17, 18]. In the literature related to three-phase parameter estimation of transmission systems, a method combining SE and parameter tracking for estimating three-phase parameters of untransposed lines has been proposed [19]. This method is based on dynamic parameters associated with multi-scan snapshots using PMU data. The method assumes that some elements of the shunt admittance and the series impedance matrices of a three-phase transmission line model are equal to each other to decrease the number of unknown states and make the system observable. In [20], a method for detection, identification, and estimation of parameter errors using PMUs in three-phase line parameters has been presented. A normalized Lagrange multiplier test based on the modal domain has been developed to determine and estimate errors in the self and mutual parameters for untransposed lines. While the methods in [21] and [22] have been developed to determine the accurate parameters of an interesting line without considering the voltage state variables of buses. The simultaneous estimation of three-phase line parameters and tap changer ratios has been performed in [23]. This method is based on PMU measurements using multiple time instants to increase measurement redundancy and make an observable system. Since these three-phase SE methods require the voltage and the current phasor information on both ends of each line, they may not be appropriate for estimating the parameters of all transmission lines in practice. Furthermore, they assume that the shunt admittance and the series impedance matrices of each line are symmetric to decrease unknown parameters in those matrices. In [24], incorporating the conductor temperatures of the overhead lines into a three-phase SE problem has been studied. The line thermal model, based on weather data and current magnitude measurements, is used to estimate the conductor temperatures of all lines. However, many additional state variables along the transmission lines and a technique of line segmentation are required, particularly for long-length lines, in order to obtain the reliability of the SE method. In [25], the concept of an alternating optimization approach has been applied to the three-phase SE method for estimating the conductor temperatures of the overhead lines and the underground cables. The method is based on a constrained optimization using weather-based models of conductors. The SE problem is formulated and divided into two sub-problems to reduce computation burden. Since the methods in [24] and [25] can provide the conductor

temperature states of all lines, the obtained estimated states can be used to assess the thermal operating points of the lines [26]-[28]. Additionally, using the three-phase line models based on the conductor temperature states, the temperature-dependent admittance matrices of lines can be non-symmetrical. Nonetheless, variation of the transformer's parameters has not been taken into account by the temperature-dependent SE methods [24] and [25].

This paper addresses a three-phase SE problem in which the parameters of transformers and transmission lines are considered as additional state variables. The tap ratio and the reactance of the transformer, together with the conductor temperature of the transmission line, are estimated in this work. The measured values obtained from both SCADA and PMUs are employed. The validation tests are carried out on two three-phase power networks. The results demonstrate that the proposed SE method can provide the estimated parameters successfully and significantly improve SE accuracy in the three-phase test networks.

The contributions of this paper are summarized as follows:

- (1) A three-phase power system SE algorithm is proposed for estimating the system's state variables, i.e., voltage phasors of all buses, all transformers' tap ratios and leakage reactances, and conductor temperatures of all lines. The proposed SE computes the series-current magnitudes and applies them to form the line's heat balance constraints. There is no need to install additional current magnitude measurements.
- (2) A proposed constrained SE method can be applied to both RTU and PMU measurements and permits no branch measurements on some transmission lines while all estimated states are still available.
- (3) The effects of the transformer parameters and the line's conductor temperature on SE results have been investigated. The inclusion of line conductor temperature shows better estimation accuracy. In addition, it is found that bad data detection cannot be done correctly when conductor temperature is not considered for three-phase SE integrating transformer parameter states.

The paper is organized as follows: The mathematical modelling of transformers and transmission lines is briefly described in Section 2. The proposed SE algorithm is presented in the next section. Test systems are explained in Section 4. Simulation results are illustrated in Section 5. Finally, conclusions are provided in the final section.

2. POWER COMPONENTS MODELLING

2.1 Tap-Changing Transformer

Details of two-winding three-phase transformer models, which consider tap ratio and winding configurations, have been reported in [29, 30]. In this subsection, the three-phase tap-changing transformer model based

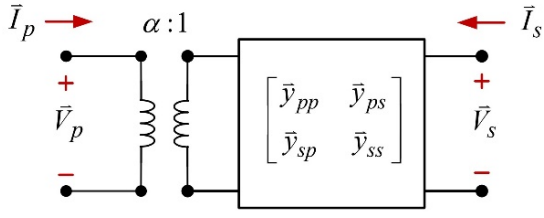


Fig. 1: Two-port network representation of three-phase transformer.

on a node admittance matrix is explained in terms of phase coordinates for the tap and the leakage reactance of the transformer. Fig. 1 shows an equivalent two-port network of a two-winding, three-phase transformer. For the tap-changing transformer, where the tap ratio between primary and secondary windings is $\alpha:1$, the three-phase node admittance equations can be given as follows:

$$\begin{bmatrix} \vec{I}_p \\ \vec{I}_s \end{bmatrix} = \begin{bmatrix} \frac{\vec{y}_{pp}}{\alpha^2} & \frac{\vec{y}_{ps}}{\alpha} \\ \frac{\vec{y}_{sp}}{\alpha} & \vec{y}_{ss} \end{bmatrix} \begin{bmatrix} \vec{V}_p \\ \vec{V}_s \end{bmatrix}, \quad (1)$$

where

\vec{I}_p and \vec{I}_s represent *abc*-component of the branch current phasor at the primary and the secondary sides, respectively,

\vec{V}_p and \vec{V}_s are *abc*-component of the voltage phasor at the primary and the secondary sides, respectively,

\vec{y}_{pp} , \vec{y}_{ps} , \vec{y}_{sp} , and \vec{y}_{ss} denote sub-admittance matrices corresponding to the node admittance matrix of the model. Their values depend on leakage admittance and winding configurations of the three-phase transformer. Elements of each sub-admittance matrix corresponding to the winding configurations are given in Appendix A.

In this work, the transformer's leakage reactance and tap are also considered as the state variables. The injected current measurement quantities in (1) can be used to form a measurement function. The injected power measurement equations, derived from the product of the corresponding bus voltage phasors and the conjugation of the injected current phasors, can be used to form the measurement functions in terms of power measurements. A complex power flow from the sending bus p to the receiving bus s , \vec{S}_p , can be calculated by

$$\vec{S}_p = P_p + jQ_p = \vec{V}_p \vec{I}_p^*, \quad (2)$$

where P_p and Q_p are the real and the reactive power flows from bus p to bus s , respectively. Note that the superscript $*$ denotes a complex conjugation. One can notice that the current phasor and the power measurements are dependent on both the tap ratio and the leakage reactance of the transformer.

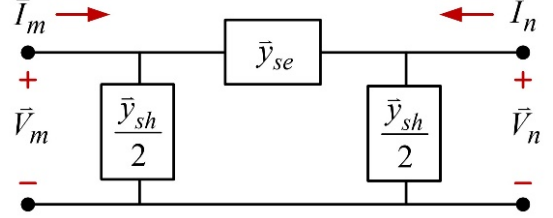


Fig. 2: The equivalent π circuit of transmission line.

2.2 Transmission Line

Fig. 2 shows an equivalent π circuit of a three-phase transmission line. The current phasor flowing from bus m to bus n can be obtained from

$$\begin{bmatrix} \vec{I}_m \\ \vec{I}_n \end{bmatrix} = \begin{bmatrix} 0.5 \vec{y}_{sh} + \vec{y}_{se} & -\vec{y}_{se} \\ -\vec{y}_{se} & 0.5 \vec{y}_{sh} + \vec{y}_{se} \end{bmatrix} \begin{bmatrix} \vec{V}_m \\ \vec{V}_n \end{bmatrix}, \quad (3)$$

where

\vec{I}_m and \vec{I}_n represent *abc*-component of the branch current phasor at the sending bus and the receiving bus, respectively,

\vec{V}_m and \vec{V}_n are *abc*-component of the voltage phasor at the sending bus and the receiving bus, respectively,

\vec{y}_{se} and \vec{y}_{sh} denote the series and the shunt admittance matrices of the transmission line, respectively.

In this work, a phase frame of reference is used for a three-phase power network. The series and the shunt admittance parameters of the transmission line are 3×3 matrices whose values depend on conductor data, the geometry of the tower, the earth's resistivity, and the conductor temperature. These admittance matrices are usually asymmetric matrices due to conductor arrangement and matrix reduction to 3×3 dimensional matrices consisting of *abc*-phase components. Both matrices can be calculated using the three-phase model of the line [17]. The phase conductor temperature of each line can be evaluated based on the concept of the heat balance equation according to IEEE standard 738-2006 [31]. In this work, the equality constraint in (4) is employed to obtain the constraint for phase conductor temperature, T_c ,

$$I_{eff}^2 R_{ac}(T_c) = P_c + P_r - P_s, \quad (4)$$

where I_{eff} stands for the effective value per phase conductor of the series current magnitude flowing through the transmission line. $R_{ac}(T_c)$ is a temperature-dependent AC resistance per unit length of the conductor. P_c is the convection heat loss per unit length of the conductor, P_r represents the radiation heat loss per unit length of the conductor, and P_s denotes the solar heat gain per unit length of the conductor. The measured values of the weather parameters corresponding to P_c , P_r , and P_s are available from the meteorological data and the weather stations. The weather environmental variables, which consist of the wind direction, the wind

speed, the solar radiation, and the ambient temperature, are also added to the state variable vector. While the earth wire conductor temperature is assumed to be equal to the ambient temperature since only small current magnitudes are flowing through the earth wire conductors in a steady state condition. In this work, \vec{y}_{se} and \vec{y}_{sh} matrices are pre-computed within certain temperature ranges of conductor temperature, T_c , and ambient temperature, T_a . The values of admittance parameters at any T_c and T_a are approximated based on a 2-dimensional interpolation of the pre-computed admittance data.

3. PROPOSED STATE ESTIMATION

The proposed temperature-dependent SE using the weighted least squares (WLS) criterion for three-phase power systems is explained in this section. The proposed SE, which includes bus voltage phasors, conductor temperatures, tap-changing transformer parameters, and weather environmental parameters as the state variables, can be formulated as the following nonlinear constrained optimization problem:

$$\left. \begin{aligned} \phi &= \min \frac{1}{2} r^T W r, \\ \text{s.t.} \quad r &= z - h(x), \\ f(x) &\leq 0, \\ g(x) &= 0, \\ x_{\min} &\leq x \leq x_{\max} \end{aligned} \right\}, \quad (5)$$

where $z \in \mathbb{R}^{N_z}$ denotes the measurement vector, N_z is the number of measurements, $h(\cdot)$ is the measurement function, $r \in \mathbb{R}^{N_z}$ is the residual vector. The inequality and equality constraints are represented by $f(\cdot)$ and $g(\cdot)$, respectively. $W \in \mathbb{R}^{N_z \times N_z}$ is an inverse of the measurements covariance matrix. The state variables $x = (v, t) \in \mathbb{R}^{N_x}$ consists of $v = (V_{abc}, \theta_{abc})$, a vector of magnitudes and phase angles of the three-phase voltage phasors of all buses. The vector t contains auxiliary state variables, which are transformer tap, α , leakage reactance of transformer, x_t , line conductor temperature, T_c , and weather-related variables, i.e., T_a , ambient temperature; w_s , wind speed; w_d , wind direction; and q_s , solar radiation. Therefore, $t = (\alpha, x_t, T_c, T_a, w_s, w_d, q_s)$. We let N_x be the total number of state variables.

The measurement function consists of,

$$h(x) = (h_{PMU}, h_{RTU}, h_{Wea}), \quad (6)$$

where h_{PMU} is the measurement function for the PMUs. It consists of bus voltage and current phasors measured in polar coordinates. In this work, a rectangular coordinate of the current phasor is used as a measured value. The standard uncertainty of the current phasor in the rectangular coordinate is approximated based on the uncertainty propagation technique [32]. h_{RTU} is the measurement function for the RTUs. It consists of the bus voltage magnitude and the power measurement functions. And h_{Wea} is weather-related measurements. Notice that the tap position measurement is not required.

Accordingly, the measurement vector, z , contains the data obtained from the corresponding measuring devices.

$$z = (z_{PMU}, z_{RTU}, z_{Wea}). \quad (7)$$

The inequality constraint consists of

$$f(x) = (f_{Tr}, f_{Hb}, f_{Wea}), \quad (8)$$

where f_{Tr} contains inequality related to transformer variable states, f_{Hb} denotes inequality constraints due to heat balance equations of transmission lines, and f_{Wea} contains inequality of weather variables.

For the equality constraint, three sub-constraints can be grouped as follows:

$$g(x) = (g_V, f_{Inj}, f_{Hb}), \quad (9)$$

where g_V is obtained from reference bus voltage phasor constraints, g_{Inj} represents zero injection constraints, and g_{Hb} denotes equality constraints derived from the heat-balance condition of each line where its conductor temperature is estimated.

The solution of the SE can be achieved by solving the optimization problem in (5). In this work, the method based on Mehrotra's predictor-corrector interior point [33] is applied. It should be remembered that the number of state variables can be much larger than the traditional SE, due to the inclusion of auxiliary states. Additionally, the values of series and the shunt admittance matrices have to be re-evaluated when the line's conductor temperature value is updated in each iteration. These factors considerably increase the computational burden on the solver.

4. TEST SYSTEMS

The standard IEEE 30-bus and 118-bus systems have been modified to be used as three-phase test systems. The impedance of each transmission line in both standard systems is considered a positive sequence impedance, and the length of each line is obtained based on the positive sequence reactance. The conductor properties and physical geometry of the line are assumed and used to compute the three-phase series and shunt admittance parameters of each transmission line in both test systems. The temperature of each phase conductor is assumed to be uniform, while the temperature of each earth wire conductor is assumed to be equal to the ambient temperature. A three-phase representation of each transformer is set based on available transformer parameters obtained from the standard systems. While transformer winding configurations are assumed. The weather data and three-phase unbalanced load conditions are also simulated.

The modified IEEE 30-bus system with measurement set is illustrated in Fig. 3. The transformers connecting buses 4-12, 6-9, 6-10, and 28-27 are simulated as three-phase wye-delta grounding transformers. They are the tap-changing transformers whose tap and leakage admittance parameters are included in the state variables.

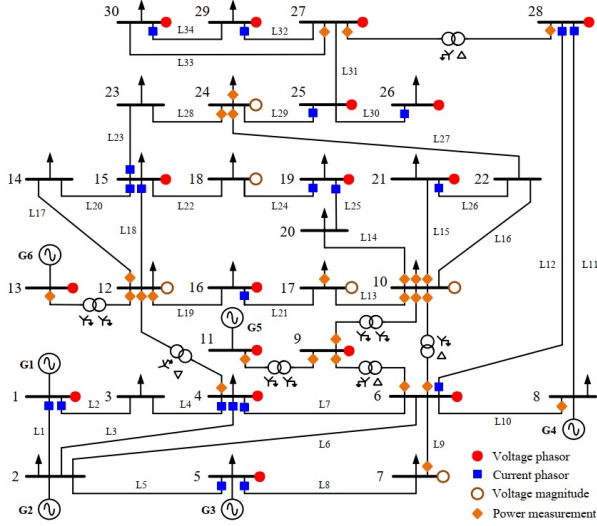


Fig. 3: The IEEE 30-bus system with measurement set.

While the three-phase grounded wye-wye connection transformers are used for the transformers at buses 9-10, 9-11, and 12-13. The parameters of these transformers are assumed to be given, and hence they are not included in the state variables.

For the modified IEEE 118-bus system, 171 overhead lines and 9 transformers are contained in the power network. All three-phase transformers are tap-changing transformers in which their configurations are grounded by wye-delta winding connections. The tap and leakage admittance parameters are included in the state variables to be estimated for each three-phase transformer.

5. SIMULATION RESULTS

The proposed method has been developed in a MATLAB environment. All numerical simulations are conducted on a computer with an Intel Core i7-7700, 3.60 GHz, and 16 GB of main memory. True values are obtained from the temperature-dependent power flow algorithm [34]. The convergence tolerance is set to 10^{-2} for the weather-related state variables and 10^{-4} for others. Two cases have been performed to investigate the effects of the line conductor's temperature and transformer parameters on the SE.

5.1 Case 1: No measurement noise

In this case, to show the effects of conductor temperature, the true values of measurements are employed for the SE. Two tests using the conventional method, which does not include line temperature in estimation, have been performed by fixing line conductor temperature at 20°C and 90°C, respectively. The obtained results are compared with our proposed method, in which the line conductor temperatures are considered as state variables.

For the modified IEEE 30-bus system, the difference of bus voltage phasors (phase *a*) between the estimated state variables and their actual values is shown in Fig. 4. The

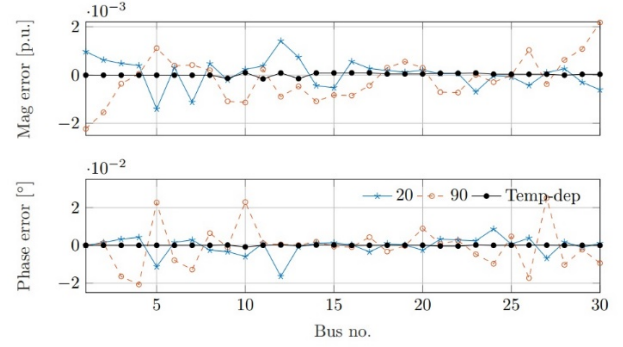


Fig. 4: Error of bus voltage phasor (phase *a*) for the IEEE 30-bus system (Case 1).

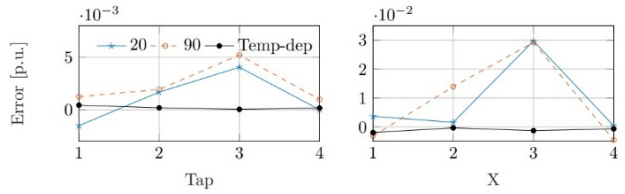


Fig. 5: Error of transformer parameters for the IEEE 30-bus system (Case 1).

results obtained through the proposed method, denoted by 'Temp-dep', show that the estimation errors of the bus voltage magnitudes and the phase angles are very close to zero. In contrast, the conventional method yields larger estimation errors even if there is no noise in the measured data. The estimation errors of tap and leakage reactance for each transformer are shown in Fig. 5. It can be noticed that the error obtained by the conventional method is greater than that of the proposed method.

For the modified IEEE 118-bus system, similar results are also obtained. The difference of bus voltage phasors (phase *a*) between the estimated state variables and their actual values is presented in Fig. 6. While the errors of tap and leakage reactance of each transformer are shown in Fig. 7. It can be seen that, using the proposed method, the estimation errors are close to zero.

As the results illustrated in this case, it can be concluded that the SE method with incorrect line tem-

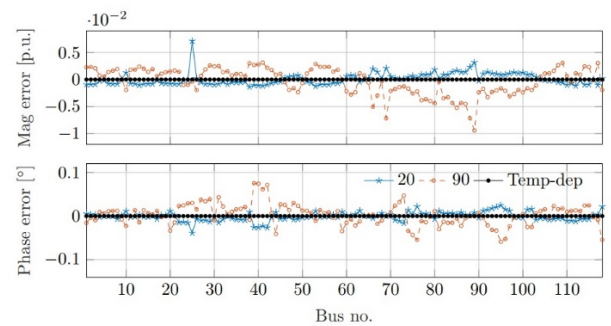


Fig. 6: Error of bus voltage phasor (phase *a*) for the IEEE 118-bus system (Case 1).

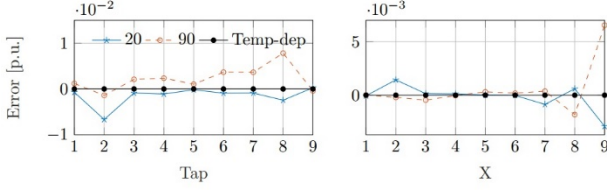


Fig. 7: Error of transformer parameters for the IEEE 118-bus system (Case 1).

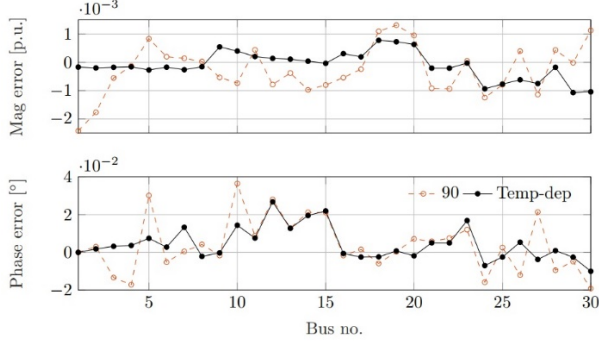


Fig. 8: Error of bus voltage phasor (phase a) for the IEEE 30-bus system (Case 2).

perature produces larger estimation errors even if there is no noise in the measurement vectors.

5.2 Case 2: Under noise conditions

This case is for investigating the measurement noise influence on the estimation accuracy. Monte-Carlo simulations have been performed by randomly varying measurement noise with a Gaussian distribution corresponding to the standard deviations of the measurements. The SE problems are solved by the conventional method, where each line temperature is fixed at 90°C and by the proposed method.

The root mean square error (RMSE) defined in the following is used as the performance index:

$$\text{RMSE} = \sqrt{\frac{1}{N_M} \sum_{i=1}^{N_M} \frac{\|x_{true} - \hat{x}_i\|_2^2}{N}}, \quad (10)$$

where N_M is the number of Monte-Carlo simulations, $\hat{x}_i \in \mathbb{R}^{N_x}$ denotes the estimated values of state variables at the i -th Monte-Carlo simulation, and $x_{true} \in \mathbb{R}^{N_x}$ is the true value. In this test case, $N_M = 100$.

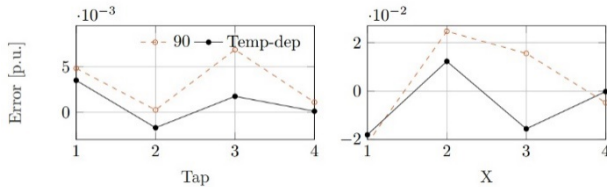


Fig. 9: Error of transformer parameters for the IEEE 30-bus system (Case 2).

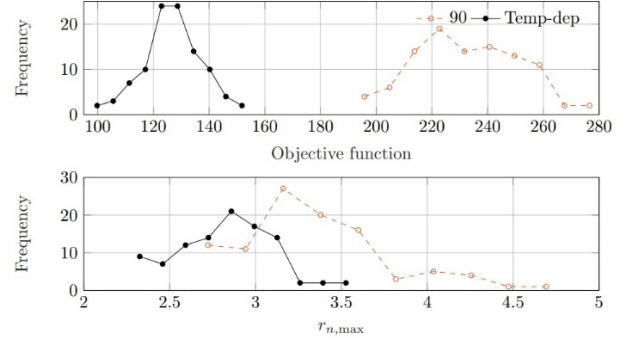


Fig. 10: Histograms of objective function and $r_{n,max}$ for the IEEE 30-bus system (Case 2).

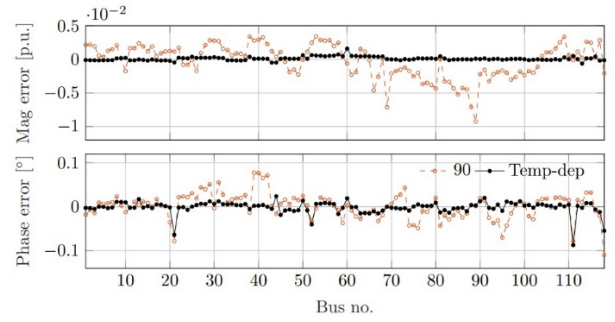


Fig. 11: Error of bus voltage phasor (phase a) for the IEEE 118-bus system (Case 2).

For the modified IEEE 30-bus system, an example of the difference of bus voltage phasors (phase a) between the estimated state variables and their actual values in one simulation is shown in Fig. 8. The errors in tap and leakage reactance of each transformer are shown in Fig. 9. In the figure, the parameters no. 1 to 4 correspond to the three-phase wye-delta grounding transformers connecting buses 4-12, 6-9, 6-10, and 28-27, respectively. The histograms of the objective function and the largest normalized residual, $r_{n,max}$, are shown in Fig. 10. It should be noted that the corresponding measurement, which provides $r_{n,max}$ greater than the detection threshold value (usually 3.0 - 4.0), is suspected to contain bad data [1]. These results indicate that about 9 percent of the estimations using the conventional method provide unreliable estimated states for the modified IEEE 30-bus system since it is determined that bad

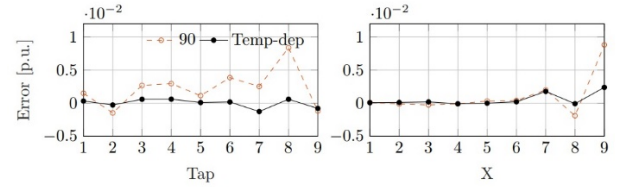


Fig. 12: Error of transformer parameters for the IEEE 118-bus system (Case 2).

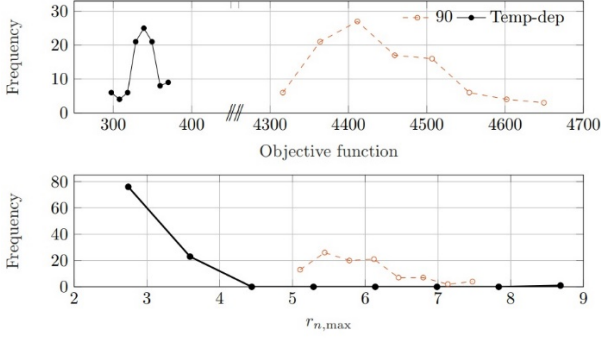


Fig. 13: Histograms of objective function and $r_{n,max}$ for the IEEE 118-bus system (Case 2).

Table 1: RMSE comparison results.

| | IEEE 30-bus | | IEEE 118-bus | |
|--|-------------|----------|--------------|----------|
| | Conv. | Proposed | Conv. | Proposed |
| Voltage magnitude ($\times 10^{-4}$ p.u.) | | | | |
| Phase a | 8.95 | 4.77 | 25.25 | 3.03 |
| Phase b | 9.58 | 3.57 | 26.91 | 2.82 |
| Phase c | 11.57 | 6.10 | 26.38 | 2.98 |
| Phase Angle ($\times 10^{-2}$ °) | | | | |
| Phase a | 1.21 | 0.67 | 2.91 | 1.28 |
| Phase b | 1.24 | 0.72 | 3.75 | 1.33 |
| Phase c | 1.58 | 1.32 | 2.90 | 1.45 |
| Transformer ($\times 10^{-4}$ p.u.) | | | | |
| Tap | 33.70 | 7.89 | 39.60 | 10.95 |
| Reactance | 172.90 | 48.38 | 22.93 | 13.65 |
| Conductor temperature (°C) | 51.61 | 0.34 | 51.41 | 1.07 |
| Air temperature (°C) | – | 0.18 | – | 0.16 |
| Wind speed (m/s) | – | 0.18 | – | 0.17 |
| Wind direction (°) | – | 0.58 | – | 0.66 |
| Solar radiation (W/m ²) | – | 2.76 | – | 3.30 |

measurement data is detected, i.e., $r_{n,max} > 4$. In contrast, the proposed method can provide reliable estimations for all Monte-Carlo simulations in this test case.

For the modified IEEE 118-bus system, the difference of bus voltage phasors (phase a) between the estimated state variables and their actual values in one simulation is shown in Fig. 11. The errors of tap and leakage reactance of each transformer are shown in Fig. 12, where the parameters no. 1 to 9 in this figure correspond to the three-phase wye-delta grounding transformers connecting buses 8-5, 26-25, 30-17, 38-37, 63-59, 64-61, 65-66, 68-69, and 81-80, respectively.

Additionally, the histograms of the objective function and $r_{n,max}$ are shown in Fig. 13. Similar to the modified IEEE 30-bus system, one can notice that the conventional method gives unreliable estimated states. Its solution is detected as there exists bad data in the measurement data in all simulations, while only about 1 percent of the estimations obtained by the proposed method show

Table 2: Average computation time and iteration numbers.

| Test system | Conven. | | Proposed. | |
|--------------|---------|-----------|-----------|-----------|
| | Time | Iteration | Time | Iteration |
| IEEE 30-bus | 3.74 | 13.00 | 10.53 | 16.63 |
| IEEE 118-bus | 12.36 | 13.89 | 32.17 | 18.16 |

incorrect bad data detection.

In this case, the comparison of RMSE for both three-phase test systems is provided in Table 1. The average computation time (in seconds) and the average number of iterations required for the proposed and the conventional methods are summarized in Table 2. From these results, it is noted that the proposed method yields better accurate estimations, but it requires more computational effort than the conventional approach. Nevertheless, one may improve the computational burden by utilizing a better computing platform.

6. CONCLUSION

This paper has presented the temperature-dependent three-phase SE considering the transformer parameter states. The proposed method uses a set of traditional measurements obtained from RTUs and PMUs without requiring additional tap and line temperature measurements. The formulation of the SE as a constrained nonlinear optimization problem has been described. Two modified three-phase test systems have been used to demonstrate the effectiveness of the proposed SE. The results achieved from the proposed method have been compared with those obtained from the conventional method, in which all line temperatures are fixed. Comparison results show that the proposed method can improve the accuracy of the estimated state variables of voltage buses and the transformer parameters. Moreover, the estimated conductor temperatures of all transmission lines can be achieved. In future works, dynamic modeling of the temperature-dependent SE for three-phase power systems operating under uncertainties of renewable energy sources and various loading conditions should be explored.

APPENDIX A

Sub-admittance matrices of three-phase transformer

The sub-admittance matrices in (1) depend on the winding configuration of both primary and secondary sides. Their values are given as shown in Table A.1 [30]. The submatrices in this table are defined as follows:

$$y_I = \begin{bmatrix} \vec{y}_t & 0 & 0 \\ 0 & \vec{y}_t & 0 \\ 0 & 0 & \vec{y}_t \end{bmatrix}, \quad y_{II} = \frac{1}{3} \begin{bmatrix} 2\vec{y}_t & -\vec{y}_t & -\vec{y}_t \\ -\vec{y}_t & 2\vec{y}_t & -\vec{y}_t \\ -\vec{y}_t & -\vec{y}_t & 2\vec{y}_t \end{bmatrix}$$

and

$$y_{III} = \frac{1}{\sqrt{3}} \begin{bmatrix} -\vec{y}_t & \vec{y}_t & 0 \\ 0 & -\vec{y}_t & \vec{y}_t \\ \vec{y}_t & 0 & -\vec{y}_t \end{bmatrix},$$

where \vec{y}_t is the leakage admittance per phase in per unit.

Table A.1: Sub-admittance matrices.

| Primary | Secondary | y_{pp} | y_{ss} | y_{ps} | y_{sp} |
|--------------|--------------|----------|-----------|-----------|-------------|
| wye-grounded | wye-grounded | y_I | y_I | $-y_I$ | $-y_I$ |
| wye-grounded | wye | y_{II} | y_{II} | $-y_{II}$ | $-y_{II}$ |
| wye | wye-grounded | y_{II} | y_{II} | $-y_{II}$ | $-y_{II}$ |
| wye | wye | y_{II} | y_{II} | $-y_{II}$ | $-y_{II}$ |
| wye-grounded | delta | y_I | y_{III} | y_{III} | y_{III}^T |
| wye | delta | y_{II} | y_{III} | y_{III} | y_{III}^T |
| delta | wye | y_{II} | y_{III} | y_{III} | y_{III}^T |
| delta | wye-grounded | y_{II} | y_{III} | y_{III} | y_{III}^T |
| delta | delta | y_{II} | y_{II} | $-y_{II}$ | $-y_{II}$ |

y_{III}^T is transpose of y_{III} .

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