Point Estimate for Optimizing Single-Phase PV Placement to Mitigate Voltage Unbalance

Bongkoj Sookananta[†] and **Mongkol Pusayatanont**, Non-members

ABSTRACT

This research addresses the challenge of mitigating voltage unbalance in low-voltage distribution networks using single-phase photovoltaic (PV) systems. Voltage unbalance significantly impacts power quality, leading to increased power losses and equipment inefficiencies. The proposed method utilizes heuristic optimization to identify the optimal placement, phase, size, and power factor of PV systems, aiming to minimize both voltage unbalance and neutral line power losses. The Point Estimate (PE) method is employed for handling demand variations, and its computational efficiency is compared to the Monte Carlo Simulation (MCS) approach.

The effectiveness of the methodology is validated through numerical case studies on 29-bus and 104-bus real-world distribution networks. In the 29-bus network, deploying two single-phase PV systems achieves a VUF reduction of up to 74.4% and a decrease in neutral line power losses of up to 81.7%, with PE requiring only 87 simulations. Similarly, in the 104-bus network, the installation of two PV systems reduces the VUF by up to 55.6% and neutral line power losses by up to 47.4%, with PE requiring 175 simulations. In contrast, MCS requires up to 400 iterations for comparable results, underscoring PE's computational efficiency in achieving effective voltage unbalance mitigation.

Keywords: Voltage unbalance, Point of estimate, Monte Carlo simulation, Optimization, Single-phase PV

1. INTRODUCTION

Voltage unbalance in distribution networks is a significant concern due to its potential to cause equipment malfunctions, reduced efficiency, and increased operational costs. It can lead to power quality issues such as power loss, reverse power flow, and voltage fluctuations [1]. This issue is further intensified by the growing penetration of single-phase photovoltaic (PV) systems and other single-phase loads, such as residential appliances, which can disrupt networks originally designed for three-phase operation. However, single-phase PV systems can also be strategically employed to mitigate these unbalance conditions. By optimally locating and phasing these systems, and adjusting their output using advanced strategies, PV systems can effectively balance the load across different phases.

Recent studies have explored various strategies to mitigate voltage unbalance in distribution systems, particularly through integrating renewable energy sources. Girigoudar et al. (2020) developed a three-phase AC optimal power flow (OPF) formulation to minimize voltage unbalance using reactive power from solar PV inverters, emphasizing the need to consider multiple objectives and constraints [2]. Strategies like deploying Battery Energy Storage Systems (BESS) to reduce curtailment also impact voltage unbalance [3]. Optimal sizing of PV and BESS influences distribution grid stability and voltage unbalance [4]. Additionally, Vanin et al. (2022) proposed an OPF-based demand management strategy to address congestion in unbalanced residential networks [5].

Mitigation strategies for voltage unbalance have traditionally included load balancing, power factor correction, and advanced monitoring techniques. Load balancing, often achieved manually through load redistribution or phase swapping, remains an effective but disruptive method, as it requires temporary disconnection of loads [6]. While automatic systems reduce these disruptions, they are not entirely free from service interruptions. Power factor correction using capacitors, synchronous condensers, and advanced devices like Static Var Compensators (SVC) or Dynamic Voltage Restorers (DVR) offers a dynamic and more seamless solution for maintaining reactive power balance [7], [8]. However, their functionality is restricted to reactive power compensation without the ability to generate energy. Additionally, these systems lack energy storage capabilities and the flexibility to adapt to long-term changes in demand, which limits their effectiveness in modern grids driven by renewable energy. In contrast, the modular nature of PV systems allows for gradual and scalable expansion to meet real-time network needs. Scaling SVC/DVR systems to address increasing voltage demands can be costly and complex, often requiring substantial infrastructure upgrades.

In response to these challenges, this paper proposes the integration of single-phase PV systems with dynamically adjustable power factors. These systems, either standalone or coupled with BESS, offer a flexible solution to mitigate voltage unbalance without service

Manuscript received on August 27, 2024; revised on December 6, 2024; accepted on January 13, 2025. This paper was recommended by Associate Editor Chawasak Rakpenthai.

The authors are with Department of Electrical and Electronics Engineering, Faculty of Engineering, Ubon Ratchathani University, Ubonratchathani, Thailand.

[†]Corresponding author: bongkoj.s@ubu.ac.th

Digital Object Identifier: 10.37936/ecti-eec.2525232.255672

disruptions. By dynamically adjusting their output in response to real-time conditions, such systems can address the time-varying nature of electricity demand and voltage unbalance. The implementation of Advanced Metering Infrastructure (AMI) is essential for monitoring and controlling these real-time variations [6].

Given the inherent uncertainty in voltage unbalance conditions, it is crucial to consider probabilistic methods in the mitigation strategy. Monte Carlo Simulation (MCS) is widely used in power systems for uncertainty analysis, including steady-state security assessment with variable generation resources [9], modeling stochastic behavior of electrical elements as probability distributions, optimizing preventive maintenance schedules in isolated systems [10] and quantifying uncertainty in stochastic economic dispatch with wind power variability [11]. Despite its accuracy, MCS is computationally expensive, limiting its practical application in real-time scenarios.

The Point Estimate (PE) method presents a more computationally efficient alternative to MCS. Though less detailed, the PE method provides quick insights into specific scenarios by using fixed representative values for variables such as load and generation. It has been effectively applied to probabilistic load flow analysis , renewable generation systems [12], [13], power tracking [14] and parameter estimation [15]. This paper applies the PE method to voltage unbalance mitigation in distribution networks, offering a simplified yet effective approach to uncertainty management.

This paper presents a comprehensive strategy for mitigating voltage unbalance using single-phase PV systems, with a focus on addressing uncertainties in distribution networks. By integrating the computational efficiency of the Point Estimate (PE) method with interval estimates and optimization techniques, this approach offers a robust alternative to the Monte Carlo Simulation (MCS). A comparative analysis of PE and MCS through numerical studies will evaluate the effectiveness and reliability of PE as a time-efficient solution for voltage unbalance mitigation.

This paper is structured as follows: Section 2 addresses the problem formulation, while Section 3 outlines the optimization techniques utilized. Section 4 details the unbalanced power flow calculation, followed by Section 5, which introduces the PE method, Section 6 explains the MCS. Section 7 presents the case studies and finally, Section 8 concludes the paper.

2. PROBLEM FORMULATION

The objective of this study is to determine the optimal location and operation of photovoltaic (PV) systems in terms of their connection bus and phase, and output power to minimize voltage unbalance in the distribution system as shown in Fig. 1.

$$\text{Minimize} \quad VUF(x) \tag{1}$$

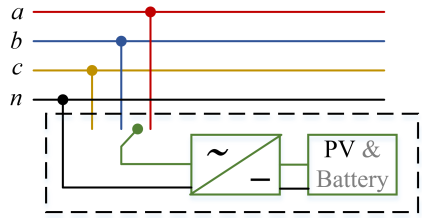

***Fig. 1**: Voltage Unbalance Mitigation System.*

where VUF is the voltage unbalance factor and x are variables including bus of connection PV, the power factor and apparent power output from PV.

The voltage unbalance factor, expressed as a percentage, can be calculated as follows.

$$VUF = \frac{max\left(\left|V_{an} - V_{avg}\right|, \left|V_{bn} - V_{avg}\right|, \left|V_{cn} - V_{avg}\right|\right)}{V_{avg}} \tag{2}$$

where V_{an}, V_{bn}, and V_{cn} are phase A, B, C voltage magnitude and V_{avg} is the average of phase voltage magnitude.

$$V_{avg} = \frac{\left|V_{an}\right| + \left|V_{bn}\right| + \left|V_{cn}\right|}{3} \tag{3}$$

This VUF calculation method relies on voltage magnitudes, minimizing computational complexity and enabling faster, real-time assessments. Although it is less precise than the ratio of the negative-sequence voltage (V2) to the positive-sequence voltage (V1), V2/V1 method (IEC 60034-26), it offers adequate accuracy for detecting moderate unbalances. This makes it particularly suitable for low-voltage networks and quick diagnostics. In practice, it is often preferred over the IEC 60034-26 method due to practical limitations, as the method requires both voltage magnitudes and phase angles data that may not always be available from typical distribution network meters [16]. It is considered most appropriate to calculate the VUF for three-phase four-wire low-voltage (LV) distribution networks [17].

In addition to mitigating voltage unbalance, the appropriate installation of single-phase PV systems can also contribute to reducing power losses caused by unbalanced conditions, particularly losses in the neutral line. To further enhance energy efficiency, the objective function can be formulated as follows:

$$\text{Minimize} \quad f(x) = VUF + P_N \tag{4}$$

$$P_N = \sum_{l=1}^{N_{br}} I_{N,l}^2 R_{N,l} \tag{5}$$

where P_N is the total power losses in neutral lines, $I_{N,l}$ is current flow in neutral line l, $R_{N,l}$ is the resistance of neutral line l.

The constraints for this study include maintaining power balance for each phase, which is determined by performing unbalanced power flow calculations, and adhering to the voltage unbalance factor (VUF) limits specified by international standards.

Various standards provide guidelines for voltage unbalance factor (VUF) limits to ensure the reliability and efficiency of electrical systems [17]. Globally adopted standards, such as IEC and IEEE 241-1990, recommend a VUF of less than 2%.

This study aims to achieve voltage unbalance mitigation strategies that maintain a maximum VUF limit of 2%, emphasizing global consistency in voltage quality and electrical safety. Collectively, these standards underscore the importance of maintaining voltage unbalance within acceptable limits to promote the stability and performance of power distribution networks.

3. OPTIMIZATION TECHNIQUE

The solution to the problem in this paper is approached using heuristic optimization techniques. Heuristic optimization, which often mimics natural evolutionary behavior, provides solutions to complex problems through methods such as genetic algorithms and particle swarm optimization [18]. These techniques have been applied across various engineering fields, including inverter control and load shedding strategy [19].

This paper utilizes Genetic Algorithm (GA) optimization [20], a method developed before many other optimization techniques, which has proven to be highly effective over the years. Despite being relatively slower, GAs offer several advantages, making them a preferred choice for complex problems. They are robust and adaptable to various optimization challenges, such as non-linear and multi-modal issues. GAs efficiently explore large solution spaces by evaluating multiple potential solutions simultaneously, increasing the likelihood of finding the global optimum and avoiding local optima. Their flexibility in handling different types of objective functions and constraints allows them to be applied across diverse fields, from engineering design to machine learning. Moreover, GAs can adapt to changing environments, making them suitable for real-time applications. Their ease of implementation and scalability further enhance their appeal, enabling them to tackle complex problems by simply adjusting parameters like population size and mutation rates. Overall, GAs consistently deliver high-quality solutions, making them a reliable and effective optimization method in the engineering domain.

The Genetic Algorithm (GA) process can be summarized in three key steps:

Step 1. Initialization and Evaluation: Generate an initial population of potential solutions and evaluate their fitness using a predefined fitness function.

Step 2. Selection, Crossover, and Mutation: Select parent solutions based on their fitness, perform crossover to combine genetic information and create offspring then apply mutation to introduce small random changes and maintain genetic diversity.

Step 3. Replacement and Termination: Generate a new population by replacing some or all of the current population with offspring, and continue this process until the termination criterion is met, which, in this paper, is the achievement of a specified maximum number of generations.

4. UNBALANCED POWERFLOW CALCULATIONS

This paper employs the algorithm described in [21] to calculate power flow in a four-wire distribution network. The nodal current at each bus is determined using demand and shunt element data, as expressed by the following equations.

$$\boldsymbol{I}_i^{(v)} = \begin{bmatrix} \left(S_{ia}/V_{ia}\right)^{(v-1)} \\ \left(S_{ib}/V_{ib}\right)^{(v-1)} \\ \left(S_{ic}/V_{ic}\right)^{(v-1)} \\ -Z_{ng}\left(I_{ia}+I_{ib}+I_{ic}\right)^{(v)} \\ -Z_{gn}\left(I_{ia}+I_{ib}+I_{ic}\right)^{(v)} \end{bmatrix} - \boldsymbol{Y}_i\boldsymbol{V}_i^{(v-1)} \tag{6}$$

where

I_{ia}, I_{ib}, I_{ic} are phase A, B, C current at bus i.

I_{in} and I_{ig} are neutral and ground current at bus i respectively.

V_{ia}, V_{ib}, V_{ic} are phase A, B, C voltages at bus i repectively.

v represents the v th iteration number.

Z_{ng} and Z_{gn} are factors that determine the division of current between the neutral line and the ground.

$\boldsymbol{V}_i$ is 5x1 matrix representing the voltages at bus i, including phase A, phase B, phase C, neutral and ground voltages.

$\boldsymbol{I}_i$ is 5x1 matrix representing the currents at bus i, including phase A, phase B, phase C, neutral, and ground currents.

$\boldsymbol{Y}_i$ is the shunt admittance diagonal matrix for bus i, containing the admittances of phase A, phase B, phase C and neutral. It is important to note that ground shunt admittance, Y_{ig} is zero.

The method based on the forward and backward sweep considering ground return of the distribution network. The backward sweep step calculates the line flow current by summation of downstream nodal current. The forward sweep step calculates the bus voltage where the receiving end bus voltage is equal to the sending bus voltage minus the voltage drop across line.

$$\boldsymbol{V}_j^{(v)} = \boldsymbol{V}_i^{(v)} - \boldsymbol{Z}\boldsymbol{J}_l^{(v)} \tag{7}$$

where $\boldsymbol{V}_j$ is 5x1 matrix representing the voltages at the receiving end bus j, including phase A, phase B, phase C, neutral, and ground voltages and $\boldsymbol{J}_l$ is 5x1 matrix representing the current flow in line l, including phase A, B, C, neutral and ground currents.

The power flow calculation process stops once the solution has converged. The condition for convergence is defined as follows:

$$\boldsymbol{V}_i^{(v)} - \mathbf{V}_i^{(v-1)} < tol \tag{8}$$

where *tol* is the tolerance level representing the acceptable difference between the voltage at bus i in the current iteration v and the previous iteration $v-1$

5. STOCHASTIC MODELING TECHNIQUES

5.1 Point of Estimate

The point estimate (PE) technique is a fundamental statistical method used to approximate population parameters from sample data by providing a single value, known as the point estimate [22]. This technique is widely used across various disciplines, including economics, engineering, and social sciences, due to its straightforward approach to making inferences about a population based on sample observations. In this study, the PE method is employed to model the stochastic behavior of demand fluctuations and renewable energy sources. It provides a computationally efficient means of representing uncertainty in these variables, allowing for the analysis of their impact on system performance, particularly on voltage unbalance and power losses in neutral lines.

This paper employs the $2M+1$ scheme of Hong's point estimate [23], which leverages statistical moments such as mean, variance, skewness, and kurtosis to effectively represent the stochastic behavior of variables. The method condenses the statistical information of each input random variable into two concentration points for each electricity demand variable. As a result, the objective function of the optimization problem requires only two evaluations for each of the M input random variables, along with one additional evaluation at the mean value of each variable.

The point or location of variables can be determined as follows.

$$p_{m,k} = \mu_{p_m} + \xi_{m,k}\sigma_{p_m} \tag{9}$$

where $p_{m,k}$ is the location of m variables at k concentration point ; m = 1, 2, ..., M and k = 1, 2.

μ_{p_m} and σ_{p_m} are the mean and standard deviation of the input random variables.

$\xi_{m,k}$ is the standard deviation multiplier.

$$\xi_{m,k} = \frac{\lambda_{m,3}}{2} + (-1)^{3-k}\sqrt{\lambda_{m,4} - \frac{3}{4}\lambda_{m,3}^2} \tag{10}$$

where $\lambda_{m,3}$ and $\lambda_{m,4}$ are skewness and kurtosis of the m input random variables respectively.

Considering demand as variables, the point of estimate is applied to determine the single phase PV placement and corresponding size to mitigate the unbalance voltage in distribution network using the following algorithm:

Step 1: Input Distribution Network Data: Begin by inputting the data related to the distribution network.

Step 2: Input Demand Characteristics: Input the mean, standard deviation, skewness and kurtosis

Step 3: Determine the optimal solution (placement and size of the single phase PV) for the mean value of variable demand

Step 4: Determine the locations of variables for k equals one and two, then find the optimal solutions for those locations

This process results in $2M+1$ optimal solutions, one from step 3 and $2M$ from step 4.

5.2 Monte Carlo Simulation

Monte Carlo Simulation (MCS) is a robust statistical method used to model and analyze complex systems through random sampling and statistical modelling [24]. The fundamental principle of this technique involves generating numerous random samples from a probability distribution that captures the uncertainty in the model's input variables. These samples are then utilized to compute the model's outcomes, yielding a distribution of possible results instead of a single deterministic outcome. This approach aids decision-makers in evaluating risk and making more informed choices by providing a comprehensive understanding of potential variability and uncertainty in the system.

To validate the performance of the PE method, this paper employs MCS to analyze the stochastic behavior of demand in a distribution network (DN). The various states of the DN are examined until the convergence criterion is satisfied. The MCS process terminates when the mean and standard deviation of the random samples fall below the specified tolerance level.

To apply MCS to this problem, the following algorithm is used:

Step 1: Input Distribution Network Data: Begin by inputting the data related to the distribution network.

Step 2: Input Demand Characteristics: Input the mean, standard deviation, and probability distribution function (pdf) of the demand at each bus and phase.

Step 3: Generate Random Demand Values: Use the pdf to generate random demand values.

Step 4: Determine Optimal PV Parameters: Identify the optimal location and phase for installation, as well as the apparent power and operational power factor of the PV inverter output.

Step 5: Check for Convergence: Verify if the convergence criterion is met. If the mean and standard deviation of the samples are below the tolerance level, the process stops.

6. CASE STUDIES

Two case studies are conducted using real-life low-voltage (LV) distribution networks to demonstrate the effectiveness of the proposed technique. The first case study utilizes a 29-bus network, which its original network data can be found in [21], while the second focuses on a 104-bus network located in the downtown area of Ubon Ratchathani, Thailand. These case studies aim to illustrate the applicability of the proposed method under varying network conditions.

In the analysis, single-phase PV systems owned by the utility are considered, functioning specifically to mitigate

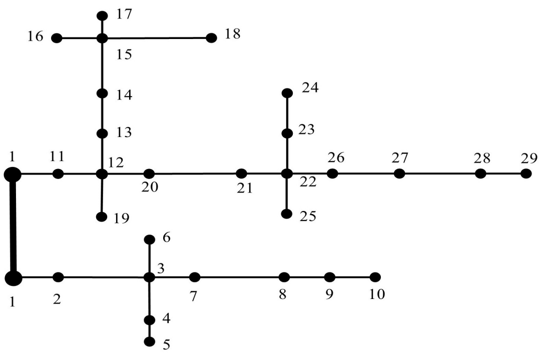

Fig. 2: The 29-bus LV Distribution Network.

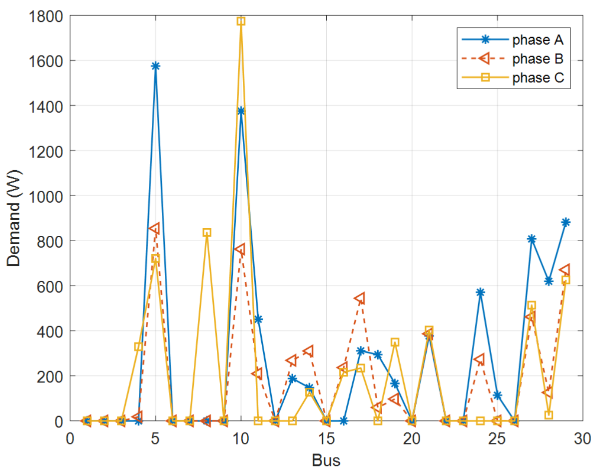

Fig. 3: Load Demand of Case Study 1.

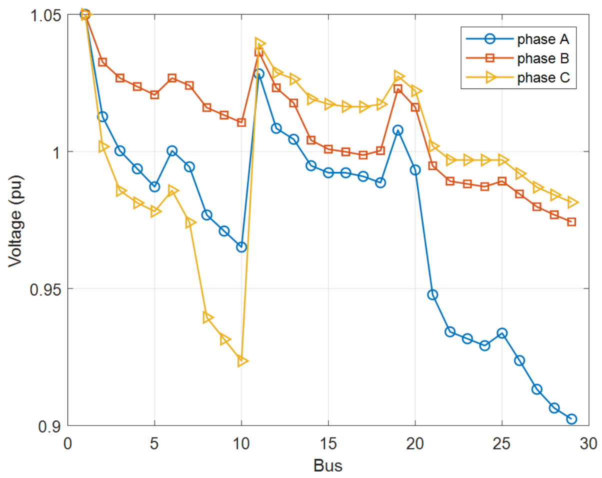

Fig. 4: Base-case Phase Voltage of Case Study 1.

voltage unbalance. According to the specifications of commercially available single-phase PV inverters, the maximum capacity for a single-phase PV installation is set at 10 kVA. However, for locations where the demand rating is less than 10 kVA, the maximum allowable PV size is limited to the demand rating of the respective phase in the distribution network. This ensures that the PV installations are appropriately sized for effective voltage unbalance mitigation while remaining within the operational limits of the network.

6.1 Case Study 1

The 29-bus LV radial distribution network, depicted in Fig. 2, is modified to operate at a base voltage of 400/230 V, with a reference voltage of 1.05 pu at the root node. A constant power load model is employed for simplicity as shown in Fig. 3. The total loads are 7.88 kW for phase A, 5.28 kW for phase B, and 6.16 kW for phase C, with a power factor of 0.95 lagging across all loads. The network is divided into two main sub-feeders: sub-feeder 1, covering buses 2 to 10, and sub-feeder 2, covering buses 11 to 29.

This case study is divided into two sub-cases: one without customer-installed single-phase PV systems and another with customer-installed PV systems, assumed to be allowed to inject power into the network. Typically, customer-installed PV systems operate at a unity power factor, generating revenue through real power injection. However, in networks experiencing low-voltage conditions or increased reactive power demands, operators may require customers to adjust their power factor below unity to improve voltage stability and support reactive power requirements. This adjustment is particularly relevant in systems with significant PV integration, where inverters can be configured to provide reactive power support. For example, the Australian Energy Market Operator (AEMO) mandates that inverters connected to the grid must have the capability to operate at power factors ranging from 0.8 leading to 0.8 lagging, enabling them to supply or absorb reactive power as needed to maintain network stability [25]. For the purposes of this study, customer-installed PV systems are assumed to operate at a unity power factor.

This case study examines the optimal placement and bus connections for utility-owned single-phase photovoltaic (PV) installations to mitigate voltage unbalance in distribution networks. The analysis assumes that utility-owned PV systems are not constrained in their reactive power injection capabilities, thereby enabling maximum flexibility in addressing voltage stability issues. Additionally, the impact of operating under a 0.8 power factor limitation is investigated to assess its influence on the effectiveness of voltage unbalance mitigation.

The voltage profile and voltage unbalance factor (VUF) percentage for the base case of 29-bus LV network are shown in Figures 4 and 5, respectively.

The voltage profile in Figure 4 shows that many buses do not fall within the standard acceptable voltage range of ±5% (0.95 to 1.05 pu). However, all remain within a broader ±10% range. Significant deviations are observed in phase C on sub-feeder 1, where the voltage at the end of the line drops below 0.95 pu. On sub-feeder 2, notable deviations occur primarily in phase A, with the end-of-line voltage dropping to nearly 0.9 pu. In contrast,

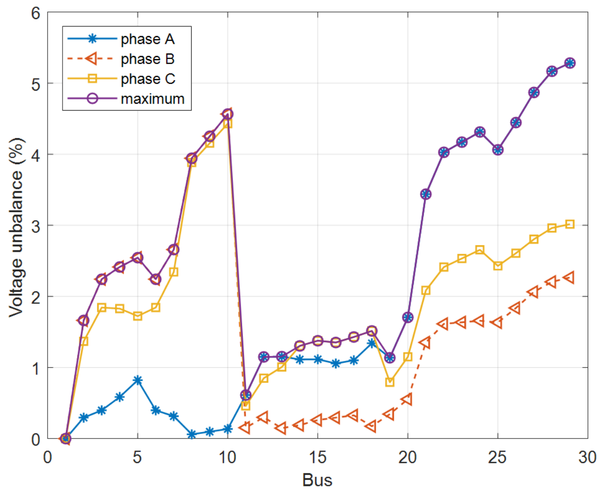

***Fig. 5**: Base-Case VUF of Case study 1.*

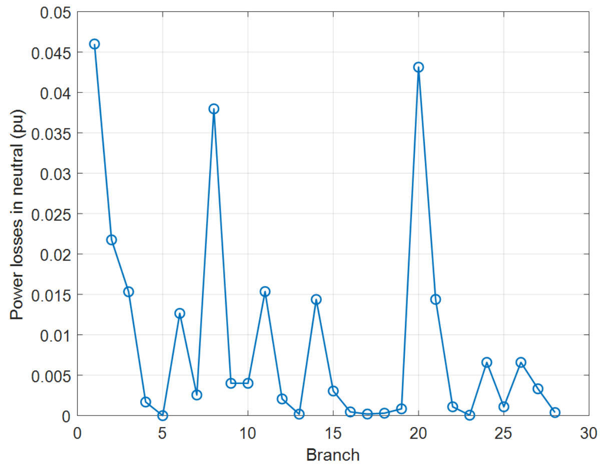

***Fig. 6**: Base-Case Power Losses in Neutral Lines of Case Study 1.*

phases B and C on sub-feeder 2 maintain voltages closer to 1 pu. The substantial voltage disparity across phases, particularly the low voltage in phase A, underscores the presence of voltage unbalance within the network.

The VUF profile of the 29-bus LV distribution network, as presented in Figure 5, reveals significant variations in unbalance levels among phases A, B, and C. Phase A consistently maintains relatively low unbalance levels, remaining below 1% throughout sub-feeder 1. In contrast, phases B and C exhibit higher VUF values on sub-feeder 1, with levels exceeding 4% toward the end of the feeder. The VUF values for all phases tend to increase progressively toward the ends of each sub-feeder.

The maximum VUF line in Figure 5, representing the highest voltage unbalance observed across all phases at each bus, exhibits an increasing trend, reaching a peak of 5.28% at Bus 29 on Sub-feeder 2. Notably, Bus 10 experiences the most significant unbalanced load, contributing to power losses in the neutral line as illustrated in Figure 6. This results in total power losses in the neutral line amounting to 0.2595 pu.

To mitigate the VUF in the network, an analysis was conducted to determine the optimal single-phase PV installations, including their sizing, phase connections and operating power factor of the inverter. The results of this approach are summarized in Table 1.

The deployment of a single-phase PV system with a minimum capacity of 0.182 kVA at Phase A of bus 29 reduces the VUF to 4.56%. However, increasing the size of the PV system at this location does not result in further reductions in VUF, as bus 29 is situated on sub-feeder 2 and has no influence on the VUF of sub-feeder 1, which remains at 4.56%.

With deployment of two single-phase PV systems, the optimal locations are identified as Phase C of bus 9 on sub-feeder 1 and phase A of bus 28 on sub-feeder 2. This configuration reduces the VUF to 2.28%. However, this value still exceeds the limit recommended by the IEC standard.

The optimal reduction in voltage unbalance is achieved through the installation of three single-phase PV systems at buses 9, 10, and 22, which successfully lower the VUF from 5.28% to 0.967%, meeting the standard limit of less than 2%. Notably, the power factors at these locations fall outside the typical operational range of standard inverters, with a power factor of 0.13 at bus 9 and 0.17 at bus 22. However, when inverter operation is constrained to a minimum power factor of 0.8, the VUF can still be effectively reduced to 0.975%. This is achieved by installing PV systems at phase A of bus 7, phase C of bus 10, and phase A of bus 26, with capacities of 1.726 kVA at 0.94 power factor, 1.795 kVA at 0.98 power factor, and 1.924 kVA at 0.81 power factor, respectively.

Although both solutions achieve comparable reductions in the VUF, the higher power factor solution provides additional benefits, primarily by requiring smaller PV capacities. The corresponding voltage profile, power losses in neutral lines, and VUF are illustrated in Figures 7 to 9, respectively.

It is observed that the minimum bus voltage of the network with the higher power factor solution is 0.97 pu, as shown in Figure 7, which is less favorable compared to the lower power factor solution that achieves a better minimum bus voltage of 0.966 pu. Additionally, the total power losses in the neutral lines are reduced to 0.051 pu.

Further increasing the number of deployed PV systems results in only marginal reductions in the VUF. However, with a higher distribution of PV systems across the network, smaller system capacities are required.

The output characteristics of a photovoltaic (PV) system in northeastern Thailand are applied to 1 kW single-phase PV systems at phase A of bus 10 and bus 29. As shown in Figure 10, the normalized probability distribution indicates a mean output of 0.48 kW, with a skewness of -0.04, a kurtosis of 1.75, and a standard deviation of 0.3 kW.

With customer-installed PV systems, the voltage

***Table 1**: The Optimal PV Systems for Case Study 1.*

No. of PV	Bus	Phase	Size (kVA)	PF	VUF (%)
1	29	A	0.182	0.88	4.56
2	9, 28	C, A	1.69, 1.912	1, 0.87	2.28
3	9, 10, 22	A, C, A	1.643, 1.90, 3.624	0.13, 0.94, 0.17	0.967
4	27, 12, 7, 8	A, B, A, C	2.063, 2.906, 2.331, 2.4	0.91, 0.14, 0.42, 0.73	0.506
5	7, 8, 11, 13, 26	A, C, A, B, A	1.723, 2.589, 0.935, 1.374, 2.076	0.88, 1.00, 0.88, 0.95, 1.00	0.387
6	5, 10, 10, 13, 20, 28	A, A, C, B, A, A	0.68, 0.771, 1.998, 1.413, 1.282, 1.43	0.91, 0.93, 0.91, 0.98, 0.97, 0.92	0.303

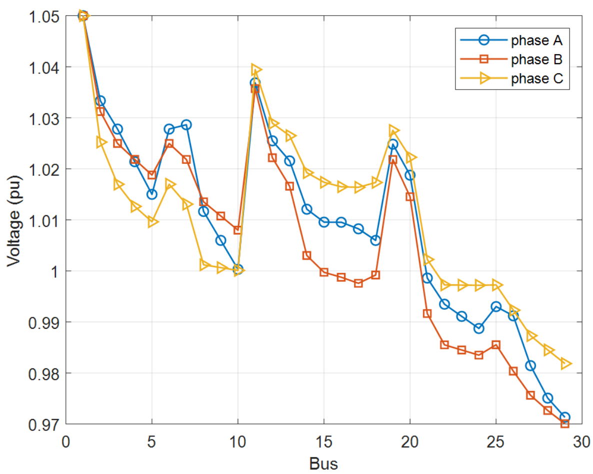

***Fig. 7**: Phase Voltage of Case Study 1 with 3 PV Systems.*

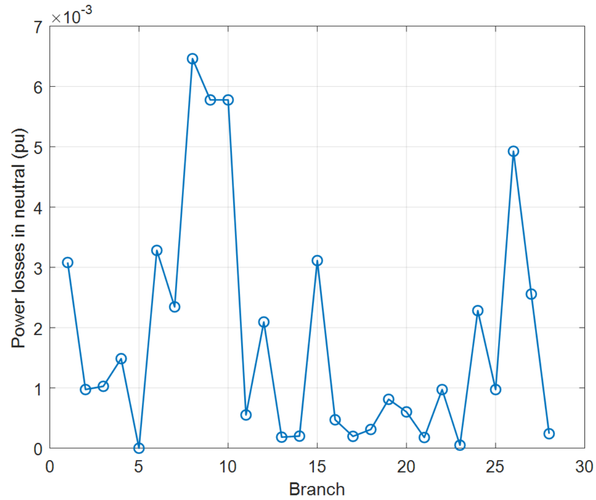

***Fig. 8**: Power Losses in Neutral of Case Study 1 with 3 PV Systems.*

profile of the network is altered, as shown in Figure 11, with the VUF reaching 5% at phase C of bus 10, as illustrated in Figure 12. The total neutral line losses are 0.196 pu.

The optimal configuration for a utility-owned PV system is revised to a 0.4 kVA system with a power factor of 0.9, installed at phase C of bus 10. This configuration reduces the VUF to 3.67% and results in neutral line losses of 0.147 pu. However, this solution is still considered suboptimal as it exceeds the standard recommended limit.

To achieve the minimum combined sum of the VUF and neutral line losses, the PV size must be increased to 1.53 kVA with a power factor of 0.85. This adjustment yields a VUF of 3.67% and reduces the total neutral line losses to 0.093 pu, as summarized in Table 2.

For a configuration involving two utility-owned PV systems, the optimal placement is determined to be at phase C of bus 10 and phase A of bus 27. The corresponding capacities are 1.694 kVA with a power factor of 0.96 and 1.28 kVA with a power factor of 0.88, respectively. This configuration effectively reduces the Voltage Unbalance Factor (VUF) to 1.28%, representing a 74.4% reduction, achieving the target standard of

***Table 2**: The Optimal Single-phase PV Systems for Case Study 1 with Customer-installed PV Systems.*

No. of PV	Bus	Phase	Size (kVA)	PF	VUF (%)	PN (pu)
1	10	C	1.53	0.85	3.67	0.093
2	10, 27	C, A	1.694, 1.28	0.96, 0.88	1.28	0.036
3	5, 8, 26	A, C, A	0.75, 1.918, 1.475	0.96, 0.94, 0.81	0.98	0.024

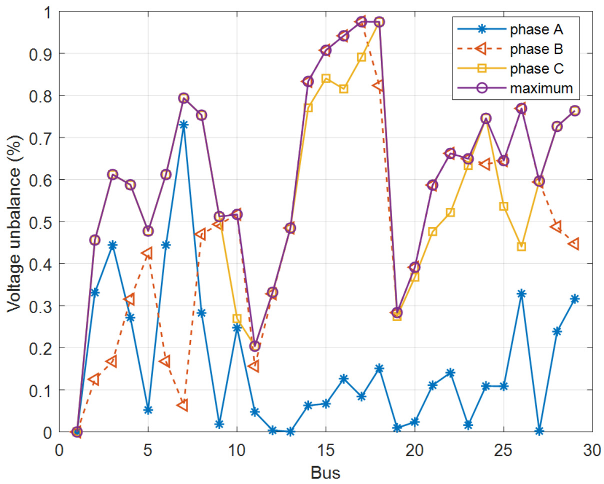

***Fig. 9**: VUF of Case Study 1 with 3 PV Systems.*

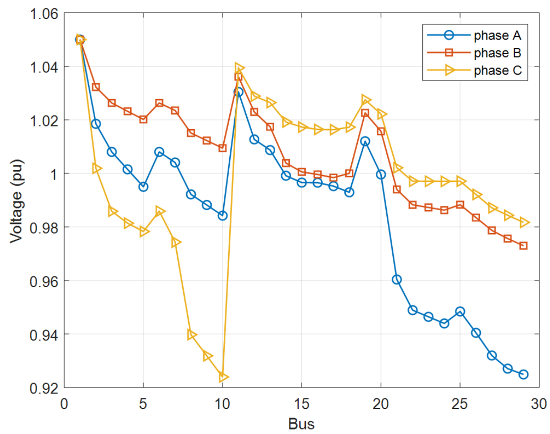

***Fig. 11**: Phase Voltage of Case Study 1 with Customer-installed PV Systems.*

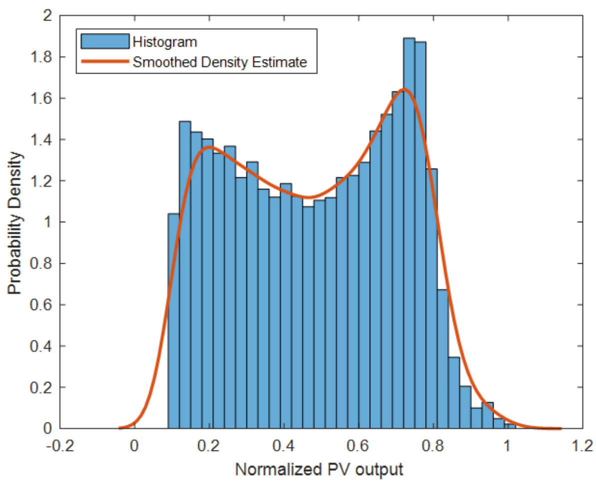

***Fig. 10**: Normalized Distribution of PV System Output.*

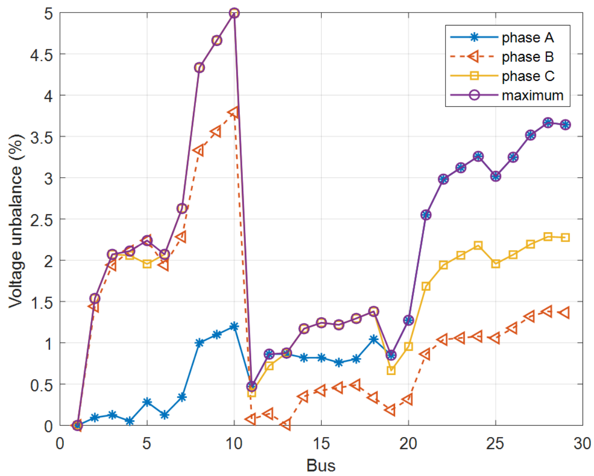

***Fig. 12**: VUF of Case Study 1 with Customer-installed PV Systems.*

VUF below 2%, in line with recommended guidelines. Additionally, this setup reduces neutral line power losses to 0.036 pu, reflecting an 81.7% reduction.

For PV systems with output characteristics depicted in Fig. 10, where the average output is 48.2% of the rated capacity, the required minimum capacities to achieve these results are 3.4 kVA and 2.6 kVA for the two PV systems, respectively.

To assess the effect of demand variation on voltage unbalance mitigation strategies using a single-phase PV system, the load demand power is modelled by adopting distribution characteristics from the Provincial Electricity Authority (PEA) of Thailand [26] as given in Table 3. The mean values of the load demand are set equal to the base case data.

It is essential to note that when evaluating a PV system without battery storage, the demand distribution characteristics are analyzed solely using daytime data. However, if sufficient battery capacity is available to

***Table 3**: The Optimal Single-phase PV Systems for Case Study 1 with Customer-installed PV Systems.*

Demand location	SD	Skewness	Kurtosis
Bus 1 − 17 phase A	0.064	-0.07	1.83
Bus 1 − 17 phase B	0.08	-0.10	1.655
Bus 1 − 17 phase C	0.07	0.71	2.94
Bus 18 − 29 phase A	0.04	1.02	3.76
Bus 18 − 29 phase B	0.09	0.36	2.28
Bus 18 − 29 phase C	0.05	-0.05	1.98

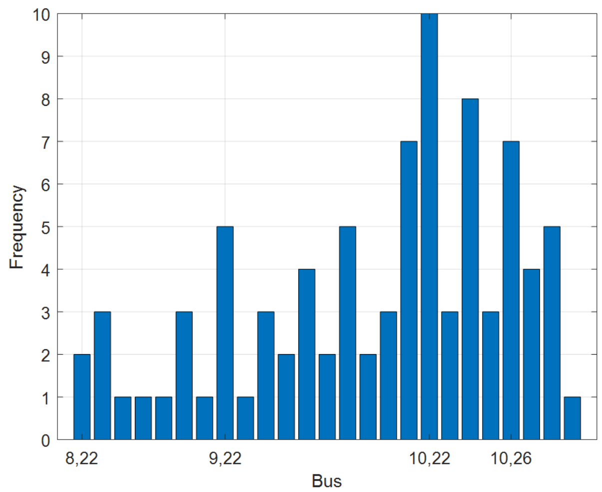

***Fig. 13**: The Optimal Solutions for Case Study 1 with Customer-installed PV Systems using PE.*

ensure continuous energy supply, a 24-hour demand profile should be considered to accurately reflect the system's performance. This consideration, however, is not included in the case studies, as the focus of this study is on analyzing the direct impact of single-phase PV systems in mitigating voltage unbalance.

To address fluctuations in electricity demand within the distribution network, this case study examines 41 variations in demand across phases A, B, and C for buses 1 through 29, focusing exclusively on non-zero demand values. Additionally, two resource variations are considered for customer-installed PV systems. Utilizing the Point Estimate (PE) method with the 2M+1 scheme requires 87 simulations. The results identify the most frequently optimal locations for the PV systems as phase C of bus 10 and phase A of bus 22, as illustrated in Fig. 13. At these locations, the PV system sizes range from 1 to 3 kVA, operating with power factors between 0.1 and 0.97.

Monte Carlo Simulation (MCS) is employed to validate the performance of the Point Estimate (PE) method. The optimal location for the single-phase PV system, as shown in Fig. 14, is determined using MCS. The convergence of MCS is assessed by monitoring the mismatch of the standard deviation (SD), which demonstrates a gradual decline, as depicted in Fig. 15. The results indicate that the optimal solution trends remain

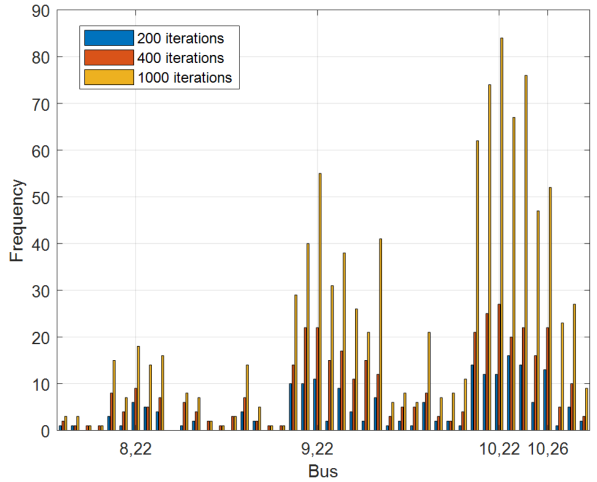

***Fig. 14**: The Optimal Solutions for Case Study 1 with Customer-installed PV Systems using MCS.*

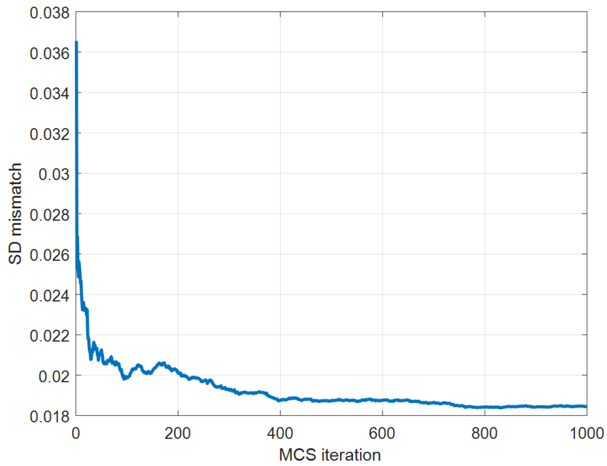

***Fig. 15**: Convergence of MCS.*

consistent across 400 and 1000 iterations, with bus 10 and bus 22 being the most frequently identified optimal locations. The system primarily operates on phase C for sub-feeder 1 and phase A for sub-feeder 2, with an operational capacity range of 0.1 to 4 kVA and a power factor between 0 and 1. The phase operation is designed to be dynamic, facilitated by switching mechanisms to adjust post-installation.

For 200 iterations, the most frequently identified solutions are at bus 10 and bus 26, which do not correspond to the most frequent solutions observed at higher iteration counts. This discrepancy aligns with the convergence curve of MCS, as shown in Fig. 15, where a significant decline in SD is observed at 400 iterations, further supporting the findings.

The results indicate that the optimal placement and phase of operation identified by the PE method align with those obtained through MCS. However, the operational range determined by MCS is broader. This is due to MCS's more comprehensive analysis, which accounts for

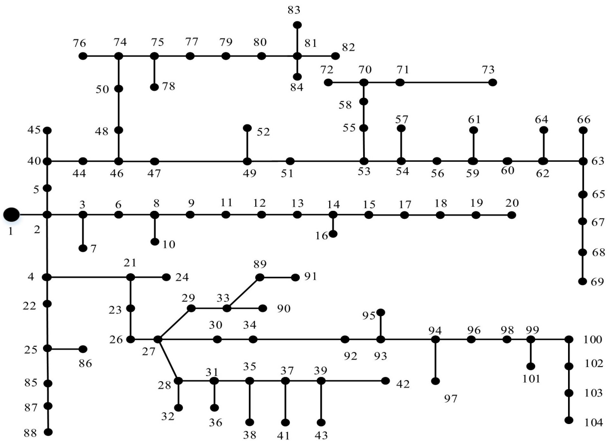

Fig. 16: PEA 104-bus LV Distribution Network.

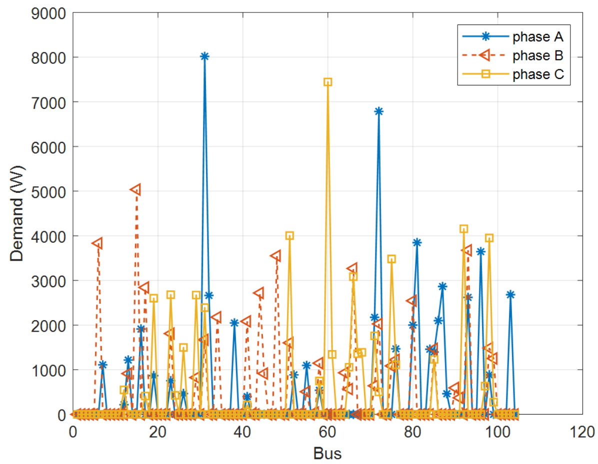

Fig. 17: Load Demand of Case Study 2.

a wide range of possible demand scenarios through random sampling, effectively capturing the variability and uncertainty in the distribution network more thoroughly than the PE method.

6.2 Case Study 2

The real-world 104-bus LV distribution network operated by the PEA of Thailand, functioning at a voltage level of 400/230V, is depicted in Fig. 16. The network's total load comprises 56.6 kW on phase A, 52.82 kW on phase B, and 50.97 kW on phase C, each operating with a lagging power factor of 0.85, as detailed in Fig. 17. Customer-installed single-phase PV systems with a capacity of 10 kW are located at phase B of bus 34 and phase C of bus 75. The probability distribution of the PV output is provided in Fig. 10, with the base-case PV output assumed to be its average value of 4.82 kW.

The phase voltages, power losses in neutral lines, and voltage unbalance for the base case are presented in Figures 18 to 20. The analysis indicates that the majority of voltage unbalance occurs in phase C, with the maximum unbalance of 3.29% observed at bus 66. The total power losses in the neutral line are calculated to be 0.786 pu.

Without considering demand variation, the optimal placement, phase, operational size, and power factor of the single-phase PV system are summarized in Table 4. Deploying a single PV system at bus 63, connected to phase C, with an operational capacity of 9.02 kVA and a power factor of 0.91, reduces the VUF to 2.86% and neutral line power losses to 0.58 pu. Greater compensation is achieved by deploying two PV systems, which further reduce the VUF to 1.46%—a 55.6% reduction—and neutral line power losses to 0.41 pu—a 47.4% reduction. However, increasing compensation beyond this configuration yields only marginal additional benefits.

Furthermore, the deployment of two single-phase PV systems not only effectively mitigates voltage unbalance but also improves voltage magnitudes, as illustrated in

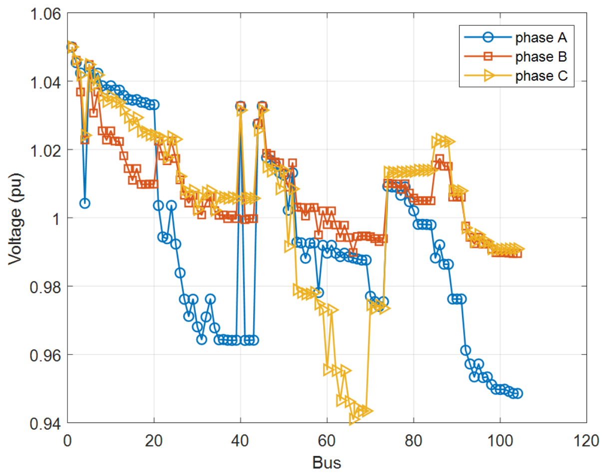

Fig. 18: Base-Case Phase Voltage of Case Study 2.

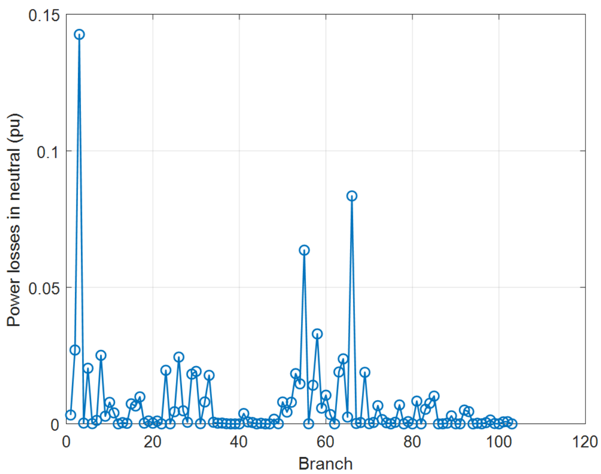

Fig. 19: Base-Case Power Losses in Neutral of Case Study 2.

Figures 21 and 22. This configuration also leads to a reduction in power losses within the neutral lines, as

Table 4*: The Optimal PV Systems for Case Study 2.*

No. of PV	Bus	Phase	Size (kVA)	PF	VUF (%)	PN (pu)
1	63	C	9.02	0.9	2.86	0.58
2	30, 63	A, C	10, 9.1	0.96, 0.91	1.46	0.41
3	31, 67, 87	A, C, A	10, 8.22, 5	1, 0.91, 0.9	1.34	0.32

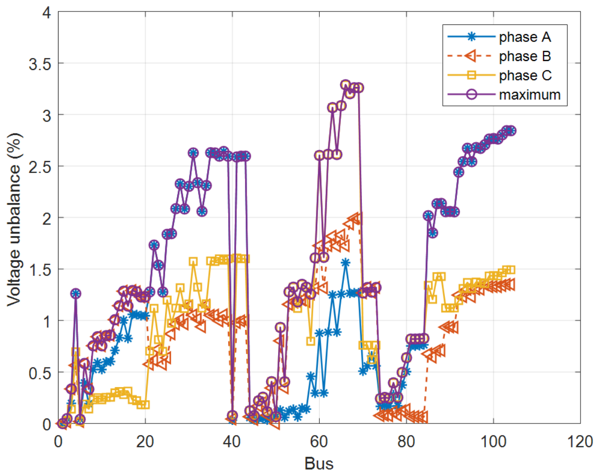

Fig. 20*: Base-Case VUF of Case Study 2.*

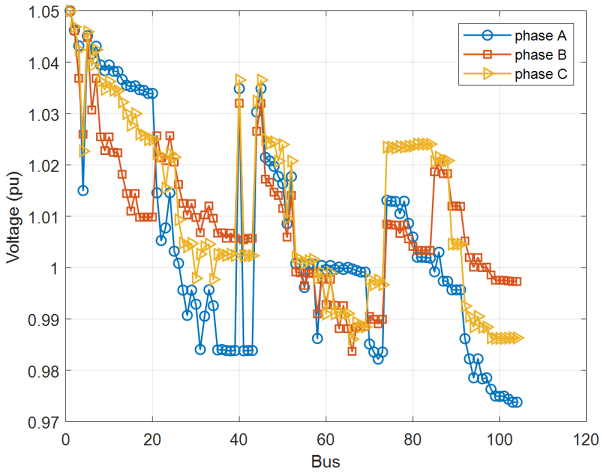

Fig. 21*: Phase Voltage of Case Study 2 with 2 PV Systems.*

detailed in Figure 23.

To present the variation of electricity demand in the 104-bus LV distribution network, the active and reactive power of the load buses are modelled with means equal to the base case data, and standard deviations set as follows: 7% for buses 1 to 36, 4% for buses 37 to 55 and 9% for buses 56 to 104. To capture the high fluctuation in electricity demand, the distribution characteristics are modelled with different skewness values: -0.7 for buses 1 to 36, 0 for buses 37 to 55 and 1 for buses 56 to 104, with a kurtosis of 5 at every bus, reflecting distributions that range from symmetrical to skewed with sharp peaks.

When accounting for demand variation, both PE and MCS consistently identify the optimal compensation, with the most frequent location being phase C of bus 63, as shown in Fig. 24. At bus 63, the operational range determined by PE is 9 to 9.38 kVA at phase C, with an identical operational power factor of 0.9. In contrast, MCS identifies an operational range of 7.98 to 9.67 kVA at phase C, with a power factor ranging from 0.87 to 0.95. Although both methods yield the same most frequently identified solution, MCS provides a broader range of alternative locations, operational PV sizes, and phases.

The results demonstrate that PE is effective in representing load variations within the optimization framework for mitigating voltage unbalance using single-phase PV systems. In this case study, with 85 points of electricity demand and 2 points of renewable resources,

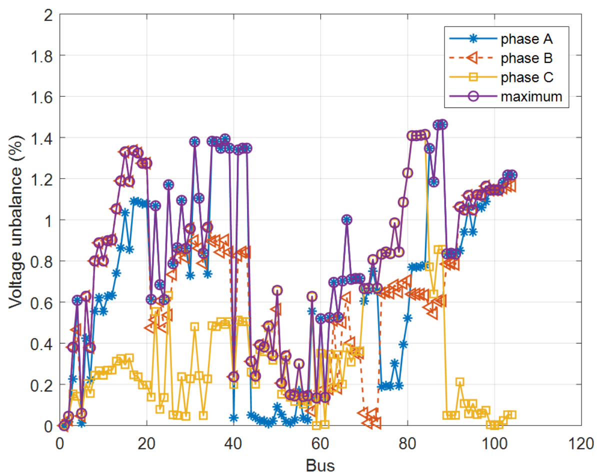

Fig. 22*: VUF of Case Study 2 with 2 PV System.*

PE requires only 175 simulations—a significant reduction compared to the number of simulations required by MCS. However, as the number of load variables increases, the computational effort required for PE may approach that of MCS, potentially reducing its efficiency advantage.

The numerical studies demonstrate the effectiveness of the proposed method in determining the optimal location, operating phase, and capacity of multiple PV systems based on their impact on VUF and neutral power losses, while accounting for dynamic demand and renewable resource behavior. The results show that deploying

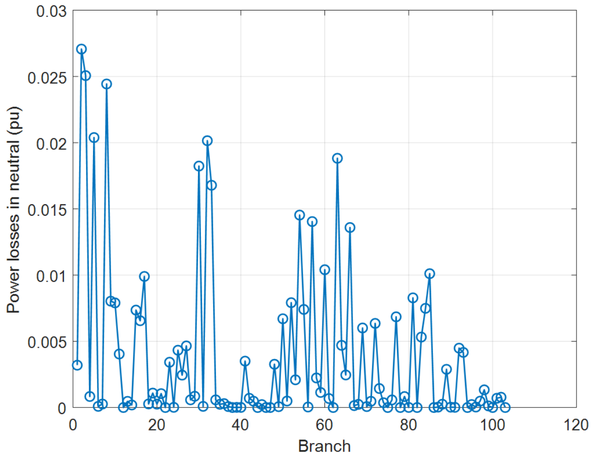

***Fig. 23**: Power Losses in Neutral of Case Study 2 with 2 PV Systems.*

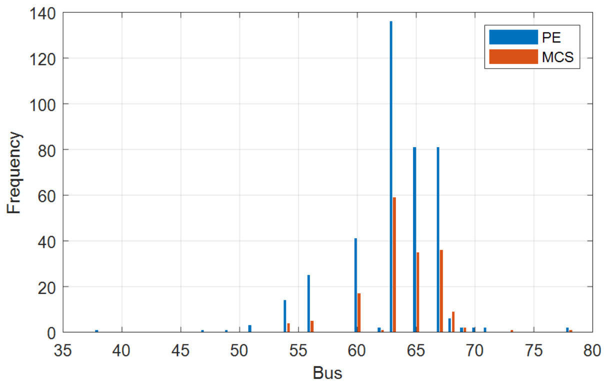

***Fig. 24**: The Optimal Solutions for Case Study 2 using PE and MCS.*

multiple PV systems leads to greater reductions in VUF, and a highly distributed PV system deployment requires smaller individual capacities, providing a more flexible and efficient solution for mitigating voltage unbalance and optimizing resource utilization.

In case study 1, the optimal operating capacities of the PV systems are below their maximum allowable limits and should correspond to the average output. To achieve the desired reduction in VUF and neutral line power losses, the installed capacity of the PV system should match the maximum output, as determined by the probability distribution of PV output in the installation area. In contrast, in Case Study 2, the optimal operating capacity is near the maximum limit of 10 kVA. Consequently, the highest probability of power output from the PV system is lower than the optimal operating capacity. As a result, installing the PV system at the optimal capacity may not lead to the expected reductions in VUF and neutral line power losses. To achieve these reductions, a higher number of PV systems would need to be deployed.

The size of the PV systems directly impacts VUF and neutral line power losses, key components of the objective function. Larger PV systems generally offer better compensation for unbalanced conditions, reducing both VUF and power losses. However, as the results show, this effect diminishes beyond a certain capacity due to network saturation. This highlights the need for optimizing PV sizes to avoid oversizing, which increases costs without proportional benefits. While this study does not include a cost analysis, it is clear that PV installation and operational costs are linked to system size. Practical implementation requires balancing the benefits of reduced VUF and power losses with the additional costs of larger PV systems. This trade-off is crucial for cost-effective deployment, particularly in financially constrained low-voltage distribution networks. Future research could incorporate cost-benefit analysis to enhance decision-making in PV system deployment.

7. CONCLUSION

This study demonstrates the effectiveness of single-phase photovoltaic (PV) systems in addressing voltage unbalance in distribution networks. A significant advantage of the proposed single-phase PV systems is their capability to operate with varying power factors and connection phases, offering enhanced flexibility in network management. Moreover, the compact size of these systems enables their installation near connection buses, simplifying their integration into existing infrastructure. The methodology employs heuristic optimization to identify optimal locations and operational parameters for the PV systems, ensuring that the minimum necessary capacity is utilized to achieve substantial reductions in voltage unbalance and power losses in neutral lines.

The PE method is utilized to account for demand variation, providing a simplified and computationally efficient approach. The performance of PE is benchmarked against MCS, which explores a broader spectrum of potential solutions by comprehensively analyzing demand fluctuations and renewable resource variations. While the PE method effectively addresses demand variation in this context, its computational advantage may diminish in larger systems with an increasing number of load variables, potentially approaching the computational effort required by MCS. Consequently, PE is more appropriate for networks with fewer variation points.

Future research will focus on advancing the proposed approach by integrating strategies to mitigate distribution network unbalance through the adoption of other low-carbon technologies, such as electric vehicle integration. This endeavour will involve the development of sophisticated optimization techniques aimed at achieving more comprehensive enhancements in the stability and operational efficiency of distribution networks.

REFERENCES

[1] B. Uzum, A. Onen, H. M. Hasanien, and S. M. Muyeen, "Rooftop solar pv penetration impacts on distribution network and further growth factors—a

comprehensive review," *Electronics*, vol. 10, no. 1, p. 55, 2020.

[2] K. Girigoudar and L. A. Roald, "On the impact of different voltage unbalance metrics in distribution system optimization," *Electric Power Systems Research*, vol. 189, p. 106656, 2020.

[3] C. Lee, S. Im, J. Jung, and B. Lee, "BESS deployment strategy in Jeju carbon-free islands for reducing renewable energy curtailment," *Energies*, vol. 13, no. 22, p. 6082, 2020.

[4] T. Weckesser, D. F. Dominković, E. M. Blomgren, A. Schledorn, and H. Madsen, "Renewable Energy Communities: Optimal sizing and distribution grid impact of photo-voltaics and battery storage," *Applied Energy*, vol. 301, p. 117408, 2021.

[5] M. Vanin, T. Van Acker, H. Ergun, R. D'hulst, K. Vanthournout, and D. Van Hertem, "Congestion mitigation in unbalanced residential networks with OPF-based demand management," *Sustainable Energy, Grids and Networks*, vol. 32, p. 100936, 2022.

[6] Y.-D. Lee, W.-C. Lin, J.-L. Jiang, J.-H. Cai, W.-T. Huang, and K.-C. Yao, "Optimal individual phase voltage regulation strategies in active distribution networks with high PV penetration using the sparrow search algorithm," *Energies*, vol. 14, no. 24, p. 8370, 2021.

[7] A. M. Nour, A. A. Helal, M. M. El-Saadawi, and A. Y. Hatata, "Voltage imbalance mitigation in an active distribution network using decentralized current control," *Protection and Control of Modern Power Systems*, vol. 8, no. 2, pp. 1–17, 2023.

[8] I. Afandi, A. P. Agalgaonkar, and S. Perera, "Integrated volt/var control method for voltage regulation and voltage unbalance reduction in active distribution networks," *Energies*, vol. 15, no. 6, p. 2225, 2022.

[9] S. Kim and J. Hur, "A probabilistic modeling based on Monte Carlo simulation of wind powered EV charging stations for steady-states security analysis," *Energies*, vol. 13, no. 20, p. 5260, 2020.

[10] Y. S. Duarte, J. Szpytko, and A. M. del Castillo Serpa, "Monte Carlo simulation model to coordinate the preventive maintenance scheduling of generating units in isolated distributed Power Systems," *Electric Power Systems Research*, vol. 182, p. 106237, 2020.

[11] X. Wang, R.-P. Liu, X. Wang, Y. Hou, and F. Bouffard, "A data-driven uncertainty quantification method for stochastic economic dispatch," *IEEE Transactions on Power Systems*, vol. 37, no. 1, pp. 812–815, 2021.

[12] A. R. Abbasi, "Probabilistic load flow based on holomorphic embedding, kernel density estimator and saddle point approximation including correlated uncertainty variables," *Electric Power Systems Research*, vol. 183, p. 106178, 2020.

[13] L. A. Gallego, J. F. Franco, and L. G. Cordero, "A fast-specialized point estimate method for the probabilistic optimal power flow in distribution systems with renewable distributed generation," *International Journal of Electrical Power & Energy Systems*, vol. 131, p. 107049, 2021.

[14] Z. Xie and Z. Wu, "Maximum power point tracking algorithm of PV system based on irradiance estimation and multi-Kernel extreme learning machine," *Sustainable Energy Technologies and Assessments*, vol. 44, p. 101090, 2021.

[15] A. M. Malkawi, A. Odat, and A. Bashaireh, "A Novel PV Maximum Power Point Tracking Based on Solar Irradiance and Circuit Parameters Estimation," *Sustainability*, vol. 14, no. 13, p. 7699, 2022.

[16] M. U. Hashmi, A. Koirala, R. Lundholm, H. Ergun, and D. Van Hertem, "Evaluation of voltage magnitude based unbalance metric for low voltage distribution networks," *in 2022 IEEE Power & Energy Society General Meeting (PESGM), IEEE, 2022*, pp. 1–5. Accessed: Oct. 30, 2024. [Online]. Available: https://ieeexplore.ieee.org/abstract/document/9916772/

[17] W. Pinthurat, B. Hredzak, G. Konstantinou, and J. Fletcher, "Techniques for compensation of unbalanced conditions in LV distribution networks with integrated renewable generation: An overview," *Electric Power Systems Research*, vol. 214, p. 108932, 2023.

[18] S. Sundarajoo and D. Soomro, "A Particle Swarm Optimization Trained Feedforward Neural Network for Under-Voltage Load Shedding," *ECTI Transactions on Electrical Engineering, Electronics, and Communications*, vol. 21, no. 2, pp. 249825–249825, 2023.

[19] N. Nimpitiwan and S. Kaitwanidvilai, "Robust Control Design for Three-Phase Power Inverters using Genetic Algorithm," *ECTI Transactions on Electrical Engineering, Electronics, and Communications*, vol. 10, no. 1, pp. 80–88, 2012.

[20] S. Mirjalili, "Genetic Algorithm," in Evolutionary Algorithms and Neural Networks, vol. 780, in Studies in Computational Intelligence, vol. 780. , Cham: Springer International Publishing, 2019, pp. 43–55. doi: 10.1007/978-3-319-93025-1_4.

[21] R. M. Ciric, A. P. Feltrin, and L. F. Ochoa, "Power flow in four-wire distribution networks-general approach," *IEEE Transactions on Power Systems*, vol. 18, no. 4, pp. 1283–1290, 2003.

[22] A. T. Arnholt, "Statistics: The Art and Science of Learning From Data," *The American Statistician*, vol. 61, no. 2, pp. 181–182, May 2007, doi: 10.1198/000313007X190808.

[23] J. M. Morales and J. Perez-Ruiz, "Point estimate schemes to solve the probabilistic power flow," *IEEE Transactions on power systems*, vol. 22, no. 4, pp. 1594–1601, 2007.

[24] R. Y. Rubinstein and D. P. Kroese, *Simulation and the Monte Carlo method.* John Wiley & Sons, 2016.

[25] R. Strahan, "Electricity Engineers' Association of NZ submission to Standards Australia for AS/NZS 4777.1 (2016)," 2016, Accessed: Nov. 11, 2024. [Online]. Available: https://ir.canterbury.ac.nz/bitstream/10092/100845/
[26] "Load Profile :: Load Research of PEA." Accessed: Aug. 08, 2024. [Online]. Available: http://peaoc.pea.co.th/loadprofile/en/

Bongkoj Sookananta is an Assistant Professor in the Department of Electrical and Electronics Engineering at Ubon Ratchathani University, Thailand. She earned her Ph.D. in Electrical Engineering from the University of Strathclyde, UK. Her research focuses on optimization and artificial intelligence in electrical power systems, with particular interests in electric vehicles, energy storage, renewable energy resources, and uncertainty in power system operations. Her work plays a crucial role in advancing sustainable and efficient energy solutions for modern power networks.

Mongkol Pusayatanont is an Assistant Professor in the Department of Electrical and Electronics Engineering at Ubon Ratchathani University, Thailand. He holds a Ph.D. in Electrical Engineering from the University of Sussex, United Kingdom. His research focuses on condition monitoring in process control, flow measurement, motor drives, renewable energy resources and AC machines. Dr. Pusayatanont's work significantly contributes to advancements in motor drive hardware and signal monitoring within drive systems.