Eliminable-Noise Dithering in MASH Digital Delta-Sigma Modulator Based on Error-Feedback Modulator

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ABSTRACT

Conventional LFSR-based dithering in the digital delta-sigma modulator (DDSM) is widely used to suppress spurious tones in the DDSM at the expense of the increased modulator's noise floor. In this work, we present a new dithering technique using the error feedback modulator (EFM) that can both suppress spurious tones and eliminate the dithering noise. The proposed technique also offers the precise frequency-tuning ability beneficial for fractional-N frequency synthesizers. Moreover, the hardware complexity of the DDSM using the proposed dithering technique can be reduced significantly with multistage dithering and a short wordlength. Mathematic proof and simulation results have been given to show that the proposed technique efficiently suppress spurious tones without raising the low-frequency noise floor. The proposed technique shows the improved performance compared to previously reported spur reduction techniques.

Keywords: Digital delta–sigma modulator (DDSM), Fractional-N synthesizer, Maximum sequence length, Dithering, multistage noise shaping (MASH), Error feedback modulator (EFM)

1. INTRODUCTION

The digital delta-sigma modulator (DDSM) is usually employed to shorten a digital wordlength by means of oversampling and quantization in a digital domain [1]. Although represented by the shorter wordlength, the good signal-to-noise ratio (SNR) can still be retained since the quantization noise which is pushed to high frequencies can be simply removed by a low pass filter. The shorter-wordlength representation proves to be beneficial in many applications such as class-D audio amplifiers [2], digital-to-analog convertors [3] and fractional-N frequency synthesizers [4] etc. In the fractional-N frequency synthesizer shown in Fig. 1, the DDSM is utilized to produce a fractional output frequency. The

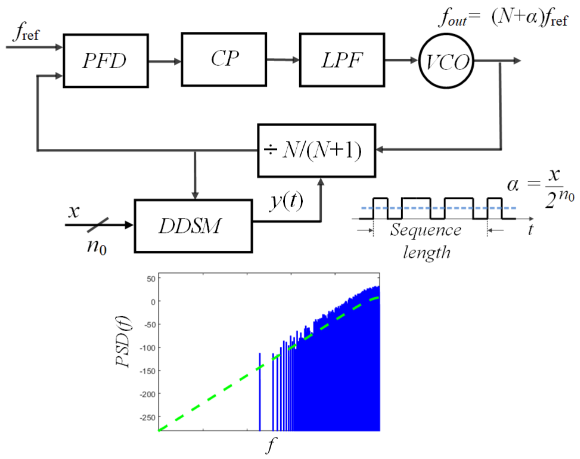

Fig. 1: Fractional-N frequency synthesizer using the DDSM.

DDSM generates a random binary control signal $y(t)$ that has the average value equal to the desired fractional $\alpha = x/2^{n_0}$ where x and n_0 are the DDSM's constant input and the modulator's wordlength, respectively. The divider's modulus is randomly switched between N and $N+1$ by the control signal, yielding the output frequency $f_{out} = (N+\alpha)f_{ref}$ where f_{ref} is the reference frequency.

Unfortunately, with a constant input, the DDSM behaves like a finite state machine (FSM) and produces the periodic output instead of the desired random output.

This causes spurious tones in the DDSM's output spectrum (see Fig. 1). The DDSM's output period or the sequence length strongly depends on the input value, the initial conditions and the wordlength of the DDSM. When these parameters are inappropriately set, the sequence length becomes short resulting in the DDSM's output power spectrum concentrating in spurious tones (solid blue line) rather than distributing over wide frequencies (dash green line). If these spurious tones locate inside the bandwidth of the frequency synthesizer, they cannot be removed by the loop filter, and will deteriorate the phase noise performance.

To suppress spurious tones, the DDSM's periodicity must be broken, or the sequence length must be maximized. There are two main approaches that have been presented in an attempt to maximize the sequence length, stochastic and deterministic approaches [5-13].

In the stochastic approach, regardless of the initial

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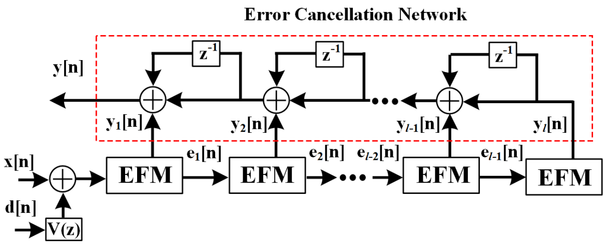


Fig. 2: Block diagram of the l^{th}-order MASH DDSM where the dither signal $d(n)$ is usually generated by the LFSR and shaped by the high pass filter $V(z)$ to reduce the dithering noise floor.

conditions, the sequence length can be increased by means of adding a pseudo-random sequence or dithering to disrupt the DDSM's periodicity. However, these approaches inherently raise the low-frequency noise floor of the output spectrum which can degrade the performance of the frequency synthesizer. The low-frequency dithering noise can be attenuated by passing the dither signal through the high pass filter $V(z) = (1 - z^{-1})^R$ before applying to the input of the DDSM as shown in Fig. 2. Unfortunately, the order of the filter is limited by the MASH's order. It has been shown in [6] that the high pass filter can efficiently attenuate the low frequency noise only if the filter's order is less than $l - 1$ for the l^{th} order modulator. If the filter's order exceeds this limit, spurious tones can reappear (see Fig. 3).

It is suggested that the LFSR dithering can be efficient without raising the noise floor if a sufficiently short LFSR's wordlength is used. Unfortunately, using a short wordlength will result in a short sequence length. Hongjia et al. [9] have presented the high order masked LFSR dithering technique to achieve the maximum sequence length without raising the noise floor. In this technique, multiple LFSR dithers are added to each stage of the modulator in the MASH to implicitly implement the high order dither shaping. When the LFSR's wordlength of each dither is appropriately chosen, spurious tones from the dithering noise can be masked by a normal quantization noise despite the order of the shaped dither is the same as that of the MASH. Nonetheless, the performance of this technique still depends on its input and initial conditions. Furthermore, since the appropriate wordlength must be obtained from the design equations, the design process of the modulator is rather complicated.

In the deterministic approach, on the other hand, the maximum sequence length can be achieved without raising the low-frequency noise floor. In the state-of-the-art deterministic technique, the HK-MASH [10], an output of the traditional MASH structure is fed back to the input such that the maximum sequence length can be achieved for all initial conditions without raising the noise floor. Nevertheless, owing to the feedback connection, the output average differs from the input average, so an extra gain is required for the HK-MASH to be able to support the full input range and avoid the average error. The idea of using the output-feedback can be extended to the multi-bit higher order EFM and the single-quantizer DDSM (SQ-DDSM) [11,12]. It is demonstrated that similar results can also be obtained. To avoid the average error in the HK-MASH, the SP-MASH [13] uses the feedforward connections between the adjacent modulator stages to maximize sequence length without increasing the hardware complexity. However, the SP-MASH is shown to still exhibit spurious tones for even initial conditions.

In the frequency synthesizer, apart from spurious tones which directly arise from the DDSM, parasitic coupling and nonlinearities in the frequency synthesizer loop can also cause unwanted fixed and wandering spurs [14-18]. Even though the DDSM's output spectrum is free of spurs, the output phase noise spectrum of the frequency synthesizer can still exhibit both fixed and wandering spurs when the DDSM's output interact with nonlinearities in the frequency synthesizer's loop. When the accumulated quantization error of the DDSM is distorted by nonlinearity of the PFD, its low-frequency noise floor is raised, and the inherent periodic behavior of the DDSM will manifest itself leading to spurious tones reappearing in the spectrum of the distorted quantization error. As a result, these low frequency noises and spurious tones cannot be hidden in the output phase noise spectrum as in the linear case and eventually will degrade the frequency synthesizer performance [15].

Solutions to alleviate spurs due to nonlinearities of the frequency synthesizer have been presented [19-21] which are based on either dithering or modification of the MASH's structure. It is shown that appropriately adding the large amplitude dither to the MASH's structure can sufficiently uncorrelated the MASH's internal signal in both linear and nonlinear cases, as a result, spurious tones due to nonlinearities can be effectively mitigated. Nonetheless, since a pseudorandom number generator is required in this technique, the complexity of the MASH is significantly increased. A more compact solution is the MASH's structure modification approach where the error signal within the MASH is passed to an extra feedback transfer function to generate the internal signal equivalent to the dither signal without employing an external dither. By appropriately choosing the feedback transfer function, wandering spurs can be effectively suppressed [19].

On the other approach, instead of using the DDSM-based divider controller, a novel divider controller, namely, the Successive reQuantizer (SQ) [20] was introduced and shown to successfully eliminate spurious tones in the presence of nonlinearity which is well modelled by a continuous polynomial. The SQ consists of the combinational logic that when appropriately designed, the spectrum of the accumulated quantization noise can be spur-free for a limited order of the polynomial nonlinearity. Unfortunately, this technique requires the significant amount of hardware than the conventional

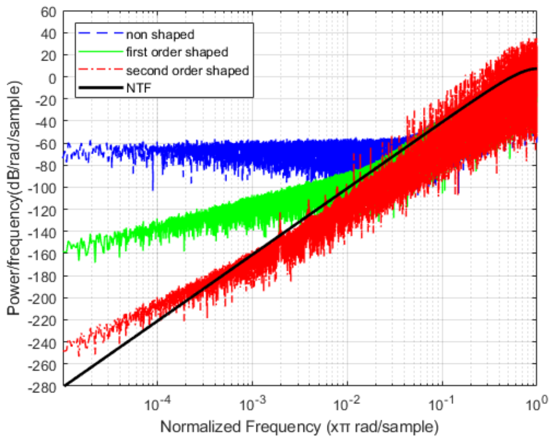


***Fig. 3**: Output PSD of the 9-bit MASH 1-1-1 DDSM with a constant input x = 1 for 3 cases: non-shaped, first-order shaped and second-order shaped dithers.*

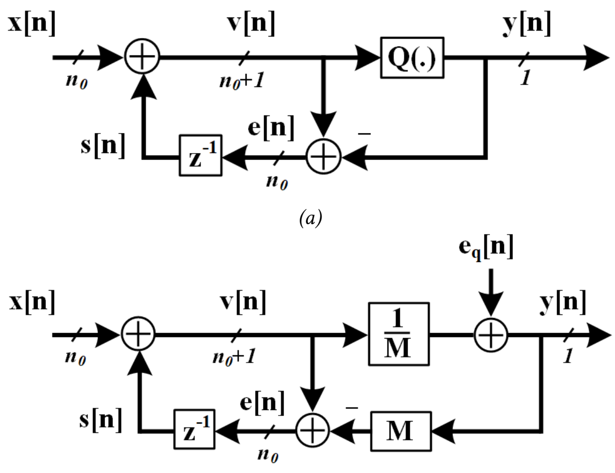


***Fig. 4**: Block diagram of a) the EFM and b) its linearized model.*

MASH DDSM. A hybrid structure between MASH DDSM and SQ, namely, nested MASH-SQ was proposed to reduce the SQ's complexity [21].

In this paper, we present a new dithering technique to suppress spurious tones which explicitly arise from the DDSM. The proposed dithering technique uses the EFM instead of the LFSR to generate the dither signal. Unlike conventional LFSR-based dithering, with EFM-based dithering, the sequence length can be maximized while the effect of the dithering noise and spurious tones can be eliminated. Hence, the order of the low-frequency noise shaping can be as high as that of the MASH.

This paper is organized as follows. In Section 2, overviews of the EFM and the MASH are presented. The MASHs with the proposed dithering technique, namely, the ED-MASH and the AED-MASH are described, and their transfer functions are derived in Section 3. The simulation results and the conclusion are discussed and summarized in Section 4 and 5, respectively.

2. EFM AND MASH STRUCTURES

The structure of the first order DDSM based on the EFM is shown in Fig. 4(a). Its operation can be described as follows. First, the EFM receives the input signal $x[n]$ which is summed with the feedback signal $s[n]$ to create the quantizer's input $v[n]$. Then, the quantizer (Q) quantises $v[n]$ producing the binary output $y[n]$ with the values of 1 and 0 when $v[n]$ is greater and less than $M = 2^{n_0}$, respectively. The error signal $e[n]$ is obtained by taking the difference between the quantizer's input and output. In fact, the error signal $e[n]$ is equivalent to the residue of dividing $v[n]$ by M. Hence, $e[n]$ can be expressed mathematically as

$$e[n] = v[n] \bmod M \tag{1}$$

where $[\cdot] \bmod M$ is a modulo operator with a modulus M. Finally, the error signal $e[n]$ is delayed providing the feedback signal $s[n]$ to the input. Since, with a constant input, the EFM is inherently a finite state machine, it is inevitable that the EFM will generate the periodic output. When the EFM has a constant input, v[n] will continue to accumulate in each clock cycle, and once its value exceeds the modulus M, the quantizer will produce the output $y[n] = 1$ which will then reset $v[n]$ and $y[n]$ in the following clock cycle. This process continues to repeat and produce a periodic output which causes problematic spurious tones in the output PSD of the EFM.

To simplify the analysis of the EFM, the model in Fig. 4(a) is linearized as shown in Fig. 4(b) where the input-output relation of the quantizer is assumed to be linear with the quantization gain of $1/M$. The effect of the quantization error is modelled by the additive quantization noise $eq(n)$. Since $y[n]$ corresponds to 0 and M when its value is 0 and 1, respectively, hence, the feedback signal from the output can be modelled by multiplying the output $y[n]$ by M. By analysing the linearized model in Fig. 4(b) using the z-transform, the output and the error signal of the EFM can be expressed as

$$Y(z) = \frac{1}{M}X(z) + 1(1 - z^{-1})E_q(z) \tag{2}$$

$$E(z) = -ME_q(z) \tag{3}$$

Equation (2) indicates that the average of the EFM's output is equal to the input scaled down by a factor of M, and the quantization noise is first-order shaped by the high pass filter $(1 - z^{-1})$.

Higher quantization-noise attenuation can be achieved by employing the higher order noise-shaping structures such as single-loop, multi-loop and multistage-noise-shaping (MASH) DDSMs. In this work, thanks to the low hardware complexity, the good stability and wide input dynamic range, the MASH DDSM shown in Fig. 2 is considered. In Fig. 2, the first order EFMs are cascaded, and the error signal of each EFM is fed forward to the input of the adjacent EFM. The output of each EFM is then fed to the error cancellation network such that the quantization noises of all EFMs are cancelled out except that of the last stage. The output of the lth order MASH DDSM can be written in the z domain as

$$Y(z) = \frac{1}{M}X(z) + 1(1 - z^{-1})^{l}E_{ql}(z) \tag{4}$$

where E_{ql} is the quantization noise of the l^{th}-stage EFM. It can be seen that the quantization noise is now shaped by the lth-order high pass filter $(1 - z^{-1})^{l}$.

3. PROPOSED EFM-BASED DITHERING

3.1 EFM Dithering MASH

Even though the MASH DDSM can provide a high-order noise shaping, its performance is yet affected by spurious tones for a constant input similar to the first order EFM. In this paper, we present using the multistage EFMs to generate the dither signal. Unlike conventional LFSR-based dithering, it can be shown that with EFM-based dithering, the maximum sequence length can be obtained without degrading a low-frequency performance of the modulator. To illustrate our approach, let us consider the MASH 1-1 with the EFM-based dithering or the ED-MASH 1-1 with m-stage dithering shown in Fig. 5 (only black part) which consists of the MASH 1-1 (EFM1 and EFM2) and m dithering EFMs (DEFMi) connected in the cascade manner to generate the dither signal d(n) for the MASH 1-1. The dither signal $d(n)$ is added to the LSB of the input $x(n)$ similar to the conventional LFSR-based dithering. The output of each DEFM ($d_i(n)$) serves as the dithering input of the adjacent DEFM in addition to the existing DEFM input (x_{di}). The z-domain representation of the output of the ED-MASH 1-1 with m-stage dithering can be found as follows.

By using (2), the DEFMi outputs can be expressed as

$$\begin{aligned} D(z) &= \frac{1}{M}\left(X_{d1}(z) + D_1(z)\right) + (1 - z^{-1})E_{qd1}(z) \\ D(z) &= \frac{1}{M}\left(X_{d2}(z) + D_2(z)\right) + (1 - z^{-1})E_{qd2}(z) \\ &\vdots \\ D_{m-1}(z) &= \frac{1}{M}\left(X_{dm}(z) + D_1(z)\right) + (1 - z^{-1})E_{qdm}(z) \end{aligned} \tag{5}$$

where $E_{qdi}(z)$ is the quantization noise of the ith dithering EFM (DEFMi) for $i = 1$ to m. Combing (5) yields

$$\begin{aligned} D(z) = &\left(\frac{1}{M}X_{d1}(z) + \frac{1}{M^2}X_{d2}(z) + \cdots + \frac{1}{M^m}X_{dm}(z)\right) \\ &+ (1 - z^{-1})\left(\frac{1}{M^0}E_{qd1}(z) + \cdots + \frac{1}{M^{m-1}}E_{qdm}(z)\right) \end{aligned} \tag{6}$$

The output $y(n)$ of the ED-MASH 1-1 with m-stage dithering can be presented in the z domain using (4) as

$$Y(z) = \frac{1}{M}\left(X(z) + D(z)\right) + (1 - z^{-1})^2 E_{q2}(z) \tag{7}$$

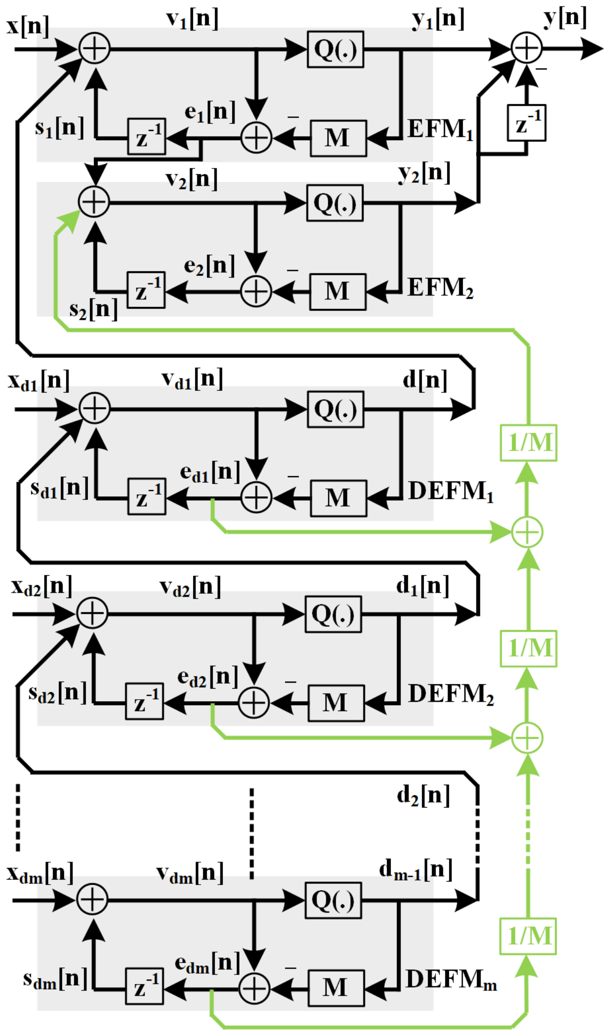


Fig. 5: Block diagram of the ED-MASH 1-1(black) and AED-MASH 1-1(black and green) with m-stage dithering.

By substituting (6) into (7), (7) can be re-expressed as

$$\begin{aligned} Y(z) = &\left(\frac{1}{M}X(z) + \frac{1}{M^2}X_{d1}(z) + \cdots + \frac{1}{M^{m+1}}X_{dm}(z)\right) \\ &+ (1 - z^{-1})\left(\frac{1}{M}E_{qd1}(z) + \cdots + \frac{1}{M^m}E_{qdm}(z)\right) \\ &+ (1 - z^{-1})^2 E_{q2} \end{aligned} \tag{8}$$

It can be seen that, besides the MASH's input, the STF in (8) contains the additional terms which are the inputs of the dithering EFMs divided by the modulus M with different orders. These terms can help to obtain a higher resolution of the frequency synthesizer. The output frequency of the fractional-N frequency synthesizer which employs the ED-MASH 1-1 with m-stage dithering can be written as

$$f_{out} = \left(\alpha + \frac{x}{2^{n_0}} + \frac{x_{d1}}{2^{2n_0}} + \cdots + \frac{x_{dm}}{2^{(m+1)n_0}}\right) f_{ref} \tag{9}$$

Compared to the conventional fractional-N frequency synthesizer, the resolution of the frequency synthesizer with the ED-MASH 1-1 with m-stage dithering can be increased by a factor of 2^{mn_0} without requiring a long wordlength. In addition, from (8) besides the MASH's quantization noise, the NTF contains additional noises resulting from the dithering EFMs divided by the modulus $\boldsymbol{M}$ with different orders. These dithering noises clearly raise the noise floor of the output PSD of the MASH. However, the effect of these dithering noises on the noise floor can be minimized by employing the long wordlength or the large $\boldsymbol{M}$.

It can be proved mathematically that the sequence length of the ED-MASH 1-1 with m-stage dithering and input wordlength n_0 is equal to

$$N_m = M^{m+1} = 2^{n_0(m+1)} \tag{10}$$

It can be seen from (10) that the sequence length of the ED-MASH 1-1 with m-stage dithering is independent of the input and the MASH's order, and is determined by both the MASH's wordlength and the number of the dithering stages. Moreover, the resulting sequence length also equals the maximum sequence length which is achievable by the undithered MASH when an appropriate input and initial conditions are employed. The result in (10) also implies that it is possible to achieve the maximum sequence length despite of a short wordlength. It can be easily shown that, by using a short word length with the multistage dithering, the hardware complexity can be significantly reduced. To illustrate this point, let us consider the n_1 and n_2-bit ED-MASH 1-1 with m_1 and m_2-stage dithering, respectively. Let us assume that these two MASHs yield the same sequence length. Hence, according to (10), we have

$$n_1(m_1 + 2) = n_2(m_2 + 2) \tag{11}$$

Since the EFM can be simply implemented by a digital accumulator, it can be assumed that the ED-MASH's hardware complexity is approximately proportional to both the MASH's wordlength and the number of modulators in the MASH. Consequently, if the n_1-bit ED-MASH 1-1 with m_1-stage dithering has the lower hardware complexity than n_2-bit ED-MASH 1-1 with m_2-stage dithering, it can be approximated that

$$n_1(m_1 + 2) < n_2(m_2 + 2) \tag{12}$$

To satisfy the sequence length and the hardware complexity conditions in (11) and (12), respectively, the following criteria must be met:

$$n_1 < n_2 \text{ and } m_1 = (n_2/n_1)(m_2 + 1) - 1 \tag{13}$$

This result implies that if the criteria in (13) are satisfied, the hardware complexity of the MASH can be lowered by using multistage dithering with a short wordlength while the maximum sequence length can still be retained. For example, the 8-bit ED-MASH 1-1 with 1-stage dithering and the 4-bit ED-MASH 1-1 with 3-stage dithering can achieve the same sequence length, however, the later MASH has a lower hardware complexity.

Even though using a short wordlength with the multistage dithering can save the hardware consumption, the noise floor and spurious tones in the PSD of the dither signal may be increased as mentioned earlier. Hence, to exploit the advantage of using the short wordlength with the multistage dithering without causing excessive spurious tones and the dithering noise, the input wordlength, the input and the number of the dithering EFMs must be carefully chosen. If the short wordlength is used, the first-order shaped dithering noise will dominate and raise the low-frequency noise floor. In fact, using the short wordlenth not only raises the noise floor but also creates spurious tones in the dither signal. Recall that the dithering EFM itself is also a finite state machine. If its wordlength becomes too short, with a constant input, the dither signal can no longer be assumed to be random, and it will become more periodic causing spurious tones in the output PSD of the MASH. Hence, the period of the dither signal will strongly depend on the input and the wordlength of the dithering EFM. Besides, it can be shown by the simulation that when the number of dithering EFMs increases, the number of spurious tones in the PSD of the dithering signal will be higher. Since spurious tones from the dithering EFM appear at the MASH's input, they cannot be suppressed by the MASH's NTF, and are only attenuated by the factor of $\boldsymbol{M}$. Consequently, these spurious tones from the dithering EFM will appear in the output PSD of the MASH. Therefore, to minimize the effects of the dithering noises and the spurious tones from the dithering EFMs in the ED-MASH with multistage dithering, the wordlength, the input and the number of the dithering stages of the dithering EFMs must be carefully chosen.

It is shown in Section 3.2 that the dithering noises and spurious tones in the ED-MASH can be efficiently eliminated by the AED-MASH.

3.2 Anti EFM Dithering MASH

As explained in the previous section, the output PSD of the ED-MASH can be deteriorated by spurious tones and the increased quantization noise floor owing to the dithering EFMs. Even though the effect of the dithering noise can be moved to very low frequencies by using a long wordlength, the MASH's PSD is yet degraded by spurious tones of the dithering EFMs. In this section, we present the anti-EFM-dithering MASH or the AED-MASH which is developed from the ED-MASH that can completely eliminate the effect of the dithering noise and spurious tones from the dithering EFM.

To illustrate the efficiency of the AED-MASH, let us consider the AED-MASH 1-1 with m-stage dithering shown in Fig. 5 (black and green parts). The structure of the AED-MASH 1-1 with m-stage dithering in Fig. 5 is similar to that of the ED-MASH 1-1 with m-stage dithering except that the error signal $e_{di}(n)$ of each dithering EFM (DEFMi) is fed forward and summed with

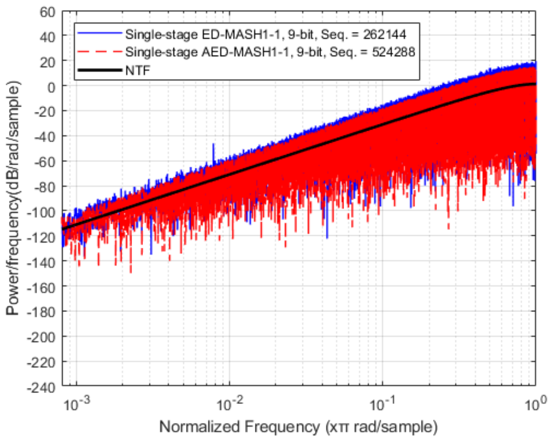


***Fig. 6**: Output PSDs of the 9-bit ED and AED-MASH 1-1s with 1-stage dithering, $x = 256$, $x_{d1} = 255$, $s_1[0] = s_2[0] = 0$ and $s_{d1}[0] = 0$.*

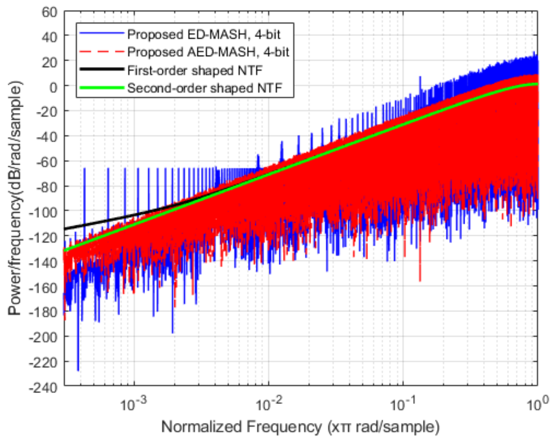


***Fig. 7**: Output PSDs of the 4-bit ED and AED-MASH 1-1s with 4-stage dithering, $x = 8$, $x_{d1} = x_{d2} = x_{d3} = 7$, $s_1[0] = s_2[0] = 0$ and $s_{d1}[0] = s_{d2}[0] = s_{d3}[0] = s_{d4}[0] = 0$.*

the signal from the previous DEFM, and is then divided by the modulus M before being fed forward to the next DEFM. The accumulation of these signals serves as the dithering input of EFM2 in the MASH 1-1 in addition to $e_1(n)$. The z-domain representation of the output of the AED-MASH 1-1 with m-stage dithering can be derived as follows.

First, from Fig. 5, applying the result from (4), $Y_1(z)$ can be written as

$$Y_1(z) = \frac{1}{M}(X(z) + D(z)) + (1 - Z^{-1})E_{q1}(z) \quad (14)$$

Since it can be noticed that the ED-MASH and AED-MASH share the signal $d(n)$. Hence, the result in (6) can be substituted in (14) which yields

$$\begin{aligned} Y_1(z) = &\left(\frac{1}{M}X(z) + \frac{1}{M^2}X_{d1}(z) + \cdots + \frac{1}{M^{m+1}}X_{dm}(z)\right) \\ &+ (1 - z^{-1})\left(\frac{1}{M}E_{qd1}(z) + \cdots + \frac{1}{M^m}E_{qdm}(z)\right) \\ &+ (1 - z^{-1})E_{q1}(z) \end{aligned} \quad (15)$$

Now applying (4) to $Y_2(z)$, we have

$$\begin{aligned} Y_2(z) = &\left(\frac{1}{M^2}E_{d1}(z) + \cdots + \frac{1}{M^{m+1}}E_{dm}(z)\right) \\ &+ \frac{1}{M}E_1(z) + (1 - z^{-1})E_{q2}(z) \end{aligned} \quad (16)$$

By expressing each error signal in (16) in term of their respective quantization noises using (3), (16) can be re-expressed as

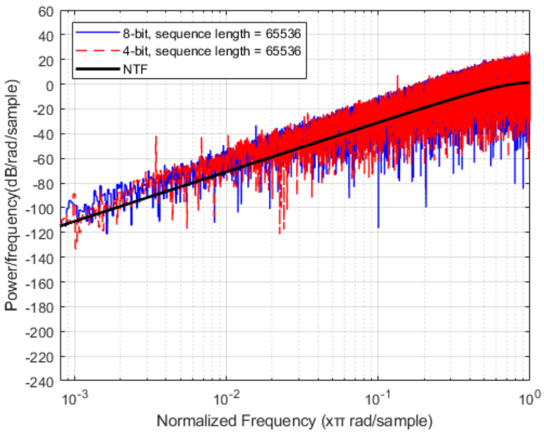
$$\begin{aligned} Y_2(z) = &-\left(\frac{1}{M}E_{qd1}(z) + \cdots + \frac{1}{M^m}E_{qm}(z)\right) \\ &- E_{q1}(z) + (1 - z^{-1})E_{q2}(z) \end{aligned} \quad (17)$$

From (15) and (17), $Y(z)$ can be obtained as

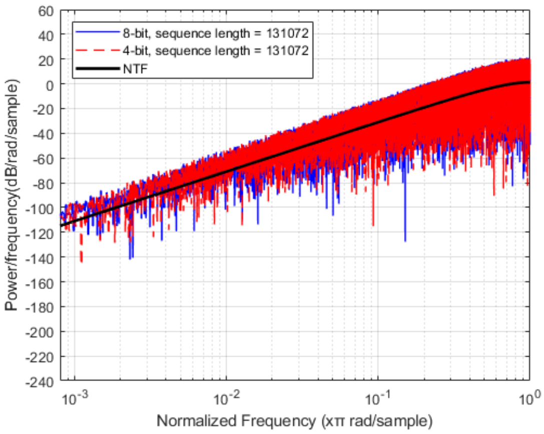
$$\begin{aligned} Y(z) &= Y_1(z) + (1 - z^{-1})Y_2(z) \\ &= \left(\frac{1}{M}X(z) + \frac{1}{M^2}X_{d1}(z) + \cdots + \frac{1}{M^{m+1}}X_{dm}(z)\right) \\ &+ (1 - z^{-1})^2 E_{q2}(z) \end{aligned} \quad (18)$$

By comparing (18) with (8), it can be seen that all the quantization noises (E_{qdi}) of the dithering EFMs are completely cancelled out and disappear from the NTF. This indicates that the AED-MASH 1-1 with m-stage dithering can efficiently eliminate the effect of the dithering noise and spurious tones from the dithering EFMs. Therefore, in the case of the AED-MASH 1-1 with m-stage dithering, the noise floor is only dominated by the MASH's second-order shaped quantization noise as desired which is in contrast to the LFSR-based dithering where its noise floor is dominated by the undesired dithering noise. Hence, it can be seen that, in the AED-MASH 1-1 with m-stage dithering, the noise shaping order at low frequencies can be as high as the MASH's order. This is a clear advantage of the proposed EFM-based dithering over the traditional LFSR-based the value of the obtained sequence length is consistent with the theoretical results in (10) and (19), and the sequence length obtained from the AED-MASH is twice as much as that from the ED-MASH as predicted.

To illustrate the dithering-noise elimination efficiency of the AED-MASH, the output PSDs of the 4-bit ED and AED-MASH 1-1s with the 4-stage dithering are compared in Fig. 7. Owing to the excessively short wordlength, besides the first-order shaped noise floor, it can be seen that spurious tones caused by the dithering EFMs are excessively pronounced in the output PSD of the 4-bit ED-MASH 1-1 with 4-stage dithering. On the dithering where the dithering noise shaping is limited by the MASH's order. Moreover, as it can be seen from the previous section, besides the dithering noise, Eqdi also



(a)



(b)

***Fig. 8**: Output PSDs of the 4 and 8-bit a) ED and b) AED-MASH 1-1s with 3 and 1-stage dithering, respectively. For the 4-bit case,* $x = 8$, $x_{d1} = x_{d2} = x_{d3} = 7$, $s_1[0] = s_2[0] = 0$ *and* $s_{d1}[0] = s_{d2}[0] = s_{d3}[0] = 0$*, and for the 8-bit case,* $x = 128, x_{d1} = 127, s_1[0] = s_2[0] = 0$ *and* $s_{d1}[0] = 0$.

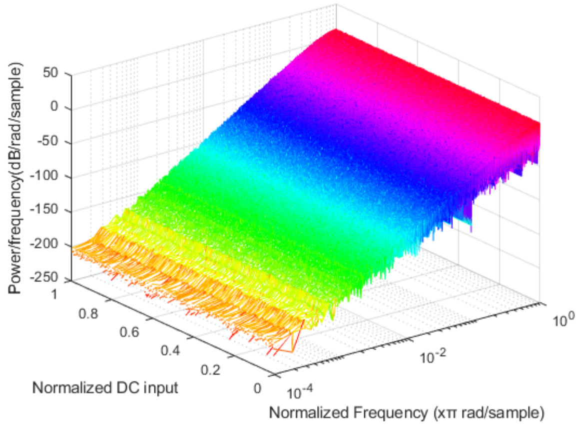
includes the effect of spurious tones arising from the dithering EFM when the short wordlength is used. These spurious tones can also be removed from the output PSD by the AED-MASH. It can be proved mathematically that the sequence length of the AED-MASH 1-1 with m-stage dithering and input wordlength n_0 is equal to

$$\bar{N}_m = 2M^{m+1} = 2^{n_0(m+1)+1} \quad (19)$$

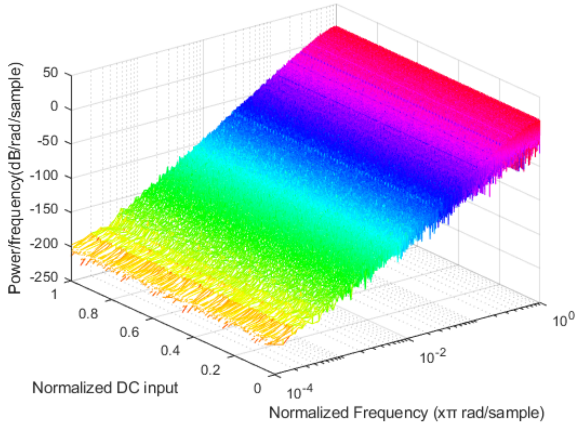
From (19), it can be seen that the sequence length of the AED-MASH with m-stage dithering is twice as much as that of the ED-MASH with m-stage dithering. Hence, the AED-MASH not only eliminates the effect of the dithering noise and spurious tones but also further improves the MASH's sequence length.

4. SIMULATION RESULTS

Fig. 6 shows the output PSDs of the 9-bit ED and AED-MASH 1-1s with 1-stage dithering. The output PSDs



(a)

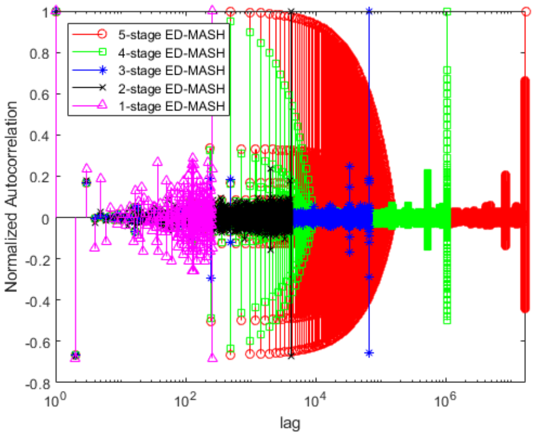


(b)

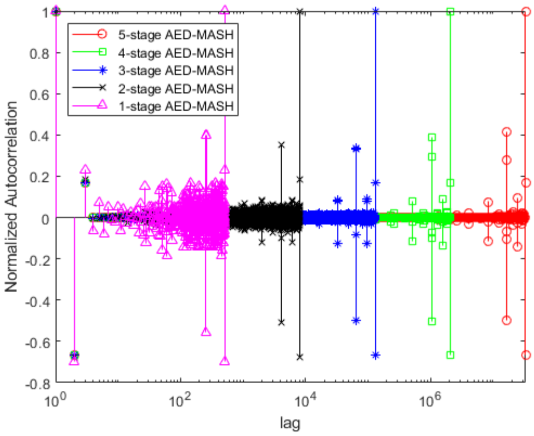
***Fig. 9**: Output PSDs of the 9-bit a) AED and b) ED-MASH 1-1s with 1-stage dithering for all input with* $x_{d1} = 255$, $s_1[0] = s_2[0] = 0$ *and* $s_{d1}[0] = s_{d2}[0] = s_{d3}[0] = 0$.

exhibit the second-order shaped quantization noises as derived in (8), and spurious tones are effectively suppressed by the proposed dithering techniques. Moreover, it can be seen that the dithering noise of the ED-MASH is unnoticeable at low frequencies thanks to the sufficiently long wordlength used in the MASHs resulting in the dithering noise pushed to low frequencies. Finally, the value of the obtained sequence length is consistent with the theoretical results in (10) and (19), and the sequence length obtained from the AED-MASH is twice as much as that from the ED-MASH as predicted.

To illustrate the dithering-noise elimination efficiency of the AED-MASH, the output PSDs of the 4-bit ED and AED-MASH 1-1s with the 4-stage dithering are compared in Fig. 7. Owing to the excessively short wordlength, besides the first-order shaped noise floor, it can be seen that spurious tones caused by the dithering EFMs are excessively pronounced in the output PSD of the 4-bit ED-MASH 1-1 with 4-stage dithering. On the other hand, the output PSD of the AED-MASH 1-1with 4-stage dithering is clean of spurious tones, and its noise floor



(a)



(b)

***Fig. 10**: Output PSDs of the 4-bit a) ED and b) AED-MASH 1-1s with 1 to 5-stage dithering,* $x = 255, x_{d1} = x_{d2} = x_{d3} = x_{d4} = x_{d5} = 7$, $s_1[0] = s_2[0] = 0$ *and* $s_{d1}[0] = s_{d2}[0] = s_{d3}[0] = s_{d3}[0] = s_{d4}[0] = s_{d5}[0] = 0$.

is dominated only by the MASH's second-order shaped quantization noise. This is thanks to the fact that the first-order shaped dithering noise and spurious tones from the dithering EFMs are eliminated by the AED-MASH as described in Section 3.2.

Fig. 8 shows the output PSD comparison between the 8-bit MASH with 1-stage dithering and the 4-bit MASH with 3-stage dithering. It can be seen that the 8-bit ED-MASH 1-1 with 1-stage dithering can suppress spurious tones and the dithering noise efficiently even though it does not have the noise elimination mechanism. This is thanks to the long wordlength used. Moreover, even though the 4-bit AED-MASH 1-1 with 3-stage dithering uses the shorter wordlength, it can still achieve the maximum sequence length of the 8-bit AED-MASH 1-1, and their output PSDs are also comparable and free of spurious tones. On the other hand, there are noticeable spurious tones in the low-frequency range of the output PSD of the 4-bit ED-MASH 1-1 with 3-stage dithering. These spurious tones result from the

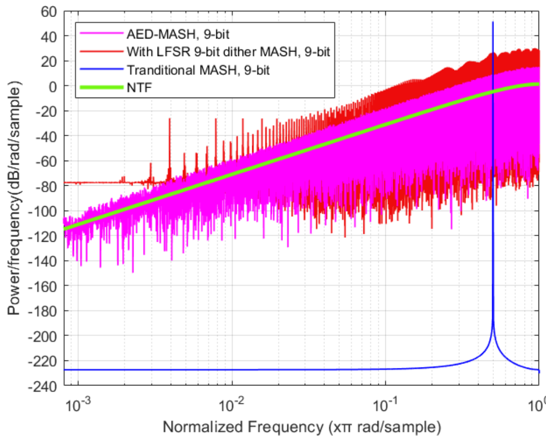
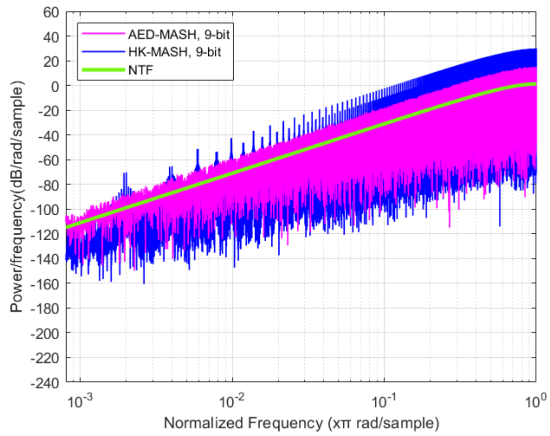
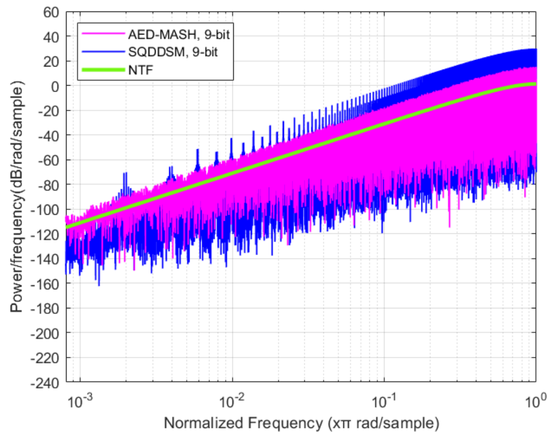


***Fig. 11**: Comparison of output PSDs of the 9-bit AED-MASH 1-1s with 1-stage dithering, traditional MASH 1-1 and traditional MASH 1-1 with LFSR dithering.*



(a)



(b)

***Fig. 12**: Comparison of output PSDs of the 9-bit AED-MASH 1-1s with 1-stage dithering, HK-MASH 1-1 [10] (a) and* 2^{nd} *order SQDDSM [11] (b).*

dithering EFMs with a short wordlength which cannot be eliminated by the ED-MASH 1-1 as explained in Section

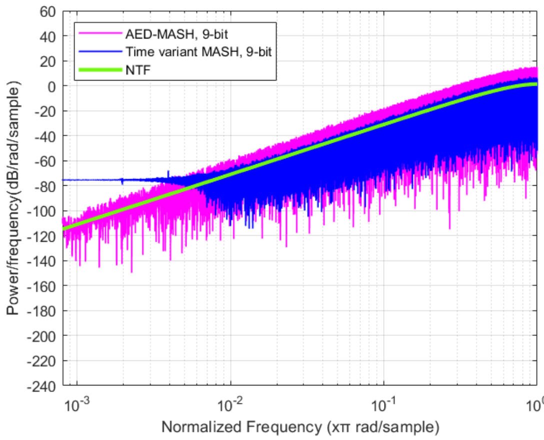


***Fig. 13**: Comparison of output PSDs of the 9-bit AED-MASH 1-1s with 1-stage dithering and 2nd order Time-variant-MASH [8].*

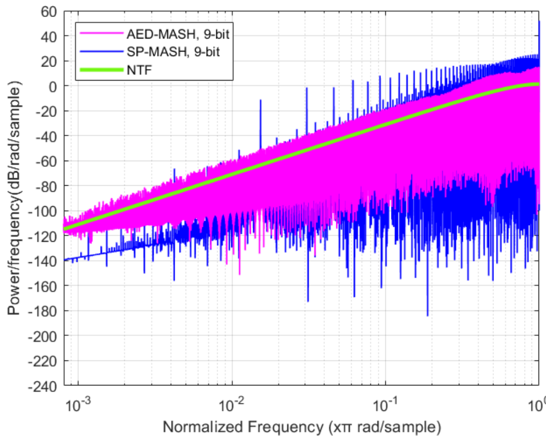


***Fig. 14**: Comparison of output PSDs of the 9-bit AED-MASH 1-1s with 1-stage dithering and 2nd SP-MASH [13].*

3.1. This is in contrast to the 4-bit AED-MASH 1-1 with 3-stage dithering where the low-frequency noise floor and spurious tones arising from the dithering EFM with the short wordlength can be efficiently eliminated.

In Fig. 9, the output PSDs of the 9-bit ED and AED-MASH 1-1s are plotted for all combinations of inputs with zero initial conditions. The output PSDs exhibit smooth surfaces without any spurious tones in both cases which indicate that both MASHs can efficiently suppress spurious tones for all inputs and the resulting sequence lengths are independent of inputs as explained in our analysis in Section 3. It should be noted, however, that although the obtained sequence length is independent of the input, it still strongly depends on the initial conditions. The brute force simulation with all combinations of initial conditions shows that the maximum sequence length can be obtained only if the initial conditions of EFM1 and DEFMi are all initialized to zero. If this criterion is not met, the sequence length may become very short, and spurious tones will manifest themself.

As indicated in (10) and (19), the sequence lengths of both the ED and AED-MASHs with m-stage dithering increase exponentially as the number of the dithering stages m linearly increases. To verify this result, the autocorrelation of the outputs of the ED and AED-MASH 1-1s with the different number of the dithering stages are shown in Fig. 10. It can be seen that the maximum locations of the autocorrelation spread linearly over the log scale of the time lag in both cases. This indicates that the sequence length indeed increases exponentially with the increased dithering stages. In addition, the autocorrelation can also give an insight about the number of spurious tones because the local maxima in the autocorrelation correspond to spurious tones in the output PSD. By comparing the autocorrelations in Fig. 10, it can be seen that the number of the local maxima in the autocorrelation of the AED-MASH 1-1 is less than that of the ED-MASH 1-1. Clearly, this suggests that the number of spurious tones in the AED-MASH 1-1 is much less than that in the ED-MASH 1-1. Moreover, apart from the 1-stage dithering in both cases, it can be seen that the number of the local maxima increases when the number of the dithering stages in both ED and AED-MASH increase. Hence, this result confirms that increasing the number of the dithering stages can result in the increased spurious tones.

To illustrate the performance of the proposed dithering technique, the output PSD of the 9-bit AED-MASH 1-1 with 1-stage dithering were compared to that of the previously reported works.

Fig.11 shows the output PSD of the traditional MASH 1-1s with and without dithering. Clearly without dithering, a single and strong spurious tone can be noticed. On the other hand, with dithering, even though the spurious tones become weaker but the number of spurious tones increases and the low-frequency noise floor is raised. Obviously, the proposed technique outperforms these traditional MASHs in term of both spurious tone suppression and low-frequency noise.

Fig.12.(a) and 12.(b) shows that the output PSDs of the HK-MASH 1-1 [10] and the second-order SQ-DDSM [11], respectively, are similar since they use the same output feedback connection technique. They both exhibit the higher number of spurious tones compared to the AED-MASH 1-1. This is due to the fact that the sequence length of the AED-MASH is not limited by the MASH's order and, instead, depends on the number of dithering stages unlike the HK-MASH and the SQ-DDSM. Consequently, even though the orders of the HK-MASH, the SQ-DDSM and the AED-MASH are the same, the sequence length of the AED-MASH can be higher and spurious tones can be efficiently eliminated.

Fig.13 shows the output PSD of the time-variant $\Sigma\Delta$ modulator [8] which is free of spurious tones. However, due to its randomization of the modulator's feedback coefficient, the zero of the NTF slightly moves away from DC causing the higher low-frequency noise floor compared to the AED-MASH. The SP-MASH [13] which is the improvement of the HK-MASH still exhibits the tonal spectrum when the modulator's initial condition is even as shown in Fig.14. The AED-MASH, in contrast yields a smooth shaped white-noise spectrum despite of the even initial condition.

5. CONCLUSION

In this paper, we have presented a new dithering technique which not only effectively suppresses spurious tones and achieves the maximum sequence length but also is able to eliminate the effect of the dithering noise for all inputs. Thus, unlike the LFSR-based dithering, the low-frequency noise shaping can be the same order as the MASH's order.

In our technique, instead of using the LFSR, the EFM is used to generate the dither signal. It is demonstrated that the MASH with a proposed dithering technique, namely the ED-MASH, can achieve the maximum sequence length while the effect of the low-frequency dithering noise becomes negligible with an appropriate wordlength. Furthermore, the ED-MASH can also provide the precise frequency-tuning ability suitable for fractional-N frequency synthesizers. It has been shown that the hardware complexities can be significantly reduced while the maximum sequence length can still be retained when the multistage dithering and the short wordlength is used. Nonetheless, it has been shown by the simulation that increasing the number of the dithering stages results in the increased spurious tones arising from the dithering EFMs. Moreover, spurious tones will be more pronounced with the inappropriate dithering EFMs' inputs and initial conditions. Hence, the appropriate number of the dithering stages, the wordlength, the dithering EFMs' inputs, and the initial conditions must be carefully chosen to ensure that the dithering noise and spurious tones from the dither signal are minimized.

To completely eliminate the effect of the dithering noise and spurious tones as well as to further improve the sequence length, the AED-MASH has been developed from the ED-MASH. We have shown mathematically that the effects of the dithering noise and spurious tones can be eliminated from the AED-MASH's transfer function. The simulation results have confirmed that the AED-MASH can efficiently eliminate the dithering noise and spurious tones while the maximum sequence length can be achieved for all inputs.

In term of hardware consideration, the basic building block of the ED and AED-MASHs is the first order EFM which can be simply implemented by a digital accumulator and a unit delay. Compared to the other MASHs based on LFSR dithering where the registers and logic gates are required for the LFSR, the proposed MASH will require slightly higher hardware consumption. Furthermore, in the AED-MASH, it is required that the error signal of each dithering EFM must be scaled by a factor of M. This can be simply achieved by discarding the LSBs of the error signal without resorting to an actual digital multiplier.

Finally, it should be noted that even though the MASH 1-1 is used to illustrate the concept of the proposed dithering technique, without loss of generality, the results discussed in this paper can also be applied to any higher order MASHs.

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