

Bi-Level DG Optimization in Distribution Networks with TOU Pricing and Demand Response Using MPA

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ABSTRACT

This study proposes a bi-level optimization model for the optimal allocation of distributed generation (DG) in active distribution networks, taking into account time-of-use (TOU) electricity pricing and demand response (DR) behavior. The upper level represents the decision-making of the distribution system operator (DSO), aiming to minimize power losses, voltage deviations, and DG investment costs. At the lower level, responsive consumers optimize their hourly demand profiles to minimize electricity costs under TOU tariffs, subject to behavioral and operational constraints. To enable tractable computation, the original bi-level structure is reformulated into a single-level nonlinear programming problem through the application of Karush-Kuhn-Tucker (KKT) optimality conditions. The resulting model is highly constrained and non-convex. This makes it suitable for solutions using metaheuristic approaches. In this work, the Marine Predators Algorithm (MPA) is adopted due to its superior global search capability and convergence characteristics. Numerical simulations on the IEEE 33-bus and 69-bus test systems confirm the effectiveness of the proposed approach in reducing both energy losses and operational costs. The results highlight the practical applicability of the MPA-based framework for intelligent DG planning in smart grids under dynamic pricing environments.

Keywords: Bi-level optimization, distributed generation, time-of-use, demand response, distribution networks

Nomenclature:

X - Composite decision vector $[x_{DG}, P_{DG}, P_L^{1:24}]$
 x_{DG} - Binary siting vector for DG placement $\{0, 1\}^{N_c}$
 P_{DG} - DG capacity [MW]
 $P_{DG}^{\max}$ - Maximum DG capacity [MW]
 $P_L^{(t)}$ - Hourly demand shaped by DR at hour t [MW]
 E_{day} - Daily energy requirement [MWh]

V_i - Voltage magnitude at bus i [p.u.]
 $V^{\min}, V^{\max}$ - Voltage limits [p.u.]
 V_{dev} - Voltage deviation index [p.u.]
 P_{loss} - Active power loss [kW]
 E_{loss} - Daily energy loss [kWh/day]
 C_{op} - TOU-based operating cost [\$]
 w_1, w_2, w_3 - Objective weights (loss, cost, voltage)
 N - Population size (number of predator agents)
 $g, g_{\max}$ - Generation index and maximum generation
 X_{elite} - Elite (best-so-far) solution
 N_c - Number of candidate buses for DG placement
 S, r - Adaptive control parameters in MPA
 λ - Weighting factor (scalarization)
 ϵ - Convergence tolerance
 δ - Small perturbation term in exploitation phase
 β - Lévy-flight exponent

1. INTRODUCTION

Power distribution systems worldwide are undergoing a profound transformation from passive infrastructures into dynamic and intelligent active distribution networks (DN). This evolution is fueled by the rapid integration of distributed generation (DG) resources such as rooftop solar panels, small-scale wind turbines, and battery energy storage systems [1]. Integrating DG offers numerous key benefits, including alleviating the burden on centralized power grids, improving energy resilience, reducing technical losses, and contributing to carbon neutrality objectives [2], [3]. However, as the penetration of DG grows, it introduces a crucial challenge: optimally determining the locations and sizes of DG units within distribution networks that are becoming more complex and dynamic. This challenge is further intensified by the increasing involvement of consumers, whose electricity usage patterns are no longer passive but adapt in real time to economic signals and time-varying pricing schemes. Among these, time-of-use (TOU) pricing has proven particularly effective in reshaping demand patterns, encouraging users to shift their consumption to off-peak hours in order to lower electricity costs [4], [5].

Capturing this interplay between system-level planning and consumer behavior calls for a more sophisticated modeling framework. Bi-level optimization has emerged as an effective mathematical framework. In such models, the upper level, typically representing the Distribution System Operator (DSO), seeks to minimize

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technical losses and investment costs through optimal DG placement and sizing. Simultaneously, the lower level representing responsive consumers adjusts hourly demand to minimize TOU-based energy bills. These two levels interact through shared constraints related to power balance, voltage limits, and behavioral flexibility, forming a tightly coupled decision-making process [6]. Figure 1 illustrates the conceptual bi-level framework used in this study, which reflects both strategic infrastructure planning and consumer-side demand response. Although prior studies have explored TOU and aware load scheduling using bi-level approaches, few have jointly considered DG planning and responsive consumption under TOU pricing in a unified optimization model [7], [8]. This gap is particularly evident in the Vietnamese context, where the adoption of TOU pricing is growing, yet integrated distribution-level planning that leverages both DG and demand-side flexibility remains limited in both academic and practical domains.

To address the emerging challenges in integrating DG with consumer side responsiveness, this study introduces a BLO optimization framework for planning DG in active DN [9]. The upper level represents the DSO, aiming to determine the optimal locations and capacities of DG units to minimize power losses and investment costs. The lower-level models consumer behavior under TOU electricity pricing, where demand response (DR) is driven by efforts to reduce electricity bills. This hierarchical decision structure is reformulated into a single-level nonlinear model using the Karush-Kuhn-Tucker (KKT) optimality conditions, enabling efficient computational handling [10].

To address the formulated nonlinear and highly constrained optimization problem, this study employs the Marine Predators Algorithm (MPA), a recent nature-inspired metaheuristic that mimics the intelligent foraging strategies of marine predators [11]. MPA features a dynamic three-phase search process that effectively balances global exploration and local exploitation, making it particularly suitable for solving large-scale, non-convex optimization problems. For benchmarking purposes, five well-established metaheuristic algorithms are selected for comparison: the Harris Hawks Optimization (HHO) [12], Slime Mould Algorithm (SDO) [18], Whale Optimization Algorithm (WOA) [13], Gradient Based Optimizer (GBO) [14], and Arithmetic Optimization Algorithm (AOA) [15]. Each of these algorithms is inspired by distinct biological or mathematical principles and has shown competitive performance in recent studies. The proposed bi-level optimization framework is implemented on the 33-bus and 69-bus DN under 24-hour TOU pricing and demand response scenarios. Simulation results demonstrate that the MPA consistently outperforms the benchmark algorithms in minimizing technical losses, reducing operational costs, and achieving more effective DG placements. These results highlight the effectiveness of integrating advanced metaheuristics with demand-side flexibility in DN planning. The proposed

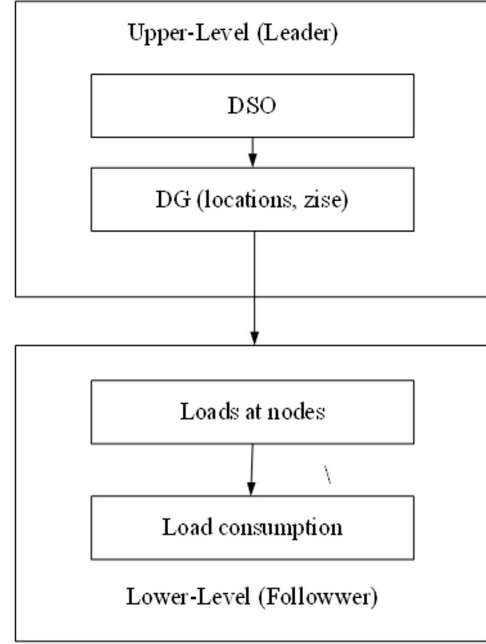


Fig. 1: Bi-level model of a DN with integrated DG.

model serves as a mathematically tractable and scalable optimization framework for future smart grid applications, where dynamic pricing and active consumer participation are expected to play an increasingly vital role.

2. MATHEMATICAL MODEL

The proposed planning framework is developed as a comprehensive bi-level optimization model that reflects the hierarchical decision-making structure between the DSO and end-use consumers within an active DN. At the upper level, the DSO is responsible for strategic decisions regarding the optimal placement and sizing of DG units, with the objective of minimizing total power losses, voltage deviations, and investment costs while enhancing overall network performance and renewable energy penetration. Simultaneously, the lower level encapsulates the behavior of responsive consumers who adjust their hourly electricity demand to minimize their individual energy expenditures in response TOU pricing schemes. This interaction captures the decentralized flexibility available at the demand side and its impact on system-wide planning decisions. Due to the inherent complexity and non convex nature of the bi-level formulation, arising from the combinatorial structure of DG placement and the coupling introduced by DR dynamics, the original model is reformulated into an equivalent single level nonlinear program. This transformation is achieved by embedding the KKT optimality conditions associated with the lower-level problem into the upper level formulation. The resulting model enables tractable computation while preserving the key decision interactions between the DSO and consumers.

2.1 Upper Level Problem

At the upper level, the DSO aims to balance three critical aspects of system performance: active energy loss, DG investment cost, and voltage deviation. To unify these conflicting objectives into a single mathematical formulation, a normalized weighted sum objective function is proposed as follows:

$$\min \left(w_1 \frac{A_{\Delta}}{A_{\Delta}^{base}} + w_2 \frac{C_{inv}}{C_{max}} + w_3 \frac{\sum_{t=1}^{24} \sum_{i=1}^{N_b} |V_i(t) - 1.0|}{V_{norm}} \right) \quad (1)$$

with $w_1 + w_2 + w_3 = 1$.

Where,

- A_{Δ} : Daily active energy loss (kWh/day).
- $C_{inv} = \sum_{i=1}^{N_{DG}} C_i \cdot P_{DG,i}$: Total DG investment cost.
- $V_i(t)$: Voltage magnitude at bus i and time t .
- w_1, w_2, w_3 : User defined weights for loss, investment, and voltage deviation.
- $A_{\Delta}^{base}, C_{max}, V_{norm}$: Normalization factors derived from benchmark scenarios.

Subject to:

$$\text{Power flow equation: } P_i = \sum_{j \in N(i)} f(P_j, V_j) \quad (2)$$

$$\text{Voltage limits: } V_{min} \leq V_i \leq V_{max}, \forall i \in B \quad (3)$$

$$\text{DG capacity: } 0 \leq P_{DG,i} \leq P_{DG,i}^{max}, \forall i \in C \quad (4)$$

At the upper level, the objective aggregates three criteria—daily energy loss, DG investment cost, and voltage deviation—using a normalized weighted sum so each term is on a comparable scale; weights are chosen via a brief sensitivity analysis (Section 4.1). Constraints enforce AC nodal power flow balance, distribution-level voltage bands, and DG capacity limits. Investment cost is modeled linearly in DG size (with an option for fixed/block charges when data are available). Feasibility during optimization is maintained through penalty/repair operators. The lower-level DR model and its KKT-based single-level reformulation are described in Section 2.3.

2.2 Lower Level Problem

In the lower level, consumers optimize their hourly energy consumption profile to minimize total electricity cost under TOU tariffs while satisfying operational constraints. Objective function:

$$\min_{\{P_L^t\}} \sum_{t=1}^{24} \rho_t \cdot P_L^t \quad (5)$$

Subject to:

$$\text{Daily energy constraint: } \sum_{t=1}^{24} P_L^t = E_{daily} \quad (6)$$

$$\text{Hourly demand bounds: } P_{min}^t \leq P_i^t \leq P_{L,max}^t, \forall t \quad (7)$$

2.3 KKT Reformulation for Lower

To transform the bi-level problem into a single-level model, the lower-level problem is replaced by its KKT conditions, assuming convexity holds. Let $P_L(t)$ be the decision variable, λ_t the Lagrange multiplier for the total demand constraint, and $\mu_i^{min}(t), \mu_i^{max}(t)$ the dual variables for inequality constraints. The KKT system includes:

- Stationarity:

$$\frac{\partial L}{\partial P_{L,i}(t)} = \rho_t - \lambda_t + \mu_i^{min}(t) - \mu_i^{max}(t) = 0 \quad (8)$$

- Primal feasibility:

$$\sum_{t=1}^{24} P_{L,i}(t) = E_i^{daily}, P_{L,i}^{min} \leq P_{L,i}(t) \leq P_{L,i}^{max} \quad (9)$$

- Dual feasibility:

$$\mu_i^{min}(t) \geq 0, \mu_i^{max}(t) \geq 0 \quad (10)$$

- Complementary slackness:

$$\begin{aligned} \mu_i^{min}(t) \cdot (P_{L,i}^{min} - P_{L,i}(t)) &= 0, \\ \mu_i^{max}(t) \cdot (P_{L,i}(t) - P_{L,i}^{max}) &= 0 \end{aligned} \quad (11)$$

For the convex lower-level DR problem, the KKT conditions are necessary and sufficient for optimality. Omitting any block (stationarity, primal feasibility, dual feasibility, complementary slackness) breaks the one-to-one equivalence between the bi-level model and its single-level KKT reformulation, potentially admitting non-optimal or infeasible lower-level responses and biasing the upper-level objective.

2.4 Final Single Level Reformulation

By embedding the KKT conditions (Equations 8–11) into the upper level model (Equations 1–4), the original bi-level framework is transformed into a single level nonlinear program. The resulting formulation is a mixed integer, non convex, and highly constrained problem due to the discrete nature of DG placement, nonlinear power flow, and DR coupling. To address this, the Marine Predators Algorithm (MPA) is adopted for its global search ability and robustness in complex solution spaces.

3. SOLUTION METHODOLOGY

Solving the proposed single-level nonlinear optimization model presents significant computational challenges due to its mixed integer nature, non convexity, and high dimensionality arising from time coupled demand response and power flow constraints. To effectively explore the complex solution space and handle the nonlinearities involved, this study adopts the MPA, a recent bio-inspired metaheuristic that has shown promising performance in solving large-scale, multi-constraint engineering problems.

3.1 Overview of The MPA

The MPA is inspired by the foraging strategies of marine predators in nature, particularly their ability to adaptively switch between active pursuit and ambush tactics. The algorithm divides the population into three functional zones, simulating distinct predator-prey interactions across the search process:

Phase I - Exploration (High speed pursuit): Global exploration is performed using Lévy flight based movement, allowing candidate solutions to explore distant regions of the search space.

Phase II - Transition (Moderate speed): A balanced exploration exploitation phase where predators adjust movement based on both random steps and elite solution guidance.

Phase III - Exploitation (Low speed trapping): Fine tuning occurs as predators close in on prey, using Brownian motion and neighborhood intensification to refine solutions.

This dynamic adjustment of search behavior across the algorithmic phases enables MPA to maintain diversity while achieving fast convergence.

3.2 Implementation Steps of MPA for Bi-Level DG Planning

To effectively apply the MPA to the proposed single level reformulation of the bi-level DG planning problem, the following structured procedure is employed:

Step 1: Problem encoding

Each candidate solution (individual) is encoded as a composite decision vector:

$$X = [x_{DG}, P_{DG}, P_L^{1:24}]$$

Where, $x_{DG} \in \{0, 1\}^{N_c}$: Binary vector indicating DG placement at candidate buses, $P_{DG} \in \mathbb{R}^+$: Real valued DG capacities (in MW), $P_L^{1:24}$: Hourly demand vector shaped by demand response at each time slot.

Step 2: Initialization of MPA population

- Initialize a population of N predator agents, each with randomly assigned values within predefined bounds: DG locations, DG sizes, load profiles.

- Evaluate the fitness of each individual using the normalized objective function defined in Equation (1), incorporating power flow simulation and penalty terms for constraint violations.

Step 3: Marine predators dynamics

For each generation $g = 1$ to $g^{\max}$, update the population based on the three MPA phases:

- Phase I (Exploration): If $\text{rand} < 0.5$, use Lévy flight to simulate fast global search:

$$X_i^{g+1} = X_i^g + S \cdot \text{Lévy}() \cdot (X_i^g - X_{\text{elite}}^g)$$

- Phase II (Transition zone): Combine elite solution and Brownian motion for balanced search:

$$X_i^{g+1} = X_{\text{elite}}^g + r \cdot \text{Brownian}() \cdot (X_i^g - X_{\text{elite}}^g)$$

- Phase III: Slow convergence around elite to refine the optimum:

$$X_i^{g+1} = X_i^g + \epsilon (X_i^g - X_j^g) + \delta$$

Here, S, r, ϵ, δ are adaptive control parameters, and X_{elite} is the best solution found so far.

Step 4: Constraint handling and repair After each update: DG sizes, voltage, demand profiles: enforce hourly min max and total energy balance. Use penalty functions or repair heuristics to re-map infeasible solutions into feasible space.

Step 5: Power flow and fitness evaluation For each individual: Perform power flow analysis, calculate (total energy loss, total investment cost, voltage deviation normalize components and compute the composite fitness using Equation (1).

Step 6: Elite selection and termination

Update the elite predator if a better one is found, repeat steps 3 to 5 until convergence criterion is met, return the best DG placement, sizing, and responsive load profile as the final solution. The proposed MPA based methodology effectively addresses the complexity of bi-level DG planning by balancing exploration and exploitation through marine predator dynamics. Its structured design ensures feasibility, convergence, and adaptability to TOU driven demand shifts. This provides a solid foundation for smart grid applications. Future improvements such as parallel computation or hybrid local refinement may further enhance performance and scalability. As shown in Figure 2, the proposed method follows an MPA based single level procedure for bi-level DG planning under TOU and demand response.

4. SIMULATION RESULTS

4.1 Simulation Setup Parameters

The proposed optimization model was tested on two benchmark radial distribution networks: the IEEE 33-bus and 69-bus systems. To reflect realistic operational conditions, hourly load demand over a 24-hour period was used, based on typical Vietnamese residential and commercial consumption profiles. The simulation time horizon was divided into 24 hourly intervals (from 00:00 to 23:00), capturing daily variations in power usage and allowing the modeling of time-sensitive responses to electricity pricing. A TOU pricing structure with three tariff levels—off-peak, standard, and peak hours—was adopted, as summarized in Table 1. This pricing model is designed to encourage demand shifting and energy cost reduction. To scale the load demand for each system, hourly load factors were applied as shown in Table 2, ensuring consistency with dynamic demand patterns observed in practice.

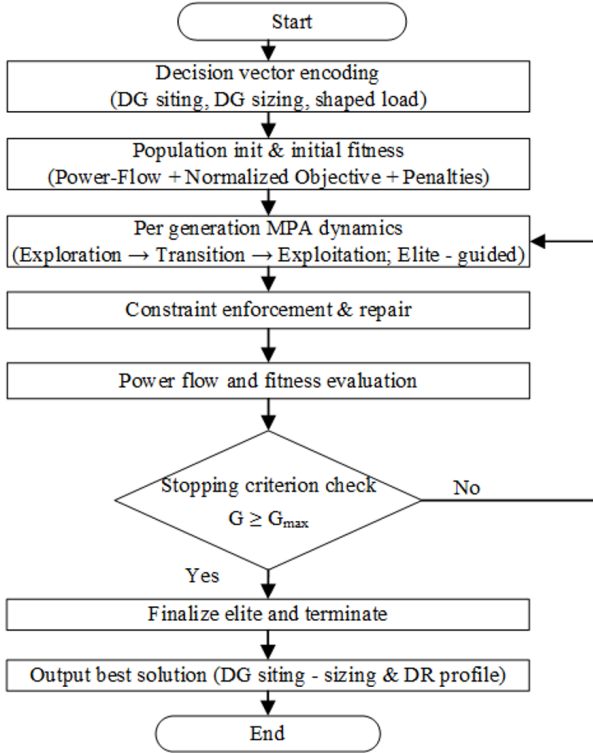


Fig. 2: Structured MPA Procedure for Bi-Level DG Planning (TOU + DR).

Table 1: Time based electricity tariffs (TOU) [5].

TOU Period	Time Interval	Electricity prices (USD/kWh)
Off-peak	22:00 to 4:00	0.0454
Standard	4:00 to 9:00; 11:00 to 17:00; 20:00 to 22:00	0.0700
Peak	9:00 to 11:00; 17:00 to 20:00	0.1289

To ensure reproducibility and a fair comparison, all algorithms were executed under the same optimization settings: a population of 30 agents and a maximum of 100 generations, with termination triggered by either the generation cap or a convergence tolerance on the composite objective; constraint handling employed a violation-proportional penalty/repair scheme. For MPA, the exploration phase uses Lévy flights with exponent $\beta = 1.5$, the transition phase combines Brownian motion with elite guidance, and the exploitation phase applies a step size decay (initial scale $c1 = 0.5$) while retaining the elite to mitigate premature convergence.

During the simulation, the following assumptions and constraints were applied: The total installed DG capacity was capped at 3.0 MW per system to reflect practical limitations in medium-scale DN. The number of DG units was limited to three per network, with optimal placement determined through the optimization

algorithm. Each DG unit was allowed a maximum penetration of 1.0 MW, ensuring compatibility with local voltage and loading limits. The investment cost for DG installation was set at 750 USD/kW based on current market data. Based on a one-factor sensitivity analysis across $\lambda \in [0.3, 0.7]$, we set $\lambda=0.6$, the knee-point that achieves the best loss cost voltage compromise with the lowest variance across 20 runs on both systems. In the upper-level objective function, a weighting factor $\lambda = 0.6$ was assigned to prioritize energy loss reduction, while the remaining weight ($1-\lambda = 0.4$) represented the cost-related objectives, including DG investment and voltage deviation minimization. This balance ensures that both technical performance and economic efficiency are addressed. Voltage magnitudes at all buses were constrained within the allowable operational range of $[0.95, 1.05]$ p.u., and power flow was calculated using the backward/forward sweep method, which is well suited for DN.

The multi objective upper-level function (Equation (1)) integrates three normalized performance indicators: energy loss, investment cost, and voltage deviation. These objectives inherently conflict with one another; minimizing energy loss may require greater investment in DG units, while reducing voltage deviation may not always align with minimizing costs. To address this trade-off, a weighted sum approach was adopted, in which each term is assigned a specific weight reflecting its relative priority in the planning objective. The weights were determined based on sensitivity analysis and practical considerations in distribution network operations. Energy loss was assigned the highest weight, $w_1 = 0.5$, because it directly impacts both technical efficiency and long-term operational costs, making it the primary objective. The investment cost was assigned a moderate weight, $w_2 = 0.3$, balancing financial feasibility with performance gains. Voltage deviation, while important for maintaining power quality, typically fluctuates within tolerable limits in radial networks and thus was assigned a lower weight, $w_3 = 0.2$. All three components were normalized using baseline values obtained from a no-DG scenario to ensure that each term contributes proportionally to the overall objective function, regardless of unit scale or magnitude. This normalization enables fair aggregation of heterogeneous objectives and improves the numerical stability of the optimization process.

To solve the single-level reformulated optimization model, six modern metaheuristic algorithms were employed: WOA, HHO, AOA, GBO, SDO, and the proposed MPA. All algorithms were executed under identical simulation settings, including a population size of 30 agents, a maximum of 100 iterations, and shared stopping criteria based on convergence thresholds or maximum generation limits. A constraint handling mechanism was also applied, with penalty factors scaled dynamically based on the degree of constraint violation, ensuring solution feasibility throughout the search process. MPA

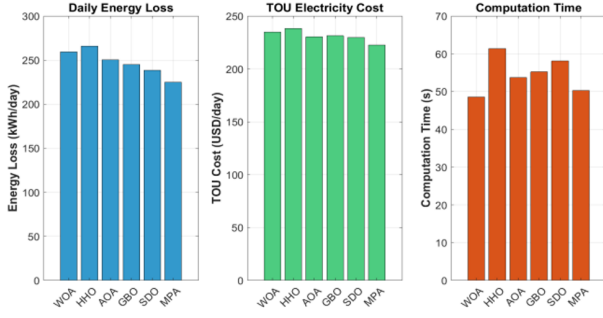


Fig. 4: Optimization algorithms for the 33-bus DN.

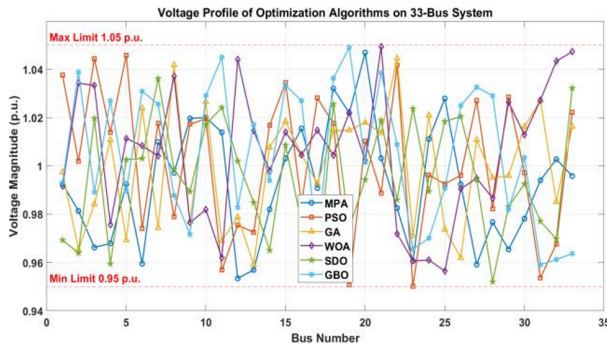


Fig. 5: Voltage profiles for the 33-bus DN.

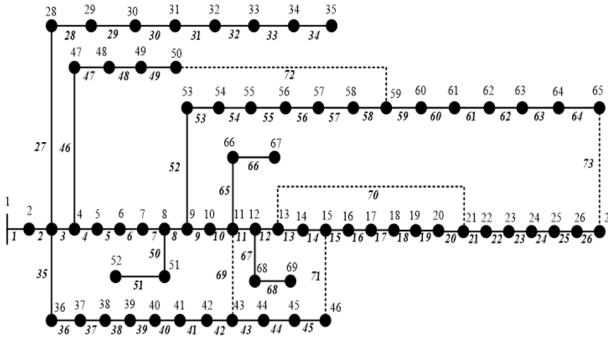


Fig. 6: Single line diagram of the IEEE 69-bus DN.

4.2.2 The 69-bus Distribution System

The 69-bus DN operates at 12.66 kV and has a detailed structure with closely connected branches. System parameters, including line impedances and loads, are taken from [17] for modeling accuracy. With a total apparent power of about $(3.80 + j2.69)$ MVA, the network represents a medium-scale real-world system. Without DG integration, it incurs a power loss of 224.88 kW. Figure 6 shows its single-line diagram used for analysis and optimization evaluation [18].

Table 4 summarizes the optimization results for the 69-bus DN using six different metaheuristic algorithms. The MPA once again achieved the best overall performance. It delivered the lowest daily energy loss of 243.9 kWh/day and a relatively low TOU-based operational cost of 229.75 USD/day. With DG units placed at buses 15, 27, and 64, and capacities close to the upper limit (1.00, 0.87,

Table 4: DG Planning optimization results for the 69-bus.

Algorithm	L _{DG} (bus)	P _{DG} (MW)	ΔΔ (kWh/day)	TOU Cost (USD/day)	Time (s)
WOA	13, 25, 63	0.95; 0.84; 0.96	268.2	239.8	48.6
HHO	12, 24, 62	0.98; 0.80; 0.95	263.1	225.88	54.3
AOA	11, 23, 61	0.99; 0.82; 0.94	258.4	236.1	60.7
GBO	14, 26, 60	0.97; 0.81; 0.93	251.7	237.6	62.9
SDO	10, 22, 59	0.96; 0.83; 0.92	248.5	238.2	58.4
MPA	15, 27, 64	1.00; 0.87; 0.98	243.9	229.75	51.1

and 0.98 MW), MPA demonstrated effective DG sizing and placement. Despite its advanced search mechanism, MPA maintained a competitive computation time of 51.1 seconds (Figure 7).

Other algorithms exhibited higher losses and/or operational costs. For example, WOA resulted in the highest energy loss (268.2 kWh/day) and cost (239.8 USD/day), while HHO achieved a lower cost (225.88 USD/day) but with significant energy loss (263.1 kWh/day). AOA, GBO, and SDO produced moderate results but did not surpass MPA in overall performance. These comparisons highlight MPA's ability to balance technical efficiency and economic effectiveness in large-scale DN. Figure 7 illustrates the voltage profiles obtained by each algorithm across the 69-bus DN. All methods maintained voltages within standard operational limits, but MPA yielded the most stable profile with reduced deviations, especially at buses far from the substation. This further confirms MPA's superiority in voltage regulation, enhancing power quality and supporting reliable network operation under TOU pricing and demand response conditions.

Beyond the baselines evaluated in this work (WOA, HHO, AOA, GBO, SDO), recent DG-planning studies frequently adopt PSO variants, DE families, and GA-based hybrids. PSO variants are simple and fast but can be prone to premature convergence under tight voltage constraints; DE typically offers strong global exploration yet may slow when the feasible region is narrow; GA hybrids enhance local refinement at the expense of additional parameter tuning and runtime. By contrast, the proposed MPA employs an elite-guided search with Lévy-Viét. On both the 33- and 69-bus systems, this yields lower energy losses and smaller voltage deviations at comparable operating cost and competitive runtime, indicating a more robust multi-objective trade-off (loss, cost, voltage) for DG planning with TOU/DR. Overall, simulation results on both the 33-bus and 69-bus DN confirm the superior performance of the MPA. Compared to other metaheuristic algorithms, MPA consistently achieved the lowest energy losses and operational costs while maintaining competitive computation time. It also delivered more stable voltage

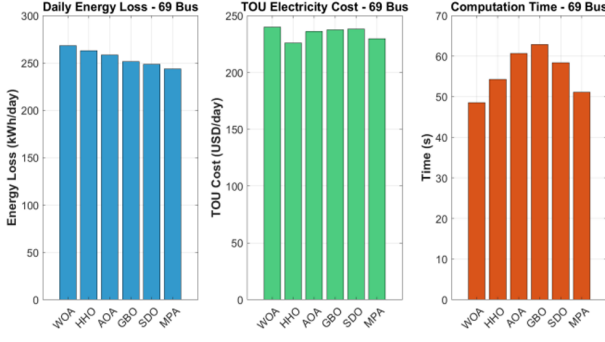


Fig. 7: Optimization algorithms for the 69-bus DN.

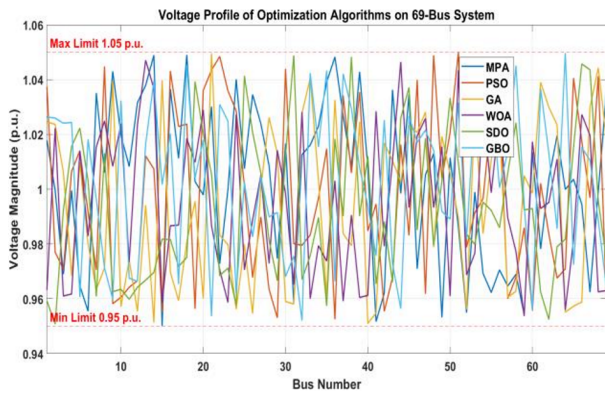


Fig. 8: Voltage profiles for the 69-bus DN.

profiles, ensuring that all bus voltages remained within permissible operational limits and exhibited minimal deviation. Furthermore, as depicted in Figure 9, MPA demonstrated faster and smoother convergence toward the optimal solution compared to other algorithms. Its ability to rapidly minimize the objective function while maintaining convergence stability highlights its effective balance between exploration and exploitation. These outcomes collectively emphasize MPA's robustness, adaptability, and suitability for complex distributed generation planning in smart DN under TOU pricing and demand response conditions.

Across 20 independent restarts, the best/mean composite objective at iteration 100 matched the converged value within the prescribed tolerance. Increasing the iteration budget did not yield any material improvement, indicating that 100 iterations are sufficient for convergence under our settings. MPA incurs a slightly higher runtime (on the order of a few seconds) because each generation performs hourly power flow evaluations and constraint repair before updating the population. Nevertheless, MPA is more stable (lower run-to-run variance), achieves greater reductions in energy loss, and maintains the voltage profile within permissible limits at comparable/competitive cost. For offline planning, this time-quality tradeoff is well justified.

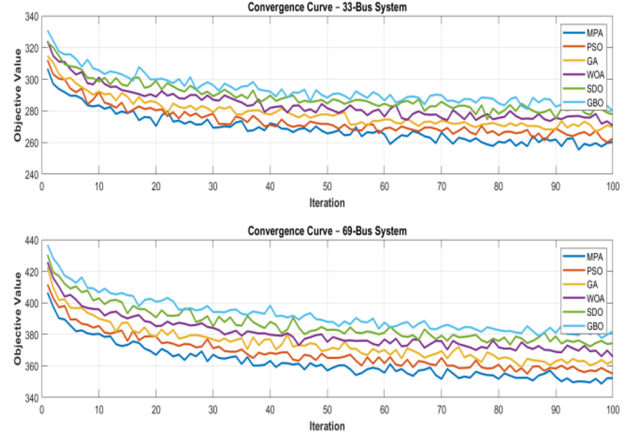


Fig. 9: Convergence curves of the optimization algorithms for the 33-bus and 69-bus DN.

5. CONCLUSION

This paper proposed a bi-level framework for DG siting and sizing in distribution networks under TOU pricing and consumer DR, recast as a single-level NLP by embedding the complete KKT conditions for the convex lower-level DR and solved with MPA, which is well suited to constrained, nonconvex spaces. On the IEEE 33-bus and 69-bus feeders, the MPA-based approach achieved lower daily energy losses and lower TOU-based operating costs than five state-of-the-art metaheuristics, kept voltages within limits, and converged more stably; the slight runtime overhead (a few seconds) is acceptable for offline planning. Limitations include idealized DR, a fixed TOU structure, and deterministic generation. Future extensions will incorporate renewable uncertainty, dynamic DR, and energy-storage integration and will explore hybrid metaheuristics and parallel computing for scalability, alongside field validation. Although calibrated for Vietnam, the framework is transferable to other TOU/DR-enabled networks by re-parameterizing core inputs (tariff vector, DR bounds/elasticities, operational limits, and feeder data); the optimization procedure remains unchanged and can be piloted after data calibration.

REFERENCES

- [1] A. Kumar, R. Verma, N. K. Choudhary, and N. Singh, "Optimal Multi-Objective placement and sizing of distributed generation in power distribution system: a comprehensive review," *Energy Sources, Part A Recover. Util. Environ. Eff.*, vol. 45, no. 3, pp. 7160–7185, 2023.
- [2] P. Siano and D. Sarno, "Assessing the benefits of residential demand response in a real time distribution energy market," *Appl. Energy*, vol. 161, no. October 2017, pp. 533–551, 2016.
- [3] W. L. Theo, J. S. Lim, W. S. Ho, H. Hashim, and C. T. Lee, "Review of distributed generation (DG) system planning and optimisation techniques: Com-

- parison of numerical and mathematical modelling methods,” *Renew. Sustain. Energy Rev.*, vol. 67, pp. 531–573, 2017.
- [4] N. Carolina, Z. Li, and D. Ph, *Market operations in electric power systems: forecasting, scheduling, and risk management*, vol. 40, no. 03. New York: IEEE Press & Wiley-Interscience, 2002.
- [5] T. N. Trieu, N. T. Thuan, T. V. Anh, and V. P. Tu, “Optimal location and operation of battery energy storage system in the distribution system for reducing energy cost in 24-hour period,” *Int Trans Electr Energ Syst*, vol. e12861, no. February, pp. 1–17, 2021.
- [6] M. S. Javadi, K. Firuzi, M. Rezanejad, M. Lotfi, M. Gough, and J. P. S. Catalão, “Optimal Sizing and Siting of Electrical Energy Storage Devices for Smart Grids Considering Time-of-Use Programs,” in *IEEE, IECON 2019-45th Annual Conference of the IEEE Industrial Electronics Society. IEEE*, 2019, pp. 4017–4022.
- [7] H. Sun, Z. Gao, and J. Wu, “A bi-level programming model and solution algorithm for the location of logistics distribution centers,” *Appl. Math. Model.*, vol. 32, no. 4, pp. 610–616, 2008.
- [8] International Energy Agency, “Renewables 2023,” 2023. doi: 10.1002/peng.20026.
- [9] H. Xie, X. Teng, Y. Xu, and Y. Wang, “Optimal Energy Storage Sizing for Networked Microgrids Considering Reliability and Resilience,” *IEEE Access*, vol. 7, pp. 86336–86348, 2019.
- [10] A. F. Izmailov and M. V. Solodov, “Karush-Kuhn-Tucker systems: Regularity conditions, error bounds and a class of Newton-type methods,” *Math. Program. Ser. B*, vol. 95, no. 3, pp. 631–650, 2003.
- [11] A. Faramarzi, M. Heidarinejad, S. Mirjalili, and A. H. Gandomi, “Marine Predators Algorithm: A nature-inspired metaheuristic,” *Expert Syst. Appl.*, vol. 152, pp. 0–43, 2020.
- [12] A. Selim, S. Kamel, A. S. Alghamdi, and F. Jurado, “Optimal Placement of DGs in Distribution System Using an Improved Harris Hawks Optimizer Based on Single- And Multi-Objective Approaches,” *IEEE Access*, vol. 8, pp. 52815–52829, 2020.
- [13] D. B. Prakash and C. Lakshminarayana, “Multiple DG placements in radial distribution system for multi objectives using Whale Optimization Algorithm,” *Alexandria Eng. J.*, vol. 57, no. 4, pp. 2797–2806, 2018.
- [14] A. M. El-Rifaie et al., “Modified Gradient-Based Algorithm for Distributed Generation and Capacitors Integration in Radial Distribution Networks,” *IEEE Access*, vol. 11, no. October, pp. 120899–120917, 2023.
- [15] J. Zhang, G. Zhang, Y. Huang, and M. Kong, “A Novel Enhanced Arithmetic Optimization Algorithm for Global Optimization,” *IEEE Access*, vol. 10, no. July, pp. 75040–75062, 2022.
- [16] M. E. Baran and F. F. Wu, “Network reconfiguration in distribution systems for loss reduction and load balancing,” *IEEE Transactions on Power Delivery*, vol. 4, no. 2, pp. 1401–1407, 1989.
- [17] T. N. Trieu, L. M. Phong, and L. M. Tan, “Multi-objective optimization strategy for enhancing distribution power system efficiency with integrated distributed generation,” *Univ. Danang - J. Sci. Technol.*, vol. 23, no. 3, pp. 7–10, 2025.
- [18] T. N. Trieu, P. H. Loc, L. M. Phong, and L. M. Tan, “Multi-Objective Optimization of Electric Distribution Systems With Integrated Distributed Generation Using Deep Reinforcement Learning,” *Eng. Technol. Appl. Sci. Res.*, vol. 15, no. 2, pp. 22166–22171, 2025.



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