Application of the Adomian Modified Decomposition Method to Free Vibration Analysis of Thin Plates with Elastic Supports

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ABSTRACT

The purpose of this study is to introduce an innovative method, the Adomian modified decomposition method, to solve the vibration problem of thin elastic plates. By applying the present method to vibration problem of thin plates, the fundamental and higher frequencies as well as their mode shapes can be obtained easily for common and complicated boundary conditions, including elastic supports. Other benefits of using this method can be realized in terms of rapid convergence, small computational expensiveness and stability in calculation. The significant effects such as effects of boundary conditions, aspect ratios and translational and rotational spring constants, which lead to considerable changes in frequency results and mode shapes, are investigated and discussed.

Keyword: *The Adomian modified decomposition method; Natural frequencies; Mode shapes; Vibration of plates*

1. INTRODUCTION

The Adomain modified decomposition method (AMDM) has been successfully applied to solve differential and integral equations of linear and nonlinear problems in the fields of mathematics, physics, biology and chemistry; while, for mechanical engineering, it is very rare. The principle of this method is to decompose a solution into an infinite series which converges to exact solutions rapidly. The standard ADM and modified ADM were reviewed and discussed in Ref. [1] – [4]

In mechanics, the standard ADM was applied to solve vibration analysis of one-stepped and multiple-stepped beams [5] – [6]. In the study of Lai et al. [7] the standard ADM was also used to analyze free vibration of Euler-Bernoulli beams. In their further investigations, they have extended their works to deal with free vibration of non-uniform Euler-Bernoulli beams and uniform Timoshenko beams using the modified version [8] – [9]. From literature survey, it is seen that the AMDM was applied for solving free vibration of isotropic beams only.

Regarding to plate problems, the classical analytical methods, such as Navier and -type solutions, for solving plate problems subjected to static and dynamic loadings are well-known, for example in Ref. [10]. The extended Kantorovich method was employed to solve bending and buckling analysis of laminated composite and isotropic plates with various edge supports [11] – [12]. An accurate analytical method for calculating the static deflection and modal characteristics of orthotropic plates with general elastic boundary supports was presented by Khov et al. [13]. Leissa [14] provided the close-form solution for analyzing free vibration of thin isotropic plates, of which two opposite edges were simply-supported. Dozio [15] used a set of trigonometric functions as admissible solutions in the Ritz method in order to solve vibration problem of rectangular Kirchhoff plates. Differential quadrature method (DQM) was developed to solve free vibration problem of variable thickness thin and moderately thick plates in the study of Malekzadeh and Shahpari [16]. A Fourier series method was conducted by Li and Daniels [17] to deal with vibrations of elastically restrained plates including added mass effect.

In present study, to the best of authors' knowledge, it is the first time to apply AMDM to solve the vibration problem of thin elastic plates supported by all of common and elastically restrained boundary conditions. The fundamental and higher frequencies and their mode shapes in relation to various kinds of boundary conditions are presented and discussed. The significant effects such as spring constant values and plate aspect ratios are also taken into account.

2. THE AMDM BACKGROUND

Consider a general differential equation, composing of linear and nonlinear parts, including any given function as follows.

$$L\Psi(x)+R\Psi(x)+N\Psi(x)=g(x) \quad (1)$$

Where L is the linear invertible operator of the highest-order derivative, R, the remainder of the linear operator, $N\Psi(x)$, the nonlinear term and $g(x)$ is defined as any given function. It is noted that the Eq. (1) is an initial-value or a boundary-value problem. The solution of Eq. (1) can take the following form

$$\Psi(x)=\Phi+L^{-1}g(x)-L^{-1}R\Psi(x)-L^{-1}N\Psi(x) \quad (2)$$

where Φ is the constant of integration. According to an initial-value problem, the inverse operator L^{-1} is regarded as a definite integration from 0 to x. The AMDM is applied to solve Eq. (2) by decomposing $\Psi(x)$ into an infinite series as follows

$$\Psi(x)=\sum_{k=0}^{\infty}C_k x^k. \quad (3)$$

And the nonlinear term $N\Psi(x)$ is decomposed as:

$$N\Psi(x)=\sum_{k=0}^{\infty}x^k\Lambda_k(C_0,C_1,C_2...,C_k) \quad (4)$$

where Λ_k are known as Adomian coefficient. For $g(x)$, it is also required to decompose as:

$$g(x)=\sum_{k=0}^{\infty}G_k x^k \quad (5)$$

Substituting Eqs. (3 - 5) into Eq. (2), one can obtain

$$\Psi(x)=\sum_{k=0}^{\infty}C_k x^k=\Phi+L^{-1}\left(\sum_{k=0}^{\infty}G_k x^k\right)-L^{-1}\left(\sum_{k=0}^{\infty}C_k x^k\right) - L^{-1}\left(\sum_{k=0}^{\infty}x^k\Lambda_k(C_0,C_1,C_2...,C_k\right). \quad (6)$$

The recurrence relation with respect to initial condition is applied to obtain the coefficient C_k in Eq. (6). However, all series in Eq. (6) cannot be determined exactly in practice so that a truncated series, $\sum_{k=0}^{K-1}C_k x^k$, is used to approximate the solution.

3. VIBRATION OF RECTANGULAR PLATES

The well-known governing differential equation for vibration of isotropic rectangular thin plates is

$$\frac{\partial^4 w(\xi,\eta,t)}{\partial\xi^4}+2\frac{\partial^4 w(\xi,\eta,t)}{\partial\xi^2\partial\eta^2}+\frac{\partial^4 w(\xi,\eta,t)}{\partial\eta^4}+\frac{\rho h}{D}\frac{\partial w(\xi,\eta,t)}{\partial t^2}=0 \quad (7)$$

Where $w(\xi,\eta,t)$ is the transverse deflection, ρ, the plate mass density, t is time and $D=Eh^3/12(1-\nu^2)$, the flexural rigidity of the plate. Assuming the solution of the problem as $w(\xi,\eta,t)=e^{i\omega t}W(\xi,\eta)$, the time independent governing equation in dimensionless form can be obtained as:

$$\frac{\partial^4 W(x,y)}{\partial x^4}+2\lambda^2\frac{\partial^4 W(x,y)}{\partial x^2\partial y^2}+\lambda^4\frac{\partial^4 W(x,y)}{\partial y^4}-\Omega^2 W(x,y)=0 \quad (8)$$

Where $x=\frac{\xi}{a}$, $y=\frac{\eta}{b}$, $\lambda=\frac{a}{b}$ and $\Omega=\omega a^2\sqrt{(\rho h/D)}$ is the frequency parameter.

In this study, we consider simply supported plates along $y=0,1$ and arbitrarily supported along $x=0,1$. A plate deflection function which satisfies these conditions is

$$W(x,y)=\Psi(x)\sin(n\pi y) \quad (9)$$

Consequently, the governing equation of the plates can be reduced to an ordinary differential equation as:

$$\frac{d^4\Psi(x)}{dx^4}-2\lambda^2n^2\pi^2\frac{d^2\Psi(x)}{dx^2}+(\lambda^4n^4\pi^4-\Omega^2)\Psi(x)=0 \quad (10)$$

Let us consider a rectangular plate simply supported along $y=0,1$ and elastically restrained along x

= 0,1 as shown in Fig. 1. The counterclockwise four-letter symbolic notation will be used for describing boundary conditions of the plates throughout this paper. For example, the ESES plate as shown in Fig. 1 has the left edge (x= 0) elastically supported (E) by translational and rotational springs, therefore the first letter in the boundary condition notation is E. The following letters are given according to the counterclockwise direction respectively.

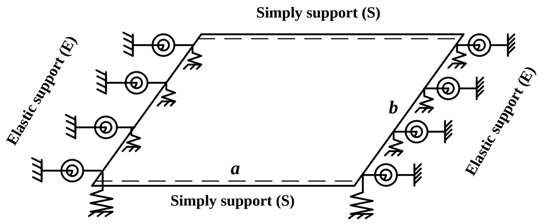

Fig. 1 *Geometry of a Rectangular Plate with ESES Boundary Condition*

The boundary conditions along the elastically restrained edges can be expressed as

$$K_{TL}W(x,y)=Q_x\,;\,K_{RL}\frac{\partial W(x,y)}{\partial x}=-M_x,\ \text{along } x=0 \tag{11}$$

$$K_{TR}W(x,y)=-Q_x\,;\,K_{RR}\frac{\partial W(x,y)}{\partial x}=M_x\ \text{ along } x=1 \tag{12}$$

where

$$M_x=-D\left(\frac{\partial^2 W(x,y)}{\partial x^2}+\nu\frac{\partial^2 W(x,y)}{\partial y^2}\right)$$
$$Q_x=-D\left(\frac{\partial^3 W(x,y)}{\partial x^3}+(2-\nu)\frac{\partial^3 W(x,y)}{\partial x\partial y^2}\right) \tag{13}$$

Where K_{TL}, K_{RL} are translational and rotational spring constants along x = 0, and K_{TR}, K_{RR}, for x = 1 respectively. Substituting the plate deflection, Eq. (9), into the boundary condition equations, Eqs. (11) and (12), the following dimensionless boundary equations are obtained.

along x = 0

$$\frac{d^3\Psi(x)}{d^3x}-(2-\nu)\lambda^2n^2\pi^2\frac{d\Psi(x)}{dx}+\beta_{TL}\Psi(x)=0 \tag{14a}$$

$$\frac{d^2\Psi(x)}{d^2x}-\nu\lambda^2n^2\pi^2\Psi(x)-\beta_{RL}\frac{d\Psi(x)}{dx}=0 \tag{14b}$$

along x = 1

$$\frac{d^3\Psi(x)}{d^3x}-(2-\nu)\lambda^2n^2\pi^2\frac{d\Psi(x)}{dx}-\beta_{TR}\Psi(x)=0 \tag{15a}$$

$$\frac{d^2\Psi(x)}{d^2x}-\nu\lambda^2n^2\pi^2\Psi(x)+\beta_{RR}\frac{d\Psi(x)}{dx}=0 \tag{15b}$$

Where, $\beta_{TL}=\frac{K_{TL}a^3}{D}$, $\beta_{RL}=\frac{K_{RL}a}{D}$, $\beta_{TR}=\frac{K_{TR}a^3}{D}$, and $\beta_{RR}=\frac{K_{RR}a}{D}$

4. APPLICATION OF AMDM TO VIBRATION PROBLEM OF PLATES

Following the AMDM outlined in section 2, the solution of the vibration problem of Eq. (10) is obtained as:

$$\Psi(x)=\Phi(x)+L^{-1}\left[2\lambda^2n^2\pi^2\frac{d^2\Psi}{dx^2}-(\lambda^4n^4\pi^4-\Omega^2)\Psi(x)\right] \tag{16}$$

where $L^{-1}=\int_0^x\int_0^x\int_0^x\int_0^x ....dxdxdxdx.$ Substituting the decomposing infinite series $\Psi(x)=\sum_{k=0}^{\infty}C_kx^k$ into Eq. (16) yields:

$$\Psi(x)=\sum_{k=0}^{\infty}C_kx^k=\Phi(x)+L^{-1}\left[2\lambda^2n^2\pi^2\sum_{k=0}^{\infty}(k+1)(k+2)C_{k+2}x^k-(\lambda^4n^4\pi^4-\Omega^2)\sum_{k=0}^{\infty}C_kx^k\right] \tag{17}$$

Integrating Eq. (17) leads to

$$\Psi(x)=\sum_{k=0}^{\infty}C_k x^k=\Phi(x)+\left\{\sum_{k=0}^{\infty}\frac{x^{k+4}}{(k+1)(k+2)(k+3)(k+4)}\times\left[2\lambda^2n^2\pi^2\sum_{k=0}^{\infty}(k+1)(k+2)C_{k+2}x^k-(\lambda^4n^4\pi^4-\Omega^2)\sum_{k=0}^{\infty}C_k x^k\right]\right\} \quad (18)$$

Where the constants of integration are:

$$\Phi(x)=\Psi(0)+\frac{d\Psi(0)}{dx}x+\frac{d^2\Psi(0)}{dx^2}\frac{x^2}{2}+\frac{d^3\Psi(0)}{dx^3}\frac{x^3}{6} \quad (19)$$

From Eq. (18), it can be seen that the initial coefficients of x^k, which are related to the initial boundary condition, can be expressed as:

$$C_0=\Psi(0),\ C_1=\frac{d\Psi(0)}{dx},\ C_2=\frac{1}{2}\frac{d^2\Psi(0)}{dx^2},\ C_3=\frac{1}{6}\frac{d^3\Psi(0)}{dx^3} \quad (20)$$

Other coefficients C_k for $k\geq 4$, can be obtained from the following recurrence relation.

$$C_k=\frac{1}{k(k-1)(k-2)(k-3)}\sum_{k=4}^{\infty}\left[2\lambda^2n^2\pi^2(k-2)(k-3)C_{k-2}-(\lambda^4n^4\pi^4-\Omega^2)C_{k-4}\right] \quad (21)$$

Hence, the coefficients that satisfy the initial boundary conditions at $x=0$ for various types of boundary conditions can be achieved as presented in Table 1

Table 1 *Initial Coefficients for Various Types of Boundary Conditions at x=0*

B.C.	C_0	C_1	C_2	C_3
Simply (S)	0	A_1	0	A_2
Clamped (C)	0	0	A_1	A_2
Free (F)	A_1	A_2	$A_1\nu\lambda^2n^2\pi^2/2$	$A_2(2-\nu)\lambda^2n^2\pi^2/6$
Elastic (E)	A_1	A_2	$A_1\nu\lambda^2n^2\pi^2/2+A_2\beta_{RL}/2$	$A_2(2-\nu)\lambda^2n^2\pi^2/6-A_1\beta_{TL}$

From the initial coefficients C_0 to C_3 tabulated in Table 1, other C_k , for $k\geq 4$, can be expressed in terms of arbitrary constants, A_1 and A_2. Now all of C_k , for k = 0 to ∞ , are ready to be inserted into the solution $\Psi(x)=\sum_{k=0}^{\infty}C_k x^k$. In practice, however, it is impossible to determine all the coefficients C_k for the whole series solution. The appropriate approach is to truncate the series for K-term approximation, ie. $\Psi(x)=\sum_{k=0}^{K-1}C_k x^k$. The next step is to substitute the solution that already satisfies the initial boundary condition into various boundary condition equations at x = 1, which are presented in Table 2

The results of the substitution yield two equations that can be written as follows

$$f_{r1}^{[K]}(\Omega)A_1+f_{r2}^{[K]}(\Omega)A_2=0,\ r=1,2. \quad (22)$$

For non-trivial solutions, the determinant of the coefficient matrix in Eq. (22) is set to zero. Thus, the frequency equation is obtained from:

$$\begin{vmatrix} f_{11}^{[K]}(\Omega) & f_{12}^{[K]}(\Omega) \\ f_{21}^{[K]}(\Omega) & f_{22}^{[K]}(\Omega)\end{vmatrix}=0 \quad (23)$$

Solving the frequency equation, one can obtain the i^{th} approximated frequency, $\Omega_i^{[K]}$ corresponding to K terms. The appropriate value of K is determined from convergence study with the following criterion,

$$\left|\Omega_i^{[K]}-\Omega_i^{[K-1]}\right|\leq\delta \quad (24)$$

Where δ is a given error tolerance. The amplitudes of vibrating plates can be obtained from either equation of Eq. (22), that is:

$$A_2=-\frac{f_{r1}^{[K]}(\Omega_i^{[K]})}{f_{r2}^{[n]}(\Omega_i^{[K]})}A_1,\ r=1\text{ or }2. \quad (25)$$

Setting the value of A_1 to unity yields the result of A_2 for substituting back into all coefficients C_k. Thus, the mode shapes corresponding to any frequency can be obtained as:

$$W(x,y)=\sum_{k=0}^{K-1} C_k x^k \sin(n\pi y). \quad (26)$$

Table 2 *Equations of Various Types of Boundary Conditions at x = 1*

B.C.	Boundary condition equations
Simply (S)	$\Psi(1)=0$ $\frac{\partial^2\Psi(1)}{\partial^2 x}-\nu\lambda^2 n^2\pi^2\Psi(1)=0$
Clamped (C)	$\Psi(1)=0$ $\frac{\partial\Psi(1)}{\partial x}=0$
Free (F)	$\frac{\partial^2\Psi(1)}{\partial^2 x}-\nu\lambda^2 n^2\pi^2\Psi(1)=0$ $\frac{\partial^3\Psi(1)}{\partial^3 x}-(2-\nu)\lambda^2 n^2\pi^2\frac{\partial\Psi(1)}{\partial x}=0$
Elastic (E)	$\frac{\partial^2\Psi(1)}{\partial^2 x}-\nu\lambda^2 n^2\pi^2\Psi(1)+\beta_{RR}\frac{\partial\Psi(1)}{\partial x}=0$ $\frac{\partial^3\Psi(1)}{\partial^3 x}-(2-\nu)\lambda^2 n^2\pi^2\frac{\partial\Psi(1)}{\partial x}-\beta_{TR}\Psi(1)=0$

5. NUMERICAL RESULTS AND DISCUSSION

5.1 RESULTS FOR COMMON BOUNDARY CONDITIONS

To demonstrate the effectiveness and rate of convergence of AMDM in solving vibration problems of thin plates, fundamental and higher frequencies, as well as their mode shapes of square plates, regarding to three types of boundary conditions are presented in Table 3. Boundary conditions considered in this demonstration are SSSS, SSCS and SSFS. According to the tabulated frequency results, AMDM gives accurate results with small number of approximated terms (K=25). However, to guarantee an accuracy, subsequent calculations for K=35 will be used throughout this paper. To validate our method, the present results are compared with available results in Ref. [15] for the case of simply supported boundary condition (SSSS). We also found out that AMDM shows very good computational stability for any boundary conditions. Some mode shapes of the square plates, corresponding to frequency results in Table 3, are plotted in Fig.2 using Eq.(26)

Table 3 *Convergence Study for Frequency Results of Square Plates with Three Boundary Conditions*

		Mode (*x*,*y*)							
B.C.	*K*	(1,1)	(1,2)	(2,1)	(2,2)	(1,3)	(2,3)	(3,3)	(1,4)
SSSS	25	19.729	49.308	49.338	78.892	98.606	128.215	181.789	167.627
	30	19.729	49.308	49.338	78.917	98.606	128.215	177.553	167.623
	33	19.729	49.308	49.338	78.917	98.606	128.215	177.563	167.623
	35	19.729	49.308	49.338	78.917	98.606	128.215	177.563	167.623
[15]		**19.739**	**49.348**	**49.348**	**78.960**	**98.697**	**128.305**	-	-
SSCS	25	23.637	51.636	58.636	86.099	100.188	133.756	-	168.859
	30	23.637	51.635	58.637	86.096	100.181	133.704	188.026	168.802
	33	23.637	51.635	58.637	86.096	100.181	133.704	188.029	168.802
	35	23.637	51.635	58.637	86.096	100.181	133.704	188.025	168.804
SSFS	25	11.675	41.158	27.745	59.029	90.306	109.018	146.085	159.925
	30	11.675	41.157	27.745	59.025	90.207	108.834	145.569	159.029
	33	11.675	41.157	27.745	59.025	90.205	108.829	145.551	158.960
	35	11.675	41.157	27.745	59.025	90.205	108.828	145.550	158.933

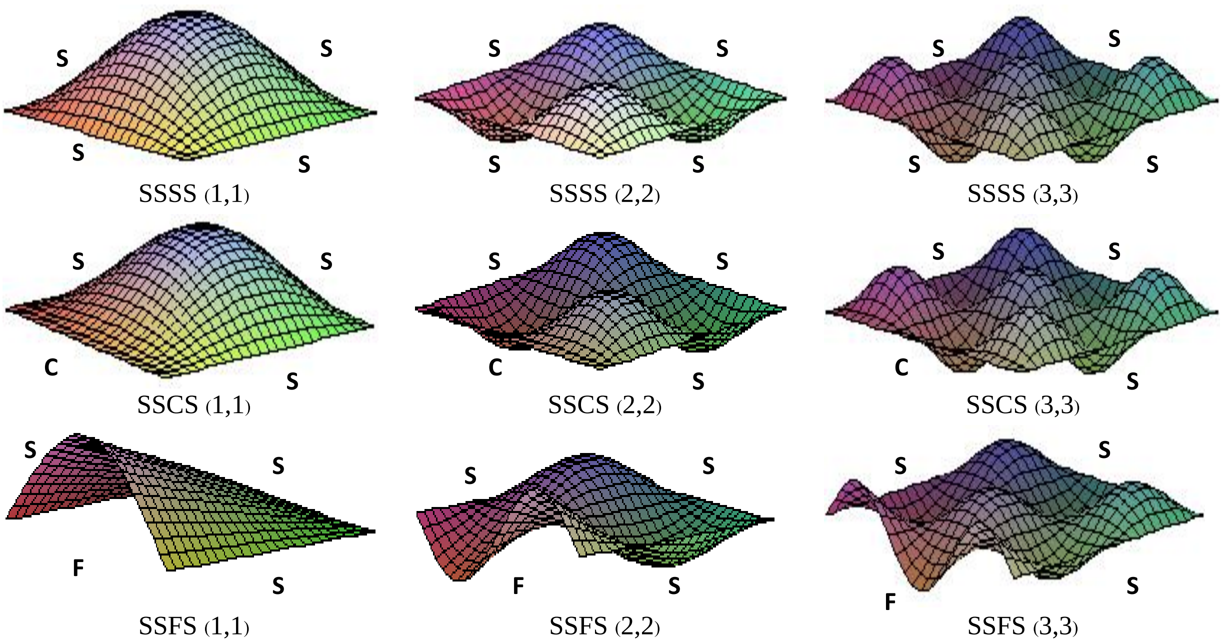

Fig.2 *Mode Shapes of Square Plates with Different Boundary Conditions*

***Table 4** Frequency Results of Rectangular Plates for Six Boundary Conditions*

	SSSS				SSCS		
(Mode)	(1,1)	(2,1)	(3,1)	(Mode)	(1,1)	(2,1)	(3,1)
λ=0.5	12.335	41.943	91.291	λ=0.5	17.330	52.096	106.476
λ=1.0	19.729	49.338	98.686	λ=1.0	23.637	58.637	113.218
[14]	19.739	49.348	98.696	[14]	23.646	58.646	113.228
λ=1.5	32.054	61.663	111.010	λ=1.5	35.030	69.892	124.612
[14]	32.076	61.685	111.033	[14]	-	-	-
λ=2.5	71.492	101.163	150.450	λ=2.5	73.377	107.359	161.924
[14]	71.556	101.163	150.512	[14]	-	-	-
	SSFS				FSFS		
(Mode)	(1,1)	(2,1)	(3,1)	(Mode)	(1,1)	(2,1)	(3,1)
λ=0.5	4.031	18.818	53.022	λ=0.5	2.376	6.877	26.369
λ=1.0	11.675	27.745	61.849	λ=1.0	9.622	16.124	36.713
[14]	11.685	27.756	61.861	[14]	9.631	16.135	36.726
λ=1.5	23.988	41.150	75.794	λ=1.5	21.799	29.185	51.620
λ=2.5	63.225	81.543	117.679	λ=2.5	60.924	69.016	92.218
	FSCS				CSCS		
(Mode)	(1,1)	(2,1)	(3,1)	(Mode)	(1,1)	(2,1)	(3,1)
λ=0.5	5.702	24.941	64.399	λ=0.5	23.814	63.533	122.911
λ=1.0	12.678	33.055	72.387	λ=1.0	28.943	69.319	129.032
[14]	12.687	33.065	72.398	[14]	28.951	69.327	129.096
λ=1.5	24.672	45.732	85.375	λ=1.5	39.070	79.506	139.021
λ=2.5	63.617	84.993	122.604	λ=2.5	75.772	116.616	140.450

Table 4 presents frequency results of rectangular plates with six different boundary conditions. The aspect ratio $\lambda = a/b$ is varied from 0.5 to 2.5. The existing results of Leissa [14] are used to validate our results. Of all boundary conditions considered in this table, CSCS plates provide the greatest values of frequencies for all aspect ratios, whereas, the FSFS plates give the lowest ones. Increase the values of the aspect ratio leads to increase of frequencies for every boundary condition.

5.2 RESULTS FOR ELASTICALLY RESTRAINED BOUNDARY CONDITIONS

To validate results for elastic edge supports, only the available frequencies results for the square plate of Ref. [15] are shown in Table 5. It is seen that all modes of frequencies agree very well.

***Table 5** Comparison of Frequency Results of a Square Plate with Elastic Boundary Condition*

B.C.	β_{TR}	β_{RR}	Source	Mode (1,1)	(1,2)	(2,1)	(2,2)	(3,1)	(3,2)
CSES	10	0	Present	13.923	42.050	33.666	63.261	72.672	102.592
			[15]	13.932	42.089	33.677	63.298	72.684	-
	100	10	Present	19.391	44.773	40.710	67.011	81.042	107.567
			[15]	19.398	44.810	40.719	67.043	81.052	-

To investigate effects of aspect ratios on frequencies of elastically restrained edges, the frequencies for the cases of CSES, SSES, FSES and ESES are shown in Table 6. All spring constant values are set to be the same: $\beta TL = \beta RL = \beta TR = \beta RR = 100$. From observation, it is seen that all frequencies increase as the aspect ratios.

***Table 6** Frequency Results with Elastic Edge Supports for Different Aspect Ratios.*

	CSES				SSES		
(Mode)	(1,1)	(2,1)	(3,1)	(Mode)	(1,1)	(2,1)	(3,1)
λ=0.5	15.505	35.598	77.619	λ=0.5	12.430	29.051	65.357
λ=1.0	19.472	41.423	84.177	λ=1.0	17.339	35.329	72.572
λ=1.5	29.116	52.000	95.336	λ=1.5	27.782	46.656	84.675
λ=2.5	66.051	90.525	95.336	λ=2.5	65.465	84.914	123.715
	FSES				ESES		
(Mode)	(1,1)	(2,1)	(3,1)	(Mode)	(1,1)	(2,1)	(3,1)
λ=0.5	5.222	16.145	36.562	λ=0.5	12.671	23.003	46.354
λ=1.0	11.620	22.297	44.652	λ=1.0	16.185	27.716	52.996
λ=1.5	23.195	33.464	57.763	λ=1.5	25.818	37.515	64.477
λ=2.5	61.788	71.631	95.657	λ=2.5	63.046	74.390	98.677

Consider ESES plate, as all spring constants are set to zero the mode shapes become those of FSFS plate and as spring constants are large the mode shapes assume those of CSCS plate as shown in Fig. 3

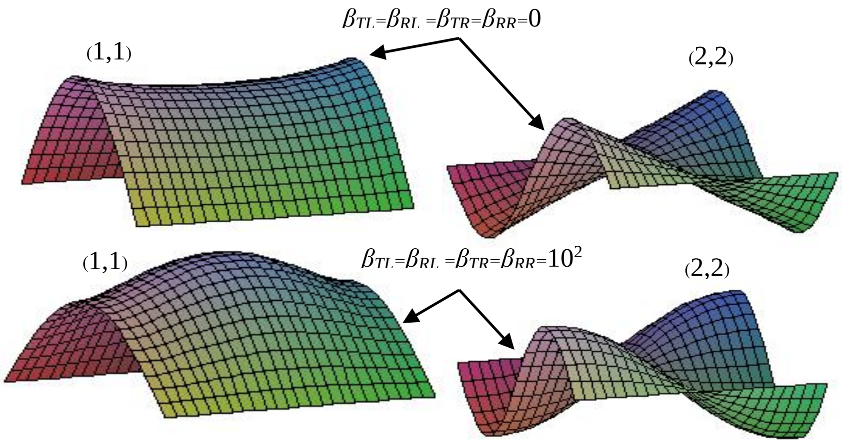

***Fig. 3** Mode Shapes of ESES Square Plates with Special Values of Spring Constants*

Table 7 *Effects of Spring Constants on Fundamental Frequencies of Square Plates*

		β_{RL}=β_{RR}						
β_{TL}=β_{TR}	B.C.	0	1	10	10^2	10^3	10^4	10^5
0	CSES	12.678	12.884	13.401	13.640	13.673	13.676	13.676
	SSES	11.675	11.814	12.152	12.304	12.325	12.327	12.327
	FSES	9.622	9.646	9.700	9.723	9.726	9.726	9.726
	ESES	9.262	9.672	9.795	9.851	9.859	9.860	9.860
1	CSES	12.814	13.010	13.500	13.728	13.759	13.763	13.763
	SSES	11.797	11.927	12.243	12.386	12.405	12.407	12.408
	FSES	9.683	9.704	9.750	9.769	9.771	9.771	9.772
	ESES	9.747	9.792	9.902	9.953	9.959	9.960	9.960
10	CSES	13.923	14.039	14.337	14.480	14.500	14.502	14.503
	SSES	12.777	12.841	13.004	13.080	13.090	13.091	13.091
	FSES	10.125	10.127	10.131	10.132	10.133	10.133	10.133
	ESES	10.746	10.759	10.791	10.807	10.809	10.809	10.809
10^2	CSES	19.212	19.256	19.391	19.472	19.484	19.485	19.485
	SSES	16.920	16.990	17.208	17.339	17.359	17.361	17.361
	FSES	11.241	11.303	11.500	11.620	11.638	11.640	11.640
	ESES	15.322	15.470	15.923	16.185	16.225	16.229	16.230
10^3	CSES	23.062	23.532	25.417	26.983	27.262	27.292	27.295
	SSES	19.380	19.762	21.249	22.427	22.631	22.652	22.655
	FSES	11.624	11.749	12.187	12.487	12.536	12.541	12.542
	ESES	19.054	19.795	22.810	25.425	25.905	25.957	25.962
10^4	CSES	23.579	24.137	26.432	28.387	28.736	28.773	28.777
	SSES	19.694	20.132	21.867	23.266	23.509	23.535	23.538
	FSES	11.669	11.803	12.276	12.604	12.658	12.663	12.664
	ESES	19.659	20.542	24.321	27.857	28.533	28.607	28.615
10^5	CSES	23.632	24.199	26.536	28.529	28.884	28.922	28.926
	SSES	19.726	20.170	21.930	23.351	23.598	23.625	23.627
	FSES	11.674	11.809	12.285	12.616	12.670	12.676	12.676
	ESES	19.722	20.621	24.485	28.127	28.825	28.902	28.909

Table 7 presents the fundamental frequencies of square plates supported by four types of elastic boundary conditions. As expected, the frequencies become higher as the translational and rotational spring constants increase. To understand this behavior more clearly, the case of ESES plates in Table 7 is chosen to display in a 3-D plot as shown in Fig. 4.

Fig. 5 plots a graph of the fundamental frequency results of ESES plates with respect to changes in aspect ratios and spring constant values. It is observed that increase in the values of aspect ratios and spring constants leads to higher frequency results. The considerable change in frequency results is seen in the range of β_{TL}=β_{RL}= β_{TR}=β_{RR}=10 to 1000

6. CONCLUDSIONS

In this study, the Adomian modified decomposition method (AMDM) is implemented to do vibration analysis of thin plates with various common and elastic edge supports. The innovative method demonstrates plenty of benefits as seen with rapid convergence, small computational expensiveness and stability in calculation as well as accuracy. The fundamental and higher frequencies, including their corresponding mode shapes, for several types of boundary conditions are determined easily. Numerical results have revealed that translational and rotational spring constants have great impact on both natural frequencies and mode shapes. Increasing the values of the spring constants usually causes substantial changes in frequencies and mode shapes. Additionally, boundary conditions and aspect ratios also have great influence on frequencies and mode shapes of plates

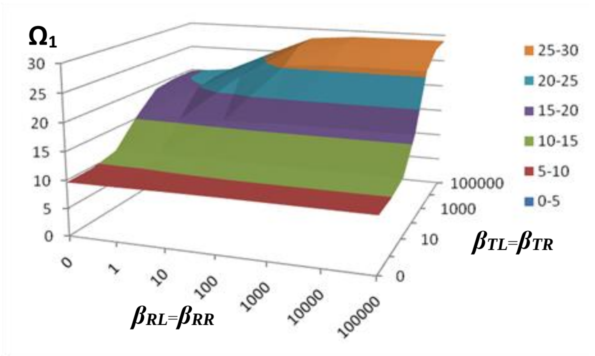

***Fig. 4** Fundamental Frequencies of ESES Plates with Variable Translational and Rotational Spring Constants*

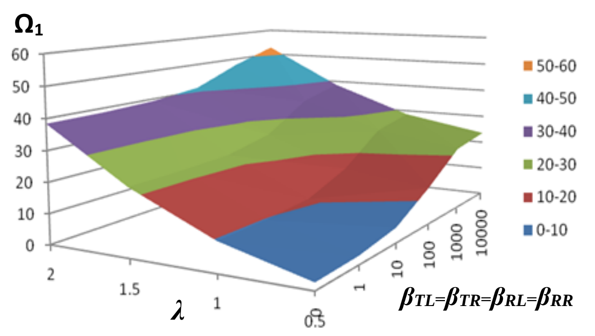

***Fig. 5** Fundamental Frequencies of ESES Plates with Different Spring Constants and Aspect*

REFERENCES

[1] G. Adomian, Solving frontier problems of physics: The decomposition method, Kluwer Academic Publishers, Dordrecht, 1994.

[2] G. Adomian and R. Rach, "Modified decomposition solution of nonlinear partial differential equations", Computer and mathematics with applications, vol. 23, pp. 17-23, 1992.

[3] A. M. Wazwaz, " A reliable modification of Adomian decomposition method", Applied mathematics and computation, vol. 102, pp. 77-86, 1999.

[4] B. Jang, "Exact solutions to one dimensional non-homogeneous parabolic problems by the homogeneous Adomian decomposition method", Applied mathematics and computation, vol. 189, pp. 969-979, 2007.

[5] Q. Mao and S. Pietrzko, " Free vibration analysis of stepped beams by using Adomian decomposition method" , Applied mathematics and computation, vol. 217, pp. 3429-3441, 2010.

[6] Q. Mao, " Free vibration analysis of multiple-stepped beams by using Adomian decomposition method," Mathematical and computer modeling, vol. 54, pp.756-764, 2011.

[7] H.Y. Lai, J.C. Hsu and C.K. Chen, " An innovative eigenvalue problem solver for free vibration of Euler-Bernoulli beam by using the Adomian decomposition method" , Computers and mathematics with application, vol. 56, pp. 3204-3220, 2008.

[8] J.C. Hsu, H.Y. Lai and C.K. Chen, "Free vibration of non-uniform Euler-Bernoulli beams with general elastically end constraints using Adomian modified decomposition method," Journal of Sound and vibration, vol. 318, pp. 965-981, 2008.

[9] J.C. Hsu, H.Y. Lai and C.K. Chen., " An innovative eigenvalue problem solver for free vibration of uniform Timoshenko beams by the Adomian modified decomposition method" , Journal of Sound and vibration, vol. 325, pp. 451-470, 2009.

[10] J.N. Reddy, Theory and analysis of elastic plates and shells, 2nd Ed., CRC Press, U.S.A, 2007.

[11] V. Ungbhakorn, and N. Wattanasakulpong, "Bending analysis of symmetrically laminated rectangular plates with arbitrary edge supports by the extended Kantorovich method", Thammasat Int. J. Science and technology, vol. 11, pp. 33-44, 2006.

[12] V. Ungbhakorn and P. Singhatanadgid, " Buckling analysis of symmetrically laminated composite plates by the extended Kantorovich method" , Composite Structures, vol. 73, pp. 120-128, 2006.

[13] H. Khov, W.L. Li, and R.F. Gibson, "An accurate solution method for static and dynamic deflections of orthotropic plates with general boundary conditions," Composite structures, vol. 90, pp. 474-481, 2009.

[14] A.W. Leissa, "The free vibration of rectangular plates", Journal of Sound and vibration, vol. 31, pp. 257-293, 1973.

[15] L. Dozio, " On the use of the trigonometric Ritz method for general vibration analysis of rectangular Kirchhoff plates", Thin-walled structures, vol. 49, pp. 129-144, 2011.

[16] P. Malekzadeh and S.A. Shahpari, " Free vibration analysis of variable thickness thin and moderately thick plates with elastically restrained edges by DQM" , Thin-walled structures, vol. 43, pp. 1037-1050, 2005.

[17] W.L. Li and M. Daniels, " A fourier series method for the vibrations of elastically restrained plates arbitrarily loaded with springs and masses", Journal Sound and vibration, vol. 252, pp. 768-781, 2002.

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