Higher Frequencies of Square Plates with Symmetrically Mixed of Simply Supported-Clamped Edge Conditions

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ABSTRACT

Since the vibration problem of plates with mixed edge conditions is of academic and technological importance, the need of knowing natural frequencies is then required. This paper attempts to deal with an accurate value for some higher frequencies expressed in terms of frequency parameters of square plates having mixed edges between the simple and clamped supports. Two plate configurations are considered. The first is of the clamped plate with varying corner simply supported lengths; while the second is the simply supported plate having an equal angle type of clamped support placed at all corners. The analysis is made by means of finite element code with a dense net of element mesh. The first twenty frequencies and their vibratory modes are presented, which could serve as a benchmark for comparison with other methods.

Keywords : *Dimensionless frequency parameter, Finite element analysis, Higher natural frequencies, Mixed edge conditions, Square plate*.

1. INTRODUCTION AND UNDERLYING THEORY

Within the scope of classical plate theory, the history of plate theory development was started by Euler, who in 1766 formulated the first mathematical approach to the membrane theory of plates [1]. By practicing uses, plates are basic components in engineering design and application with widespread uses in aircraft, ship, and building structures. Importantly, as is well-known that plate structures are generally recognized and proved to be useful models for many complex structures, because the plate action behaviors result in lighter structures and offer economical advantages.

At the present time, there exists numerous analyses both analytical and numerical methods for the plates. However, a significant contribution and extensive study in the area of plate bending analysis together with its application have been comprehensively collected and also summarized in a monograph by Timoshenko and Woinowsky-Krieger [2]. In addition, plate vibrational behaviors are also of great interest, especially the free vibration characteristics (natural frequencies and their associated vibratory shapes) [3], [4]. Since the analysis of free vibration problems is of basic and applied interest in several fields of science and technology [5,6], therefore, an exhaustive summary of the published literature on the free vibrations of various shaped plates is available in Leissa [7], [8].

In order to analytically determine the solution of partial differential equation of plates, exact analytical solutions are the most desirable, but not always easily attainable. This is due to the difficulties in trying to obtain the solutions satisfied both the plate's governing equation and all of the boundary conditions exactly.

Consequently, with these causes as mentioned above, Leissa [9] has attempted to investigate and present the comprehensive and obtainable accurate analytical results for the different twenty-one cases of free vibration problem of rectangular plates with various aspect ratios and Poisson's ratios. It is, however, interesting and significant to note that exact characteristic equations involving frequency determinations can explicitly be given and expressed in analytical closed-forms only for the six specific cases of plate with two opposite simply supported edges. Thus, enormous research works have been conducted and treated numerically using a wide range of approximate mathematical techniques.

Before considering further to deal with the numerical treatment of the titled problems, the method of analytical solution is reviewed and presented for the plates being assumed to be isotropic, free from applied external loads and of uniform thickness. The theoretical equations that involved and governed with the behaviors of free vibration problem of plate in the rectangular coordinates together with the common boundary condition equations of which being clamped, simply supported, and free edges are mathematically explained and given in the following equations below.

With limited to consideration of the problems of undamped free flexural vibratory rectangular plate that are basically eigenvalue problems of the mathematical physics, the classical governing two-dimensional partial differential equation of plate motion for the transverse displacement (w) at any point (x,y) and perpendicular to the plane of the plate can be written as [3]

$$D\nabla^4 w + \rho \frac{\partial^2 w}{\partial t^2} = 0 , \tag{1}$$

where ρ is mass density per unit area of plate surface, t is time, and D is flexural rigidity of the plate defined by

$$D = \frac{Eh^3}{12(1-\nu^2)} , \tag{2}$$

in which E is Young's modulus, h is thickness of plate, v is Poisson's ratio, and ∇^4 is the biharmonic differential operator that given by

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} . \tag{3}$$

Since free vibrations are considered and assumed to be a sinusoidal time response, therefore, the transverse displacement for the plate motion is expressible as [3], [5]

$$w(x, y, t) = W(x, y)\,\mathrm{e}^{i\omega t} , \tag{4}$$

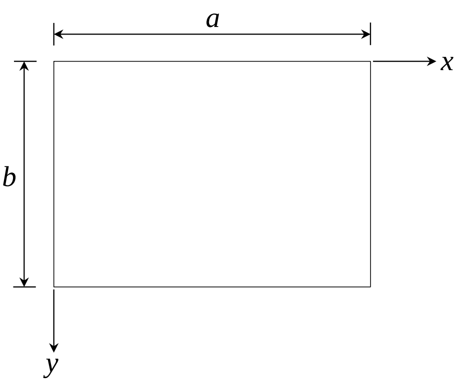

Fig. 1 *Dimensions and coordinates of rectangular plate.*

and

$$\mathrm{e}^{i\omega t} = \cos(\omega t) + i\sin(\omega t) , \tag{5}$$

where W is a function only of the position coordinates, ω is the circular frequency, and $i = \sqrt{-1}$.

Substituting the transverse displacement of Eq.(4) into plate's differential equation of motion as given by Eq.(1) and eliminating out the time dependence together with introducing a parameter k defined by

$$k^4 = \frac{\rho\omega^2}{D} , \tag{6}$$

hence, the governing differential equation of the plate can be cast in the form

$$(\nabla^4 - k^4)W = 0 . \tag{7}$$

Considering a rectangular plate with dimensions of length a and width b along the direction of x- and y-axes as shown in Fig. 1, respectively, and supposing an edge parallel to the y-axis to be clamped edge, the boundary conditions are given by, independent of time t,

$$W = 0 , \tag{8a}$$

$$\frac{\partial W}{\partial x} = 0 . \tag{8a}$$

For a simply supported edge, the condition equations can be expressed as

$$W = 0, \tag{9a}$$

$$\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2} = 0, \tag{9b}$$

and for a free edge,

$$\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2} = 0, \tag{10a}$$

$$\frac{\partial^3 W}{\partial x^3} + (2-\nu)\frac{\partial^3 W}{\partial x \partial y^2} = 0. \tag{10b}$$

If the plate has two opposite edges simply supported at $x = 0$ and $x = a$, the general solutions to Eq.(7) can be assumed to be a Fourier series form following the Levy-type solution as [8] – [10]

$$W(x, y) = \sum_{m=1}^{\infty} Y_m(y) \sin \alpha x, \tag{11}$$

and

$$\alpha = m\pi / a. \tag{12}$$

It can immediately be seen that Eq.(11) is a series solution involving the Fourier trigonometric functions of sinαx whereas Y_m are functions of the variable y only. Significantly, Eq.(11) exactly satisfies Eq.(7) and the simply supported edge conditions of Eqs.(9a) and (9b) along $x = 0$ and $x = a$.

Substitution of Eq.(11) into Eq.(7) yields

$$\sum_{m=1}^{\infty}\left[\frac{d^4 Y_m(y)}{dy^4} - 2\alpha^2 \frac{d^2 Y_m(y)}{dy^2} + (\alpha^4 - k^4) Y_m(y)\right]\sin \alpha x = 0. \tag{13}$$

Noted that the quantities inside the bracket must be identically zero for each value of m leading to

$$\frac{d^4 Y_m(y)}{dy^4} - 2\alpha^2 \frac{d^2 Y_m(y)}{dy^2} + (\alpha^4 - k^4) Y_m(y) = 0. \tag{14}$$

This shows that Eq. (14) is now an ordinary fourth-order homogeneous differential equation with constant coefficients.

The solution to Eq.(14) for Y_m is well known in mathematical point of views [11], [12] and its solution form is, however, seriously depended upon whether the third coefficient term of parenthesis gives the result to be negative or positive values.

With the assumption of $k^2 > \alpha^2$, the solution Y_m is found to be

$$Y_m = A_m \sin\sqrt{k^2 - \alpha^2}\, y + B_m \cos\sqrt{k^2 - \alpha^2}\, y + C_m \sinh\sqrt{k^2 + \alpha^2}\, y + D_m \cosh\sqrt{k^2 + \alpha^2}\, y, \tag{15}$$

where the arbitrary coefficients A_m, B_m, C_m, and D_m can be determined from the prescribed boundary conditions along the edges at $y = 0$ and $y = b$.

If k^2 is assumed to be less than α^2, the solution Y_m given in Eq.(15) has to be rewritten in the new form as, with $k^2 < \alpha^2$,

$$Y_m = A_m \sinh\sqrt{\alpha^2 - k^2}\, y + B_m \cosh\sqrt{\alpha^2 - k^2}\, y + C_m \sinh\sqrt{k^2 + \alpha^2}\, y + D_m \cosh\sqrt{k^2 + \alpha^2}\, y. \tag{16}$$

As is mentioned earlier [9] for the plates having two opposite edges simply supported along $x = 0$ and $x = a$, there are six cases of all possible combinations among clamped, simply supported or free along two remaining opposite edges. These lead to the existence of exact characteristic equations for determination of frequencies with their associated vibratory mode shapes.

Substituting Eq.(11) together with using Eqs.(15) or (16) into the two appropriate boundary conditions along each edge of $y = 0$ and $y = b$ in Eqs.(8) through (10) and interchanging the variables between x and y in their expressions yields a characteristic determinant equation of the fourth order for each m in corresponding case of six different plates. After that expanding the determinant and rearranging terms yields a complex transcendental characteristic equation, which has already presented in the previous works [8], [9].

In the problem fields of free vibration analysis, it is, however, very useful to express the circular frequency (ω) in terms of dimensionless frequency parameters (λ^2). By introducing $\lambda = ka$ where a is a dimension of plate length and using Eq.(6), the frequency parameter can be taken as in the form

$$\lambda^2 = \omega a^2 \sqrt{\frac{\rho}{D}}. \tag{17}$$

2. PROBLEMS CONSIDERATION

From the observation on a search of open available published literature is remarkable that the literature on free vibration problems of plate is vast, and most of them have considered the plates supported by regular or common boundary conditions along the plate edges. Relatively few published research is available about the vibrations of plate with mixed boundary conditions. By "mixed boundary conditions" it is referred to situations where there are discontinuities in the type of support supplied to one or more of the plate edges.

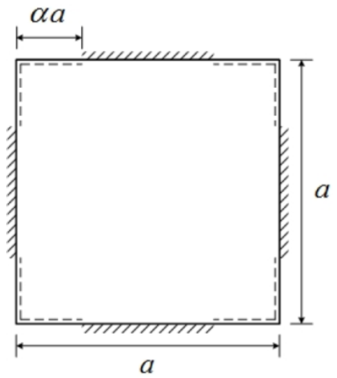

Fig. 2 *Clamped square plate with varying equal angle-leg corner simple supports.*

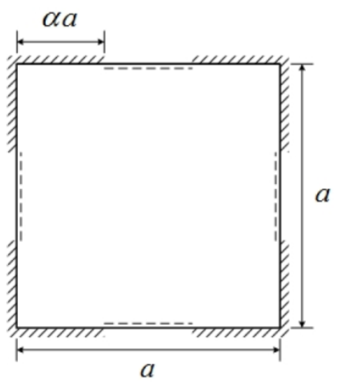

Fig. 3 *Simply supported square plate with varying equal angle-leg corner clamped supports.*

It is also noted that among all the possible shape of plates of which are circular, triangular, trapezoidal, and rectangular plates, etc., the rectangular plate is of the greatest importance and interest in vibration analysis.

Since free vibration characteristics of the plate that involved with frequencies and their corresponding mode shapes are of important and most interests, free vibration results are then certainly useful in which knowledge of the natural frequencies can help the designer to avoid the peak resonance.

Of particular interest in the present paper is mainly focused on and limited to the case of both rectangular and square plates having mixed boundary conditions. Although there have been some vibration analyses of such plates documented in the published literature, but the authors have found no publications that consider the determination of higher frequencies of the plates. One of the reasons that may be caused by the difficulties in formulating and deriving the characteristic equation of problem satisfied all boundary conditions as well as the governing differential equation exactly.

However, vibration problems of rectangular plates with mixed edge conditions have been analyzed by some investigators with the use of various solution techniques to overcome the technical and mathematical difficulties. Therefore, analyses have been carried out by means of analytical methods, namely, the energy method [13] – [15], Fourier series-type method [16], [17], conjugate series equations method [18], integral equation method [19] – [21], superposition method [22], [23], Rayleigh quotient method [24], and the Ritz method [25] – [27].

With the advent of very efficient high performance of computer speed that allowed solving the solutions of a large number of algebraic equations, among a number of numerical methods found in the literature, such as finite element method [28], [29], finite strip method [30], finite strip element method [31], differential quadrature method [32], [33], quadrature element method [34], [35], the methods based on discrete singular convolution [36] – [38], and recently the method of Hencky bar-net model [39], the finite element method is a convenient approach and is one of the powerful numerical methods to analyze the complex problems or the domain irregularities.

Based on the finite element method [40,41] together with the use of appropriate consistent and conforming plate bending elements, the governing partial differential equation presented in Eq.(7) can be arranged in a form of matrix equation to determine the natural frequencies (ω) and their associated vibratory mode shapes $\{W\}$ of the plate as

$$\{[K]-\omega^2[M]\}\{W\}=\{0\}, \tag{18}$$

where $[K]$ and $[M]$ are the matrices of global stiffness and consistent mass for the system, respectively.

The aim of this paper is to numerically determine and provide some accurate values of higher frequency parameter for free vibratory square plates with mixed edge conditions by making use of an available well-known finite element code [42], [43], which have been successfully analyzed the free vibration of square plates with twenty-one different common boundary conditions along the plate edges [44], and more recently the free vibration of circular plates with mixed edge conditions [45]. Two different kinds of square plate configuration are then considered as demonstrated in Figs. 2 and 3 in accordance with the cases of clamped plate with varying corner simply supported lengths and simply supported plate having an equal angle type of clamped support length that can be varied at all corners, respectively. It is notable as illustrated in Figs. 2 and 3 that the parameter α has the values ranging from 0.0 to 0.5.

3. NUMERICAL RESULTS AND DISCUSSION

For analyzing the plate vibration problems at hand, the ANSYS finite element program [42] is implemented to model the plates by making use of quadrilateral shape of SHELL181 element type [43] as demonstrated in Fig. 4. This element has four corner nodes with six degrees of freedom (three translations along and three rotations about the *x*, *y*, and *z*-axis) specified at each node, which is suitable for analyzing problems of thin to moderately-thick plates. Additionally, it has also well-suited for the analysis of both linear and nonlinear problems.

In order to achieve the numerical results accurately for the frequency parameters, the finite element model of both plates (Figs. 2 and 3) is discretized identically using a large element number of uniformly meshed with 10000 square elements (resulted from a 100×100 mesh used in the complete plate) for the problems considered as shown in Fig. 5.

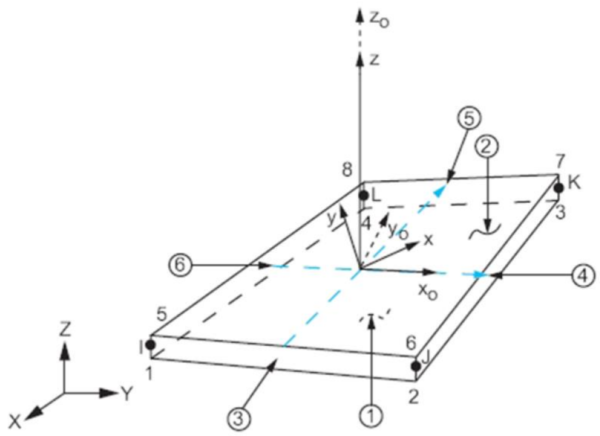

Fig. 4 *Quadrilateral SHELL181 element [43].*

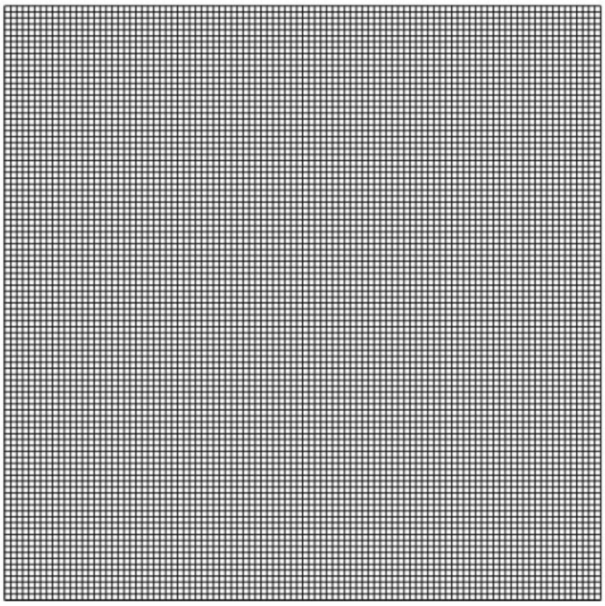

Fig. 5 *Elements mesh of square plate model*.

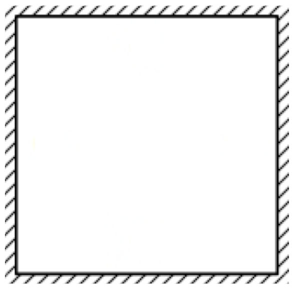

Fig. 6 *A fully clamped square plate.*

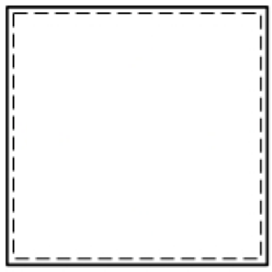

Fig. 7 *A fully simply supported square plate.*

Before solving the equation as presented in Eq.(18) to determine the required frequencies that corresponded to each mode of vibrations, the proposed criterion to be used in applying significant boundary points (the points having two different boundary support types) due to the plate having mixed boundary support conditions can be described that higher constraint condition than another is selected to represent the boundary conditions at those transition points. For example, one considers the square plate that illustrated in Fig. 2, condition equations for clamped support are then used at all transition points of discontinuity in the finite element support model as well as the plate shown in Fig. 3.

Results for the first twenty frequency parameters (λ^2) of square plates having mixed boundary conditions are prepared and tabulated in Tables 1 and 2 for the value of Poisson's ratio taken to be 0.3.

As can be seen in Tables 1 and 2 that there are two limiting cases existed for α = 0.0 and 0.5. Since α equals 0.0, the plates as shown in Figs. 2 and 3 become the fully clamped (Fig. 6) and fully simply supported (Fig. 7) square plates, respectively. In the case of α is taken to be 0.5, they are changed to be of fully simply supported and fully clamped square plates in accordance with Figs. 7 and 6, respectively.

It is interesting to notice from the obtainable results presented in Tables 1 and 2 that there are many repeated values of frequency parameters in each parameter α for the adjacent modes. These characteristics are due to the existence of degeneracy of frequencies for general plates having symmetric vibrations [6] – [10].

Table 1 *Frequencies (λ^2) for clamped square plates with varying equal angle-leg corner simple supports.*

Mode	***α***					
	0.0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**
1	35.985	35.985	35.905	35.005	31.851	19.739
2	**73.391**	**73.389**	**72.953**	**69.694**	**62.507**	**49.347**
3	**73.391**	**73.389**	**72.953**	**69.694**	**62.507**	**49.347**
4	108.214	108.187	106.237	96.251	83.929	78.957
5	131.576	131.576	129.841	118.697	107.150	**98.693**
6	132.199	132.167	131.566	130.953	125.472	**98.693**
7	**164.994**	**164.906**	**160.140**	**144.054**	**134.322**	**128.301**
8	**164.994**	**164.906**	**160.140**	**144.054**	**134.322**	**128.301**
9	**210.510**	**210.469**	**208.316**	192.664	**189.984**	**167.779**
10	**210.510**	**210.469**	**208.316**	**202.162**	**189.984**	**167.779**
11	220.025	219.758	209.669	**202.162**	190.612	177.648
12	242.130	242.130	226.997	202.343	197.495	**197.385**
13	243.135	242.704	241.904	235.240	211.802	**197.385**
14	**296.306**	**295.752**	**278.117**	**262.881**	**261.262**	**246.723**
15	**296.306**	**295.752**	**278.117**	**262.881**	**261.262**	**246.723**
16	308.875	308.875	301.760	292.922	273.032	**256.607**
17	309.142	308.937	308.486	298.889	287.591	**256.607**
18	**340.557**	**340.126**	**331.370**	**315.909**	**294.214**	**286.196**
19	**340.557**	**340.126**	**331.370**	**315.909**	**294.214**	**286.196**
20	371.315	370.146	346.011	337.542	329.586	315.806

Table 2 *Frequencies (λ^2) for simply supported square plates with varying equal angle-leg corner clamped supports.*

Mode	***α***					
	0.0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**
1	19.739	20.698	23.527	28.137	33.432	35.985
2	**49.347**	**50.842**	**54.984**	**61.546**	**69.464**	**73.391**
3	**49.347**	**50.842**	**54.984**	**61.546**	**69.464**	**73.391**
4	78.957	82.632	92.025	102.661	107.758	108.214
5	**98.693**	98.712	99.564	105.158	120.019	131.576
6	**98.693**	101.861	109.254	118.701	128.309	132.199
7	**128.301**	**133.023**	**143.015**	**153.545**	**161.639**	**164.994**
8	**128.301**	**133.023**	**143.015**	**153.545**	**161.639**	**164.994**
9	**167.779**	**169.502**	**175.146**	**185.380**	**199.685**	**210.510**
10	**167.779**	**169.502**	**175.146**	**185.380**	**199.685**	**210.510**
11	177.648	185.232	199.362	204.229	209.401	220.025
12	**197.385**	197.523	202.118	220.598	239.642	242.130
13	**197.385**	207.921	228.028	241.307	243.118	243.135
14	**246.723**	**254.961**	260.913	273.526	**286.950**	**296.306**
15	**246.723**	**254.961**	265.658	275.649	**286.950**	**296.306**
16	**256.607**	256.750	**266.720**	**276.065**	291.175	308.875
17	**256.607**	259.477	**266.720**	**276.065**	295.047	309.142
18	**286.196**	**292.736**	**311.198**	**329.097**	**338.567**	**340.557**
19	**286.196**	**292.736**	**311.198**	**329.097**	**338.567**	**340.557**
20	315.806	327.224	338.338	343.499	367.018	371.315

Additionally, an another observation can also be seen in both tables for the adjacent modes of vibration that there are some repeated frequency parameters for all values of parameter α varied such as the adjacent modes of 2-3, 7-8, 14-15, and 18-19 shown in Table 1, and the adjacent modes of 2-3, 7-8, 9-10, and 18-19 in Table 2.

To the authors' opinion, however, these vibration characteristics cannot be concluded exactly using the same reason as described previously that caused by the occurrence of degeneracy frequency possibilities. If the changes in parameter α are small enough, the adjacent modes of the plate vibrations may not be resulted in obtaining the repeated frequencies for all values of parameter α changed. This is observable primarily from two adjacent modes of 9 and 10 for α = 0.3 as shown in Table 1.

Figs. 8 and 9 demonstrate the modal patterns for the plate vibrations that corresponded to the cases of fully clamped and fully simply supported edges of square plates, respectively.

For the modal vibration patterns of clamped square plates with varying corner simply supported lengths, they are presented in Figs. 10 to 13. In the case of simply supported square plates with varying equal angle-leg corner clamped supports, the vibratory modal patterns are prepared and given in Figs. 14 to 17.

4. CONCLUSION AND RECOMMENDATION

As it has been mentioned earlier in Section 2, the authors have found no publications on free vibration analysis for determining higher frequencies of the plates with mixed boundary conditions. Thus, to the best knowledge of the authors, the present results that provided the higher frequency parameters of square plates with symmetrically mixed between the simply supported and clamped edges have yet never been presented and reported in the scattering scientific or technical published literatures. The obtained results are numerically given in the table form for easy reference, which could be served as a benchmark for comparison by other future works.

However, it is worth noting that there is no study on higher frequencies determination for other shapes of free vibratory plates with mixed edge conditions and the problem of plates with cracks [46] – [52] up to the present time. Therefore, additional works by the authors on these follow-on problems are forthcoming.

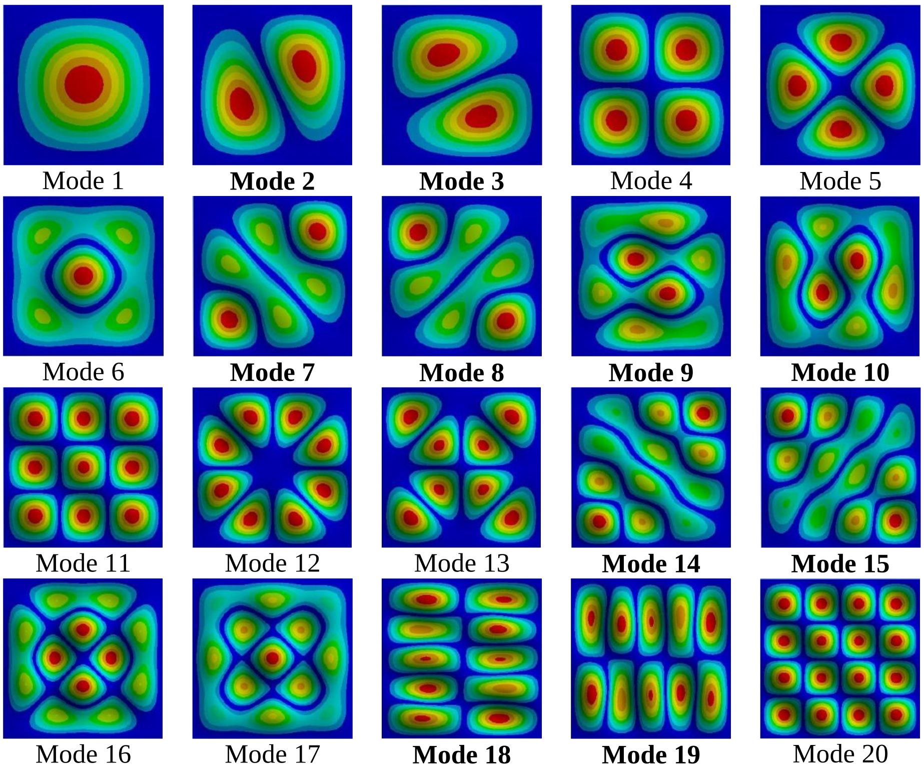

Fig. 8 *Modal patterns for fully clamped square plate.*

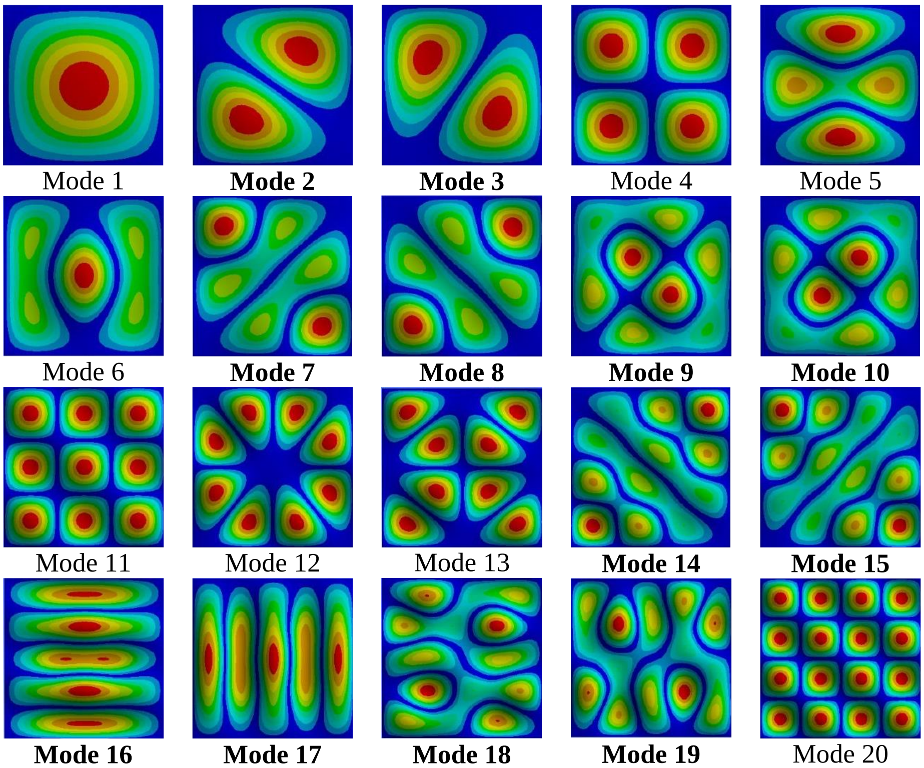

Fig. 9 *Modal patterns for fully simply supported square plate.*

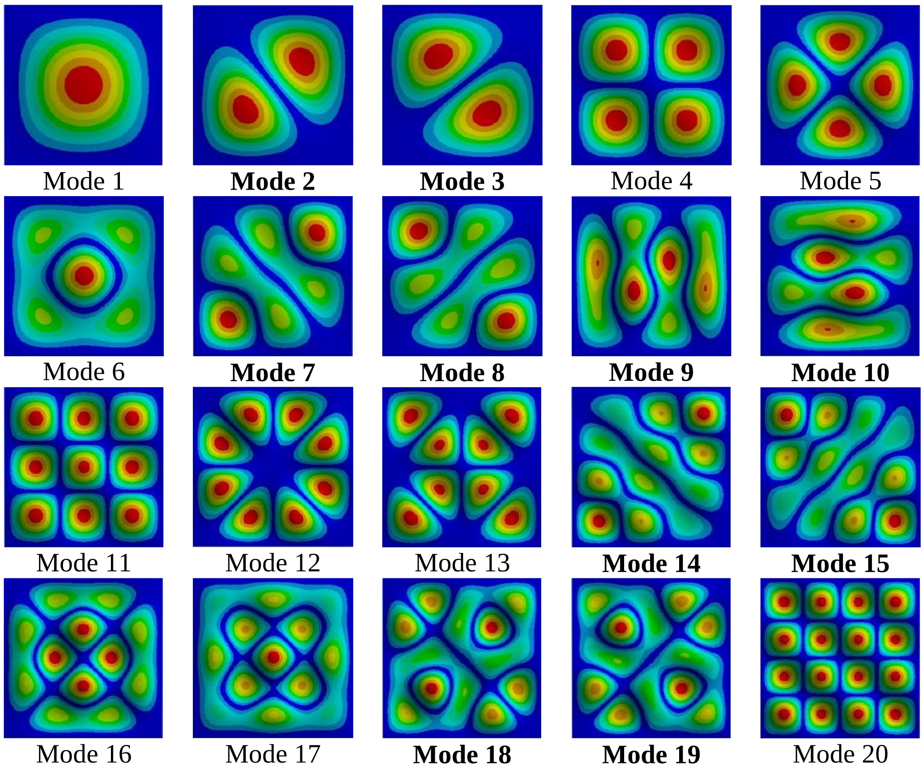

Fig. 10 *Modal patterns for clamped square plate with varying equal angle-leg corner simple supports (α = 0.1).*

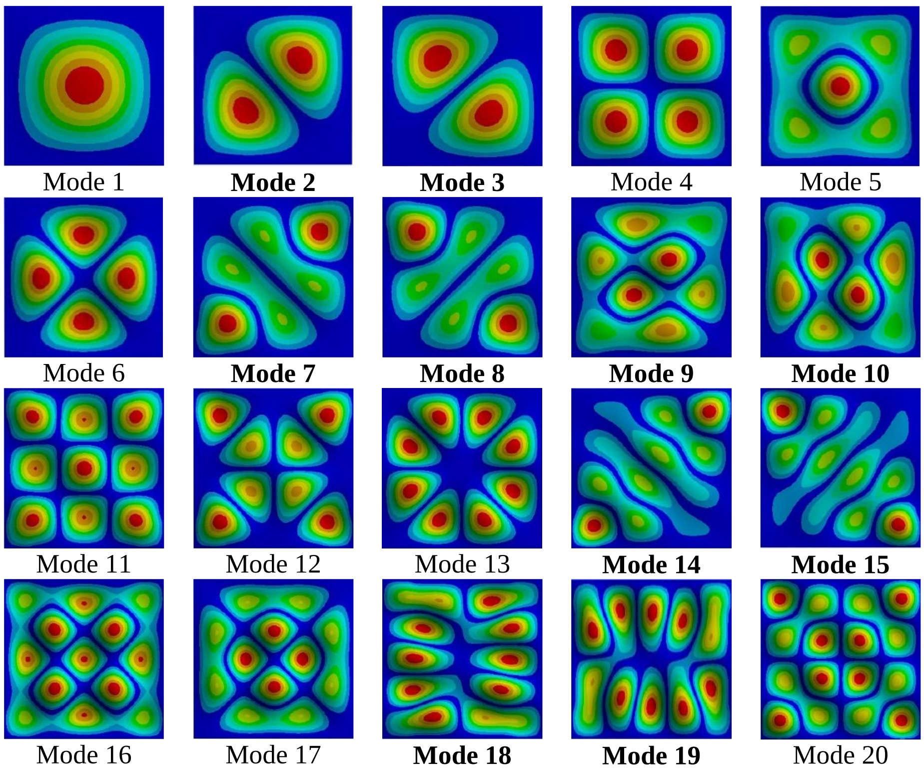

Fig. 11 *Modal patterns for clamped square plate with varying equal angle-leg corner simple supports (α = 0.2).*

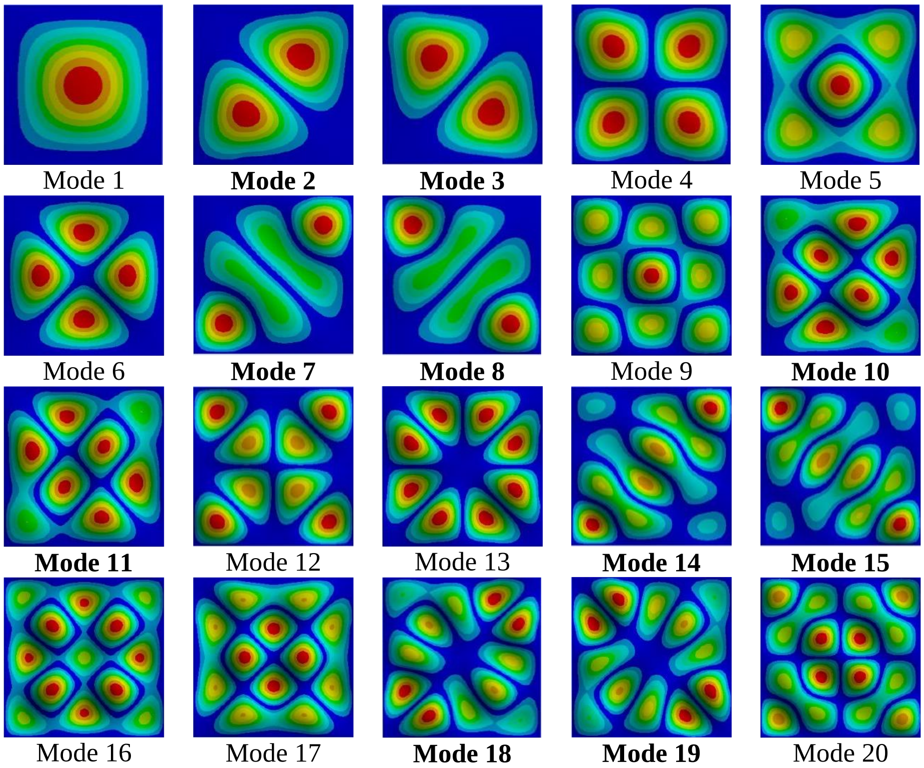

***Fig. 12** Modal patterns for clamped square plate with varying equal angle-leg corner simple supports (α = 0.3).*

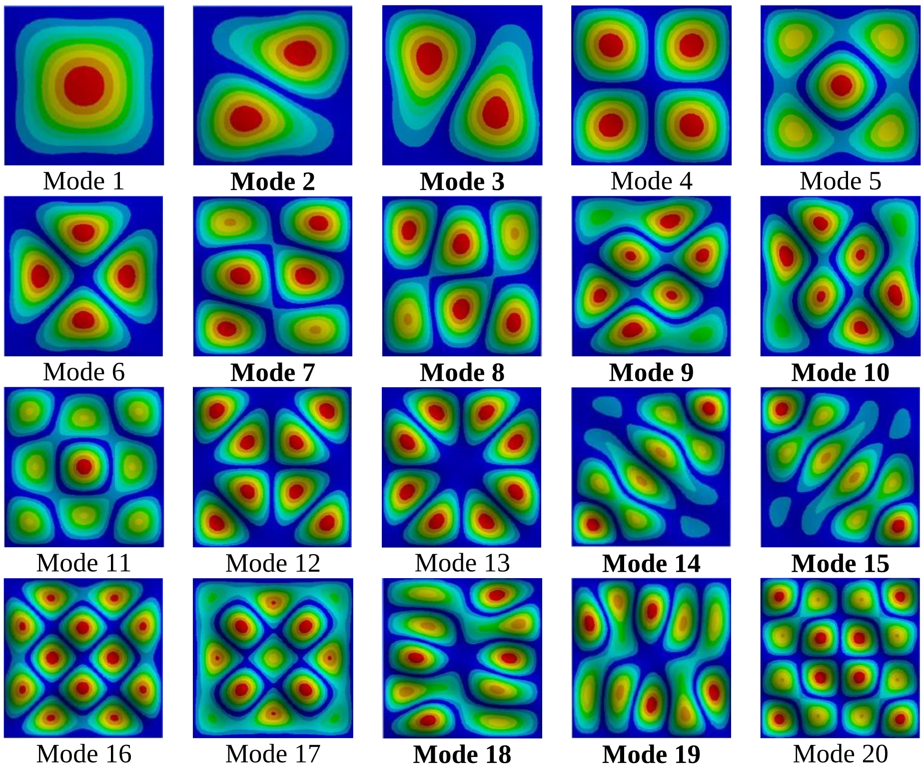

***Fig. 13** Modal patterns for clamped square plate with varying equal angle-leg corner simple supports (α = 0.4).*

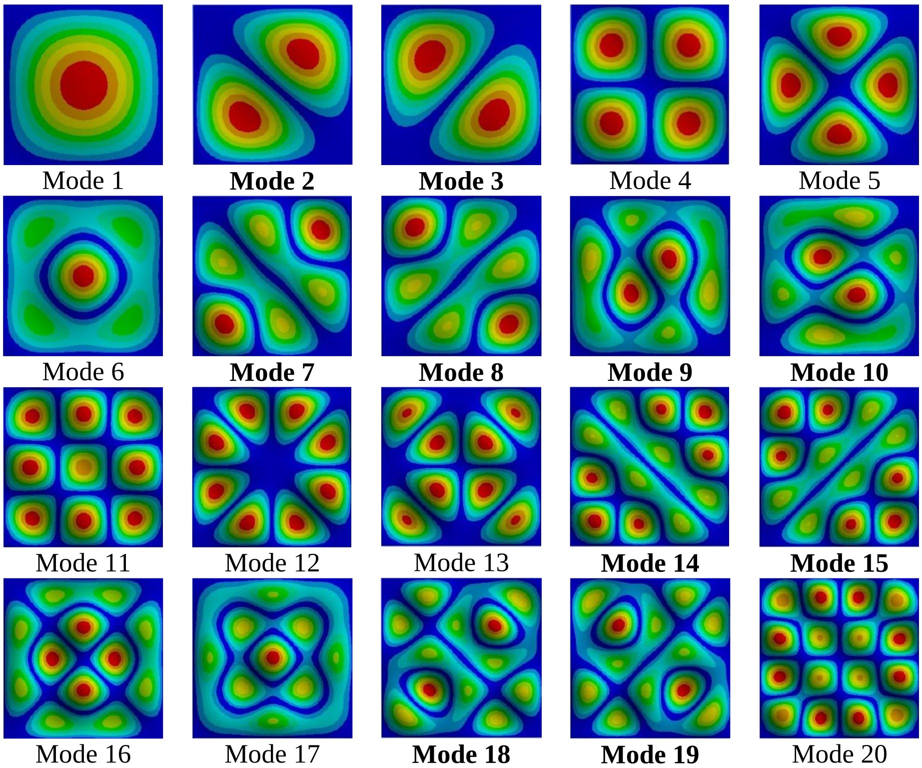

Fig. 14 *Modal patterns for simply supported square plate with varying equal angle-leg corner clamped supports (α = 0.1).*

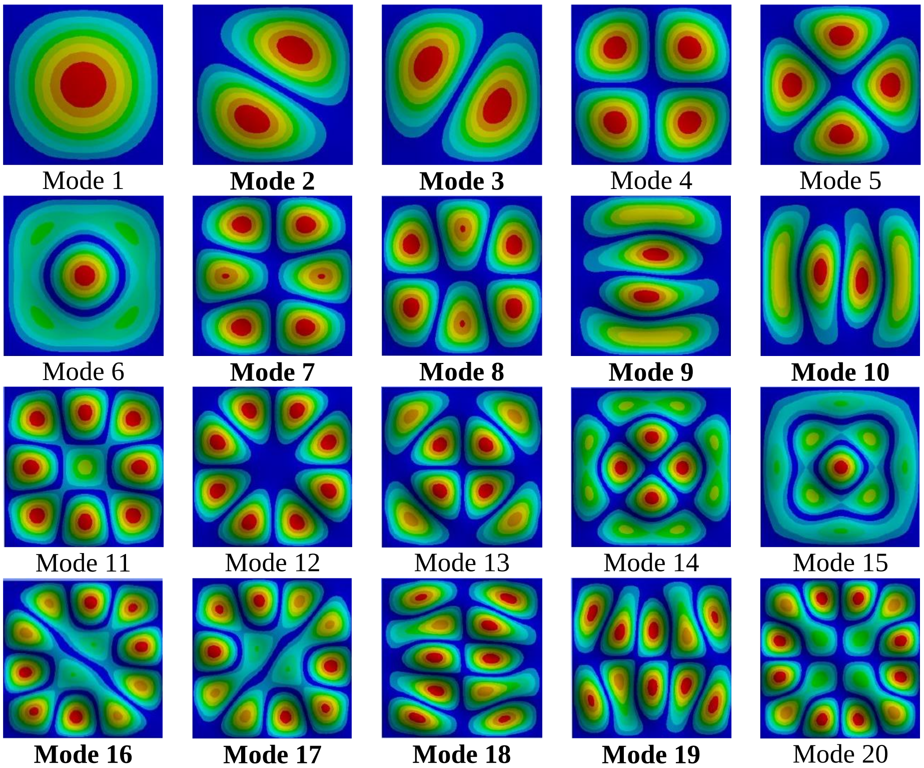

Fig. 15 *Modal patterns for simply supported square plate with varying equal angle-leg corner clamped supports (α = 0.2).*

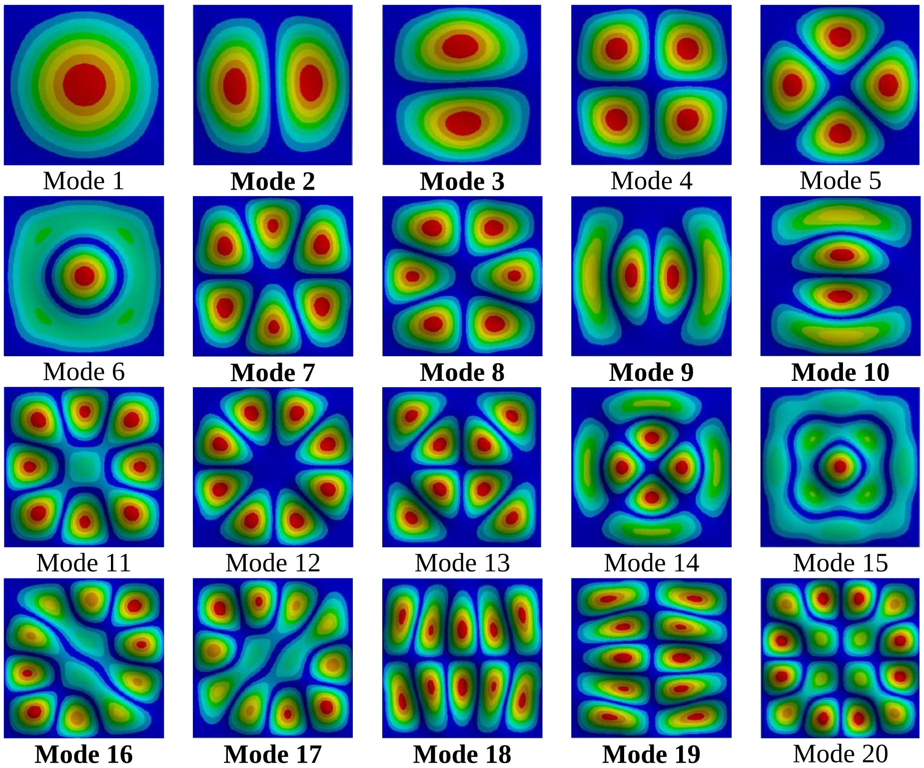

Fig. 16 *Modal patterns for simply supported square plate with varying equal angle-leg corner clamped supports (α = 0.3).*

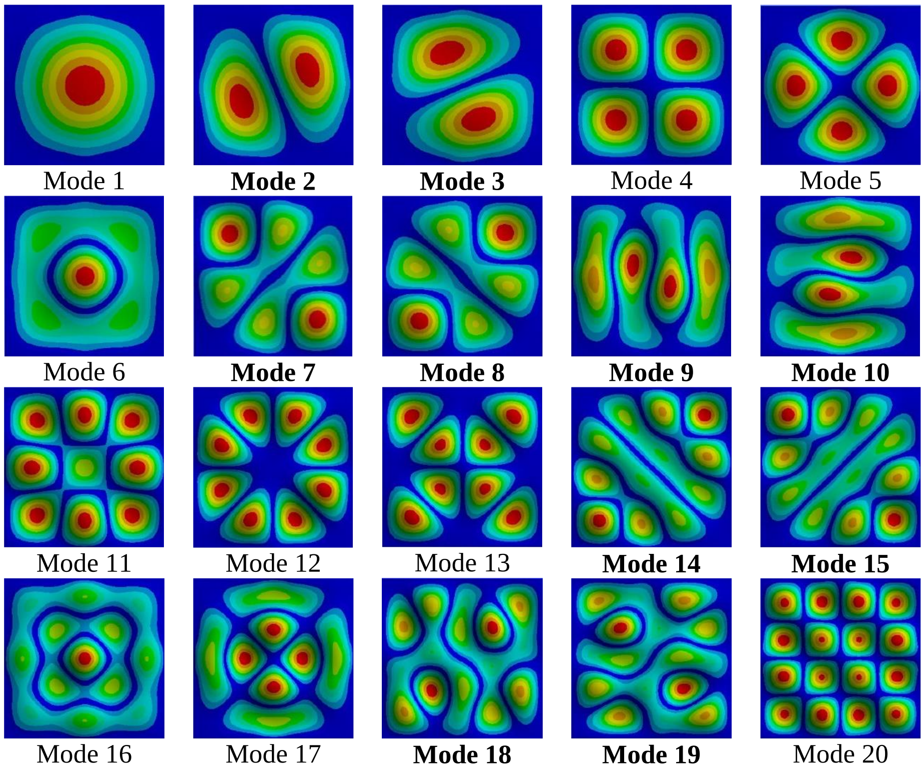

Fig. 17 *Modal patterns for simply supported square plate with varying equal angle-leg corner clamped supports (α = 0.4).*

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