Numerical Determination for Higher Frequencies of Simply Supported but Partially Clamped Square Plates

Yos Sompornjaroensuk

Department of Civil Engineering, Faculty of Engineering and Technology
Mahanakorn University of Technology, Bangkok 10530, Thailand
Email: yos@mut.ac.th (corresponding author)

Damang Dy

General Department of Immigration, Ministry of Interior
National Road Number 1, S/K Nirouch, Chbar Ampov, Phnom Penh 12355, Cambodia
Email: dydamang@gmail.com

Manuscript received December 12, 2020
Revised December 30, 2020

ABSTRACT

Owing to the advent of high speed computers that permitted computing solutions of a large number of algebraic equations in a relatively short time, the paper utilizes the advantage of this performance to evaluate some accurate higher natural frequency parameters for vibratory simply supported square plates with partially clamped segments on the plate boundaries. Analysis is focused on two cases of the plate. The first is a simply supported plate having an equal angle-leg corner partially clamped and the second is a simply supported plate having some segments adjacent to a corner equally the lengths of partial simple support and clamped on the remainders. To carry out the frequencies accurately together with their associated modes of vibration, a dense net of representative finite element model is performed. The first twenty frequency parameters are then determined and presented in a tabular form for easy reference by other alternative methods.

Keywords : *Finite element model*, *Frequency parameter*, *Partial edge supports*, *Square plate*.

1. INTRODUCTION AND LITERATURE REVIEW

The flat plate is a very common component in engineering practices and has been extensively used in civil, mechanical and aerospace structures [1], [2]. Based on the mathematical analytical approaches [3], because exact analytical solutions can exist only for rectangular plates having at least simply supported edge conditions placed at two opposite edges and either circular or annular plates supported by regular boundary conditions along the circumferential edges [4] – [8], very little amount of published research is, nevertheless, available about the bending, free vibration, and buckling problems of plates with mixed boundary conditions [9] – [13].

Utilization of the energy method together with the method of equivalent distributed loads corresponding to the edge moments, Ota and Hamada [14] dealt with the bending and vibration problems of a simply supported but partially clamped rectangular plate. In addition, an extension of the method was also made to analyze the fundamental vibrations for various cases of plate with mixed boundary conditions between simply supported and clamped edges [15].

An important research is that of Keer and Stahl [16] who applied the analytical method of finite Hankel integral transform techniques to analytically determine the fundamental frequency of rectangular plates with mixed edge conditions. The problems were formulated as dual series equations and reduced to finding the solution of homogeneous Fredholm integral equations of the second kind.

The high precision triangular plate bending element has been used to numerically determine the fundamental frequency of rectangular plates with mixed boundary conditions by Venkateswara Rao et al. [17]. The obtained results were in good agreement compared with Keer and Stahl [16], with a maximum difference of about 3.5%.

Narita [18] applied a series-type method to the free vibration of an orthotropic rectangular plate with mixed boundary conditions in which the plate considered is elastically constrained produced by locating rotational springs along parts of the edge or clamped along a few parts of its edge and simply supported on the remainder. By the use of this method, a comprehensive study of the variations of the frequency parameters with the change of clamped portion was presented and given with four significant figures for the lowest six modes of vibration and only the fundamental frequencies have compared with the results obtained by Keer and Stahl [16] and Venkateswara Rao et al. [17].

Gorman [19] developed a comprehensive analytical method for analyzing free vibration of rectangular plates with discontinuities along the boundaries based on the superposition technique. Accurate numerical results for the fundamental frequency values of rectangular plate with varying the side ratios of the plate were presented.

Cheng [20] has attempted to analytically evaluate the lower and upper limits of fundamental frequency of simply supported rectangular plates having partially clamped segments along the edges by the method of releasing boundary conditions and the Ritz method, respectively. The solution of the eigenvalue problems which is the plate vibratory displacement is expressible in the form of Fourier's series type following the Levy approach.

An approximate method for analyzing free vibration of rectangular Mindlin plates having mixed boundary conditions was proposed by Sakiyama and Matsuda [21] for the fundamental mode of vibration. The solutions of the partial differential equations were obtainable in discrete form by transforming the differential equations into integral equations together with applying numerical integration techniques. Numerical results for variation of fundamental frequency parameters of rectangular plates with fixed length ratio of mixed boundaries were carried out and presented. For a specific case of square plates, the obtained results were also compared with analytical solution results given by Keer and Stahl [16].

Another approximate numerical method was made by Leung and Au [22] who numerically developed the spline functions that can be expressed in terms of physical coordinates at the boundary resulting in the required shape functions in finite element method. Seven different cases of rectangular plate with mixed boundary conditions have analyzed for the lowest three frequency parameters.

Lee and Lim [23] applied a simple numerical method that based on the Rayleigh principle for predicting the natural frequencies of a simply supported rectangular plate having an edge or two opposite edges partially clamped. Results were carried out and presented for isotropic and orthotropic plates, and were also in good agreement with other available analytical results.

The substructure method that involved partitioning of the entire plate domain into appropriate elements to approximate the displacement function of each element by a set of admissible orthogonal polynomials was used to analyze the free vibration of a rectangular plate with discontinuous boundary conditions by Liew et al. [24]. By minimizing the total energy of the system of the plate resulting Rayleigh quotient with respect to the unknown coefficients leads to the governing eigenvalue equation for the entire plate. The computed lowest four frequency parameters for three cases of considered plate having mixed boundary conditions between the simply supported and clamped supports were numerically given and compared favorably with other previously published values.

Laura and Gutierrez [25] dealt with the application of the differential quadrature technique to numerically determine the fundamental frequency of free vibration of rectangular thin elastic plates in the case of non-uniform boundary conditions and mixed boundary conditions. Two cases of square plate with discontinuously varying edge conditions were considered and their fundamental frequency parameters were given and also compared with available published literature.

Cheung and Kong [26] developed and described a new finite strip element method to investigate the free vibration of four different plates with mixed boundary conditions. The lowest three dimensionless frequency parameters were provided and also compared with other finite strip and spline strip methods.

A highly accurate and rapidly converging hybrid approach for the quadrature element method solution of rectangular plate free vibration problems was proposed by Striz et al. [27]. Two cases of square plate with mixed edge conditions between simply supported and clamped were investigated for determining the first five natural frequency parameters and validated with both analytical and numerical methods.

Singhal and Gorman [28] described an analytical procedure based on the method of superposition [29] for obtaining the free vibration frequencies and their mode shapes of partially clamped cantilevered rectangular thin plates with and without rigid point supports. In order to verify their proposed analytical procedure, comparison of computed and experimental results for the first six frequencies of rectangular plates with varying three side ratios of length to width of the plate were demonstrated.

The method of generalized differential quadrature has been used by Shu and Wang [30] to the treatment of mixed and nonuniform boundary conditions for the free vibration analysis of rectangular plates. The advantage of the proposed approach can be stated that it overcomes the drawbacks of conventional approaches such as the δ-technique in treating the boundary conditions. The first five frequency parameters for two different square plates having mixed boundary conditions were carried out with different meshes used in their analyses.

Liu and Liew [31] considered and analyzed the free vibration problem of thick rectangular plates with mixed boundary conditions that based on the first-order theory of shear deformable plate by the use of differential quadrature element method. Six different cases of square plate with mixed edge conditions were investigated for obtaining the lowest eight frequency parameters.

Wei et al. [32] numerically computed and dealt with some new results for square plates with combinations of the mixed and nonuniform edges by making use of the discrete singular convolution approach. The lowest five natural frequencies were given in tabular form and also validated with existing results in the literature.

A procedure of finite strip element method combined with a spring system was proposed and employed by Huang and Thambiratnam [33] to numerically treat the free vibration of rectangular plates. The spring system can be used to model point supports, line supports, locally distributed supports and also complex boundary conditions. Results of the first three natural frequencies for three cases of square plate having mixed boundary conditions between simply supported and clamped edges were carried out and compared well with other available results found in the published literature.

Su and Xiang [34] analyzed the free vibration and buckling problems of rectangular plates with mixed edge support conditions based on a non-discrete approach that is a domain decomposition method. With this method, the entire plate is decomposed into multiple rectangular subdomains along the change of discontinuous support conditions. Compared to other domain decomposition technique, it can be stated that a set of non-discretized interconnecting boundaries is established to maintain the completeness of the entire plate domain in which the convergence rate and the accuracy of the results in the plate analysis can be improved well. The lowest six accurate frequency parameters were given for the eight cases of square plates with mixed boundary conditions.

A comprehensive comparison study of free vibration analysis between the discrete singular convolution and the global generalized differential quadrature methods was presented by Ng et al.[35]. Five rectangular plates of mixed supporting edges over a range of aspect ratios and non-dimensional spring coefficients have considered and their first five frequency parameters were computed. Numerical experiments have showed that the results of the discrete singular convolution with employing the Lagrange kernels agree well with the existing literature, whereas the results obtained by generalized differential quadrature are generally more accurate. However, for higher order frequencies, it is interesting to note that the discrete singular convolution method produces more accurate results than those of generalized differential quadrature.

For three-dimensional free vibration analysis, Zhou et al. [36] considered two kinds of rectangular plates with mixed boundary conditions, based on the small strain linear elasticity theory. The Ritz method was employed successfully to derive the governing eigenvalue equation by minimizing the energy functional of the plate. The first six dimensionless frequency parameters of square plates were carried out in correspondence with both antisymmetric and symmetric modes in the thickness direction.

Xiang et al. [37] developed a new approach called the DSC-Ritz element method that based on the concept of finite element method together with the implementation of discrete singular convolution as a trial function in the Ritz method, to analyze the free vibration of moderately thick plates following the Mindlin first-order shear deformation plate theory. Ten cases of rectangular plate with various edge support discontinuities were treated numerically to obtain the lowest eight frequencies.

Problem of symmetrical free vibration of rectangular plate with discontinuous simple supports was considered and formulated by Chaiyat and Sompornjaroensuk [38]. The obtained dual series equations that resulted from the mixed boundary conditions were derived and reduced to homogeneous integral equation of Fredholm-type using the method of finite Hankel integral transforms similar to that work of Keer and Stahl [16].

Zhang et al. [39] proposed the development of the Hencky bar-net model (HBM) to analyze free vibration of rectangular plates with mixed boundary conditions and point supports. It is describable that the model is a two-dimensional discrete net system composed of rigid segments connected by frictionless hinges and rotational springs, which is able to handle any boundary conditions of plates including mixed boundary conditions and also internal point supports. The first three natural frequency parameters were determined for three different cases of the plates having mixed boundary conditions with and without an interior point support.

Recently, the authors [40] have numerically analyzed free vibration of square plates with symmetrically mixed boundary edges between simple and clamped supports using the reliable finite element program [41,42]. The higher frequencies and their corresponded mode shapes of plate vibrations were presented. In addition, the free vibrations of circular plates with partially clamped or partially simply supported along circumferential edge were already treated to determine the first thirty accurate frequency parameters [43].

The primary purpose of this paper is to numerically deal with the reasonably accurate results for higher free vibration frequencies, expressed in terms of frequency parameters, of simply supported but partially clamped square plates by means of an available ANSYS finite element code [41,42] with a dense net of element model. Some interested observations of the obtainable results are addressed and discussed in details. Furthermore, the associated modes of plate vibrations are also graphically provided.

2. PROBLEM STATEMENT

Two different cases of square plate configuration are then illustrated in Figs. 1 and 2, which have never been considered in the past technical literature.

One begins by considering the plate as shown in Fig. 1. This plate has an equal angle-leg partially clamped segment placed at one corner where the remaining parts of the plate edges are all simply supported.

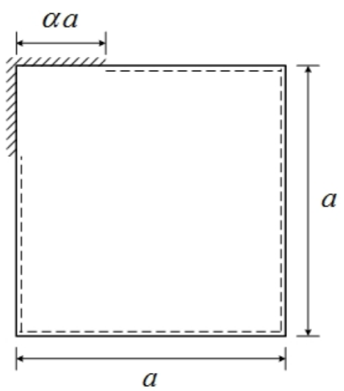

Fig. 1 *Simply supported square plate with an equal angle-leg corner partially clamped***.**

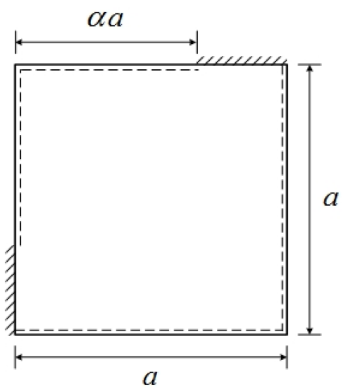

Fig. 2 *Simply supported square plate with an equal angle-leg corner partially simply supported***.**

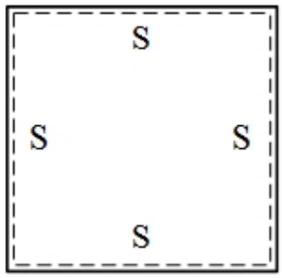

Fig. 3 *A square plate with all edges simply supported (S-S-S-S plate).*

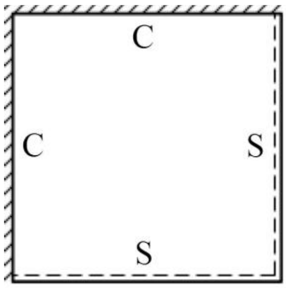

Fig. 4 *A square plate with two adjacent edges clamped and two adjacent edges simply supported (C-S-S-C plate).*

On an another plate as demonstrated in Fig. 2, is that corner support of the plate as given in Fig. 1 has to be replaced by partial simple supports and the remaining parts of two adjacent edges to the corner are partially clamped. Other two remaining edges are fully simply supported along the plate edges.

In both cases of the plate considered (Figs. 2 and 3), the parameter α is changeable starting from zero to unity in order to define the length of partial supports.

If the values of α are taken to be zero and unity for the plate illustrated in Fig. 1, two limiting cases exist for the plates as shown in Figs. 3 and 4, respectively. On the other hand of the plate presented by Fig. 2, two limiting cases become the plates as shown in Figs. 4 and 3 in correspondence with α-values taken as zero and unity, respectively.

It can also be noted that the letters S and C as seen in both Figs. 3 and 4 are symbolically designated to be the conditions of simple and clamped supports, respectively.

The free vibration analysis can be made in the same manner of authors' previous work [40] by making use of the efficient finite element code [41].

In order to model the plates that based on the finite element procedure [44,45], the quadrilateral shaped of SHELL181 element type [42] as shown in Fig. 5 and also together with the representative refinement of finite element discretization for the entire plate as depicted in Fig. 6 are then used for this purpose.

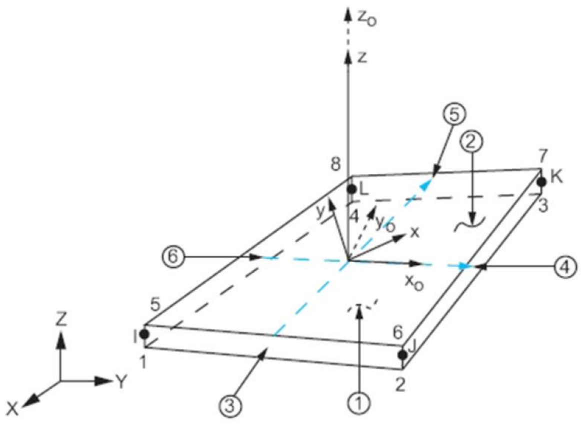

Fig. 5 *ANSYS SHELL181 element [42].*

The final important step is to identify the boundary conditions of problem modelling properly. In earlier works [40,43], the first author has provided the detailed descriptions of representing the points of discontinuous support in which higher constraint support condition than another at the points of discontinuity is chosen to represent the boundary conditions at those points.

3. PRESENTATION OF RESULTS

After performing the analysis by ANSYS computer program, the obtainable results are given in terms of natural frequencies (f) of the plates. It is, however, very convenient to express them in dimensionless form of frequency parameters (λ^2) as

$$\lambda^2 = 2\pi f a^2 \sqrt{\frac{\rho}{D}}\,, \tag{1}$$

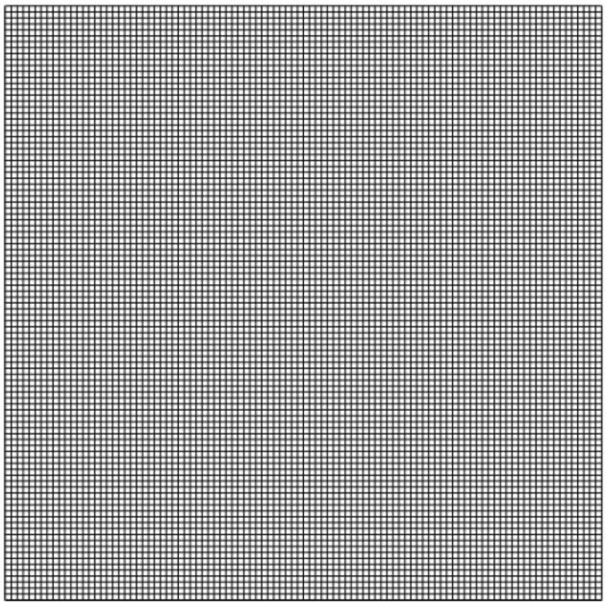

Fig. 6 *Mesh of 100×100 elements in square plate model.*

and the plate's flexural rigidity (D) defined by

$$D = \frac{Eh^3}{12(1-\nu^2)}\,, \tag{2}$$

where a is the plate length, ρ is mass density per unit area of the plate surface, h is the plate thickness, and the plate's material properties E and ν are Young's modulus and Poisson's ratio, respectively.

The first twenty frequency parameters (λ^2) for square plates shown in Figs. 1 and 2 with varying of parameter α are numerically tabulated and listed in Tables 1 and 2, respectively. All the presented values are given only for the Poisson's ratio taken to be 0.3.

It is obviously seen in Table 1 for the parameters α taken to be 0.0 and 1.0 that the plates are corresponded to the cases of fully simply supported square plate (Fig. 3) and a square plate as shown in Fig. 4, respectively. Their modal patterns for the first twenty modes of two associated vibratory plates are graphically presented in Figs. 7 and 8.

As shown in Figs. 9 to 12, they are demonstrated for the first twenty modal patterns of square plates as shown in Fig. 1 with varying parameters α as 0.2, 0.4, 0.6, and 0.8, respectively.

Similarly, Figs. 13 to 16 have prepared and provided for the modal patterns of the plate shown in Fig. 2 with variation of α's to the corresponded plates with varying of the length of partial simple supports.

4. DISCUSSION

Let us to first consider all the obtainable results of frequency parameters (λ^2) for α taken to be 0.0 as given in Table 1 that corresponded to the simply supported square plate shown in Fig. 3 (S-S-S-S plate), it is clearly found to have many repeated values of the frequency parameter for neighbouring modes of free vibration.

These characteristics can then certainly be described by the existence of degeneracy frequency possibilities that found in general plates having symmetric vibrations [7,10,29] along the centrelines of the plate with respect to the x- and y-axes. These vibration characteristics do not appear to the plate having α taken to be 1.0 (Fig. 4) and also other plates with mixed edge conditions ($\alpha \neq 0$).

Therefore, the repeated frequency parameters that seen in Table 2 for α taken as 1.0 are corresponded to the vibration behaviors of simply supported square plate (Fig. 3), which can be explained in the same manner of results given in Table 1 for $\alpha = 0$.

Table 1 *Frequencies (λ^2) for simply supported square plate with an equal angle-leg corner partially clamped.*

Mode	α					
	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
1	19.739	20.643	22.981	25.644	26.936	27.053
2	**49.347**	49.379	50.507	55.096	59.889	60.539
3	**49.347**	51.941	56.964	60.313	60.783	60.785
4	78.957	81.772	84.583	85.840	91.723	92.835
5	**98.693**	98.907	103.120	109.959	113.677	114.554
6	**98.693**	101.341	106.313	111.392	114.522	114.700
7	**128.301**	128.529	131.744	133.582	142.034	145.776
8	**128.301**	135.741	141.019	143.746	146.037	146.076
9	**167.779**	168.365	173.637	180.281	187.054	188.452
10	**167.779**	169.349	174.533	182.686	188.340	188.545
11	177.648	182.180	183.572	190.489	194.110	198.098
12	**197.385**	198.537	208.275	211.351	215.309	219.205
13	**197.385**	206.204	211.535	211.823	218.897	219.430
14	**246.723**	247.215	248.158	255.069	261.200	270.653
15	**246.723**	253.428	260.523	265.752	270.756	270.961
16	**256.607**	257.694	264.624	273.319	280.434	282.136
17	**256.607**	262.287	265.014	276.415	281.890	282.198
18	**286.196**	288.944	299.873	301.822	307.809	312.607
19	**286.196**	293.435	302.498	303.934	311.603	312.792
20	315.806	320.584	323.188	327.843	334.630	342.833

Table 2 *Frequencies (λ^2) for simply supported square plate with an equal angle-leg corner partially simply supported and clamped on the remaining parts.*

Mode	*α*					
	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
1	27.053	27.047	26.549	24.035	20.963	19.739
2	60.539	60.537	56.688	50.561	49.373	**49.347**
3	60.785	60.670	60.475	58.775	52.900	**49.347**
4	92.835	92.525	86.454	84.389	82.363	78.957
5	114.554	114.308	109.147	104.144	98.868	**98.693**
6	114.700	114.552	113.605	107.365	102.692	**98.693**
7	145.776	144.353	133.899	132.528	128.527	**128.301**
8	146.076	145.774	144.554	140.504	137.193	**128.301**
9	188.452	187.497	179.389	174.101	168.211	**167.779**
10	188.545	188.440	184.007	177.148	171.183	**167.779**
11	198.098	196.009	192.674	183.449	181.956	177.648
12	219.205	216.293	211.228	209.157	198.588	**197.385**
13	219.430	219.164	212.643	210.982	208.275	**197.385**
14	270.653	263.148	254.495	248.692	247.277	**246.723**
15	270.961	270.612	264.563	261.938	255.335	**246.723**
16	282.136	281.295	276.005	263.620	259.211	**256.607**
17	282.198	282.075	278.732	267.741	260.093	**256.607**
18	312.607	307.604	301.698	299.463	289.067	**286.196**
19	312.792	312.423	303.482	302.539	296.224	**286.196**
20	342.833	336.209	330.099	323.639	320.092	315.806

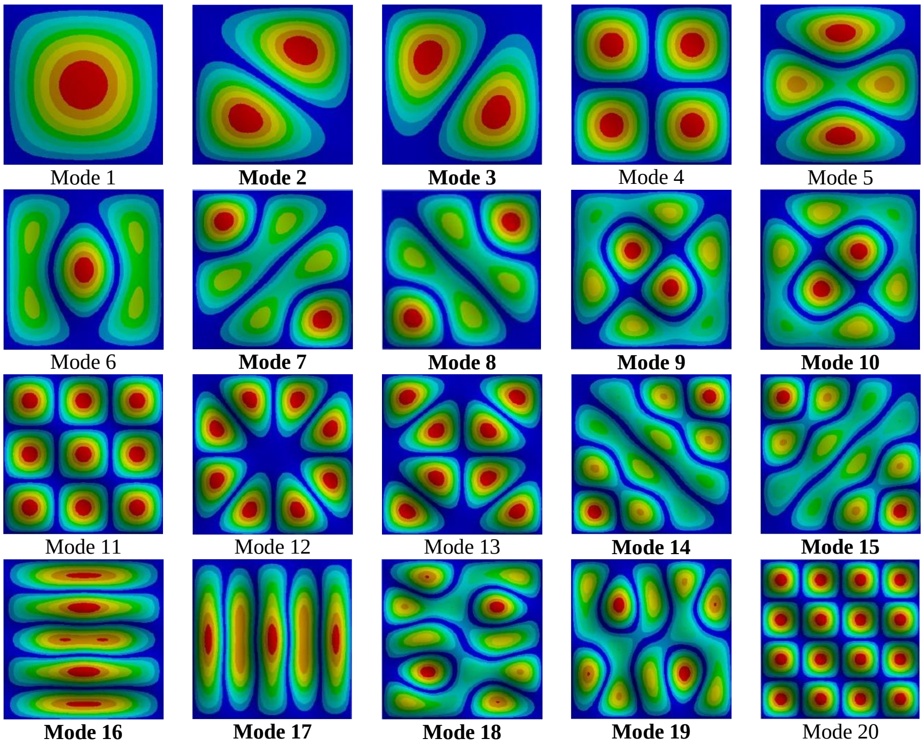

Fig. 7 *Modal patterns for square plate with all edges simply supported.*

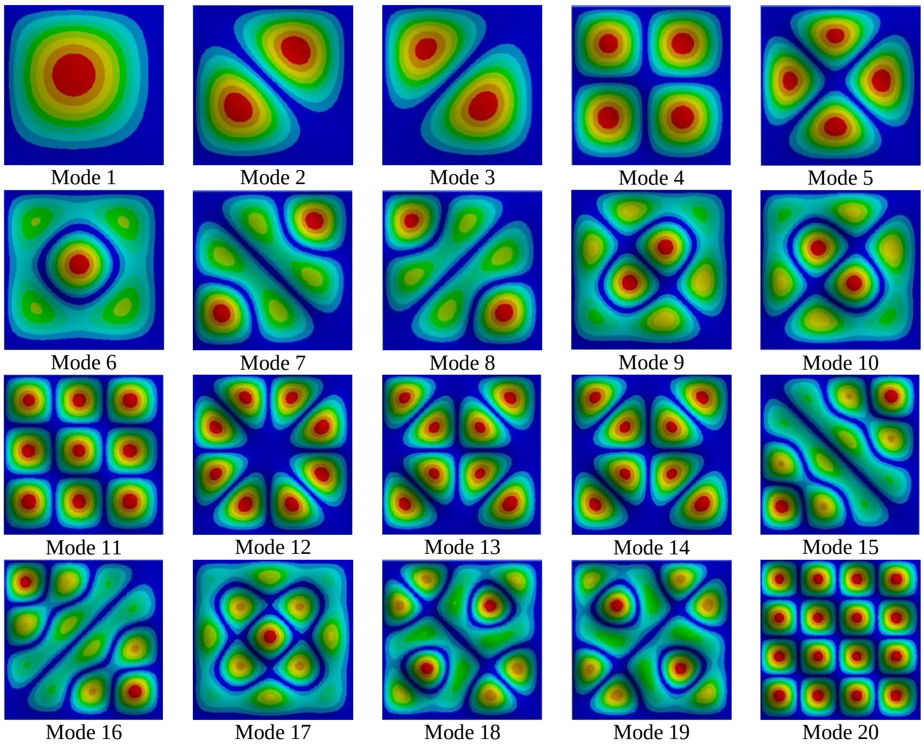

Fig. 8 *Modal patterns for square plate with two adjacent edges clamped and two adjacent edges simply supported*.

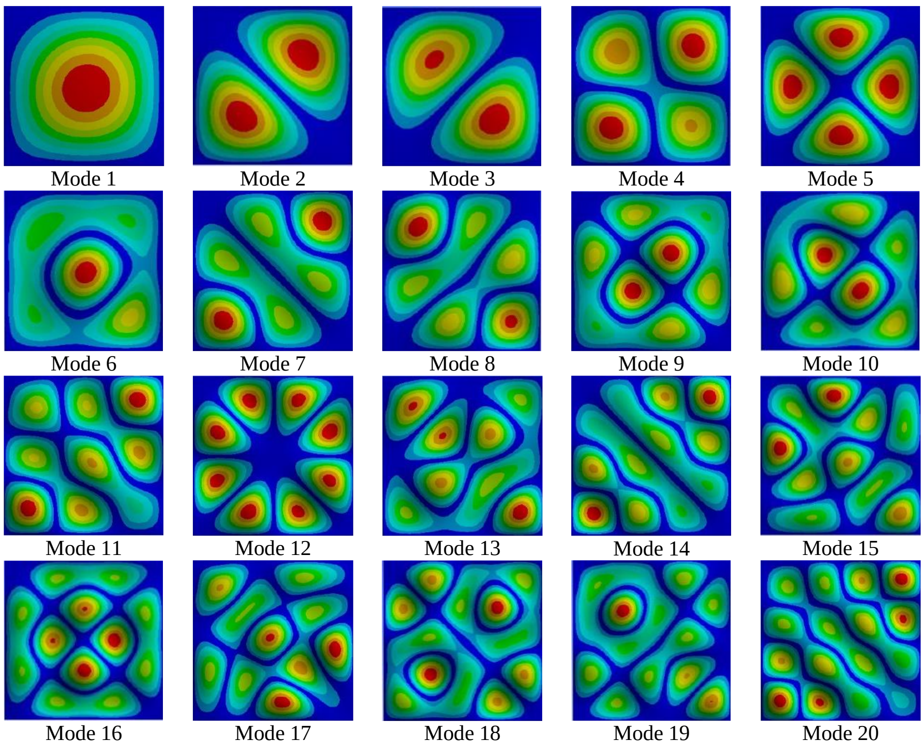

Fig. 9 *Modal patterns for simply supported square plate with an equal angle-leg corner partially clamped (α = 0.2).*

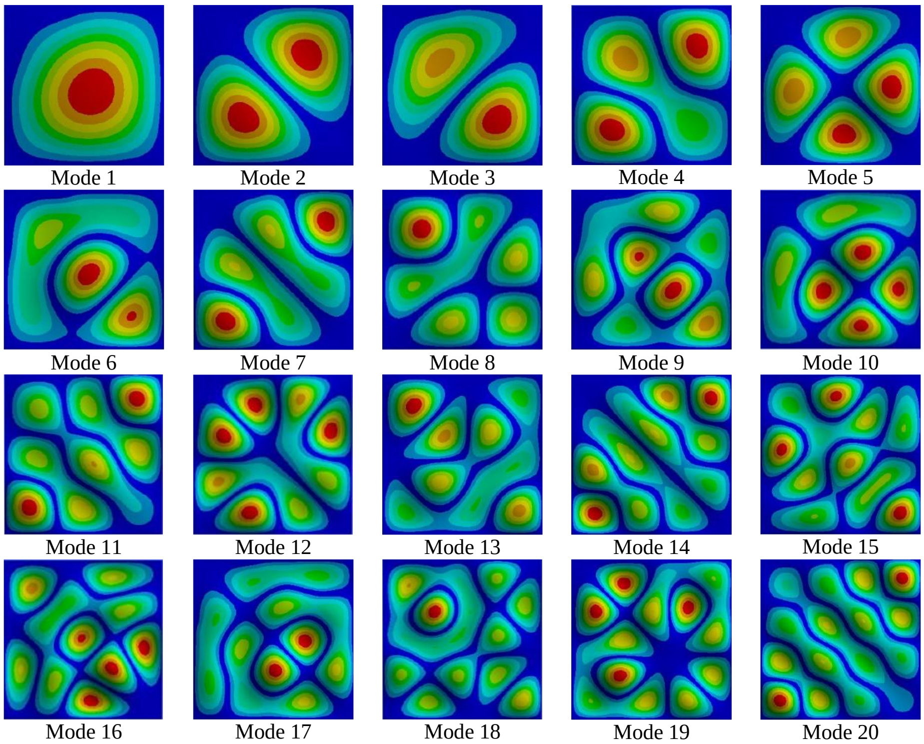

Fig. 10 *Modal patterns for simply supported square plate with an equal angle-leg corner partially clamped (α = 0.4).*

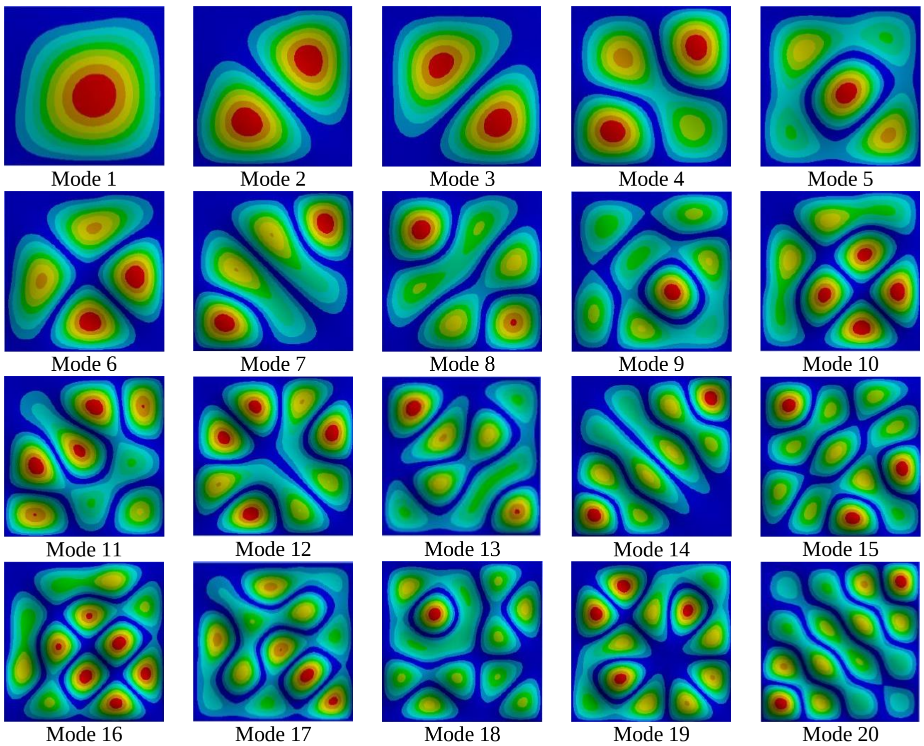

Fig. 11 *Modal patterns for simply supported square plate with an equal angle-leg corner partially clamped (α = 0.6).*

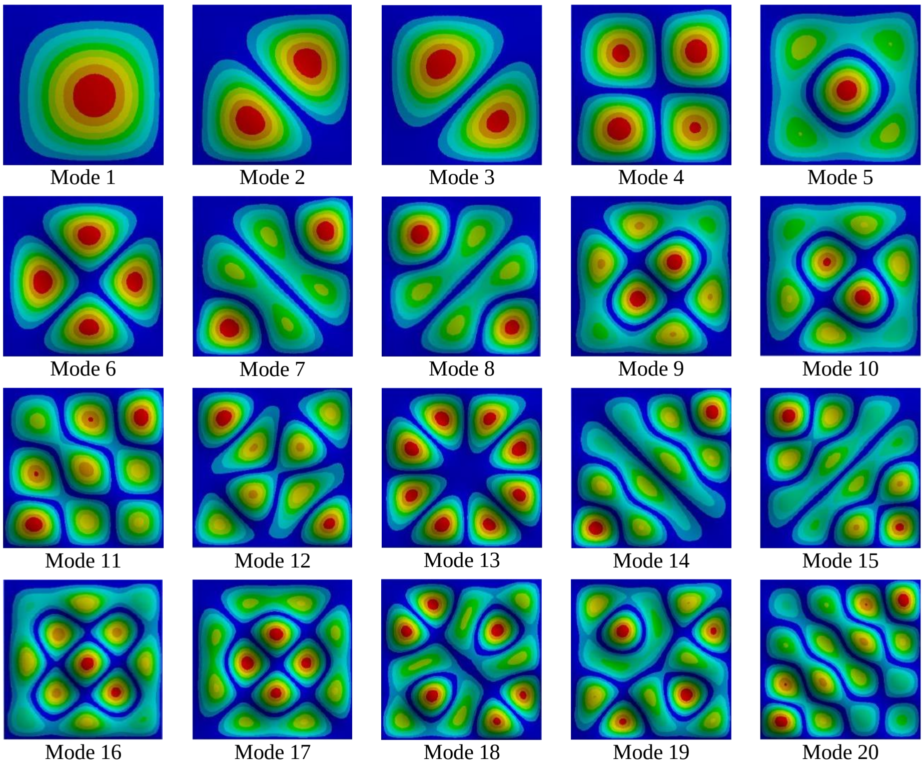

Fig. 12 *Modal patterns for simply supported square plate with an equal angle-leg corner partially clamped (α = 0.8).*

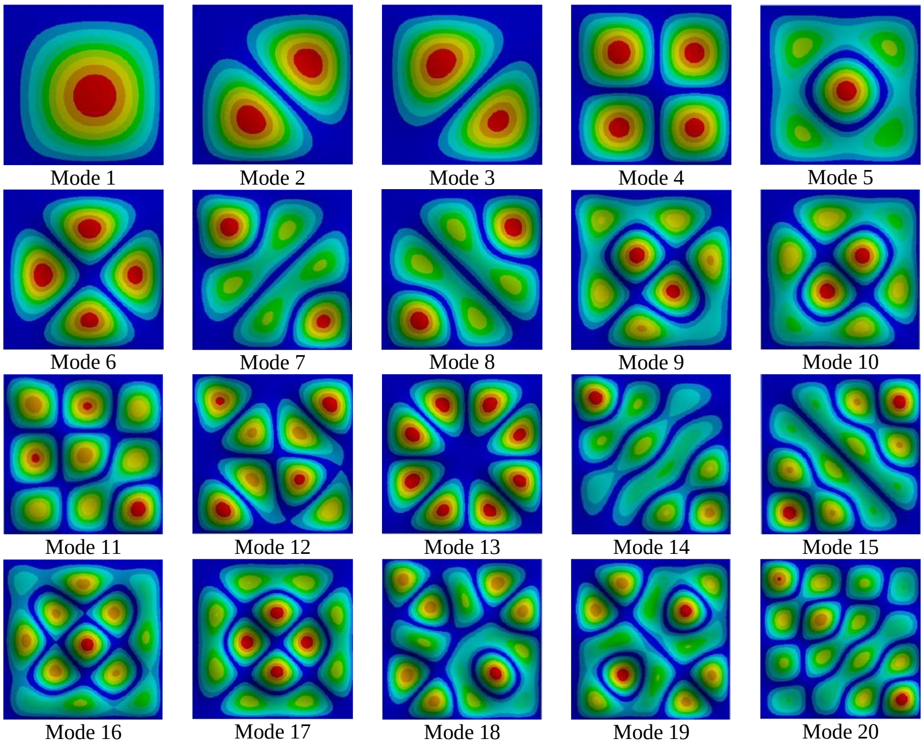

Fig. 13 *Modal patterns for simply supported square plate with an equal angle-leg corner partially simply supported (α = 0.2).*

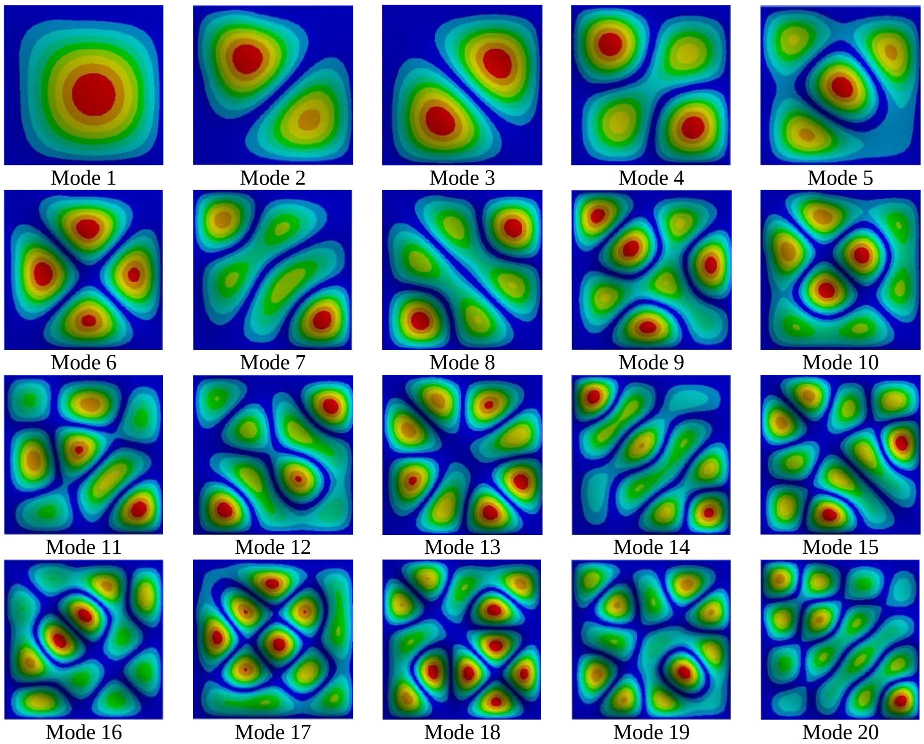

Fig. 14 *Modal patterns for simply supported square plate with an equal angle-leg corner partially simply supported (α = 0.4).*

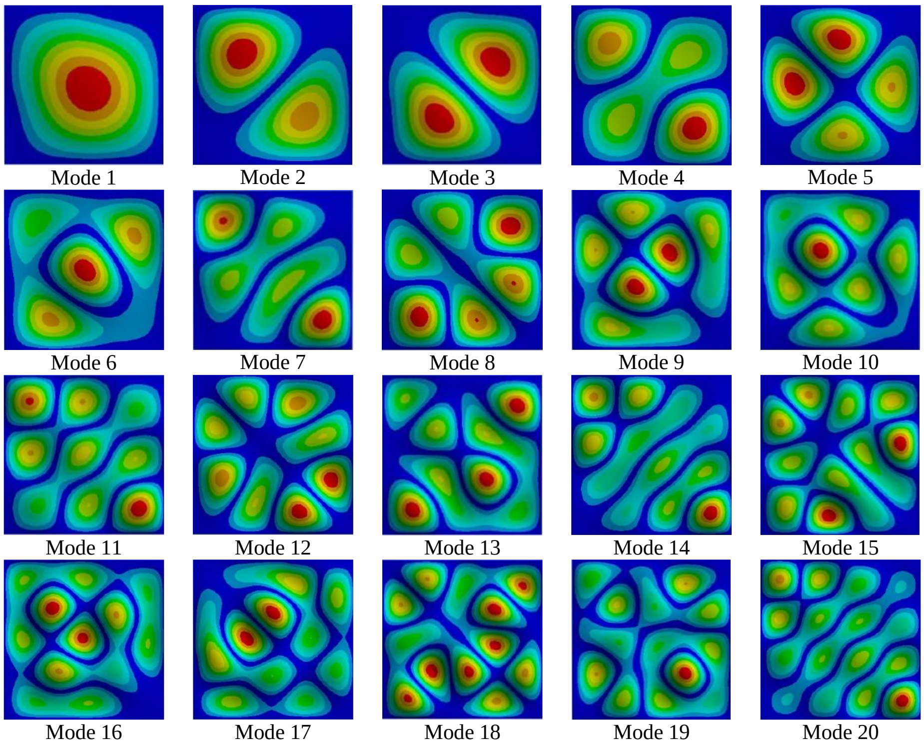

Fig. 15 *Modal patterns for simply supported square plate with an equal angle-leg corner partially simply supported (α = 0.6).*

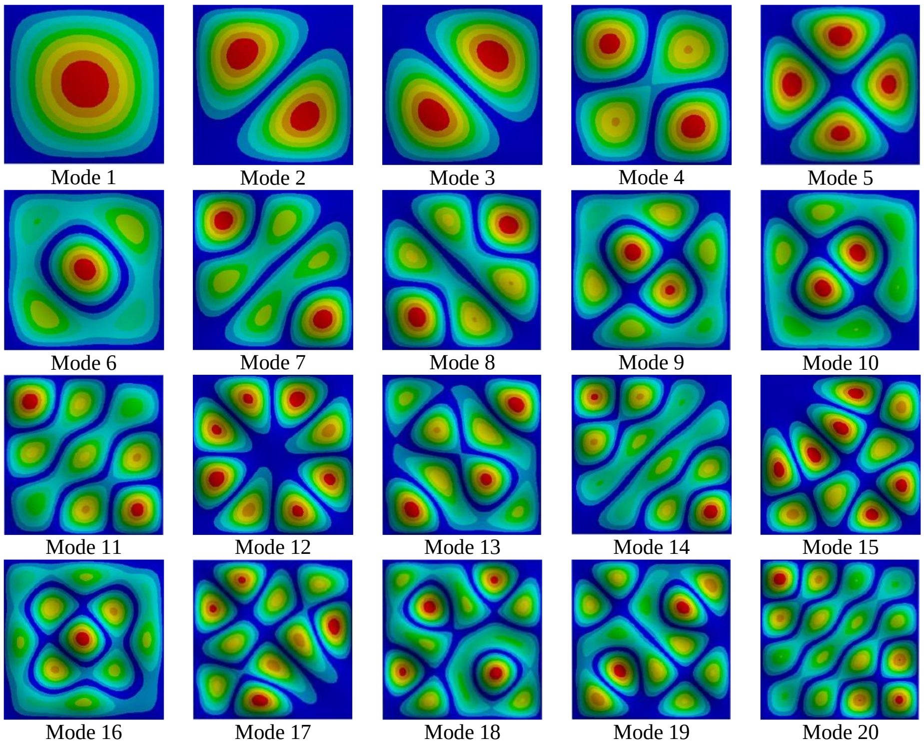

Fig. 16 *Modal patterns for simply supported square plate with an equal angle-leg corner partially simply supported (α = 0.8).*

It is remarkable that all numerical results carried out in present work have no considering the characteristic of stress singularities occurred at the points of boundary discontinuities [46], [47]. Significantly, these local stresses actually tend to infinity in the vicinity of the ends of discontinuous supports.

Furthermore, stress singularities are very important to practical engineering design process that may cause a local change in plate's stiffness due to some existence of damages near those end points of supports, and may also alter the dynamic characteristics of the plate. Thus, the necessities of considering the singularity in solutions have been studied and confirmed by Chen and Pickett [48], Leissa et al. [49], and Leissa [50].

Since the order of stress singularities in problem of plates with mixed edge conditions is in the same order with the problem of cracked plates, then Huang and his colleagues have analyzed the free vibration and buckling problems of plates with cracks using the methods based on the Ritz approach [51] – [54], which included the correct singularity order at the roots of the crack.

For other numerical methods such as the boundary element and finite element methods can, respectively, be found in Sun and Wei [55] and Ayatollahi et al. [56]. It is interesting to notice that the Ritz method is very suitable for analyzing the present problems because the geometry of plate under consideration is simple, so that the area integration required in the Ritz method is easy to set up. This method will then be considered for future research.

5. CONCLUSION

In summary, to the authors' knowledge, the results shown that provided the higher frequency parameters of square plates with two edges mixed boundary supports adjacent to one corner of the plates are the first ones available in the published literature. They are, however, prepared in the tabular form for easy reference, which could serve as the benchmark values for other future researches in plate vibrations involving mixed boundary conditions.

REFERENCES

[1] E. Ventsel and T. Krauthammer, Thin Plates and Shells: Theory, Analysis, and Applications, Marcel Dekker, Inc., New York, 2001.

[2] R. Szilard, Theories and Applications of Plate Analysis: Classical, Numerical and Engineering Methods, John Wiley & Sons, Inc., New Jersey, 2004.

[3] L. Meirovitch, Analytical Methods in Vibrations, Macmillan Publishing Co., Inc., New York, 1967.

[4] S.P. Timoshenko and D.H. Young, Vibration Problems in Engineering, 3rd ed., D. Van Nostrand Company, Inc., Princeton, 1955.

[5] S.P. Timoshenko and S. Woinowsky-Krieger, Theory of Plates and Shells, 2nd ed., McGraw-Hill, Singapore, 1959.

[6] A.W. Leissa, "Free vibrations of elastic plates", Proceedings of the AIAA 7th Aerospace Science Meeting, 20-22 January 1963, New York City, New York, AIAA Paper No. 69-24, 1963.

[7] A.W. Leissa, "The free vibration of rectangular plates", Journal of Sound and Vibration, vol. 31, pp. 257-293, 1973.

[8] J.S. Rao, Dynamics of Plates, Narosa Publishing House, New Delhi, 1999.

[9] W. Nowacki, Dynamics of Elastic Systems, Chapman & Hall Ltd., London, 1963.

[10] A.W. Leissa, Vibration of Plates, Reprinted ed., Acoustical Society of America, Washington, D.C., 1993.

[11] Y. Sompornjaroensuk and K. Kiattikomol, "Solving problems of plate with mixed boundary conditions using Hankel integral transforms: Part 1-Solution method and static bending problems", Proceedings of the 12th National Convention on Civil Engineering (NCCE12), Pisanulok, Thailand, May 2-4, vol. 7, pp. 64-72, 2007.

[12] Y. Sompornjaroensuk and K. Kiattikomol, "Solving problems of plate with mixed boundary conditions using Hankel integral transforms: Part 2-Free vibration and buckling problems", in Proceedings of the 12th National Convention on Civil Engineering (NCCE12), Pisanulok, Thailand, May 2-4, vol. 7, pp. 73-84, 2007.

[13] Y. Sompornjaroensuk and P. Chantarawichit, "Vibration of circular plates with mixed edge conditions. Part I: Review of research", UTK Research Journal, vol. 14, 2020. (Accepted for publication).

[14] T. Ota and M. Hamada, "Bending and vibration of a simply supported but partially clamped rectangular plate", Transactions of the Japan Society of Mechanical Engineers, vol. 25, pp. 668-674, 1959 (in Japanese).

[15] T. Ota and M. Hamada, "Fundamental frequencies of simply supported but partially clamped square plates", Bulletin of JSME, vol. 6, pp. 397-403, 1963.

[16] L.M. Keer and B. Stahl, "Eigenvalue problems of rectangular plates with mixed edge conditions", Journal of Applied Mechanics, vol. 39, pp. 513-520, 1972.

[17] G. Venkateswara Rao, I.S. Raju and T.V.G.K. Murthy, "Vibration of rectangular plates with mixed boundary conditions", Journal of Sound and Vibration, vol. 30, pp. 257-260, 1973.

[18] Y. Narita, "Application of a series-type method to vibration of orthotropic rectangular plates with mixed boundary conditions", Journal of Sound and Vibration, vol. 77, pp. 345-355, 1981.

[19] D.J. Gorman, "An exact analytical approach to the free vibration analysis of rectangular plates with mixed boundary conditions", Journal of Sound and Vibration, vol. 93, pp. 235-247, 1984.

[20] Z.-Q. Cheng, "The application of Weinstein-Chien's method-the upper and lower limits of fundamental frequency of rectangular plates with edges are the mixture of simply supported portions and clamped portions", Applied Mathematics and Mechanics, vol. 5, pp. 1399-1408, 1984.

[21] T. Sakiyama and H. Matsuda, "Free vibration of rectangular Mindlin plate with mixed boundary conditions", Journal of Sound and Vibration, vol. 113, pp. 208-214, 1987.

[22] A.Y.T. Leung and F.T.K. Au, "Spline finite elements for beam and plate", Computers and Structures, vol. 37, pp. 717-729, 1990.

[23] H.P. Lee and S.P. Lim, "Free vibration of isotropic and orthotropic rectangular plates with partially clamped edges", Applied Acoustics, vol. 35, pp. 91-104, 1992.

[24] K.M. Liew, K.C. Hung and K.Y. Lam, "On the use of the substructure method for vibration analysis of rectangular plates with discontinuous boundary conditions", Journal of Sound and Vibration, vol. 163, pp. 451-462, 1993.

[25] P.A.A. Laura and R.H. Gutierrez, "Analysis of vibrating rectangular plates with non-uniform boundary conditions by using the differential quadrature method", Journal of Sound and Vibration, vol. 173, pp. 702-706, 1994.

[26] Y.K. Cheung and J. Kong, "The application of a new finite strip to the free vibration of rectangular plates of varying complexity", Journal of Sound and Vibration, vol. 181, pp. 341-353, 1995.

[27] A.G. Striz, W.L. Chen and C.W. Bert, "Free vibration of plates by the high accuracy quadrature element method", Journal of Sound and Vibration, vol. 202, pp. 689-702, 1997.

[28] R.K. Singhal and D.J. Gorman, "Free vibration of partially clamped rectangular plates with and without rigid point supports", Journal of Sound and Vibration, vol. 203, pp. 181-192, 1997.

[29] D.J. Gorman, Free Vibration Analysis of Rectangular Plates, Elsevier North Holland, Inc., New York, 1982.

[30] C. Shu and C.M. Wang, "Treatment of mixed and nonuniform boundary conditions in GDQ vibration analysis of rectangular plates", Engineering Structures, vol. 21, pp. 125-134, 1999.

[31] F.-L. Liu and K.M. Liew, "Analysis of vibrating thick rectangular plates with mixed boundary constraints using differential quadrature element method", Journal of Sound and Vibration, vol. 225, pp. 915-934, 1999.

[32] G.W. Wei, Y.B. Zhao and Y. Xiang, "The determination of natural frequencies of rectangular plates with mixed boundary conditions by discrete singular convolution", International Journal of Mechanical Sciences, vol. 43, pp. 1731-1746, 2001.

[33] M.-H. Huang and D.P. Thambiratnam, "Free vibration analysis of rectangular plates on elastic intermediate supports", Journal of Sound and Vibration, vol. 240, pp. 567-580, 2001.

[34] G.H. Su and Y. Xiang, "A non-discrete approach for analysis of plates with multiple subdomains", Engineering Structures, vol. 24, pp. 563-575, 2002.

[35] C.H.W. Ng, Y.B. Zhao and G.W. Wei, "Comparison of discrete singular convolution and generalized differential quadrature for the vibration analysis of rectangular plates", Computer Methods in Applied Mechanics and Engineering, vol. 193, pp. 2483-2506, 2004.

[36] D. Zhou, Y.K. Cheung, S.H. Lo and F.T.K. Au, "Three-dimensional vibration analysis of rectangular plates with mixed boundary conditions", Journal of Applied Mechanics, vol. 72, pp. 227-236, 2005.

[37] Y. Xiang, S.K. Lai, L. Zhou and C.W. Lim, "DSC-Ritz element method for vibration analysis of rectangular Mindlin plates with mixed edge supports", European Journal of Mechanics A/Solids, vol. 29, pp. 619-628, 2010.

[38] S. Chaiyat and Y. Sompornjaroensuk, "Integral equation for symmetrical free vibration of Levy-plate having discontinuous simple supports", Procedia Engineering, vol. 14, pp. 2933-2930, 2011.

[39] H. Zhang, Y.P. Zhang and C.M. Wang, "Hencky bar-net model for vibration of rectangular plates with mixed boundary conditions and point supports", International Journal of Structural Stability and Dynamics, vol. 18, 1850046, 2018.

[40] Y. Sompornjaroensuk and D. Dy, "Higher frequencies of square plates with symmetrically mixed of simply supported-clamped edge conditions", Engineering Transactions, 2020. (Submitted for publication).

[41] ANSYS, Inc., ANSYS Mechanical APDL Theory Reference, Release 14.5, 2012.

[42] ANSYS, Inc., ANSYS Mechanical APDL Element Reference, Release 14.5, 2012.

[43] Y. Sompornajroensuk, "Vibration of circular plates with mixed edge conditions. Part II: Numerical determination for higher frequencies", UTK Research Journal, vol. 14, 2020. (Accepted for publication).
[44] T.J.R. Hughes, The Finite Element Method: Linear Static and Dynamic Finite Element Analysis, Prentice-Hall, Inc., New Jersey, 1987.
[45] K.J. Bathe, Finite Element Procedures, Prentice-Hall, Inc., New Jersey, 1996.
[46] M.L. Williams, "Surface stress singularities resulting from various boundary conditions in angular corners of plates under bending", Proceedings of the First U.S. National Congress of Applied Mechanics, American Society of Mechanical Engineers, June 1952, Illinois Institute of Technology, Chicago, vol. 1, pp. 325-329, 1952.
[47] P. Kongtong and Y. Sompornjaroensuk, "On the Williams' solution in plates having stress singularities", Proceedings of the 15th National Convention on Civil Engineering (NCCE15), 12-14 May 2010, Ubonratchatani, Thailand, STR025, 2010.
[48] S.S.-H. Chen and G. Pickett, "Bending of plates of any shape and with any variation in boundary conditions", Journal of Applied Mechanics, vol. 34, pp. 217-218, 1967.
[49] A.W. Leissa, W.E. Clausen, L.E. Hulbert, and A.T. Hopper, "A comparison of approximate methods for the solution of plate bending problems", AIAA Journal, vol. 7, pp. 920-928, 1969.
[50] A.W. Leissa, "Singularity considerations in membrane, plate and shell behaviors", International Journal of Solids and Structures, vol. 38, pp. 3341-3353, 2001.
[51] C.S. Huang and C.W. Chan, "Vibration analyses of cracked plates by the Ritz method with moving least-squares interpolation functions", International Journal of Structural Stability and Dynamics, vol. 14, 1350060, 2014.
[52] H.C. Zeng, C.S. Huang, A.W. Leissa and M.J. Chang, "Vibrations and stability of a loaded side-cracked rectangular plate via the MLS-Ritz method", Thin-Walled Structures, vol. 106, pp. 459-470, 2016.
[53] C.S. Huang, M.C. Lee and M.J. Chang, "Vibration and buckling analysis of internally cracked square plates by the MLS-Ritz approach", International Journal of Structural Stability and Dynamics, vol. 18, 1850105, 2018.
[54] C.S. Huang, H.T. Lee, P.Y. Li, K.C. Hu, C.W. Lan and M.J. Chang, "Three-dimensional buckling analyses of cracked functionally graded material plates via the MLS-Ritz method", Thin-Walled Structures, vol. 134, pp. 189-202, 2019.
[55] L. Sun and X. Wei, "A frequency domain formulation of the singular boundary method for dynamic analysis of thin elastic plate", Engineering Analysis with Boundary Elements, vol. 98, pp. 77-87, 2019.
[56] M.R. Ayatollahi, M. Nejati, and S. Ghouli, "The finite element over-deterministic method to calculate the coefficients of crack tip asymptotic fields in anisotropic planes", Engineering Fracture Mechanics, vol. 231, 106982, 2020.

Yos Sompornjaroensuk received his B. Eng. (Civil), M. Eng. and Ph.D. in the field of structural engineering from King Mongkut's University of Technology Thonburi. He is an Assistant Professor in the Department of Civil Engineering, Mahanakorn University of Technology. His research interests are in the area of Structural Engineering and Mechanics, Fracture Mechanics, Contact Mechanics, and Mathematical Modeling.

Damang Dy received his B. Eng. (Civil) and M.Eng. from Sirindhorn International Institute of Technology, Thammasat University and King Mongkut's University of Technology Thonburi, respectively. Currently, he served as Police Major General, Deputy Director General in the General Department of Immigration, Phnom Penh, Cambodia. His current research interests include structural engineering and mechanics.