Analysis of Reduced Complexity Widely-Linear Adaptive Forgetting-Factor Inverse Square-Root Recursive Least Squares algorithm

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ABSTRACT

Based on widely-linear approach, the proposed reduced complexity inverse square-root recursive least squares algorithm is presented with the methods of adaptive forgetting-factor algorithm. The proposed reduced complexity widely-linear approaches based on inverse square-root recursive least squares algorithm is introduced for a relation between widely-linear and reduced complexity mechanism. By using mean square deviation approach, the proposed optimal forgetting-factor mechanism for optimal gain sequence is presented. Adaptive forgetting-factor inverse square-root recursive least squares algorithm is considered with regard to an optimal forgetting-factor algorithm. A reduced-complexity widely-linear inverse square-root recursive least squares algorithm with the adaptive inverse square-root mechanism, called QR-decomposition for single-carrier frequency-domain equalization systems is presented. Simulation results show that the performance of proposed algorithm is shown that similar to widely-linear approach compared with the conventional algorithm.

Keywords: *Widely-Linear approach, Reduced complexity scheme, adaptive inverse square-root recursive least squares algorithm, adaptive forgetting-factor algorithm.*

1. INTRODUCTION

Channel equalization is approximately an inverse filtering of the linear distortion to the channel with the multipath propagation. From the linear time-invariant systems, the linear filtering in the time-domain is the convolution operation and a point-wise multiplication operation is in the frequency-domain. Hence, the time-domain signal is transformed by the Fourier transform to frequency-domain signals, which can be equalized by dividing it point-by-point. The frequency-domain equalization can also be implemented using the digital signal processing. Transformation should not increase its size linearly with the channel response, so the discrete Fourier transform is practical for equalization.

Adaptive recursive least squares algorithm (RLS) filtering has been shown the faster convergence rate than that of a least mean square algorithm. Usage of widely-linear (WL) scheme has been used as the equalization, modulation and beamforming in the applications of complex form of signal processing [1]. Performance of higher complexity widely-linear filter has been reported in [2]. In [3], the improvement of widely-linear scheme with the methods of the circular input has been proposed the advantages compared with linear filtering. To adjust the optimal forgetting-factor for RLS algorithm, it is difficult to reach because of the large model variation [4]. In order to attain its performance, the optimal forgetting-factor RLS algorithm can be adjusted adaptively in [5].

Moreover, the reduced computational complexity for implementation of WL-RLS approach to model with the low complexity has been shown in [6]. To overcome the problem of RLS scheme, some applications in signal processing have been recommended with lower properties of inverse matrix of Hermitian symmetry [7]. Based on RLS algorithm with the inverse QR-composition that can achieve on the inverse autocorrelation matrix as shown in [8],[9], which can improve the stability considerably [10]. In [11], some applications of RLS algorithm has been operated on parallel implementation.

In this paper, we descirbe briefly the applying widely-linear approach onto the inverse square-root RLS approach in Section 2. An adaptive forgetting-factor mechanism with the optimal gain sequence is based on iQR-RLS algorithm with the method of widely-linear approach in Section 3. Then, the reduced-complexity scheme is applied for the widely-linear adaptive forgetting-factor iQR-RLS (WL-AFiQRRLS) algorithm in Section 4. Computational complexity of proposed algorithms are introduced in Section 5. Simulation results show the proposed iQRRLS-based algorithms perform in forms of the performance of symbol error rate (SER) in Section 6 and Section 7 concludes the paper.

2. Widely-Linear inverse Square-Root Recursive Least Squares Algorithm

In this section, we describe shortly how to derive the widely-linear approach based on recursive least squares algorithm and inverse square-root method by QR-decomposition.

2.1 WIDELY-LINEAR APPROACH

By following [12], the minimised cost function of least-squares approach in case of complex variables by means of widely-linear approach can be expressed as

$$J_k = \frac{1}{2} \sum_{k=1}^{K} \lambda^{K-k} \parallel \varepsilon_k \parallel^2, \quad (1)$$

where ε_k is defined as

$$\varepsilon_k = d_k - \boldsymbol{p}_k^H \cdot \boldsymbol{s}_k \,, \quad (2)$$

where d_k is the k-th element symbol on each subcarrier and λ is the forgetting-factor parameter. The vector $\boldsymbol{p}_k$ is a tap-weight estimated vector with the size of $2L \times 1$ complex-valued tap-weight vector and L is the number of tap-weight vector.

The vector $\boldsymbol{y}_k$ is the $L \times 1$ discrete Fourier transform (DFT) output as

$$\boldsymbol{y}_k = \boldsymbol{y}_{\Re_k} + j\boldsymbol{y}_{\Im_k} \,, \quad (3)$$

where subscript $\Re$ is used for real number and $\Im$ is defined for imaginary number.

That requires knowledge of these matrices as

$$R_{\Re\Re} = \mathrm{E}\{\boldsymbol{y}_{\Re_k}\boldsymbol{y}_{\Re_k}^{\mathrm{T}}\} \,, \quad (4)$$

$$R_{\Im\Im} = \mathrm{E}\{\boldsymbol{y}_{\Im_k}\boldsymbol{y}_{\Im_k}^{\mathrm{T}}\} \,, \quad (5)$$

$$R_{\Re\Im} = \mathrm{E}\{\boldsymbol{y}_{\Re_k}\boldsymbol{y}_{\Im_k}^{\mathrm{T}}\} \,. \quad (6)$$

Since, we arrive at

$$R_{yy} = \mathrm{E}\{\boldsymbol{y}_k\boldsymbol{y}_k^{\mathrm{T}}\} = (R_{\Re\Re} + R_{\Im\Im}) + j(R_{\Re\Im} - R_{\Im\Re}) \,. \quad (7)$$

Then, the augmented vector $\boldsymbol{s}_k$ is defined by

$$\boldsymbol{s}_k = \begin{bmatrix} \boldsymbol{y}_k \\ \boldsymbol{y}_k^* \end{bmatrix} \,, \quad (8)$$

where $\boldsymbol{s}_k$ is the augmented vector constituted using the DFT output $\boldsymbol{y}_k$ and its conjugate.

Therefore, the autocorrelation $R_{\boldsymbol{ss}}$ can be expressed as

$$R_{\boldsymbol{ss}} = (R_{\Re\Re} - R_{\Im\Im}) + j(R_{\Re\Im} + R_{\Im\Re}) \,, \quad (9)$$

$$R_{\boldsymbol{ss}} = \mathrm{E}\{\boldsymbol{s}_k\boldsymbol{s}_k^H\} = \begin{bmatrix} R_{yy} & R_{\boldsymbol{ss}} \\ R_{\boldsymbol{ss}}^H & R_{yy}^H \end{bmatrix}. \quad (10)$$

2.2 ADAPTIVE INVERSE SQUARE-ROOT RLS ALGORITHM

Following [12], we explain an adaptive forgetting-factor λ_k based on adaptive inverse Squre-Root RLS (iQR-RLS) algorithm. The recursive equation for updating the least-squares estimate $\boldsymbol{p}_k$ for the tap-weight vector as

$$\boldsymbol{p}_k = \boldsymbol{p}_{\mathrm{k}-1} + \boldsymbol{\kappa}_k \cdot \xi_{\mathrm{k}}^* \,, \quad (11)$$

where the Kalman gain vector $\boldsymbol{\kappa}_k$ and the inverse autocorrelation $\boldsymbol{\Psi}_k$ is given as [11]

$$\boldsymbol{\kappa}_k = \frac{{\lambda_k}^{-1}\boldsymbol{\Psi}_{k-1}s_k}{{\lambda_k}^{-1}s_k^H\boldsymbol{\Psi}_{k-1}s_k+1} \,, \quad (12)$$

$$\boldsymbol{\Psi}_k = {\lambda_k}^{-1}\boldsymbol{\Psi}_{k-1} - {\lambda_k}^{-1}\boldsymbol{\kappa}_k\boldsymbol{s}_k^H\boldsymbol{\Psi}_{k-1} \,, \quad (13)$$

where λ_k is a forgetting-factor parameter and $\boldsymbol{s}_k$ is defined in Eq. (8).

To support the QR-decomposition, the inverse autocorrelation $\boldsymbol{\Psi}_k$ is reorganised by

$$\boldsymbol{\Psi}_k = {\lambda_k}^{-1}\boldsymbol{\Psi}_{k-1} - {\lambda_k}^{-2}\boldsymbol{\Psi}_{k-1}\boldsymbol{s}_k\gamma_k^{-1}\boldsymbol{s}_k^H\boldsymbol{\Psi}_{k-1} \ , \qquad (14)$$

where

$$\tilde{\gamma}_k = {\lambda_k}^{-1}\boldsymbol{s}_k^H\ \boldsymbol{\Psi}_{k-1}\ \boldsymbol{s}_k + 1 \ . \qquad (15)$$

According to the Cholesky factorisation, the block matrix $\boldsymbol{\mathcal{M}}$ is given as

$$\boldsymbol{\mathcal{M}} = \boldsymbol{\mathcal{A}}\,\boldsymbol{\mathcal{A}}^H, \qquad (16)$$

and its result consists of the matrix product of right handed on Eq.(14).

Then, the pre-array matrix $\boldsymbol{\mathcal{A}}$ to generating post-array matrix $\boldsymbol{\mathcal{B}}$ transformation using QR-decomposition is based on the update QR approach as

$$\boldsymbol{\mathcal{A}}\,\Theta = \boldsymbol{\mathcal{B}} \ , \qquad (17)$$

$$\boldsymbol{\mathcal{A}} = \begin{bmatrix} a_{11} & \boldsymbol{a}_{12} \\ a_{21} & \boldsymbol{A}_{22} \end{bmatrix} = \begin{bmatrix} 1 & {\lambda_k}^{-\frac{1}{2}}\boldsymbol{s}_k^H\widetilde{\boldsymbol{\Psi}}_{k-1}^{\frac{1}{2}} \\ 0 & {\lambda_k}^{-\frac{1}{2}}\widetilde{\boldsymbol{\Psi}}_{k-1}^{\frac{1}{2}} \end{bmatrix} , \qquad (18)$$

$$\boldsymbol{\mathcal{B}} = \begin{bmatrix} b_{11} & \boldsymbol{b}_{12} \\ \boldsymbol{b}_{21} & \boldsymbol{B}_{22} \end{bmatrix} = \begin{bmatrix} \tilde{\gamma}_k^{\frac{1}{2}} & 0^T \\ \tilde{\gamma}_k^{\frac{1}{2}}\cdot\tilde{\boldsymbol{\kappa}}_k & \widetilde{\boldsymbol{\Psi}}_k^{\frac{1}{2}} \end{bmatrix} , \qquad (19)$$

where Θ is a unitary rotation.

The Kalman gain vector $\tilde{\boldsymbol{\kappa}}_k$ and inverse autocorrelation square root matrix $\widetilde{\boldsymbol{\Psi}}_k^{\frac{1}{2}}$ are defined from the entries of the first and second column at the second row of the post-array $\boldsymbol{\mathcal{B}}$ in Eq. (19) as

$$\tilde{\boldsymbol{\kappa}}_k = \frac{\boldsymbol{b}_{21}}{b_{11}} = \frac{\tilde{\gamma}_k^{\frac{1}{2}}\cdot\tilde{\boldsymbol{\kappa}}_k}{\tilde{\gamma}_k^{\frac{1}{2}}} \ . \qquad (20)$$

where $\tilde{\gamma}_k$ is given in Eq. (15).

Thus, the approximation of autocorrelation $\widetilde{\boldsymbol{\Psi}}_k$ from QR-decompostion scheme is given as

$$\widetilde{\boldsymbol{\Psi}}_k = \boldsymbol{B}_{22}\cdot\boldsymbol{B}_{22} = \widetilde{\boldsymbol{\Psi}}_k^{\frac{1}{2}}\cdot\widetilde{\boldsymbol{\Psi}}_k^{\frac{1}{2}} \ , \qquad (21)$$

2.3 ADAPTIVE WIDELY-LINEAR INVERSE-SQUARE ROOT RLS ALGORITHM

Following the Wiener approach, the optimal tap-weight vector for widely-linear scheme $p_{\mathrm{WL}}^{\mathrm{opt}}$ is given as

$$p_{\mathrm{WL}}^{\mathrm{opt}} = R_{ss}^{-1}\boldsymbol{\rho}_{WL} \ , \qquad (22)$$

where $\boldsymbol{\rho}_{WL}$ and R_{ss} are defined by

$$\boldsymbol{\rho}_{\mathrm{WL}} = E\{d_k\ \ \boldsymbol{s}_k\} \ , \qquad (23)$$

$$R_{ss} = E\{\boldsymbol{s}_k\ \ \boldsymbol{s}_k^H\} \ . \qquad (24)$$

Proposed adaptive widely-linear iQR-RLS (WL-iQRRLS) algorithm is therefore derived as

$$\boldsymbol{p}_{\mathrm{WL}_k} = \boldsymbol{p}_{\mathrm{WL}_{k-1}} + \boldsymbol{\kappa}_k\cdot\xi_{\mathrm{WL}_k}^* \ , \qquad (25)$$

$$\xi_{\mathrm{WL}_k} = d_k - p_{\mathrm{WL}_{k-1}}^H\cdot\boldsymbol{s}_k \ , \qquad (26)$$

where $\xi_{\mathrm{WL}_k}^*$ is the conjugate of *a priori* estimated error and the augmented vector $\boldsymbol{s}_k$ and the Kalman vector $\boldsymbol{\kappa}_k$ are given in Eq.(8) and Eq.(12), respectively.

3. WIDELY-LINEAR ADAPTIVE FORGETTING-FACTOR IQR-RLS ALGORITHM

Widely-linear adaptive forgetting-factor inverse square-root recursive least squares (WLAF-iQRRLS) is introduced for frequency domain equalisation.

Following [5], a proposed adaptive forgetting-factor algorithm λ_k concerned with the inverse autocorrelation in Eq. (14) based on WL-iQRRLS algorithm can be introduced with the method of the update gain sequence $\breve{\varphi}_k$ as

$$\lambda_k = (\ 1 - \varphi_{k-1}\ )\cdot\frac{\varphi_{k-1}}{\varphi_k} \ , \qquad (27)$$

and

$$\varphi_k = \varphi_{k-1} - \beta\ \frac{\boldsymbol{\alpha}_k}{\boldsymbol{\alpha}_k^2 + C} \ , \qquad (28)$$

where the adaptation rate β is a small positive number, C is a small positive constant and $\|\ \boldsymbol{\alpha}_k\ \|^2$ is defined as

$$\boldsymbol{\alpha}_k = \boldsymbol{\zeta}_{k-1}^H\ \frac{\zeta_k\,\zeta_{k-1}^H}{\zeta_{k-1}^H\,\zeta_k}\ \boldsymbol{\zeta}_{k-1} \ , \qquad (29)$$

and

$$\boldsymbol{\zeta}_k = \boldsymbol{s}_k \cdot \widetilde{\boldsymbol{\Psi}}_k \cdot \xi^*_{\mathrm{WL}_k} \ , \tag{30}$$

where $\boldsymbol{s}_k$ and $\widetilde{\boldsymbol{\Psi}}_k$ are defined in (8) and (21), respectively. The estimated errror ξ_{WL_k} is described in (15).

4. REDUCED-COMPLEXITY WIDELY-LINEAR ADAPTIVE FORGETTING-FACTOR IQRRLS ALGORITHM

We show briefly the reduced-complexity widely-linear estimation as decribed in [6] based on adaptive forgetting-factor approach.

Using a transformation matrix U, we get

$$U = \begin{bmatrix} I_{L\times L} & j\ I_{L\times L} \\ I_{L\times L} & -j\ I_{L\times L} \end{bmatrix} \ , \tag{31}$$

$$UU^H = 2I \ , \tag{32}$$

onto the s_k, the transformed vector s_k is given as

$$\bar{\boldsymbol{s}}_k = U^H \boldsymbol{s}_k = \begin{bmatrix} \Re\{\boldsymbol{y}_k\} \\ \Im\{\boldsymbol{y}^*_k\} \end{bmatrix} \ , \tag{33}$$

where $\Re\{\cdot\}$ and $\Im\{\cdot\}$ are the real and imaginary operators. The vector s_k is defined in (2) and $I_{L\times L}$ is the $L \times L$ identity matrix, where L is the number of taps of equalisers.

Optimal tap-weight vector $p^{\mathrm{opt}}_{\mathrm{RC}}$ following the optimal Wiener approach based on the reduced complexity widely-linear mechanism can be computed as

$$p^{\mathrm{opt}}_{\mathrm{RC}} = R^{-1}_{ss} \boldsymbol{\rho}_{\mathrm{RC}} \ , \tag{34}$$

where R^{-1}_{ss} is the inverse autocorrelation matrix of augmented transformation vector s_k and

$$\boldsymbol{\rho}_{\mathrm{RC}} = E\{d_k\ U^H\ \bar{\boldsymbol{s}}_k\} = E\{d_k\ \bar{\boldsymbol{s}}_k\} \ , \tag{35}$$

$$R_{\bar{\boldsymbol{s}}\bar{\boldsymbol{s}}} = E\{\bar{\boldsymbol{s}}_k\ \bar{\boldsymbol{s}}_k\} = E\{U^H\ R_{ss}\ U\} \ , \tag{36}$$

$$R_{\boldsymbol{ss}} = E\{\boldsymbol{s}_k\ \boldsymbol{s}^H_k\} \ . \tag{37}$$

where $E\{\cdot\}$ denotes as the expectation operator and R_{ss} is the autocorrelation matrix of augmented vector s_k.

Moreover, the tap-weight vector $p^{\mathrm{opt}}_{\mathrm{RC}}$ based on the reduced-complexity widely-linear mechanism using optimum Wiener solution is calculated as

$$p^{\mathrm{opt}}_{\mathrm{RC}} = \frac{1}{2}(U\ R_{\boldsymbol{ss}})^{-1} \cdot (U^H \boldsymbol{\rho}_{\mathrm{WL}})$$

$$= \frac{1}{2} U^H\ p^{\mathrm{opt}}_{\mathrm{WL}} \ , \tag{38}$$

where $p^{\mathrm{opt}}_{\mathrm{WL}}$ is the estimated tap-weight vector using optimum Wiener solution for widely-linear approach.

Therefore, the reduced-complexity widely-linear estimated error ξ_{RC_k} is defined by applying (33) and (38) as

$$\xi_{\mathrm{RC}_k} = d_k - \tfrac{1}{2} p_{\mathrm{WL}_k} U \cdot U^H \bar{\boldsymbol{s}}_k \ ,$$

$$= d_k - p_{\mathrm{RC}_k} \cdot \bar{\boldsymbol{s}}_k \ , \tag{39}$$

and the proposed reduced-complexity tap-weight estimated vector p_{RC_k} is defined as

$$p_{\mathrm{RC}_k} = p_{\mathrm{RC}_{k-1}} + \widetilde{\boldsymbol{\kappa}}_k \cdot \xi^*_{\mathrm{RC}_k} \ , \tag{40}$$

where $\widetilde{\boldsymbol{\kappa}}_k$ is given in (20).

4.1 REDUCED COMPLEXITY WL-AFIQRRLS ALGORITHM

We introduce an adaptive forgetting-factor parameter λ_{RC_k} based on the reduced-complexity widely-linear iQR-RLS algorithm can be written as

$$\lambda_{\mathrm{RC}_k} = \frac{\tilde{\gamma}_{\mathrm{RC}_{k-1}}}{\tilde{\gamma}_{\mathrm{RC}_k}} \left(1 - \tilde{\gamma}_{\mathrm{RC}_{k-1}}\right), \tag{41}$$

$$\tilde{\gamma}_{\mathrm{RC}_k} = \tilde{\gamma}_{\mathrm{RC}_{k-1}} - \beta\ \frac{\Phi_{\mathrm{RC}_k}}{\Phi_{\mathrm{RC}_k} + C} \ , \tag{42}$$

$$\Phi_{\mathrm{RC}_k} = \boldsymbol{\zeta}^H_{\mathrm{RC}_{k-1}}\ G_k\ \boldsymbol{\zeta}_{\mathrm{RC}_{k-1}} \ , \tag{43}$$

where G_k is the orthogonal projection of $\boldsymbol{\zeta}_{\mathrm{RC}_k}$ as

$$G_k = \frac{\zeta_{\mathrm{RC}_k}\ \zeta^H_{\mathrm{RC}_k}}{\zeta^H_{\mathrm{RC}_k}\ \zeta_{\mathrm{RC}_k}} \ , \tag{44}$$

$$\boldsymbol{\zeta}_{\mathrm{RC}_k} = \boldsymbol{\Psi}_{\mathrm{RC}_k}\ \bar{\boldsymbol{s}}_k\ \xi^*_{\mathrm{RC}_k} \ , \tag{45}$$

where $\boldsymbol{\Psi}_{\mathrm{RC}_k}$ is the inverse autocorrelation of $\bar{\boldsymbol{s}}_k$ and $\bar{\boldsymbol{s}}_k$ is computed in (33). Estimated errror ξ_{RC_k} can be expressed in (39).

4.2 RECURSIVE INITIALISATION BASED ON QR-DECOMPOSITION

We propose the reduced-complexity widely-linear adaptive forgetting-factor iQRRLS (RCWL-AFiQRRLS) algorithm based on update QR-decomposition.

Based on the QR decomposition and iQR-RLS algorithm that recursive initialisation can keep and go forward the upper triangular of square-root inverse matrix $\boldsymbol{L}_k$, where

$$\boldsymbol{\Omega}_{\mathrm{RC}_k} = \boldsymbol{L}_k\,\boldsymbol{L}_k^H\ , \tag{46}$$

Where

$$\boldsymbol{L}_k = \left(\boldsymbol{\Psi}_{\mathrm{RC}_k}\right)^{1/2}\ . \tag{47}$$

The following pseudocode employs the proposed mechanism as shown in Table 2.

5 COMPUTATIONAL COMPLEXITY

Computational complexity of proposed algorithms based on the iQR-RLS algorithm are considered. We consider 1-multiplication of 2-complex numbers is given as two-real additions and four-real multiplications. A multiplication of a real number with a complex number can be expressed using two real multiplications. So, the computational complexity of proposed RCWL-AFiQRRLS, AF-iQRRLS and WL-AFiQRRLS algorithms compared with RCWL-iQRRLS, WL-iQRLS and iQR-RLS algorithms are shown in Table 1.

For reduced complexity approach, we notice that the computational complexity of proposed RCWL-AFiQRRLS and RCWL-iQRRLS algorithms are reduced by 4-time of the amount of multiplications of WL-AFiQRRLS and WL-iQRRLS algorithms, respectively. Meanwhile, the WL-AFiQRRLS and WL-iQRRLS algorithm are increased the computational complexity by 4-time of AF-iQRRLS and iQR-RLS algorithms, respectively.

Therefore, the computational complexity of proposed RCWL-AFiQRRLS and RCWL-iQRRLS approaches are similar to AF-iQRRLS and iQR-RLS algorithms.

Table 1 *Computational complexity per symbol.*

Algorithm	Amount of multiplications
RCWL-AFiQRRLS	$6.5L^2 - 5.5L + 9$
RCWL-iQRRLS	$2.5L^2 - 15.5L + 3$
WL-AFiQRRLS	$26L^2 - 11L + 9$
WL-iQRRLS	$10L^2 - 31L + 3$
AF-iQRRLS	$6.5L^2 - 5.5L + 9$
iQR-RLS	$2.5L^2 - 15.5L + 3$

Table 2 *Adaptive RCWL-AFiQRRLS algorithm*

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Initialise the independent parameters: $\lambda_{\mathrm{RC}}(0)$, $\tilde{\gamma}_{\mathrm{RC}}(0)$ and $L(0) = \vartheta \cdot I$.

- Initial vectors: $p_{\mathrm{RC}}(0) = [1\ ;\ 0 \cdots\ 0]^T$.
- Initial parameters: β and C are a small positive number.

For $k = 1, \ldots, K$

1. Generate the matrix as
$$U = \begin{bmatrix} I_{L\times L} & j\ I_{L\times L} \\ I_{L\times L} & -j\ I_{L\times L} \end{bmatrix}$$
2. Form the vectors as
$$\boldsymbol{s}_k = \begin{bmatrix} y_k \\ y_k^* \end{bmatrix}$$
$$\bar{\boldsymbol{s}}_k = U^H \bar{\boldsymbol{s}}_k$$
3. Deploy the product of matrix-vector as
$$a = \lambda_{\mathrm{RC}_k}^{-\frac{1}{2}}\ \bar{\boldsymbol{s}}_k^H\ L_{k-1}$$
4. Consider Givens rotation in [11] Q_t^a for $t = 1, \ldots, L$ as
$$\begin{bmatrix} 0_{(L-1)\times 1} \\ \delta \end{bmatrix} = Q_L \cdot Q_{L-1} \ldots Q_1 \begin{bmatrix} a \\ 1 \end{bmatrix}$$
5. Update L_k as
$$\begin{bmatrix} L_k \\ \delta\cdot\boldsymbol{\kappa}_k \end{bmatrix} = Q_L \cdot Q_{L-1} \ldots Q_1 \begin{bmatrix} \boldsymbol{D} \\ 0_{1\times(L-1)} \end{bmatrix}$$
$$\boldsymbol{D} = \lambda_{\mathrm{RC}_k}^{-\frac{1}{2}} L_{k-1}$$
6. Update the inverse autocorrelation $\boldsymbol{\Omega}_{\mathrm{RC}_k}$ from the upper triangular of square-root inverse matrix $\boldsymbol{L}_k$ as
$$\boldsymbol{\Omega}_{\mathrm{RC}_k} = \boldsymbol{L}_k \boldsymbol{L}_k^H$$
7. Create ξ_{RC_k} as
$$\xi_{\mathrm{RC}_k} = d_k - p_{\mathrm{RC}_{k-1}}^H \bar{\boldsymbol{s}}_k$$
8. Update p_{RC_k} as
$$p_{\mathrm{RC}_k} = p_{\mathrm{RC}_{k-1}} + \begin{bmatrix} \frac{\delta\cdot\boldsymbol{\kappa}_k}{\delta} \end{bmatrix}\ \xi_{\mathrm{RC}_k}^*$$
9. Update λ_{RC_k} as
$$\Phi_{\mathrm{RC}_k} = \boldsymbol{\zeta}_{\mathrm{RC}_{k-1}}^H\ G_k\ \boldsymbol{\zeta}_{\mathrm{RC}_{k-1}}$$
$$G_k = \frac{\zeta_{\mathrm{RC}_k}\,\zeta_{\mathrm{RC}_k}^H}{\zeta_{\mathrm{RC}_k}^H\,\zeta_{\mathrm{RC}_k}}$$
$$\boldsymbol{\zeta}_{\mathrm{RC}_k} = \boldsymbol{\Omega}_{\mathrm{RC}_k}\ \bar{\boldsymbol{s}}_k\ \xi_{\mathrm{RC}_k}^*$$
$$\tilde{\gamma}_{\mathrm{RC}_k} = \tilde{\gamma}_{\mathrm{RC}_{k-1}} - \beta\ \frac{\Phi_{\mathrm{RC}_k}}{\Phi_{\mathrm{RC}_k} + C}$$
$$\therefore \lambda_{\mathrm{RC}_k} = \frac{\tilde{\gamma}_{\mathrm{RC}_{k-1}}}{\tilde{\gamma}_{\mathrm{RC}_k}}\ \left(1 - \tilde{\gamma}_{\mathrm{RC}_{k-1}}\right)$$

End

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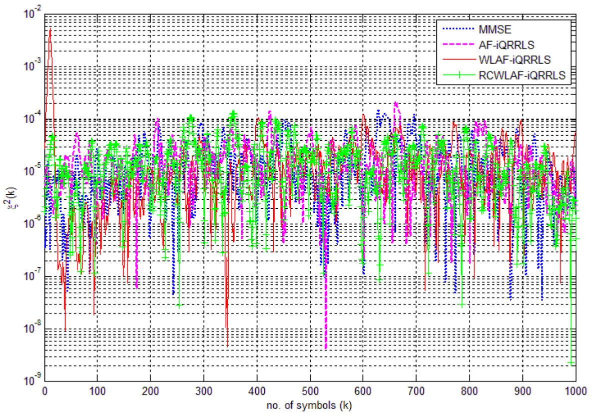

***Fig. 1** Curves of mean square errors (MSE) of proposed WLAF-iQRRLS, RCWLAF-iQRRLS algorithms compared with AF-iQRRLS algorithm, where L = 5.*

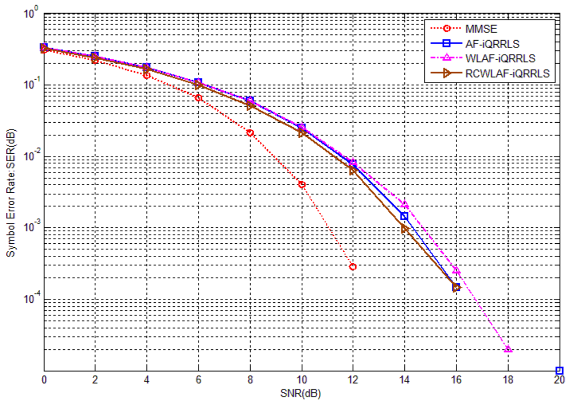

***Fig. 2** Symbol error rate (SER) in dB of AF-iQRRLS, WLAF-iQRRLS and proposed RCWLAF-iQRRLS algorithms compared with minimum mean square error (MMSE) approach, where L = 5.*

6. Simulation results

A link level simulation for single carrier frequency domain equalisation is performed using the ITU channel delay profile as Pedestrian A channel [13] with additive white Gaussian noise. Assumptions and parameters for simulation are as follows [14]: the system bandwidth is fixed at 5MHz, the rate of sampling is set at 5M-samples/s and length of cyclic prefix is equal to 20. Format of data modulation is as the size of quadrature phase shift keying approach, discrete Fourier transform (DFT) and Inverse DFT are fixed at 1024, the number of iterations is set to 100, the signal to noise ratio (SNR) is used between zero to 20dB. Simulation presents a symbol error rate (SER) to adjust the performance of proposed algorithms.

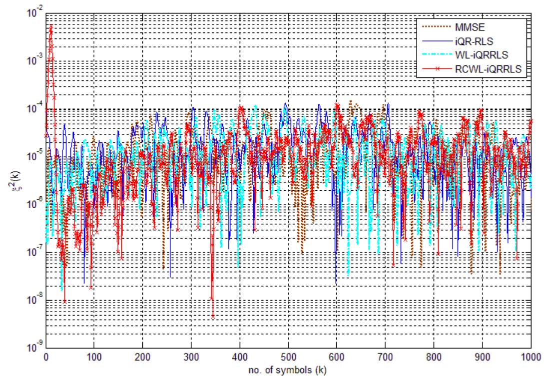

***Fig. 3** Curves of mean square errors (MSE) of proposed WL-iQRRLS, RCWL-iQRRLS algorithms and the conventional iQR-RLS algorithm, where L = 5.*

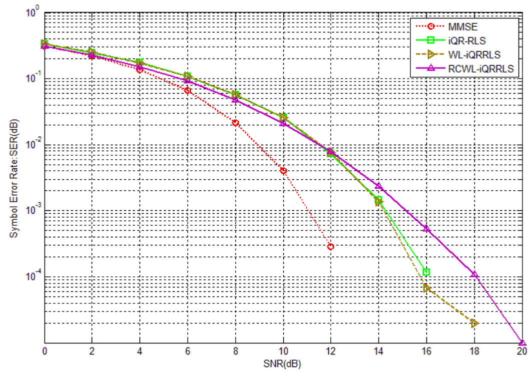

***Fig. 4** Symbol error rate (SER) in decibel (dB) of iQR-RLS, WL-iQRRLS and RCWL-iQRRLS algorithms compared with minimum mean square error (MMSE) approach, where L = 5.*

Initial parameters of proposed frequency-domain equalisers as iQR-RLS, WL-iQRRLS and RCWL-iQRRLS algorithms are as follows: $\lambda = 0.995$, $\vartheta = 1.5 \times 10^{-7}$, $L(0) = \vartheta \cdot I$, $p_{\mathrm{WL}}(0) = p_{\mathrm{RC}}(0) = [1\ ;\ 0 \cdots\ 0]^T$ and $\boldsymbol{\kappa}(0) = [1\ 0 \cdots\ 0]^T$, where L is the number of taps of equalizer and I is given as the identity matrix. And the initial parameters for the adaptive forgetting-factor mechanism as AF-iQRRLS, WLAF-iQRRLS and RCWL-AFiQRRLS algorithms are in the same conditions as $\beta = 1.95 \times 10^{-2}$, $\tilde{\varphi}(0) = \tilde{\gamma}_{\mathrm{RC}}(0) = 7.5 \times 10^{-3}$, $\lambda(0) = \lambda_{\mathrm{RC}}(0) = 0.9$, $\vartheta = 5 \times 10^{-5}$, $L(0) = \vartheta \cdot I$ and $L = 5, 7$.

Curves of mean square errors (MSEs) $\xi^2(k)$ of iQR-RLS, AF-iQRRLS, WL-iQRRLS, WLAF-iQRRLS, RCWL-iQRRLS, RCWLAF-iQRRLS algorithms and the minimum mean square error (MMSE) are compared with the MMSE approach shown in Figs. 1, 3 with the different of number of taps of equalizer as L = 5 and in Fig. 5, when L = 7, respectively.

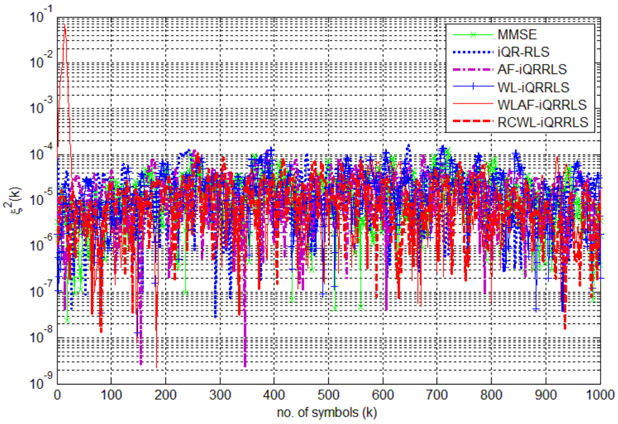

Fig. 5 *Curves of mean square errors (MSE) of AF-iQRRLS, WLAF-iQRRLS, RCWLAF-iQRRLS algorithms and iQR-RLS algorithm, where L = 7.*

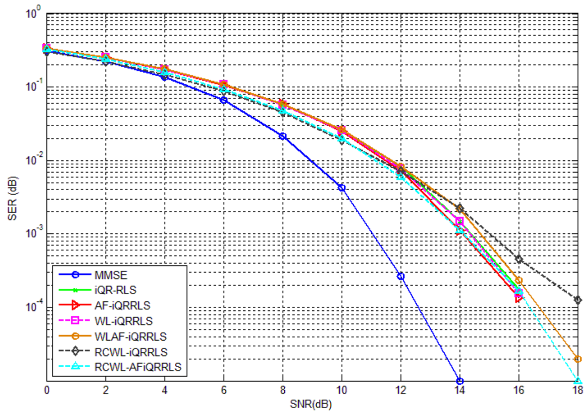

Fig. 6 *Symbol error rate (SER) in dB of proposed algorithms based on iQR-RLS algorighms shown in forms of minimum mean square error (MMSE) approach, where L = 7.*

It is noticed that the MSE curves at approximately 50 symbols of WLiQR-RLS, WLAF-iQRRLS, proposed RCWL-AFiQRRLS and RCWL-iQRRLS, algorithms are converged rapidly to steady-state conditions as shown in Figs. 1, 3, 5.

Performance of symbol error rate (SER) of proposed RCWL-AFiQRRLS, WLAF-iQRRLS, RCWL-iQRRLS, WL-iQRRLS, AF-iQRRLS algorithms based on iQR-RLS algorithm are compared with MMSE approach shown in Figs. 2, 4 with the different of number of taps of equalizers as L =5 and in Fig. 6, when L = 7.

It is noted that SER curves based on inverse square-root algorithm as RCWL-AFiQRRLS, WLAF-iQRRLS, RCWL-iQRRLS, WL-iQRRLS, AF-iQRRLS algorithms as shown in Figs. 2, 4, 6 can achieve to the performance similar to the iQR-RLS algorithm with the different of number of taps of equalizer.

7. Conclusion

By using the method of the update of optimal gain sequence, the proposed adaptive forgetting-factor approach has been presented based on the inverse square-root algorithm and the widely-linear mechanism with the reduced-complexity scheme. According to the low computational complexity, the proposed reduced-complexity scheme is used, while the widely-linear filtering based on adaptive forgetting-factor iQR-RLS (WLAF-iQRRLS) and reduced complexity widely-linear adaptive forgetting-factor iQR-RLS (RCWL-AFiQRRLS) algorithms have been presented by means of iQR-RLS algorithm for SC-FDE systems. To constitute the estimated tap-weight vector, we have described the recursive initialisation based on QR-decomposition for frequency domain equalisers. According to the simulation, they have been obtained the good performance. Proposed WLAF-iQRRLS and RCWL-AFiQRRLS algorithms have attained the achievement compared to the iQR-RLS algorithm for complexity reduction.

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