Analysis of Modified Adaptive Forgetting-Factor RLS-based Frequency Domain Equalization

Suchada Sitjongsataporn
Department of Electronic Engineering
Mahanakorn Institute of Innovation (MII)
Faculty of Engineering, Mahanakorn University of Technology
140 Cheumsamphan Rd., Nongchok, Bangkok, Thailand
Email : ssuchada@mut.ac.th

Piyaporn Nurarak
School of Science and Technology
Sukhothai Thammathirat Open University
9/9 Chaengwattana Rd., Nonthaburi, Thailand
Email : piyaporn.nur@stou.ac.th

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ABSTRACT

Analysis of a new modified adaptive forgetting-factor algorithm based on the recursive least squares algorithm (RLS) is presented for frequency domain equalization (FDE). By means of mean square deviation approach, an optimal forgetting-factor scheme is proposed by using the optimal gain sequence. A modified adaptive forgetting-factor RLS-based FDE is presented with regard to an optimal forgetting-factor algorithm for orthogonal frequency division multiplexing systems. We present simulation results to compare the proposed algorithm on the basis of its performance and rate of convergence.

Keywords: *Forgetting-factor, Recursive Least Squares (RLS) algorithm, adaptive algorithm, frequency domain equalization.*

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) isan efficient multicarrier modulation to fight against delay spread or frequency-selective fading of wireless and wireline channels. This approach has been adopted in standards for several high-speed wireless and wireline data applications, including digital audio and video broadcasting and local area networks [1].

For broadband channels, the conventional time-domain equalization is impractical, according to long channel impulse response in time domain. This approach for FDE is based on the discrete Fourier transforms (DFT) and its inverse DFT between the time and frequency domains.

In order to avoid inter-symbol interference and inter-carrier interference, the cyclic prefix is added between OFDM-symbols in the transmitter. An OFDM receiver transforms the received signal to frequency domain by applying a DFT. It performs a separate FEQ for each subcarrier.

Because of the numerical instability of the conventional RLS algorithm, the inverse QR-decomposition RLS algorithm has been used in parallel implementation in forms of the fast algorithm [2], [3]. However, it is difficult to set the optimal forgetting-factor for RLS algorithm in dealing with the large model variation [4]. Therefore, the optimal forgetting-factor RLS-based FEQ should be adapted automatically in order to achieve its performance. In [3], the adaptive forgetting-factor iQR-RLS per-tone equalization in discrete multitone systems has been presented in the form of conventional stochastic approximation by means of exponentially weighted least squares criterion. In [5], the variable step-size approach has been proposed with regard to the mean square deviation (MSD) criterion that indicates the adaptive filtering to optimal performance by means of faster convergence rate and lower misadjustment error than existing schemes.

The purpose of this paper is two-fold. First, we introduce how to derive the optimal forgetting-factor

RLS algorithm in terms of optimal gain sequence by defining the MSD approach. Second, we apply an adaptive implementation to show how the solution of L-tap complex-valued FDE can be achieved in OFDM systems.

2. SYSTEM MODEL AND NOTATION

In this section, we explain shortly the baseband OFDM system. At transmitter, the input binary bit stream is fed into a serial-to-parallel converter, then each data stream modulates the corresponding subcarrier.

The modulated data symbols are transformed by the iDFT (inverse DFT). The output symbols x_k are given by

$$x_k = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \mathbf{x}_m e^{j2\pi(km/M)} \ , 0 \le k \le M-1 \quad (1)$$

where M denotes as the number of subcarriers in OFDM system. The channel model can be described by

$$y_k = \sum_{l=0}^{L-1} h_l \, x_{k-l} + \eta \, , \ 0 \le k \le M-1 \quad (2)$$

where h_l denotes as the channel impulse response (CIR). The parameter L is the length of CIR and complex-valued Gaussian random variables η is included with zero mean and variance.

3. AN OPTIMAL FORGETTING-FACTOR RLS ALGORITHM

Following [6], the recursive equation for updating the least squares estimate tap-weight vector p_k at symbol k as

$$\boldsymbol{p}_k = \boldsymbol{p}_{k-1} + R_k^{-1} \cdot \boldsymbol{y}_k \cdot e_k^* \, , \quad (3)$$

where R_k is autocorrelation as

$$R_k = \lambda_k \, R_{k-1} + \boldsymbol{y}_k \cdot \boldsymbol{y}_{k-1}^H \, , \quad (4)$$

and e_k^* is complex conjugate of *a priori* estimated error as

$$e_k = x_k - \boldsymbol{p}_{k-1}^H \cdot \boldsymbol{y}_k, \quad (5)$$

And λ_k is update forgetting-factor parameter

$$\lambda_k = \frac{\gamma_{k-1}}{\gamma_k} (1 - \gamma_k) \, , \quad (6)$$

where γ_k denotes as the gain sequence.

Consequently, a general sequence γ_k corresponds to minimizing the least squares criterion. In particular, we note that a small positive number γ_0 will be chosen between tracking capability; if γ_0 is large, and noise sensitivity when γ_0 is small.

So, a constant gain γ_0 corresponds to an exponential forgetting-factor λ_0 as $\lambda_0 = 1 - \gamma_0$.

With the criteria for tracking assessment, the deviation vector ε_k between the optimal tap-weight frequency-domain equalizer (FDE) vector p_{opt} and adaptive tap-weight estimated vector p_k can be defined by

$$\varepsilon_k = p_{opt} - p_k \, , \quad (7)$$

where the tap-weight estimated FDE vector p_k at each symbol k is given in (3).

Then, the update tap-weight estimated FDE with the recursion method can be obtained with the method of weight deviation vector ε_k as

$$\varepsilon_{k+1} = \varepsilon_k + \gamma_k R_k^{-1} \cdot \boldsymbol{y}_k \cdot e_k^* \, , \quad (8)$$

where R_k^{-1} is the inverse autocorrelation.

On the basis of deviation vector, the mean square deviation (MSD) between the optimal tap-weight FEQ vector and tap-weight estimated FDE vector is formulated by

$$E\{\|\varepsilon_k\|^2\} = E\left\{\left\|\boldsymbol{p}_{opt} - \boldsymbol{p}_k\right\|^2\right\}, \quad (9)$$

where $E\{\cdot\}$ denotes as the expectation operator. The notation $\|\cdot\|^2$ is the Euclidean norm of a vector.

It is noted that the MSD should be small for a good tracking performance.

In the proposed algorithm, we introduce to find the optimal gain sequence γ_{opt} first. By using these assumptions above, this leads us to assuming that the MSD converges to a steady-state value. Thus, the objective function is to find the particular value of γ_{opt} that minimize the difference of MSD
δ_k at symbol k, we get

$$\gamma_k = E\{\|\varepsilon_k\|^2\} - E\{\|\varepsilon_{k+1}\|^2\} \, . \quad (10)$$

Differentiating the difference of MSD δ_k with respect to the gain sequence γ_k and setting to zero, this leads us to the optimal gain sequence γ_{opt} as

$$\gamma_{opt_k} \approx \mathcal{R}\left\{\frac{E\left\{\left\|\varepsilon_k^H \cdot g_k\right\|^2\right\}}{E\left\{\left\|\boldsymbol{g}_k^H \cdot \boldsymbol{g}_k\right\|^2\right\}}\right\}. \quad (11)$$

Where g_k is defined as

$$g_k = R_k^{-1} \cdot \boldsymbol{y}_k \cdot e_k^* \, . \quad (12)$$

4. MODIFIED ADAPTIVE FORGETTING-FACTOR RECURSIVE LEAST SQUARES ALGORIGHM FOR CONVENIENCE, LET

$$Q_k = R_k^{-1}, \tag{13}$$

$$\varepsilon_k \approx Q_k \cdot \boldsymbol{y}_k \cdot e_k^* . \tag{14}$$

Following [7], the tap-weight estimated FDE vector p_k in the recursion form may be computed by

$$\boldsymbol{p}_k = \boldsymbol{p}_{k-1} + \boldsymbol{\kappa}_k \cdot e_k^*, \tag{15}$$

$$\boldsymbol{\kappa}_k = \frac{\lambda_k^{-1}\boldsymbol{Q}_k\boldsymbol{y}_k}{1+\lambda_k^{-1}\boldsymbol{y}_k^{\boldsymbol{H}}\boldsymbol{Q}_k\boldsymbol{y}_k}, \tag{16}$$

$$Q_k = \lambda_k^{-1}Q_{k-1} - \lambda_k^{-1}\boldsymbol{\kappa}_k\boldsymbol{y}_k^H Q_{k-1}, \tag{17}$$

Therefore, the proposed modified adaptive forgetting-factor RLS can be obtained with the method of update gain sequence as

$$\lambda_k = \frac{\gamma_{k-1}}{\gamma_k}(1-\gamma_{k-1}), \tag{18}$$

$$\gamma_k = \gamma_{k-1} + \beta\frac{\|g_{k-1}^H \cdot g_{k-1}\|^2}{\|g_k^H \cdot g_k\|^2}, \tag{19}$$

where the adaptation rate β is a small positive number.

5. SIMULATION RESULTS

We simulate an OFDM system with the FFT size of 64, CP length of 16, and 16-QAM modulation. The entire channel bandwidth is of 20MHz, and is divided into the 52 used subcarriers. The symbol duration is chosen as 3.2 µs. The total OFDM frame length is about 100µs. The multipath channel model was followed by the ITU-Pedestrian A [8]. The receiver processing consists of frequency domain equalization, hard symbol decisions and decoding. The fading gains are randomly generated by complex Gaussian distributed random variables with zero mean and unit variance.

The initial parameters of proposed MAFF-RLS FDE are as: L = 7 and $\boldsymbol{p}_k(0) = \boldsymbol{k}_k(0) = [1\ 0 \cdots 0]^T$, $Q_k = \varsigma \cdot I$, where $\varsigma = 0.265$ and I is the identity matrix. The summary of proposed MAFF-RLS and conventional RLS algorithms for the computation are shown in Table I and Table II, respectively.

Fig.1 illustrates the trajectories of forgetting-factor parameters λ_k of proposed MAFF-RLS algorithm at two different values of initial forgetting-factors. It is shown to converge adaptively to its equilibrium despite large variations in initial settings of the adaptation rate (β).

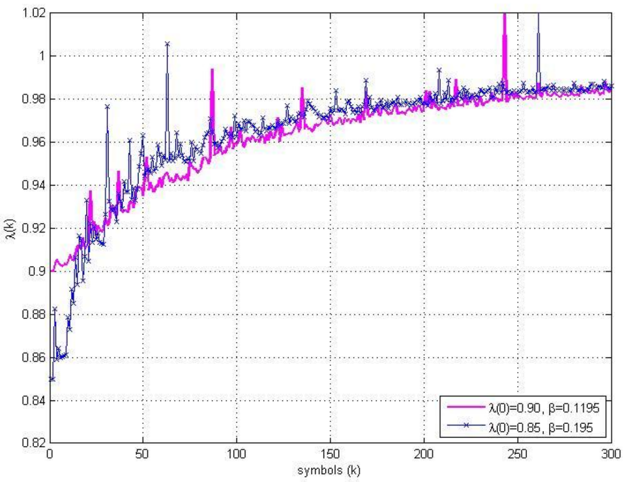

Fig. 1 *Trajectories of forgetting-factor λ_k parameter of proposed MAFF-RLS FDE with different setting of λ_0 and β.*

TABLE I Summary of proposed modified adaptive forgetting-factor recursive least squares (MAFF-RLS) algorithm

Starting with $\boldsymbol{p}_k(0) = \boldsymbol{\kappa}_k(0) = [1\ 0\ ...\ 0]^T$
$\boldsymbol{Q}_k = \zeta \cdot I$, where ζ is a small constant and I is the identity matrix.
$e(0) = \sigma_\eta^2$ and β is a small constant.
L is the number of tap-weight vector.

for k = 1, 2, . . . , m.

1) To compute $\boldsymbol{p}_k$ as:

$$\boldsymbol{p}_k = \boldsymbol{p}_{k-1} + \boldsymbol{\kappa}_k \cdot e_k^*,$$

2) To compute $\boldsymbol{\kappa}_k$ as:

$$\boldsymbol{\kappa}_k = \frac{\lambda_k^{-1}\boldsymbol{Q}_k\boldsymbol{y}_k}{1+\lambda_k^{-1}\boldsymbol{y}_k^{\boldsymbol{H}}\boldsymbol{Q}_k\boldsymbol{y}_k},$$

3) To compute e_k as:

$$e_k = x_k - \boldsymbol{p}_{k-1}^H \cdot \boldsymbol{y}_k,.$$

4) To calculate $\boldsymbol{Q}_k$ as:

$$\boldsymbol{Q}_k = \boldsymbol{\lambda}_k^{-1}\boldsymbol{Q}_{k-1} - \boldsymbol{\lambda}_k^{-1}\,\boldsymbol{\kappa}_k\,\boldsymbol{y}_k^{\boldsymbol{H}}\,\boldsymbol{Q}_{k-1},$$

5) To calculate λ_k as:

$$\lambda_k = \frac{\gamma_{k-1}}{\gamma_k}(1-\gamma_{k-1}),$$

6) To calculate γ_k as:

$$\gamma_k = \gamma_{k-1} + \beta\frac{\|g_{k-1}^H g_{k-1}\|^2}{\|g_k^H g_k\|^2}.$$

end

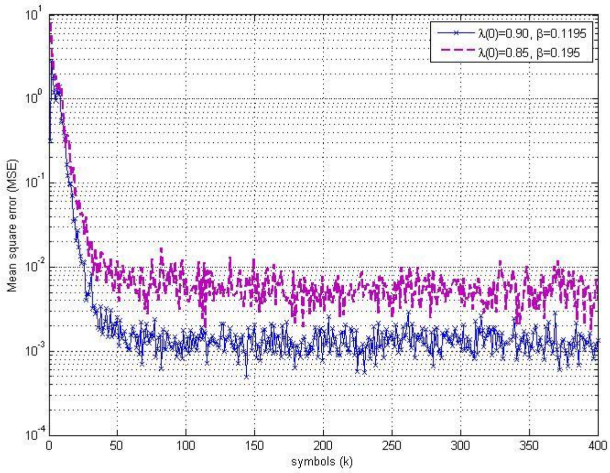

Fig. 2 *Learning curves of mean square errors (MSE) of proposed MAFF-RLS FDE with different setting of λ_0 and β.*

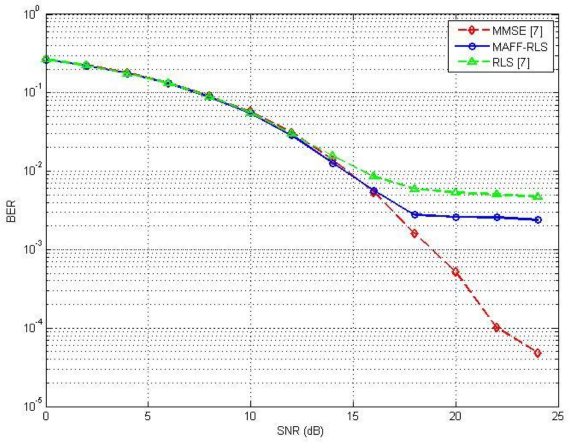

Fig. 3 *Bit error rate (BER) of proposed MAFF-RLS algorithm compared with the MMSE and fixed forgetting-factor of RLS at λ = 0.99.*

Fig.2 depicts the learning curves of mean square errors of proposed MAFF-RLS FDE with using β of γ_k are fixed at 0.195, 0.1195 and λ(0) = 0.85, 0.90, respectively. The proposed MAFF-RLS algorithm can converge to steady-state condition with the different settings of initial forgetting-factor parameters.

Fig.3 shows the bit error rate (BER) performance of proposed algorithm, where λ(0) = 0.935, β = 0.1725, compared with the minimum mean square error (MMSE) method [7] and fixed forgetting-factor of RLS [7] at λ = 0.99.

6. CONCLUSION

In this paper, we have introduced the adaptive MAFF-RLS algorithm for frequency domain equalization in OFDM systems. We have described how to investigate the optimal forgetting-factor parameter of RLS-based algorithm as a solution of mean square deviation approach. The optimal forgetting-factor parameter can be obtained by mean of optimal gain sequence. The trajectories of forgetting-factors are also shown to converge to their equilibrium. The proposed MAFF-RLS algorithm can achieve the good performance compared to the existing algorithms as MMSE and RLS algorithm.

TABLE II Summary of standard recursive least squares (RLS) algorithm [7]

Starting with $\boldsymbol{p}_k(0) = \boldsymbol{\kappa}_k(0) = [1\ 0\ ...\ 0]^T$
$\boldsymbol{Q}_k = \zeta \cdot I$, where ζ is a small constant and I is the identity matrix.
$e(0) = \sigma_\eta^2$ and λ is forgetting-factor parameter.
L is the number of tap-weight vector.

for k = 1, 2, . . . , m.

1) To compute $\boldsymbol{p_k}$ as:

$$\boldsymbol{p}_k = \boldsymbol{p}_{k-1} + \boldsymbol{\kappa}_k \cdot e_k^*,$$

2) To compute $\boldsymbol{\kappa}_k$ as:

$$\boldsymbol{\kappa}_k = \frac{\lambda_k^{-1}\boldsymbol{R}_k\boldsymbol{y}_k}{1+\lambda_k^{-1}\boldsymbol{y}_k^{\boldsymbol{H}}\boldsymbol{R}_k\boldsymbol{y}_k},$$

3) To compute e_k as:

$$e_k = x_k - \boldsymbol{p}_{k-1}^H \cdot \boldsymbol{y}_k,.$$

4) To calculate $\boldsymbol{R_k}$ as:

$$\boldsymbol{R_k} = \lambda^{-1}\boldsymbol{R_{k-1}} - \lambda^{-1}\,\boldsymbol{\kappa}_k\,\boldsymbol{y_k^H}\,\boldsymbol{R_{k-1}}\,,$$

end

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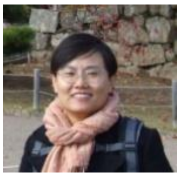

Suchada Sitjongsataporn received the B.Eng. (First-class honours) and D.Eng. degrees in Electrical Engineering from Mahanakorn University of Technology, Bangkok, Thailand in 2002 and 2009, respectively. She has worked as lecturer at department of Electronic Engineering, Mahanakorn University of Technology since 2002. Currently, she is an Assistant Professor in Mahanakorn University of Technology since 2012. Her research interests are mathematical and statistical models in the area of adaptive signal processing for communications, image and video processing.

Piyaporn Nurarak received the B.Eng. (Electronics) degree in Electrical Engineering from King Mongkut's Unversity of Technology Thonburi and M.Sc. (Electrical Engineering) degrees from University of Massachusetts, Lowell, USA. She is now a lecturer at school of Science and Technology, Sukhothai Thammathirat Open University, Nonthaburi, Thailand.